Global Sensitivity Analysis: An Introduction

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Abstract: This presentation aims to introduce global sensitivity analysis (SA), targeting an audience unfamiliar with the topic, and to give practical hints about the associated advantages and the effort needed. To this effect, we shall review some techniques for sensitivity analysis, including those that are not global, by applying them to a simple example. This will give the audience a chance to contrast each method's result against the audience's own expectation of what the sensitivity pattern for the simple model should be. We shall also try to relate the discourse on the relative importance of model input factors to specific questions, such as "Which of the uncertain input factor(s) is so non-influential that we can safely fix it/them?" or "If we could eliminate the uncertainty in one of the input factors, which factor should we choose to reduce the most the variance of the output?" In this way, the selection of the method for sensitivity analysis will be put in relation to the framing of the analysis and to the interpretation and presentation of the results. The choice of the output of interest will be discussed in relation to the purpose of the model based analysis. The main methods that we present in this lecture are all related with one another, and are the method of Morris for factors' screening and the variance-based measures. All are model-free, in the sense that their application does not rely on special assumptions on the behaviour of the model (such as linearity, monotonicity and additivity of the relationship between input factor and model output). Monte Carlo filtering will be also be discussed to demonstrate the usefulness of global sensitivity analysis in relation to estimation.

Keywords: global sensitivity analysis, factor prioritisation, main effects, second-order interaction effects, nonlinear models

INTRODUCTION

The material in this presentation is taken from a primer on global sensitivity analysis entitled "*Sensitivity Analysis in Practice: A Guide to Assessing Scientific Models*" by Andrea Saltelli, Stefano Tarantola, Francesca Campolongo and Marco Ratto. This will appear with John Wiley & Sons by early 2004, and we shall refer to it as to Saltelli et al., 2004 in the following. The primer aims at guiding a non-expert user in the choice of the method to adopt for the user own problem. The methods recommended include the variance based measures, the method of Morris, and Monte Carlo filtering, e.g. some effective methods for global sensitivity analysis.

Global sensitivity analysis is the study of how the uncertainty in the output of a model (numerical or otherwise) can be apportioned to different sources of uncertainty in the

model input". Global could be an unnecessary specification here, were it not for the fact that most analysis met in the literature are local or one-factor-at-a-time.

All models have use for sensitivity analysis. Applications worked by the Joint Research Centre group for Applied Statistics include: Atmospheric chemistry (Campolongo et al., 1999a), transport emission modelling, fish population dynamics (Campolongo et al. 1999b), composite indicators (Tarantola et al. 2002), portfolios, oil basins models (Saltelli, 2002), capital adequacy modelling (for Basle II), macroeconomic modelling, radioactive waste management (Saltelli and Tarantola, 2002). Applications from several practitioners can be found in Saltelli et al. Eds. 2000, a multi-author book.

Prescriptions have been issued for sensitivity analysis of models when these used for policy analysis.
In Europe, the European Commission recommends sensitivity analysis in the context of the extended impact assessment guidelines and handbook (2002). Similar recommendation in the United States EPA's White Paper on model use acceptability (1999)

The EC handbook for extended impact assessment, a working document by the European Commission, 2002, states: "A good sensitivity analysis should conduct analyses over the full range of plausible values of key parameters and their interactions, to assess how impacts change in response to changes in key parameters". The EPA paper (1999) is less prescriptive, but insists on the need for uncertainty and sensitivity analysis.

Even leaving prescriptions aside, one cannot ignore that models have not escaped the post-modern critique of the role of science in society. Specific critiques of simulation modelling and model validation have been frequent in recent years. One example: <<*…most simulation models will be complex, with many parameters, state-variables and non linear relations. Under the best circumstances, such models have many degrees of freedom and, with judicious fiddling, can be made to produce virtually any desired behaviour, often with both plausible structure and parameter values.*>>, Hornberger and Spear 1981.

Also, from within the modelling community reminders of the problem were frequent: Konikov and Bredehoeft, 1992, proclaims: "*Groundwater models cannot be validated*". This cry of alarm was taken up by Oreskes et al. 1994, in an article on Science entitled "*Verification, Validation and Confirmation of numerical models in the earth sciences*", both works focusing on the impossibility of model validation. Two established laboratory, IIASA and RIVM, had considerable trouble with the perceived quality of their models, see Mac Lane 1989, and van der Sluijs 2002 respectively. The post-modern French thinker Jean Baudrillard (1990) presents 'simulation models' as unverifiable artefact which, used in the context of mass communication, produce a fictitious hyper realities that annihilate truth. Science for the post modern age is discussed in Funtowicz and Ravetz 1990, 1993, 1999, mostly in relation to Science for policy use, a settings which Gibbons (1994) calls "mode 2" scientific production.

Faced with these critiques, the modelling community may consider that a quality check as that which is provided by a careful sensitivity analysis is worth its effort.

Before we discuss the methods for sensitivity analysis, we would like to say a few words about the output Y of interest. In our experience, the target of interest should not be the model output per se, but the question that the model has been called to answer. To make an example, if a model predicts contaminant distribution over space and time, it is the total area where a given threshold is exceeded at a given time which would play as output of interest, or the total health effects per time unit.

