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THEORETICAL ESTIMATE OF MAXIMUM POSSIBLE NUCLEAR EXPLOSION

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THEORETICAL ESTIMATE OF MAXIMUM POSSIBLE NUCLEAR EXPLOSION

(Preliminary Draft)

H. A. Bethe

Preface: The following report is preliminary only. While the results are believed to be correct, it is intended to study in more detail Case B referred to in Chapter 1. Moreover, it is intended to incorporate in the report the considerations presented in the section, "Influence of Sodium Content in Reflector and Blanket." This section is being added now; the remainder represents the report as originally written. Since there was pressure to issue the report in time for the meeting of the Reactor Safeguards Committee on February 8, the report is issued in this preliminary form.

Influence of Sodium Content in Reflector and Blanket

In the main part of the report, it is assumed that the reflector is solid beryllium and the blanket is solid uranium. Actually, the reflector as presently designed contains about 15 per cent by volume of sodium when averaged over all parts of the reflector. The hot blanket region contains about 30 per cent sodium by volume. The presence of highly compressible sodium will materially reduce the sound velocity and the acoustic impedance of both regions. This will reduce the maximum pressures which could occur in a nuclear explosion.

The mean compressibility of a mixture of two substances is

$$-\frac{1}{v_o}\frac{dv}{dp} = \frac{v_{10}}{v_o}\left(-\frac{1}{v_{10}}\frac{dv_1}{dp}\right) + \frac{v_{20}}{v_o}\left(-\frac{1}{v_{20}}\frac{dv_2}{dp}\right) \qquad (a)$$

where v_{10} / v_o and v_{20} / v_o are the fractional volumes originally (at 0 pressure) occupied by the two components, and $-\frac{1}{v_{10}} \frac{dv_1}{dp}$ the compressibility of the first component. Using the compressibility of Na near 12 kilobars which is 10×10^{-12}, one gets for the reflector (85% Be, 15% Na)

$$-\frac{1}{v_o} \frac{dv}{dp} = (0.85 \times 0.56 + 0.15 \times 10) \times 10^{-12} = 2.0 \times 10^{-12} \quad \text{(b)}$$

Similarly, the hot blanket consists of 30% Na and 70% U by volume, giving

$$-\frac{1}{v_o} \frac{dv}{dp} = (0.7 \times 1.08 + 0.3 \times 10) \times 10^{-12} = 3.7 \times 10^{-12} \quad \text{(c)}$$

The sound velocity is given by

$$c^2 = \frac{dp}{d\rho} = -v^2 \frac{dp}{dv} \quad \text{(d)}$$

where v is the specific volume.

The acoustic impedance is calculated from

$$(\rho c)^2 = \frac{c^2}{v^2} = -\frac{dp}{dv} = \frac{\rho_o}{-(1/v_o)\ (dv/dp)} \quad \text{(e)}$$

where ρ_o is the uncompressed density of the mixture

$$\rho_o = \rho_{10} \frac{v_{10}}{v_0} + \rho_{20} \frac{v_{20}}{v_0} \quad \text{(f)}$$

with ρ_{10} and ρ_{20} the uncompressed densities of the two components.

This gives for the reflector

$$\rho_{or} = 0.85 \times 1.8 + 0.15 \times 0.97 = 1.68$$

$$(\rho c)_r = 0.92 \times 10^6 \quad \text{(g)}$$

whereas for pure Be, the acoustic impedance is 1.80×10^6.

Similarly, for the blanket

$$\rho_{o_b} = 0.7 \times 18.9 + 0.3 \times 0.97 = 13.5$$

$$(\rho c)_b = 1.91 \times 10^6$$

while pure uranium has 4.15×10^6. Thus, in both cases the acoustic impedance is cut in half. As a result, the pressures calculated in the paper should all be divided by 2, which further reduces the seriousness of the accident.

THEORETICAL ESTIMATE OF MAXIMUM POSSIBLE NUCLEAR EXPLOSION

1. Qualitative Description of Phenomena

The most serious accident will occur if, by mistake or by sabotage, the control rods are moved in very fast, so that the pile becomes prompt critical. It may be possible to move the controls, and increase k, faster than the neutron level increases as long as the period of the pile is appreciable, let us say 1/100 of a second or more: Then no appreciable heating and expansion of the pile occurs while k increases. However, when k exceeds prompt critical, the period (e-folding time) becomes of the order 10^{-4} sec., and the motion of the controls obviously becomes negligible during one pile period. We can then consider the position of the controls as fixed, and the value of k is determined completely by temperature and thermal expansion of the pile.

