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In the preceding papers establishing the existence of a new $T=1$, $Y=0$ hyperon resonance at 1660 MeV, Y_1^* (1660),[1] it is noted that the baryons and the many low-lying baryon-meson resonances may be included within just four unitary-symmetry supermultiplets:[2] the $j=1/2^+$ α octet, the $j=1/2^-$ β singlet, the $j=3/2^-$ γ octet, and the $j=3/2^+$ δ decuplet.[3] All members of these supermultiplets are seen except Ω^-_δ (sometime called Z^-) and Ξ_γ, whose discoveries would give crucial tests of the eightfold way. Each of the four supermultiplets may be viewed as the ground state of a different Regge trajectory, and each such trajectory may have several manifestations with different physical values of angular momentum. These "Regge recurrences" (should they occur at all) must characterize all the members of a given supermultiplet, and must be spaced by two units of angular momentum. Identifying the $N^*_{3/2}$ (1920) resonance with $\Delta_\delta(j=7/2^+)$, the Y_0^* (1815) with $\Lambda_\alpha(j=5/2^+)$, and the $N^*_{1/2}$ (1688) resonance with N_α $(j=5/2^+)$, we find that all presently known baryon states are described with only four supermultiplet Regge trajectories. Because the first Regge recurrences of some members of the α and δ supermultiplets have appeared, one predicts the existence of Σ_α and Ξ_α states of $j=5/2^+$ and of the remaining members of the $j=7/2^+$ δ decuplets. These four supermultiplets are displayed in Fig. 1.

Beyond the mere classification of particles and resonances, there must be more quantitative, dynamical verifications of the "broken eightfold way."[4]

In this note, we show that the partial widths for the various two-body decay modes of the γ octet and of the δ decuplet are compatible with the approximate unitary symmetry of strong interactions. In Table I the experimental partial widths for decay into meson plus baryon are summarized. Two of these are used as input variables determining the eightfold-way D and F decay-coupling constants for the γ octet; the remaining five partial widths are calculated after adjustment of a radius of interaction.[5] With the same form factor, the calculation is repeated for the δ decuplet, for which there is a unique invariant coupling to meson plus baryon. In each case, agreement with experiment is excellent, in general to within the accuracy with which the partial widths are known.

Some of the agreement may be due merely to our use of a reasonable form factor. On the other hand, the theoretical result for $\Gamma(\Sigma_\gamma \to \bar{K}N):\Gamma(\Sigma_\gamma \to \pi\Lambda):\Gamma(N_\gamma \to \pi N)$ of about 1:3:16 (compared with the experimental values about 1:4:17) refers to modes of roughly the same momenta. Hence we see that the eightfold way supplies nontrivial coefficients. Moreover, the δ coupling strengths are consistently greater by an order of magnitude than the γ coupling strengths (e.g., $\Gamma(\Sigma_\delta \to \pi\Lambda) \approx 4\Gamma(\Sigma_\gamma \to \pi\Lambda)$, even though the γ mode has three times as much available energy as the δ mode).[6] These considerations preclude any purely kinematical explanation for the agreement between theory and experiment.[7]

Three-body decay modes of baryon resonances are less of a test of the eightfold way than two-body modes because there are many possible forms of symmetrical interaction involving two final-state mesons. However, it is instructive to consider the possible two-body decays of one baryon resonance into another baryon resonance plus a meson (giving, eventually, a three-body mode), e.g., $N_\gamma \to N_\delta\pi$, $\Sigma_\gamma \to \Sigma_\delta\pi$, $\Sigma_\gamma \to \Lambda_\beta\pi$.

Only inequalities for these partial widths have thus far been experimentally determined:

$$\Gamma(N_\gamma \to N_\delta \pi) \lesssim \Gamma(N_\gamma \to N\pi\pi) = 50 \text{ MeV} , \qquad (1)$$

$$\Gamma(\Sigma_\gamma \to \Sigma_\delta \pi) \lesssim \Gamma(\Sigma_\gamma \to \Lambda\pi\pi) = 8 \text{ MeV} , \qquad (2)$$

$$\Gamma(\Sigma_\gamma \to \Lambda_\beta \pi) \lesssim \Gamma(\Sigma_\gamma \to \Sigma\pi\pi) = 8 \text{ MeV} . \qquad (3)$$

There is again only a single invariant coupling strength for modes (1) and (2) so that their ratio is determined in the eightfold way:

$$\frac{\Gamma(N_\gamma \to N_\delta \pi)}{\Gamma(\Sigma_\gamma \to \Sigma_\delta \pi)} = 6.5 .$$

The existence of the mode (3), $\Sigma_\gamma \to \Lambda_\beta \pi$, is a test of the correctness of the assignments of resonances to unitary symmetry supermultiplets: if Λ_β is a unitary singlet, then it can be shown that unitary symmetry allows this mode if Σ_γ is a member of an octet, but forbids it if it is a member of either the 10- or 27-plet.