One should seek from the analyses conclusions of relevance to the question put to the model, as opposed to relevant to the model, e.g.

- Uncertainty in emission inventories [in transport] are driven by variability in driving habits more than from uncertainty in engine emission data.
- In transport with chemical reaction problems, uncertainty in the chemistry dominates over uncertainty in the inventories.
- Engineered barrier count less than geological barriers in radioactive waste migration.

This remark on the output of interest clearly applies to model use, not to model building, where the analyst might have interest in studying a variety of intermediate outputs.

FIRST EXAMPLE: THE OBVIOUS TEST CASE

We move now to a self-evident problem, to understand the methods as applied to it. This is a simple linear form:

$Y = \sum_{i=1}^{r} \Omega_i Z_i$

Y is the output of interest (a scalar), Ω_i are fixed coefficients, Z_i are uncertain input factors distributed as

$Z_i \sim N(\bar{z}_i, \sigma_{Z_i}), \quad \bar{z}_i = 0, \quad i = 1,2,...r$.

Y will also be normally distributed with parameters:

$\sigma_Y = \sqrt{\sum_{i=1}^{r} \Omega_i^2 \sigma_{Z_i}^2}$

$\bar{y} = \sum_{i=1}^{r} \Omega_i \bar{z}_i$

To make our point we stipulate as additional assumptions:

$\sigma_{Z_1} < \sigma_{Z_2} < .... < \sigma_{Z_r}$

$\Omega_1 > \Omega_2 > .... > \Omega_r$

According to most of the existing literature, SA should be done by taking derivatives, such as: $S_{Z_i}^d = \frac{\partial Y}{\partial Z_i}$, which would give for our model of Y: $S_i^d = \frac{\partial Y}{\partial Z_i} = \Omega_i$.

Hence the factors' ordering by importance would be $Z_1 > Z_2 > ... > Z_r$, based on our previous assumption that $\Omega_1 > \Omega_2 > .... > \Omega_r$, and this in spite of the fact that $\sigma_{Z_1} < \sigma_{Z_2} < .... < \sigma_{Z_r}$. This would seem to suggest that if our purpose is to rank input factors in terms to their contribution to the variability of the output, then simple derivatives such as $S^d_{Z_i} = \frac{\partial Y}{\partial Z_i}$ are not the best instrument to use.

A better measure could a normalised derivative of the type: $S^{\sigma}_{Z_i} = \frac{\sigma_{Z_i}}{\sigma_Y}\frac{\partial Y}{\partial Z_i}$, which, applied to our model, gives $S^{\sigma}_{Z_i} = \Omega_i \frac{\sigma_{Z_i}}{\sigma_Y}$

Comparing this with our previous expression $\sigma_Y = \sqrt{\sum_{i=1}^{r} \Omega_i^2 \sigma_{Z_i}^2}$, we obtain $\sum_{j=1}^{r} \left(S^{\sigma}_{Z_i}\right)^2 = 1$.

This is a nice result: the terms add to 1, and each of them gives the fractional contribution of the factor to the variance of the output. Unfortunately this only works for linear models.

If we want to tackle nonlinear models as well, we have to abandon derivatives and move into "exploration" of the input factors space, e.g. via Monte Carlo.

We generate a sample

$$\mathbf{M} = \begin{matrix} z_1^{(1)} & z_2^{(1)} & ... & z_r^{(1)} \\ z_1^{(2)} & z_2^{(2)} & ... & z_r^{(2)} \\ ... & ... & ... & ... \\ z_1^{(N)} & z_2^{(N)} & ... & z_r^{(N)} \end{matrix}$$

and run our computer program estimating the corresponding model output

$$\mathbf{y} = \begin{matrix} y^{(1)} \\ y^{(2)} \\ ... \\ y^{(N)} \end{matrix}$$

A natural thing to do at this point is to regress the y' s on the z_i's to obtain a regression model

$y^{(i)} = b_0 + \sum_{i=1}^{r} b_{Z_i} z_i^{(i)}$, where asymptotically $\hat{b}_0 \cong 0, \hat{b}_{Z_i} \cong \Omega_i$, $i = 1,2,...r$. Most regression packages will already provide the regression in terms of standardised regression coefficients $\hat{\beta}_{Z_i} = \hat{b}_{Z_i}\sigma_{Z_i}/\sigma_Y \cong \Omega_i \sigma_{Z_i}/\sigma_Y$. Comparing $\hat{\beta}_{Z_i} \cong \Omega_i \sigma_{Z_i}/\sigma_Y$ with

$S^{\sigma}_{Z_i} = \Omega_i \frac{\sigma_{Z_i}}{\sigma_Y}$,

it is easy to conclude that for linear models $\beta_{Z_i} = S^{\sigma}_{Z_i}$.