Increased power development in the pile will necessarily lead to heating of the fuel. Because of the very short times involved, there will be no appreciable heat conduction through the stainless steel and to the sodium. Even if there were, there would not be enough time for the sodium to carry the heat away. Therefore, in the first stage of the process, k is determined simply by the fuel temperature, with the rest of the pile remaining at normal operating temperature. The change of k under these conditions was described in Chapter 3 of KAPL-237.

Three possible cases can occur:

a) Temperature coefficient dk/dT negative

b) dk/dT positive, but total change of k from operating temperature T, to infinite temperature (Δ k) negative

c) dk/dT and Δ k positive

Case a) appears appropriate to SAPL-5 and case c) to the fastest loading of the KAPL reactor.

In case a), the heating of the pile will always remain moderate. Case b) leads to a heating of the pile from T_1 to the temperature T_2 at which k_{prompt} has again become equal to 1: Further heating will lead to a decay of the prompt neutrons, and, according to a general theorem, the temperature will only rise by an additional amount $T_2 - T_1$. When this is reached, only delayed neutrons can cause further increase of the power level: But before the delayed neutrons can take part in the reactions, general thermal expansion will occur and shut down any further reaction. This situation will not lead to a nuclear "explosion".

The important case is, therefore, c). In this case, the temperature will continue to increase and a final value $k_{prompt} = 1 + \Delta k$ will be reached. Then the neutron level, and the total energy developed, E, will continue to increase as $e^{\alpha t}$ where the multiplication rate, α, is given by

$$\alpha = \Delta k/\tau \qquad (1)$$

with τ the generation time. Calculations by R. Ehrlich have given

$$\tau = 4 \times 10^{-7} \text{ sec.} \qquad (2)$$

for the fast loading. Estimates of Δk by G. Roe have given about 0.01 for the same pile, so that $1/\alpha = 40$ microseconds. We shall use, in the following, this figure and also $\Delta k = 0.02$ which is likely to represent an upper limit.

The energy is all developed in the fuel. We shall find (see Section 3) that heat conduction is negligible, so that sodium and structural material remain relatively cool. We shall further find that the temperature of the fuel will be between 5000 and 10,000°C; therefore, the uranium will evaporate and may, to some extent, be ionized. Because of the high temperature, it will be sufficient to consider the fuel as an ideal gas.

Due to its high temperature, the fuel will exert a pressure on the surrounding sodium and structural material: The sodium will be appreciably compressed, the structural material very little. This compression will make more room for the fuel, and thereby reduce the pressure at any given temperature. However, at high pressures sodium becomes rapidly less compressible; therefore, the volume available to the fuel is definitely limited. There is a definite relation between the energy developed and the pressure, and the latter is essentially constant throughout the core (Section 3).

The pressure of the core acts on the beryllium reflector and causes a compression wave to go out into the latter. Because of the small thickness of the reflector (6.5 cm) and the high sound velocity in Be (about 10^6 cm/sec), the compression wave reaches the uranium blanket in a very short time (6.5 μ sec), short even compared with the characteristic time of the reaction (e-folding time = 40 μ sec). The compression wave is then reflected, leading to an increase of pressure and decrease of outward velocity, etc. In first approximation, we may neglect the Be reflector, and calculate as if the core were immediately pushing the blanket. This obviously requires greater pressures than the pushing of Be, and thus leads to an overestimate of the severity of the accident. In section 5, we correct for this. It is obviously not necessary to take into account reflection at the interface of blanket and shield, because the sound velocity in uranium is low (2.2×10^5 cm/sec) and the blanket thick (25 cm).

In an outgoing compression wave, the velocity is proportional to the pressure. The process must continue until the core has expanded enough so that the prompt multiplication of neutrons stops; we shall calculate the pressure necessary to bring this about. Subsequently, the neutron generation stops but the pressure developed in the fuel will be released by a large further expansion of the core. The compression wave through reflector, blanket and finally shield is thus sustained much longer than the nuclear reaction. The entire energy developed must be taken into account when estimating the damage.