Finally, we consider decays of the two higher-energy resonances supposed to be first Regge recurrences of the α octet. The experimental situation is summarized in Table II: $N_\alpha(1688)$ and $\Lambda_\alpha(1815)$ are known to be mainly "elastic" resonances, i.e., for $N_\alpha(1638)$ we know $\Gamma(N\pi)/\Gamma \approx 80\%$,[8] for $\Lambda_\alpha(1815)$, $\Gamma(N\bar{K})/\Gamma \approx 75\%$;[9] perhaps the reason that $\Sigma_\alpha(187?)$ has not yet been found is because its coupling to $N\bar{K}$ is small; in any case it is not strongly formed by 1.2-GeV/c K^- on protons. Also in Table II are given the calculated partial widths for all decays of the Regge recurrences of the α octet into two-body states of meson plus stable baryon, according to the eightfold way. [Once again, there are two invariant coupling constants. They are chosen to fit the largest partial widths of $N_\alpha(1685)$ and $\Lambda_\alpha(1815)$, with the same form factor as before.] With satisfaction and relief we find that the calculated results are completely compatible with experiment.

FOOTNOTES AND REFERENCES

*Work done under the auspices of the U. S. Atomic Energy Commission.

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1. L. W. Alvarez et al., Phys. Rev. Letters, this issue; P. L. Bastien and J. P. Berge, Phys. Rev. Letters, this issue.
2. Reference 1 contains the appropriate citations; for further detail see J. J. Sakurai, in Proceedings of the International Summer School at Varenna, 1962, to be published in Nuovo Cimento, and S. L. Glashow, in Proceedings of the International Summer School in Theoretical Physics, Istanbul, 1962 (Gordon and Breach, London, to be published).
3. Words signifying sets of similar particles find their origins in musical terminology. Thus, a trio, quartet, . . . , octet, nonet, decimet, . . . is a composition for 3, 4, . . . , 8, 9, 10, . . . voices or instruments; but a triplet, quadruplet, . . . , octuplet, nonuplet, decuplet, . . . refers to 3, 4, . . . , 8, 9, 10, . . . notes played in one beat. After triplet (of pions) and quadruplet (of Δ isobars), we use "decuplet" for the 10. Because of an unfortunate earlier misuse, "octet" has become commonplace for the 8, rather than the more appropriate "octuplet." [See P. Scholes, Oxford Companion to Music 9th. Ed. (Oxford University Press, London, 1955).]
4. The success of Gell-Mann's mass formula and its generalizations is certainly one such verification. More important is to understand why an octet resonates at $J = 3/2^-$ and a decuplet resonates at $J = 3/2^+$, and why there seem to be no resonances in the 27-plet and other decuplet channels. Some progress in this direction has been reported by Cutkosky, Kalcar, and Tarjanne, Phys. Letters 1, 93 (1962), and by R. H. Capps (preprint).
5. The momentum dependence used included a form factor* and (p/M), i. e.,

$$\Gamma \propto \left| \frac{p^2}{p^2 + X^2} \right|^{\ell} \frac{p}{M} ,$$

where p = momentum of decay products of a resonance of mass M, and X is related to the size of the interaction. The two coupling constants D and F were adjusted along with X^2 to fit the three input data shown in Table I. We found X = 350 MeV.

6. In this connection, there also seems to be evidence that the δ resonances are produced more copiously than the γ resonances.

7. Also relevant are the two-body decay modes of the vectons. Using coupling constants of the eightfold way, and with the same form factor as above, we find $\Gamma(\rho \to 2\pi)/\Gamma(K^* \to K\pi) = 2.4$, in rough agreement with experiment.

8. From summary prepared by R. Omnes and G. Valladas, at the Aix-en-Provence, International Conference on Elementary Particles, 1961 (published by C. E. N. Saclay, France), p. 472.

9. W. F. Beall et al., in Proceedings of the 1962 International Conference on High Energy Physics at CERN, (CERN, Geneva, 1962) p. 368.

Table I. Two-body partial widths for the γ octet and δ decuplet.