In summary, $\sum_{j=1}^{r} \left(S^{\sigma}_{Z_i}\right)^2 = \sum_{j=1}^{r} \left(\beta_{Z_i}\right)^2 = 1$, but only for linear models. Yet the regression coefficients are better than the derivatives in several respects.

Although for nonlinear models $\sum_{j=1}^{r} \left(\beta_{Z_i}\right)^2 \leq 1$, at least we now know how much linear the model is. This is given by the model coefficient of determination $R^2_y = \frac{\sum_{i=1}^{N} \left(\hat{y}^{(i)} - \bar{y}\right)^2}{\sum_{i=1}^{N} \left(y^{(i)} - \bar{y}\right)^2}$.

We now know that we can decompose a fraction R^2_y of the model variance using the β_{Z_i}. Furthermore the coefficients β_{Z_i} offer a measure of sensitivity that is multi-dimensionally averaged, unlike the $S^{\sigma}_{Z_i}$. For linear model this does not matter but it does, and a lot, for nonlinear ones. The drawback is when $R^2_y << 1$; typically R^2_y can be zero or near it for non-monotonic models.

In summary, we like the idea of decomposing the variance of the model output according to source (the input factors), but would like to do this for all models, independently from their degree of linearity or monotonicity. We would like a model-free approach.

In order to get there, we take a somehow twisted path and start asking ourselves the question: If I could determine the value of an uncertain factor, e.g. one of our Z_i and thus fix it, how much would the
variance of the output decrease? E.g. imagine the true value is z_i^* and hence we fix Z_i to it obtaining a "reduced" conditional variance: $V\left(Y \middle| Z_i = z_i^*\right)$. There are two problems with this quantity being a good measure of sensitivity. First I do not know where to fix the factor, and secondly for nonlinear model one could have $V\left(Y \middle| Z_i = z_i^*\right) \geq V(Y)$.

This difficulty can be overcome by averaging this measure over the distribution of the uncertain factors obtaining $E\left(V\left(Y \middle| Z_i\right)\right)$, or $E_{Z_i}\left(V_{\mathbf{Z}_{-i}}\left(Y \middle| Z_i\right)\right)$ where we have made explicit the variables over which mean and variance operators are applied. This measure has the property that $E\left(V\left(Y \middle| Z_i\right)\right) \leq V(Y)$ always, and in particular
$E\left(V\left(Y \middle| Z_i\right)\right) + V\left(E\left(Y \middle| Z_i\right)\right) = V(Y)$, where the term $E\left(V\left(Y \middle| Z_i\right)\right)$ is called a residual, and the term $V\left(E\left(Y \middle| Z_i\right)\right)$ is known as the first order effect of Z_i on Y. A nice property of the main

effect is that it is large when a factor is influential. Furthermore it is easy to verify that for linear models $S_{Z_i} = \frac{V\left(E\left(Y|Z_i\right)\right)}{V(Y)} = \beta_{Z_i}^2$.

We have made a real progress, as while $\sum_{j=1}^{r}\left(\beta_{Z_i}\right)^2 = 1$ only holds for linear models, $\sum_{j=1}^{r}\left(S_{Z_i}\right) = 1$ holds for a much larger class of models: that of the additive models. For non-additive models, $\sum_{j=1}^{r}\left(S_{Z_i}\right) \leq 1$, which is also a way to define non-additive models. Yet the measure S_{Z_i} is very useful for <u>all</u> models, as it provides a rigorous answer to a precise sensitivity analysis setting: <u>setting FP</u>, for factors prioritisation. Let us then make a digression here, and describe this setting.

FACTORS' PRIORITISATION (FP) SETTING

Imagine that I must bet on a factor that, once "discovered" in its true value and fixed, would reduce the most V(Y). Of course I do not know where the true values are for the factors, hence I cannot compare the $V\left(Y\middle|Z_i = z_i^*\right)$ for the various factors. Hence the best choice I can make is, by definition, to choose the factor with the highest $V\left(E\left(Y\middle|Z_i\right)\right)$ or , which is the same, the highest $S_{Z_i} = \frac{V\left(E\left(Y|Z_i\right)\right)}{V(Y)}$, whether the model is additive or not (Saltelli and Tarantola, 2002).

To complete all this, we must say something about non-additive model treatment, so let us complicate our model $Y = \sum_{i=1}^{r}\Omega_i Z_i$ by allowing both the Ω_i and Z_i to be uncertain, i.e. $Z_i \sim N\left(\bar{z}_i, \sigma_{Z_i}\right)$, $\bar{z}_i = 0$, $i = 1,2,...r$ as before and $\Omega_i \sim N\left(\bar{\omega}_i, \sigma_{\omega_i}\right)$, $\bar{\omega}_i = ci, i = 1,2,...r$, where *c* is a constant greater than zero (note: if the mean of the Ω_i were also null as that of the Z_i , then the model would be fully non-additive, as we shall see in a moment).

Our set of uncertain input factors is now $\mathbf{X} \equiv (\Omega_1, \Omega_2, ...\Omega_r, Z_1, Z_2, ...Z_r)$. We start crunching number estimating the sensitivity measures and we obtain the following results:

All S_{Ω_i} are zero.