2. Main Formula

We shall first develop a simple formula, making very simplified assumptions. Afterwards, we shall successively correct this formula.

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According to the qualitative picture of the first section, the Be reflector is pushed out without being much compressed, and the inner part of the uranium blanket is also pushed out. At the same time, the core is expanded so that its mean density decreases. In these conditions, k will be proportional to some inverse power of the core volume Ω, let us say Ω^{-n}, where n may be about 1 (see Chapter). Thus, in order to decrease k_{prompt} from $1+\Delta k$ to 1, we must increase the radius R by

$$\Delta R = \frac{R}{3n}\Delta k \qquad (4)$$

Taking R = 27 cm, n = 1, k = 0.01, we get $\Delta R = 0.9$ millimeters, which is rather small.

As already mentioned (see Eq. 7, below), the velocity u of the interface between core and reflector is proportional to the pressure and therefore increases exponentially as $e^{\alpha t}$, with α given by (1). Therefore, the displacement ΔR increases also as $e^{\alpha t}$, and we have

$$u = \dot{R} = \alpha \Delta R \qquad (5)$$

Inserting (1) and (4), the velocity at which the prompt neutron multiplication stops, is

$$u_o = \frac{R}{3n\tau}\Delta k^2 \qquad (6)$$

and inserting our numbers

$$u_o = 2.2 \times 10^7 \Delta k^2 \qquad (6a)$$

For $\Delta k = 0.01$, this gives 22 meters/sec (50 miles per hour), for our extreme value, $\Delta k = 0.02$, we get 90 meters/sec = 200 miles per hour. These velocities are quite moderate and about two

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orders of magnitude smaller than detonation velocities in high explosives. This shows already that the accident will be much milder than an H.E. explosion, let alone an atomic bomb explosion.

The pressure will be related to the outward velocity by the condition that we have an outgoing compression wave in the uranium blanket. Since the material velocity (6a) is small compared with the sound velocity in uranium, it is legitimate to use acoustical theory. If, in addition, we consider a plane rather than a spherical problem (cf. Section 5), we have

$$p = \rho_U c_U u \qquad (7)$$

where ρ_U is the density of uranium (= 19) and c_U its sound velocity (2.2×10^5 cm/sec), giving for the acoustic impedance

$$\rho_U c_U = 4.2 \times 10^6 \qquad (8)$$

and for the pressure when multiplication stops

$$p_o = \frac{1}{3n} \rho_U c_U \frac{R}{\tau} \Delta k^2 = 0.95 \times 10^{14} \Delta k^2 \text{ dynes/cm}^2 \qquad (9)$$

Thus the probable maximum pressure, for $\Delta k = 0.01$, is 10^{10} dynes/cm^2 = 10 kilobars (1 bar = 10^6 dynes/cm^2 = about 1 atmosphere). Our extreme assumption, $\Delta k = 0.02$, gives 40 kilobars, about a factor 10 below the pressures in high explosives.

Corrections discussed in Chapter 5 reduce the expected pressure to about 50 to 60 per cent of this; i.e., 20 to 26 kilobars.

The energy per unit volume of fuel is

$$E_1 = \frac{p}{\gamma - 1} \qquad (10)$$

where γ is the ratio of the specific heats at constant pressure and constant volume. Probably, $1/(\gamma-1)$ is about 4.5 (see Section 3). The energy in Na and structural material is negligible compared with that in the fuel. Now let V_o be the fraction of the core volume occupied by fuel at high temperature; it is about 1/4. Then $E_1 V_o$ is the energy per unit core volume, and

$$E_{tot} = \Omega \; V_o E_1 \Omega p V_o/(\gamma-1) \tag{11}$$

is the total energy developed, where Ω is the total volume. With $R = 27$ cm, we have $\Omega = 82$ liters $= 8.2 \times 10^4$ cm^3. Taking $V_o = 1/4$, $1/(\gamma-1) = 4.5$, we get

$$E_{tot} = 9 \times 10^{18} \Delta k^2 \text{ ergs} = 2.2 \times 10^{18} \Delta k^2 \text{ k cal} \tag{12}$$

For $\Delta k = 0.01$, we thus get 22,000 kilo-calories, equivalent to 50 pounds of TNT; for $\Delta k = 0.02$, we have 200 pounds of TNT equivalent. However, because of the much smaller pressures involved, the accident is in reality much milder than an explosion of these quantities of TNT.