Resonance and total width Γ	Decay mode	Momentum (MeV/c)	Width, Γ(MeV)	
			Experimental[a]	Calculated[b]
γ octet				
Ξ(1600?)	$\Xi\pi$	220	?	0.6[c]
Σ(1660)	$\bar{K}N$	406	3[d]	3
$\Gamma = 40$ MeV	$\Lambda\pi$	441	11	Input = 11
	$\Sigma\pi$	386	13	Input = 13
Λ(1520)	$\bar{K}N$	244	5	6
$\Gamma = 16$ MeV	$\Sigma\pi$	267	9	Input = 8
N(1512)	$N\pi$	450	80	67
$\Gamma = 100$ MeV				
δ decuplet				
Ω^-(1676?)	Decays weakly into $\Xi\pi$, $\Lambda\bar{K}$, or leptonically			
Ξ(1530)	$\Xi\pi$	148	<7	12
$\Gamma < 7$ MeV				
Σ(1385)	$\Lambda\pi$	210	50	35
$\Gamma = 50$ MeV	$\Sigma\pi$	119	$\lesssim 4$	5
Δ(1238)	$N\pi$	233	100	Input = 100
$\Gamma = 100$ MeV				

[a] For references to the data, see Barkas, and Rosenfeld, Lawrence Radiation Laboratory Report UCRL-8030 Rev., Feb. 1963.

[b] The D-F mixing ratio giving best fit between experiment and theory is $\alpha = 0.655$ (in Gell-Mann's notation) or $\theta = 35^0$ (in the notation of R. Cutkosky's Carnegie Inst. of Technology preprint). This value is in good agreement with dynamical considerations given in preprints by Capps, by Cutkosky, and by Martin and Wali. Moreover, Cutkosky shows that a value of θ near 33^0 is probably demanded for the bootstrap appearance of the γ octet and the δ decuplet. We thank Prof. Cutkosky for telling us of his result after our calculation was completed.

[c] The very low predicted partial width for $\Xi_\gamma \to \Xi\pi$ suggests that an alternative decay mode may predominate (such as $\Xi + \gamma$; $\Xi\pi\pi$ or, $\Lambda\bar{K}$ if Ξ_γ is sufficiently heavy).

[d] There is some discrepancy in the measurement of $\Gamma(\Sigma_\gamma \to \bar{K}N)$. According to Alvarez et it is $\lesssim 2$ MeV; according to Bastien and Berge it is $\gtrsim 4$ MeV.

Table II. Two-body decays of the α recurrences into stable baryons + mesons.

Resonance, total width, threshold	Decay mode	Momentum (MeV/c)	Width, Γ (MeV) Experimental	Width, Γ (MeV) Calculated[a]
$N_\alpha(1688)$	$N\pi$	572	80	Input = 80
$\Gamma = 100$ MeV	$N\chi$("η")	387	<20	0.5
$P_\pi = 1.00$ GeV/c	ΛK	235	< 2	1
$\Lambda_\alpha(1815)$	$N\bar{K}$	538	70	41
$\Gamma = 120$ MeV	$\Sigma\pi$	504	<40	29
$P_{K^-} = 1.05$ GeV/c	$\Lambda\chi$("η")	345	<1.3	3
$\Sigma_\alpha(1875)$	$N\bar{K}$	586		5
	$\Lambda\pi$	595		15
$P_{K^-} = 1.20$ GeV/c	$\Sigma\pi$	548		20
	ΞK	208		1
	$\Sigma\chi$("η")	322		2
$\Xi_\alpha(1972)$	$\Xi\pi$	531		5
	$\Xi\chi$("η")	290		2
$P_{K^-} = 1.40$ GeV/c	$\Lambda\bar{K}$	540		1
	$\Sigma\bar{K}$	479		41

[a] There are again two invariant coupling constants. We use the same D-F coupling constant ratio as we have determined from the decays of the γ octet, and we adjust the strength to fit the $N\pi$ decay mode of $N_\alpha(1688)$ (theoretical arguments of Cutkosky suggest that the same "self-consistent" value of θ should apply to all couplings of mesons to baryon octets). The same form factor is used as earlier (reference 6).

FIGURE LEGEND

Fig. 1. Baryons. The four unitary multiplets and their Regge recurrences. Spin and parity assignments J^P are written beside each particle if they are supporte by any experimental evidence; if not, J^P have been conjectured by assigning one known resonance to each set of quantum numbers. The notation was introduced in the *Proceedings of the 1962 International Conference on High Energy Physics at CERN,* pp. 783 and 325. Observe that the families so defined coincide with the unitary multiplets of the eightfold way. Heavy bars show stable or metastable particles; light lines show resonances. States predicted by the eightfold way but not yet seen are indicated by question marks. The masses of Ξ_γ and Ω_δ^- follow from the mass formulae alone; those of the $5/2^+$ Σ_α and Ξ_α also require the assumption of nearly parallel Regge trajectories.