All S_{Z_i} are > zero.

S_{Ω_i} is zero because the distribution of Z_i is centred in zero, and hence for any fixed value ω_i^* of Ω_i

$E\left(Y\middle|\Omega_i = \omega_i^*\right) = 0$, and *a fortiori* $V\left(E\left(Y\middle|\Omega_i\right)\right) = 0$.

Given that $\sum_{j=1}^{r}\left(S_{Z_i}\right)\le 1$ where is the remaining variance? To find it out we must compute sensitivity indices on more than one factor. If we do that, we find that $\frac{V\left(E\left(Y\middle|Z_i,Z_j\right)\right)}{V_Y}=S_{Z_i}+S_{Z_j}$, while, instead: $\frac{V\left(E\left(Y\middle|\Omega_i,Z_i\right)\right)}{V_Y}>S_{\Omega_i}+S_{Z_i}$. The difference $S_{\Omega_i Z_i}=\frac{V\left(E\left(Y\middle|\Omega_i,Z_i\right)\right)}{V_Y}-S_{\Omega_i}-S_{Z_i}$ is the second order (or two-way) effect of the two factors. We have discovered that our model is additive with respect to S_{Z_i}, S_{Z_j}, and non-additive with respect to S_{Ω_i}, S_{Z_i}.

Adding all the non-zero first order terms and all the non-zero second order terms gives back 1, i.e. 100% of the variance of Y is accounted for.
I.e. $\sum_{i=1}^{r}S_{Z_i}+S_{\Omega_i Z_i}=1$

For our model, all other terms of whatever order (1,2,3…2r) is zero. In general, if k is the total number of independent factors, then $\sum_i S_i+\sum_i\sum_{j>i}S_{ij}+\sum_i\sum_{j>i}\sum_{l>j}S_{ijl}+....S_{12...k}=1$ (Sobol', 1993).

It is quite rare that in practical applications one computes all terms in the development above. The number of terms grows exponentially with k.

We are customarily happy with computing all the S_i plus a full set of synthetic terms called S_{Ti} which give for each factor X_i, the effect of all terms including that factor.

What are the total effect terms S_{Ti} and why do we need them? Let us compute one of them, by starting with the measure $\frac{V\left(E\left(Y\middle|\mathbf{X}_{-\Omega_i}\right)\right)}{V_Y}=\frac{V\left(E\left(Y\middle|\Omega_1,\Omega_2,...\Omega_{i-1}\Omega_{i+1},...\Omega_r Z_1,Z_2,...Z_r\right)\right)}{V_Y}$. We have taken factor Ω_i as an example. Analogy with previous formulae should suggest that, by definition, this is the [first order] effect of all-but- Ω_i. Hence $S_{T\Omega_i}\equiv 1-\frac{V\left(E\left(Y\middle|\mathbf{X}_{-\Omega_i}\right)\right)}{V_Y}$ will be the effect of all terms [any order] that include Ω_i; for our model this is simply $S_{T\Omega_i}=S_{\Omega_i}+S_{\Omega_i Z_i}$, provided we remember that the S_{Ω_i} are zero as well, so that $S_{T\Omega_i}=S_{\Omega_i Z_i}$. Note that because of an algebraic relation already mentioned

$1-\frac{V\left(E\left(Y|\mathbf{X}_{-\Omega_i}\right)\right)}{V_Y}=\frac{E\left(V\left(Y|\mathbf{X}_{-\Omega_i}\right)\right)}{V_Y}$., so that the right hand expression is often used for the S_{Ti}.

There is a considerable symmetry between the S_i and S_{Ti}. Both indices can be computed in a single
shot at the cost of about N(k+2) simulations, where N is between 100 and 1000, to give an idea. In Saltelli, 2002, we use an extension of the method of Sobol', 1993. Both indices can also be computed using the Fourier based FAST method, as extended in Saltelli et al., 1999.

Furthermore S_i is ideal for factor prioritisation setting, already described, while S_{Ti} is ideal for the "factors fixing" setting (of which more in a moment).

A nice property of S_{Ti} is that if one is desperate for less expensive simulations, a rough estimate of these can be obtained via the method of Morris, at less than 1/10 of the cost, see Morris 1991. (We prefer to compute a "modulus" version of the test statistics, as described in Chapter 4, Campolongo et al., in Saltelli et al. Eds., 2000).

Finally one last useful property of variance based methods is their application "by groups", e.g.
$S_{\mathbf{\Omega}}+S_{\mathbf{Z}}+S_{\mathbf{\Omega},\mathbf{Z}}=1$, where $\mathbf{\Omega}=\Omega_1,\Omega_2,...\Omega_r$. The computational cost of this is just 3N. Or I can regroup as $\sum_{i=1}^{r}S_{\mathbf{A}_i}=1$, where $A_i=(\Omega_i,Z_i)$. The computational cost of this is kN. Note that in this latter expression all higher order terms are zero because there are interactions only within $A_i=(\Omega_i,Z_i)$.