3. Temperature and Pressure in the Core

As has been stated, we shall assume that the fuel behaves as an ideal gas with a given value of $\gamma = c_p/c_v$. Then the energy contained in this gas is

$$p V/(\gamma-1)$$

where V is the total volume occupied by the gas. We may write $V = \Omega V_o$ where Ω is the core volume and V_o the fraction of the volume occupied by fuel at the time when the catastropic nuclear reaction takes place. We thus want to calculate V_o.

The volume composition of the planned core for the fast core is approximately

9% fuel

6% void in the pins

36% structural material

49% sodium

Obviously, at the time of the reaction the fuel will occupy not only its original volume but also the volume which originally was void. Moreover, since there is appreciable pressure, the other materials will be compressed, and the fuel will be able to expand into the volume thus vacated.

The compression modulus of the structural materials will be about 2×10^{12} dynes/cm^2. A pressure of 40 kilobars will therefore cause a compression of only 2% of the initial volume of this material, or 0.7% of the total initial volume. This is negligible.

The compressibility of sodium is large but decreases with increasing pressure. According to the measurements of Bridgman (1), 10 kilobars pressure causes a compression (volume decrease) of (solid) Na by 13%; at 25 kilobars (maximum expected pressure) one may estimate by extrapolation that the compression is 26%. We shall assume the compression to be 20% of the Na volume which corresponds to 10% of the total volume; adding this to the 15% originally occupied by fuel and void gives

$$V_o = 0.25 \qquad (13)$$

We shall now calculate the fuel temperature. In the core, there is a fuel density of about $\rho_1 = 1.5$ g/cm^3. This amount of fuel occupies a volume V_0 during the reaction. If A = 235 is the atomic weight, and V_m the volume per mole, we have

1) Bridgman, Physics Review 27, 68 (1928)

$$p\,V_m = p\,A\,V_0/\rho_1 = 40\,p = 4 \times 10^{15} \quad \Delta k^2 \text{ ergs} \tag{14}$$

Now, if there were no ionization, $p\,V_m$ would be equal to RT, with R the gas constant; if on the average z - 1 electrons are split off each uranium atom, we have

$$RT = p\,V_m/z \tag{15}$$

Inserting $R = 8 \times 10^7$ ergs/degree, we get

$$T = 5 \times 10^7 \,\Delta k^2/z \quad \text{degrees} \tag{16}$$

which gives 5000/z for $\Delta k = 0.01$, and 20,000/z for $\Delta k = 0.02$. At the lower temperature, ionization is not likely to be important, so that $T = 5000^\circ$; at the higher temperature, one ionization may easily have occurred, giving $T = 10{,}000^\circ$. The temperatures are thus very high, so that there is surely complete vaporization of the fuel.

The energy of a monatomic gas is 3/2kT per atom. The energy of the evaporated uranium contains, in addition, the heat of vaporization which is about 4 ev., corresponding to $k \times 47{,}000^\circ$. This increases the energy content at 5000° to about 4.5 k T, making $1/(\gamma - 1) = 4.5$. If there is ionization, an additional 6 ev is expended per atom, giving a total of 8 ev., or 4 ev. per particle (ion and electron), and this energy is also available because in adiabatic cooling the ion and electron will recombine. It is easily calculated that this leads again, at 10,000°, to $1/(\gamma - 1) = 4.5$.

We have only considered the internal energy of the uranium and have not yet taken into account the work done by the expansion of the uranium. This, however, is exactly compensated by the work done on the Na and structural material by compression. Moreover, this work is very small, amounting in the case of Na per unit core volume to:

$$E_{Na} = -\int p \, d \, V_{Na} \tag{17}$$

Now, as we have mentioned, the compressibility of Na decreases appreciably with pressure. We, therefore, overestimate E_{Na} if we assume p to be linear in the compression, thus

$$E_{Na} < \frac{1}{2} p \ \Delta V_{Na} \tag{17a}$$

But ΔV_{Na} is about 10% of the total volume, or 0.4 V_o, and with

$$E_U = p V_o/(\gamma - 1) \tag{18}$$

and $1/(\gamma - 1) = 4.5$, we have

$$E_{Na} < 0.05 \ E_U \tag{18a}$$

which is negligible.