Fig. 2. Mesons. The meson unitary multiplets include a β (pseudoscalar) octet and a γ (vector) octet. There are *two* observed χ_γ states (i.e., vector isotopic singlet called ω and ϕ -- one linear combination of these is presumably the eighth member of the γ octet, the orthogonal linear combination is assigned to a unitary singlet. ϕ is seen as a K_1K_2 enhancement at 1030 MeV [see Bertanza et al., Phys. Rev. Letters *9*, 180 (1962), and J. J. Sakurai, Phys. Rev. Letter *9*, 472 (1962)]. In addition there appear to be two α singlets; a $J^{PG} = 2^{++}$ pion-pion resonance at 1250 MeV called f, and a 0^{++} K_1K_1 interaction near $\bar{K}K$ threshold. For more complete references see Barkas and Rosenfeld, Lawrence Radiation Laboratory Report UCRL-8030 Rev., Feb. 1963.

All the meson states have charge-conjugation properties such that they may couple to baryon-antibaryon states, e.g., π^0 and χ have $C = +1$ (and decay into two photons), while ρ^0 and ω have $C = -1$ (and couple to a single photon).

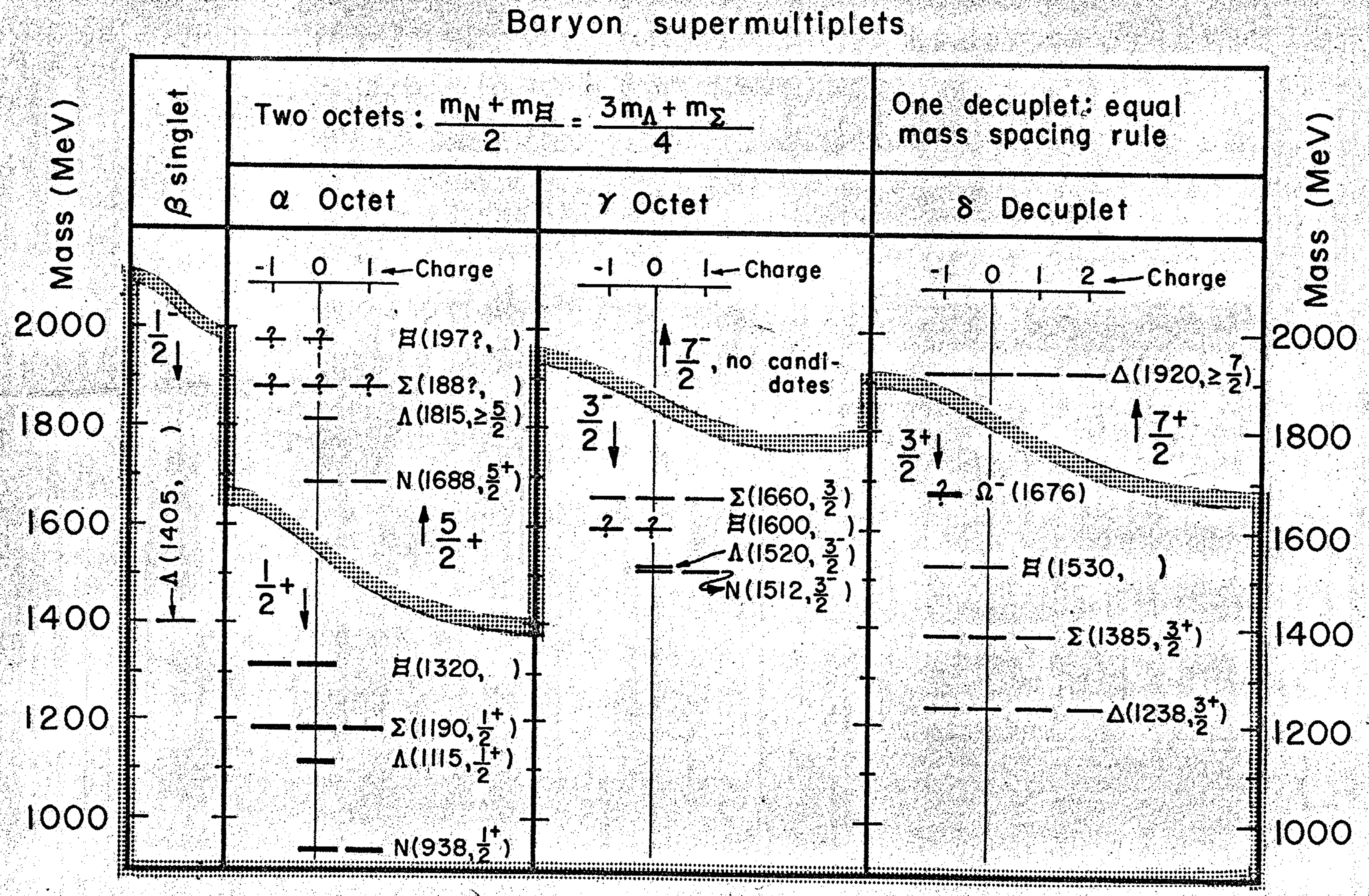

Fig. 1.

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