Although in the first regrouping we save a lot in terms of model execution, and in the second we don't, there might be reasons other than economy to regroup factors. I might want to groups factors in different submodels. In this way, if I can fix all factors in the submodels may be I can skip the submodel altogether. I might want to separate controllable factors from uncontrollable ones, and so on.

A SECOND EXAMPLE: WHAT CAN SENSITIVITY OFFER FOR PARAMETER ESTIMATION

Let us now move to an estimation/calibration problem for a computational model with six parameters. We do not know how the model is done – imagine it is a computer code. The output of interest *Y* is a measure of likelihood is obtained after comparing the model prediction Y' with data, e.g.

Y=exp(-[sum of squared residuals of the predicted Y' versus the data]).

How can we characterise the good parameter set for calibration? A scatter plots of log-likelihood (e.g. of the sum of scores) vs. parameters is not very informative (Figure 1). Even "filtering", e.g. taking the best outcomes, those with the highest log-likelihood, leaves us in the dark (Figure 2). Plotting the factors value for the input (Figure 3) as well as for the input corresponding to the best values (Figure 4) is likewise noninformative. Note that if we computed on the filtered input factors (Figure 4) the pairwise correlation coefficients we would obtain zeros. Also Principal Component Analysis would not be informative as applied to the filtered input sample, as there are no correlations among the filtered factors. Computing the first order sensitivity indices for the log-likelihood and the second order ones (Figure 5), a story starts to emerge; there are non-zero second order effects, but only within the closed groups involving factors (1,2,3) and (4,5,6). Computing the third order effect (Figure 6) again only those pertaining to (1,2,3) and (4,5,6) are non-zero. Regrouping and adding the terms up gives an interesting result:

$$S^c_{123} = S_1 + S_2 + S_3 + S_{12} + S_{13} + S_{23} + S_{123} = 0.5$$
$$S^c_{456} = S_4 + S_5 + S_6 + S_{45} + S_{46} + S_{56} + S_{456} = 0.5$$

where we have used the supescript c symbol to denote the effects closed within the indices. The variance of the problem is characterised by two groups of three factors. Higher term orders are zero.

This leads the investigator to conclude that what could be reasonably estimated are two unknown functions of two parameter sub-sets. We can now reveal that the unknown function, our computer program, was the sum of two speres.

$$f(X_1,...,X_6) =$$
$$= -\left(\sqrt{X_1^2 + X_2^2 + X_3^2} - R_1\right)^2 / A_1 - \left(\sqrt{X_4^2 + X_5^2 + X_6^2} - R_2\right)^2 / A_2$$

Were the investigator to identify this structure, by trial and error, he/she would conclude that all that estimation can provide are the two radiuses.

This concludes our illustration of sensitivity analysis as applied to a diagnostic setting, and we would now like to come back to our discussion of the settings for sensitivity analysis.

MORE ON THE SETTINGS FOR SENSITIVITY ANALYSIS

We have already mentioned that the sensitivity measure of the first order, $S_i \equiv \frac{V\left(E\left(Y|X_i\right)\right)}{V_Y}$ is the ideal measure for factor prioritisation. It is also easy to see that the total effect measure $S_{Ti} \equiv \frac{E\left(V\left(Y|\mathbf{X}_{-i}\right)\right)}{V_Y}$ is appropriate for a setting that we could call

"Factors Fixing": Can I fix a factor [or a subset of input factors] at any given value over their range of uncertainty without reducing significantly the output variance? If factor X_i is totally non-influential, then all the variance is due to $\mathbf{X}_{-i}$, and fixing this vector results in $V\left(Y|\mathbf{X}_{-i}\right)=0$. It is easy to see that the reverse is also true so that necessary and sufficient condition for X_i to be totally non-influential is $S_{Ti}\equiv 0$.

Other settings that we have found useful are the following.

Factors mapping: Which factor is mostly responsible for producing realisations of Y in the region of interest? This can be treated with Monte Carlo Filtering and related tools (described elsewhere at this workshop).

Variance cutting: Reducing the variance of the output of a prescribed amount fixing the smallest number of factors. This setting can be dealt with using a combination of the S_i and S_{Ti} measures (Saltelli and Tarantola, 2002).

Why do we need settings? One way in which a sensitivity analysis can go wrong is because its purpose is left unspecified or vague (e.g. "find the most important factors"). One throws different statistical tests and measures to the problem and obtains different factors rankings. What can then be concluded? Models can be audited and settings for sensitivity analysis can be audited as well. For this reason we believe that importance must be defined beforehand.

A FEW MORE COMMENTS ON PRACTICES

What else can go wrong in a sensitivity analysis? Two instances come to mind:

There are too many outputs of interest, as we discussed at the beginning. What is the question asked from the model? Is the model relevant to the question? The optimality of a model must be weighted with respect to the task, according to a current mode of thinking. According to Beck et al. 1997, a model is "relevant" when its input factors actually cause variation in the model response that is the object of the analysis. Model "non-relevance" could flag a bad model, or a model used out of context (e.g. a gun to kill a fly). Excess complexity could also be used to silence or to fend off criticism from stakeholders, e.g. in environmental assessment studies.