The heat flow from fuel to the other materials is also small. The heat first has to penetrate through the structural material; e.g., the pins. In heat conduction, the distance of penetration in time t is roughly given by

$$X^2 = \frac{k}{c_v \rho} \ t \tag{19}$$

where k is the heat capacity, c_v the specific heat and ρ the density. Taking the values for Fe at ordinary temperatures (which is questionable), $k \approx 0.1$, $c_v \approx 0.1$, $\rho \approx 8$, and taking $t = 1/\mathcal{L} \approx 40\mu$ sec, we get

$$X \approx 2 \times 10^{-3} \text{ cm} \approx 1 \text{ mil} \tag{19a}$$

Nearly all the heat therefore remains in the fuel, as has been assumed.

This conclusion is not changed by taking into account radiation. For, although the energy density of radiation is rather high at 10,000°, the radiation at this temperature is largely in the form of quanta with energy $h\nu$ of a few ev., and such quanta are especially strongly absorbed by any material, leading to exceedingly high opacity.

4. <u>The Rarefaction Wave at the Surface of the Core</u>

So far, we have considered the core as of uniform density and pressure. Actually, the surface of the core moves outward - this is in fact the desired effect which eventually stops the nuclear reaction - therefore there must be a volume expansion at least near the surface, and a pressure gradient. We should, therefore, obtain an equation of state for the core material.

The heat developed per unit volume is constant throughout the core. (This is true for uniform power production; if the power is non-uniform, this fact can easily be taken into account and does not change the result substantially). At any given time, therefore, all parts of the core lie very nearly on the same adiabatic so that their pressure and volume are related by the usual adiabatic equation of state

$$p \; V^{\gamma} = \text{constant} \tag{20}$$

where V is the volume occupied by fuel. In those parts of the core which have not expanded, $V = V_o$, which we have found to be about 0.25. If a volume element of the core has expanded by a factor $1 + \Delta V$, essentially all the additional volume will be occupied by fuel so that now the fuel volume is $V_o + \Delta V$ and the pressure is

$$p = p_o(t) \left(\frac{V_o}{V_o + \Delta V} \right)^{\gamma} \approx p_o(t) \left(1 - \frac{\gamma}{V_o} \Delta V \right) \tag{21}$$

where $p_o(t)$ is the pressure at the given time in the undisturbed (unexpanded part of the core. This pressure increases exponentially with time,

$$p_o(t) = P \; e^{\alpha t} \tag{22}$$

Due to the outward motion of the reflector, a rarefaction wave starts inwards from the core-reflector interface whose head moves with sound velocity. This velocity is given by

$$c(t) = \sqrt{\frac{dp}{d\rho}} = \sqrt{-\frac{1}{\rho_o} \frac{dp}{d\Delta V}} = \sqrt{\frac{P}{\rho_o V_o}} \tag{23}$$

The mean density, ρ_o, of the projected reactor is about 4, the volume $V_o = 1/4$, $\gamma = 1.2$, so that $c \approx 1.1 \sqrt{p_o}$. When the nuclear reaction stops, this is 1.1 or 2.2 x 10^5 cm/sec, corresponding to our assumptions $\Delta k = 0.01$ and 0.02, respectively. In the latter case, the sound velocity in the core is equal to that in solid uranium.

The distance by which the rarefaction wave has traveled into the core, is obtained by integration of (23), using (22), and is

$$X(t) = 2\,c\,(t)/\alpha \qquad (24)$$

The distance X at the time the reaction stops is independent of Δk and is 9 cm, with our constants. This is reasonably small compared with the radius of the reactor. Therefore we may really speak of a rarefaction wave, rather than of a uniform expansion of the core. Moreover, we may consider the rarefaction wave in good approximation as plane.