Patchy or piecewise sensitivity (performed by sub-model, or one possible model at a time, or one factor at a time): Not only conflicts with the requirement of focus just mentioned, but leads to a dangerously incomplete exploration of the uncertainties; interactions are overlooked. All uncertainties should be explored simultaneously. Also the procedure of fixing non-influential factors should be conducted in this way, as fixing factors based on their first order effect can be dangerous as discussed above. The Ω_i of our initial example all have first order equal zero.

A posteriori sensitivity: Once an analysis has been produced, its revision via sensitivity analysis by a third party is not something most modellers will willingly submit to. Sensitivity analysis should be used in the process of model development, prior and within model use in analysis.

One should never forget that an unpleasant (or pleasant, depending from the viewpoint) feature of sensitivity analysis is that it might falsify the analysis altogether, e.g. by showing that the model cannot answer the question given the uncertainties, or that the model is irrelevant, or that the variation in the output of interest (e.g. a contamination level in an estuary) is insensitive to the available policy options given the uncertainties. A nice example that shows how SA can falsify a model as applied to a policy issue is described in Chapter 20, Tarantola et al., of Saltelli et al., Eds. 2000.

CONCLUSIONS

We can itemise our main conclusions as follows. There is an increased need, scope and prescription for quantitative uncertainty and sensitivity analyses. Methods are mature for use, e.g. in terms of literature, software, computational cost, tested practice, ease of communication.

In spite of this one observes a “slow start” of quantitative methods in practical analyses

Variance based measure are concise, easy to understand and to communicate, reduce to the elementary test (the standardised regression coefficients $\beta_{i_i}^2$) for linear model, relate to the popular method of Morris.

We also like and use methods in the MC filtering family.

Whatever the method one uses, we think it important that the framing of the analysis be defensible and meaningful to its users.

Figure 1. Log-likelihood for the six input factors.

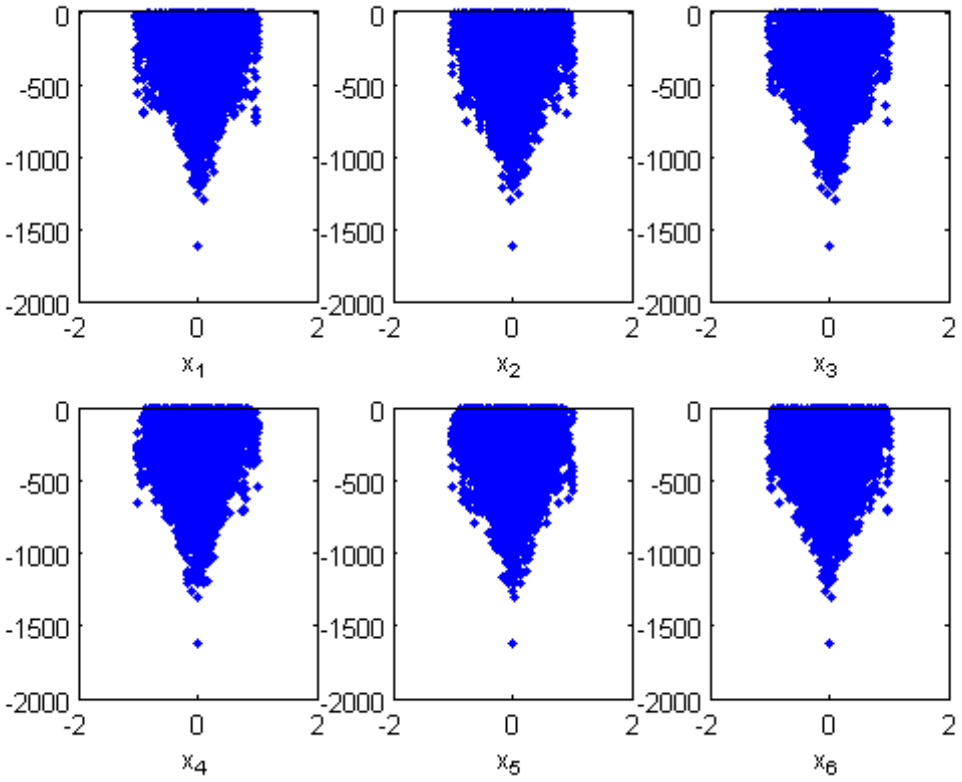


Figure 2. Same as Figure 1, for values of log-likelihood > -200.

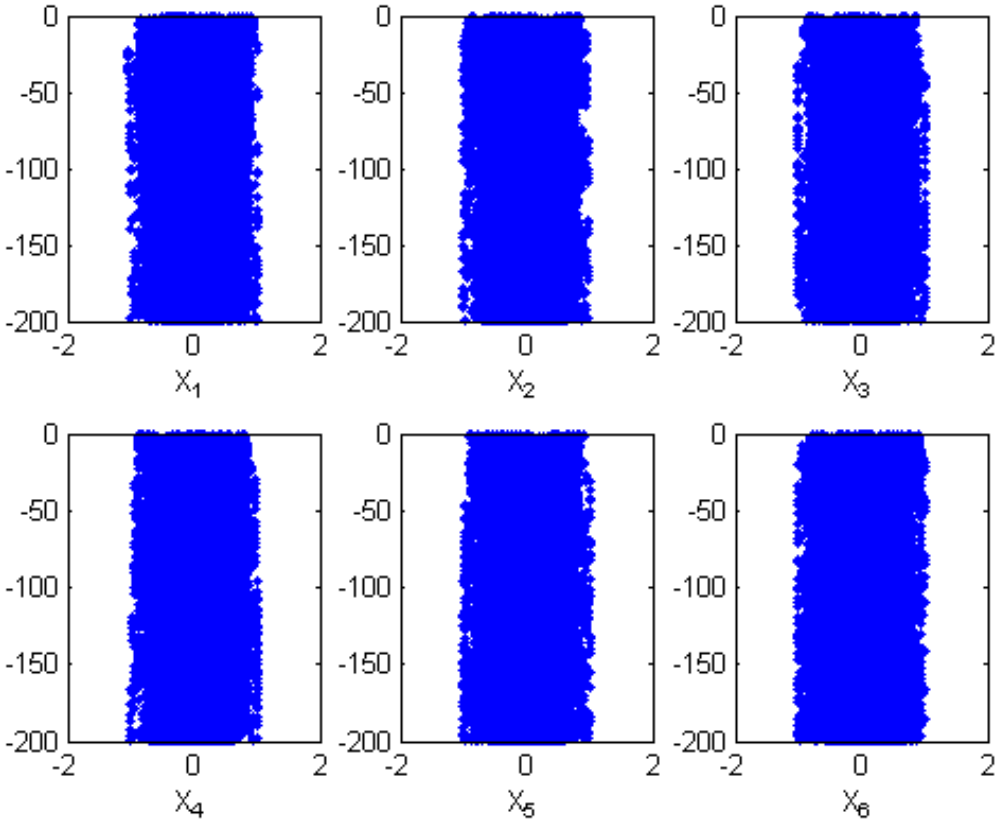


Figure 3. Pair-wise scatter plots of input factors.

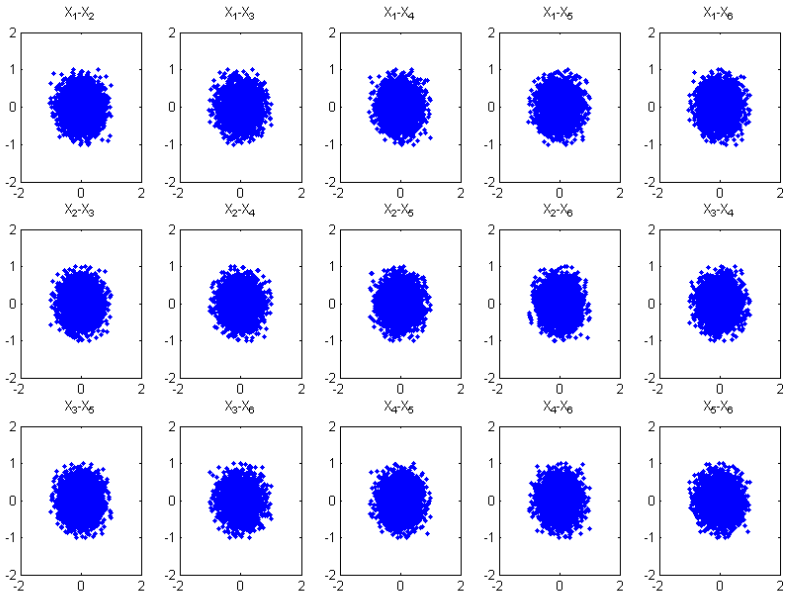


Figure 4. Same as the previous figure, for values of log-likelihood > -200.

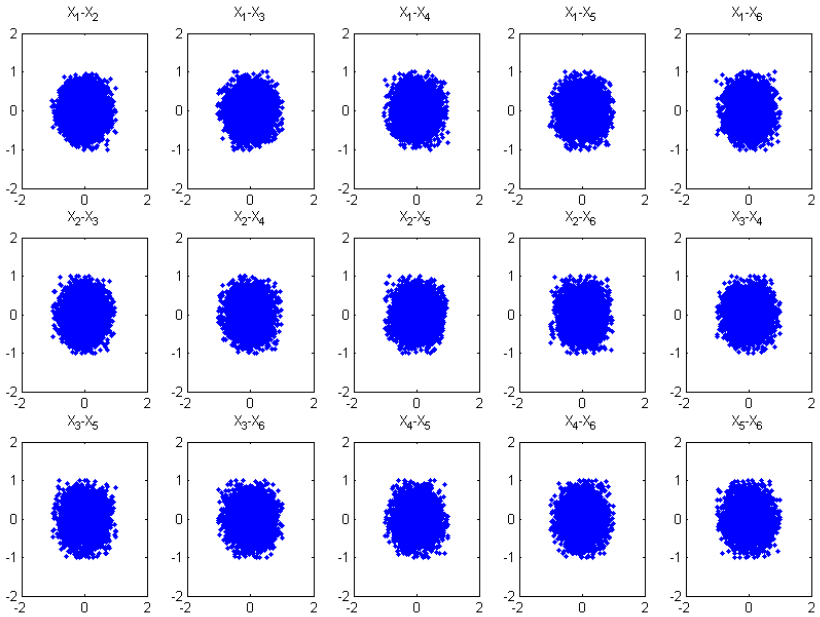


Figure 5. First- and second-order sensitivity indices for the log-likelihood.