With this assumption, the equation of motion in the rarefaction wave becomes

$$\rho_0 \frac{du}{dt} = -\frac{\partial T}{\partial r} = \frac{\gamma p_c(t)}{V_c} \quad \frac{\partial \Delta V}{\partial r} \qquad (25)$$

where u is the velocity of a material point which was originally at radius r. This equation can be solved by a similarity solution. For this purpose, we consider "similar points"; i.e., points whose distance from the interface $r = a$ is a given fraction of X(t) Eq. (24). That is, we set

$$a - r \quad X(t)\xi \qquad (26)$$

and use ξ as a new independent variable. Then, e.g., at a given ξ the material velocity u will be a given fraction of the interface velocity u_0, thus

$$u(r,t) = u_0(t)\,F(\quad) \qquad (27)$$

where $u_0(t)$ behaves as $e^{\alpha t}$, being proportional to $p_0(t)$. (Eq. 7).

It is further convenient to introduce as a dependent variable the displacement $y(r,t)$. Then we may also write

$$y(r,t) = y_o(t)\, f(\xi) \tag{28}$$

where the displacement of the interface is

$$y_o = u_o/\alpha = Y e^{\alpha t} \tag{29}$$

In terms of y, the velocity is

$$u = (\partial y/\partial t)_r \tag{30}$$

and the expansion

$$\Delta V = (\partial y/\partial r)_t \tag{31}$$

and the equation of motion (25) becomes

$$\frac{\partial^2 y}{\partial t^2} = c^2(t) \frac{\partial^2 y}{\partial r^2} \tag{32}$$

with the sound velocity c given by (23).

Inserting (28) gives, for instance

$$u = \alpha Y e^{\alpha t} f(\xi) + Y e^{\alpha t} f' \left(\frac{\partial \xi}{\partial t}\right)_r \tag{33}$$

with the prime denoting differentiation with respect to ξ.

Now, from (24), (26), (23),

$$\left(\frac{\partial \xi}{\partial t}\right)_r = -\alpha \frac{\xi}{2} \tag{34}$$

and therefore

$$u = \alpha y_o \left(f - \frac{1}{2} \xi f'\right) \tag{35}$$

Similarly,

$$\frac{\partial^2 y}{\partial t^2} = \alpha^2 y_o \left(f - \frac{3}{4} \xi f' + \frac{1}{4} \xi^2 f''\right) \tag{36}$$

$$\frac{\partial^2 y}{\partial T^2} = \frac{1}{X^2} y_o f'' = \frac{\alpha^2}{4c^2} y_o f'' \tag{37}$$

using (24). Inserting (36) and (37) into (32) shows that both sides of this equation depend in the same way on the time; namely, as y_o, which proves that the similarity solution actually works. Eq. (32) now reduces to the following differential equation in ξ:

$$f''(1-\xi^2) + 3\xi f' - 4f = 0 \tag{38}$$

This equation has a singular point at $\xi = 1$. It is convenient, therefore, to introduce

$$\eta = 1 - \xi \tag{39}$$

then (38) becomes

$$\eta(2-\eta)\frac{d^2f}{d\eta^2} - 3(1-\eta)\frac{df}{d\eta} - 4f = 0 \tag{40}$$

This equation can be solved by a power series in η whose first terms are

$$f = \eta^{5/2} + \frac{1}{28}\eta^{7/2} + \frac{1}{224}\eta^{9/2} + 0.00084\eta^{11/2} + 0.00020\eta^{13/2} + 0.00005\eta^{15/2} + 0.00001\eta^{17/2} + \ldots \tag{41}$$

The solution is especially interesting at the interface $\eta = 1$ where we get

$$f(1) = 1.0412$$

$$f'(1) = 2.6515, \quad f'/f = 2.547 \tag{42}$$

Therefore, using (31), (35), (24):

$$\frac{\Delta V}{u_o}(a) = \frac{1}{\alpha X}\frac{f'(1)}{f(1)} = \frac{1.27}{c(t)} \tag{43}$$

Further, using (7) and (23):

$$\frac{\gamma \Delta V}{V_o} = 1.27 \frac{\gamma p_o}{\rho_U c_U c(t) V_o} = 1.27 \frac{\rho_o c(t)}{\rho_U c_U} \tag{44}$$