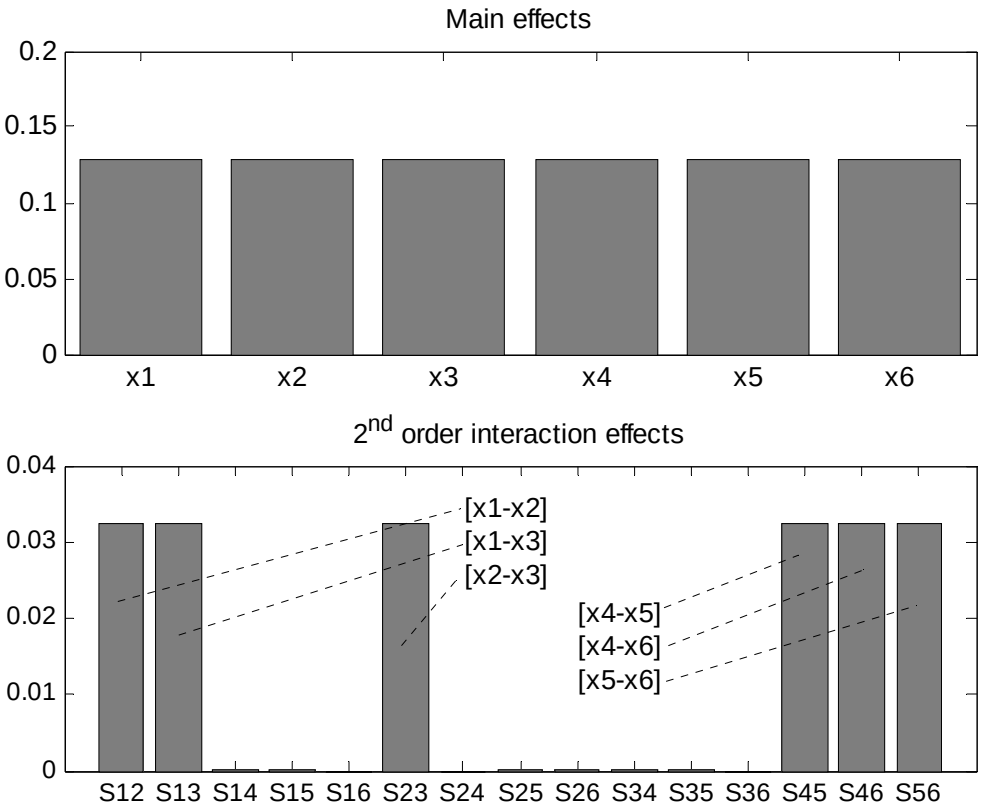
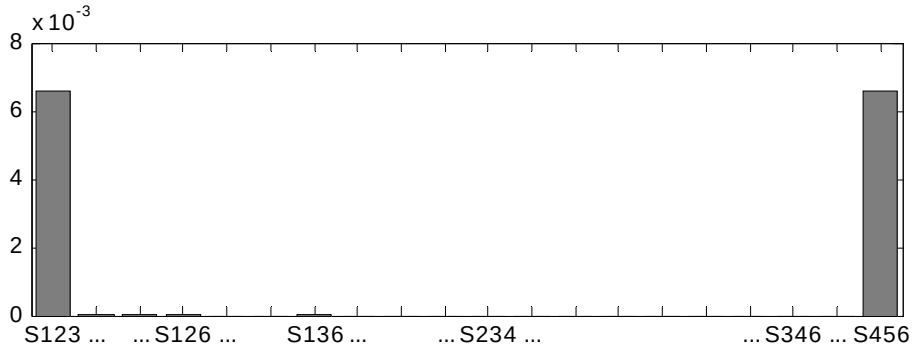


Figure 6. Third-order sensitivity indices for the log-likelihood.



NOTES

The Joint Research Centre distributes freely the software SIMLAB for uncertainty and sensitivity analysis. More information from stefano.tarantola@jrc.it. Marco Ratto (marco.ratto @jrc.it) has developed a set of scripts in Matlab to run global sensitivity analysis in diagnostic settings (e.g. with filtering plus variance based methods, see our two-sphere example). This is also available.

A forum to discuss sensitivity analysis issues is available at http://sensitivity-analysis.jrc.cec.eu.int/.
It includes a FAQ section, introduction to the main methods and a bibliography.

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