Now we have seen (below Eq. 23) that c(t) is between $\frac{1}{2} c_U$ and c_U, according to the value of Δk assumed, ρ_o is about 4 and $\rho_U = 19$, therefore

$$\frac{\gamma \Delta V}{V_0} \approx 13 \Delta k = 0.13 \text{ to } 0.26 \qquad (45)$$

Referring back to (21), we see that the pressure at the interface is 13 to 26% lower than at the center of the core. Since the calculation in Eq. (7) and in the next section gives the interface pressure, the core pressure is 15 to 35% higher, which is only a moderate amount. At the same time, the smallness of the correction $\gamma \Delta V/V_0$ justifies the linear approximation in (21), which made the solution of the differential equation possible. It also justifies the approximation that all parts of the core are on the same adiabatic, Eq. (20).

The result (43) is easily understandable: The displacement of the interface is

$$y_0 = u_0/\alpha \qquad (46)$$

therefore the <u>average</u> expansion in the rarefaction wave is

$$\langle \Delta V \rangle_{Av} = y_0/X = u_0/\alpha X \qquad (47)$$

If we assume the expansion ΔV to be a linear function of position in the rarefaction wave, we get for the expansion at the interface twice the average expansion (47). Eq. (43) shows that this plausible assumption is nearly satisfied.

5. <u>Waves in Beryllium Reflector, and Spherical Corrections</u>

So far, we have considered all the beryllium to move as a unit, and have considered the compression wave in uranium to be plane. We shall now correct these points, but we shall still retain the acoustic approximation, viz. that the change of density is small, which is very well justified.

In a spherical acoustic wave, the pressure is

$$p = \frac{\rho c}{r}\left[f'(t - r/c) + g'(t + r/c)\right] \tag{48}$$

and the outward velocity

$$u = \frac{c}{r^2}\left[f(t - r/c) + g(t + r/c)\right] + \frac{1}{r}\left[f'(t - r/c) - g'(t + r/c)\right] \tag{49}$$

where f and g are arbitary functions of their arguments, the prime means differentiation with respect to the argument, c is the sound velocity and ρ the density.

In our problem, both u and p have everywhere the time dependence $e^{\alpha t}$, therefore both f and g must have this time dependence, and in each medium

$$f' = \alpha f; \quad g' = \alpha g \tag{50}$$

In the uranium blanket, we have only outgoing waves, therefore only a function f. Then, if the subscript 2 denotes quantities at the reflector-blanket interface, and the subscript U quantities referring to uranium,

$$\frac{\rho_U c_U u_2}{p_2} = 1 + \frac{c_U}{\alpha r_2} \tag{51}$$

with $c_U = 2.2 \times 10^5$, $r_2 = 33.5$ cm, $\alpha = 5 \times 10^4$, the second term is 0.13. This represents the correction from plane to spherical wave, and is seen to decrease the pressure required for a given velocity.

In the beryllium reflector, we have both the f and the g function, and their ratio can be obtained from the ratio

u_2/p_2, viz.

$$\frac{f(t - r_2/c) - g(t+r_2/c)}{f(t - r_2/c)+g(t+r_2/c)} = \frac{\rho_B\, c_B\, u_2}{p_2} - \frac{c_B}{\alpha r_2}$$

$$= \frac{\rho_B\, c_B}{\rho_U\, c_U} - \frac{c_B}{\alpha r_2} \left(1 - \frac{\rho_B}{\rho_U}\right) \equiv K, \text{ say} \qquad (52)$$

Here (51) has been inserted and the subscript B denotes beryllium. (52) can be solved for g/f. Now we need the quantities g and f at the core-reflector interface, r_1; these can be obtained from the values at r_2 by remembering that both f and g behave as $e^{\alpha t}$; therefore

$$f(t - r_1/c) = f(t - r_2/c)\; e^{\alpha\delta/c}$$
$$g(t+r_1/c) = g(t+r_2/c)\; e^{-\alpha\delta/c} \qquad (53)$$

where c stands for c_B and $\delta = r_2 - r_1 = 6.5$ cm is the thickness of the Be reflector. This gives

$$\frac{g(t+r_1/c)}{f(t - r_1/c} = e^{-2\alpha\delta/c} \quad \frac{1 - K}{1 + K} \qquad (54)$$

We wish to obtain the ratio of velocity to pressure at r_1; we have

$$\frac{\rho_B\, c_B\, u(r_1)}{p(r_1)} = \frac{c_B}{\alpha r_1} + \frac{f(t - r_1/c - g(t+r_1/c}{f(t - r_1/c + g(t+r_1/c)}$$

$$= \frac{c_B}{\alpha r_1} \quad \frac{K+\tanh \alpha\delta/c}{1 + K \tanh\alpha\delta/c} \qquad (55)$$

The thickness of Be is small and the sound velocity is large (about 10^6 cm/sec), therefore $\alpha\delta/c$ is rather small: For $\Delta k = 0.02$, we had $\alpha = 5 \times 10^4$ and therefore $\alpha\delta/c = 0.325$. The quantity $\alpha\delta/c$ is a measure of the error made in the assumption that the Be moves as a whole. If this assumption is made,

(55) reduces to

$$\frac{\rho_B\, c_B\, u(r_1)}{p_1} = \frac{c_B}{\mathcal{L} r_1} + K \frac{\rho_B\, c_B}{\rho_U\, c_U} + \frac{c_B}{\mathcal{L}} \left(\frac{1}{r_1} - \frac{1}{r_2} + \frac{\rho_B}{\rho_U\, r_2} \right) \qquad (56)$$

using (52).

The first term on the right gives our old result, Eq. (7); the second term is due to the spherical effect and amounts to 0.47 times the first if we set $\rho_B = 1.8, \mathcal{L} = 5 \times 10^4$. This is an appreciable correction toward lower pressures in the core.

Evaluation gives the results listed in Table 1.

Table 1

$\mathcal{L}$	2.5×10^4	5×10^4
K	- 0.652	- 0.111
$\mathcal{L}\delta/c$	0.162	0.325
$\rho_B\, c_B\, u(r_1)/p(r_1)$	0.932	0.951
$G \equiv p(r_1)/\rho_U\, c_U\, u(r_1)$	0.461	0.451
$F \equiv p_{core}/\rho_U\, c_U\, u(r_1)$	0.53	0.61

The line denoted by G would be 1 according to Eq. (7). Therefore, we get a reduction of the interface pressure, by more than a factor 2, and by very nearly the same factor in either case. In the last line, F, we have given the ratio of the core pressure to that given by Eq. (7), taking into account the effect of the rarefaction wave calculated in the last section. This shows that the core pressure is 50 to 60% of that calculated from (7), thus reducing the seriousness of the accident.

6. Holes in Reflector

It may be that the reflector is not completely in place but that a control rod is missing, either in the inner or outer layer of the reflector. In either case, there will be greater expansion of the core at the point where the rod is missing than elsewhere. This may reduce k at a faster rate because the density of core material is reduced faster. However, it might happen that the reactivity is increased by displacing active material into the Be reflector; this could aggravate the accident. It will be important to insure by measurements that the reactivity is not increased by such displacement. If it is increased, two ways are open to minimize the adverse effect; namely,

1) to keep the entire inner layer of control rods in place as long as there are any rods of the outer layer in place, and to begin removing inner rods only after all outer rods have been removed, or

2) to always remove several neighboring rods of the inner layer simultaneously.

With procedure (1), the velocity of the core-reflector interface for given pressure is increased at all places where the outer rods are missing, and this will greatly reduce the core pressure at the time the nuclear reaction stops. In particular, when all outer rods are missing, the core pressure will be reduced by more than a factor 2 (See Eq. 61).

With the second procedure, one gets bulges of the core extruding into the region where the reflector is missing, but these bulges are wide and not very thick, thereby minimizing the extra moderation due to the larger contact surface between core and reflector. However, the second procedure has the possible disadvantage of making the power distribution unsymmetrical; therefore the first should be used when feasible.

It is very easy to estimate the displacement of the interface at points where only the inner layer of beryllium is present. Then it is a good approximation to consider that layer as moving as a whole: If p_1 is the interface pressure and ℓ the thickness of the Be layer (about 3.5 cm), then the interface displacement y is given by

$$\rho_B \ell \ddot{y} = p_1 \sim e^{\alpha t}$$

$$y = p_1 (\rho_B \ell \alpha^2) \tag{57}$$

and the interface velocity, $u_1 = \alpha y$. However, the pressure p_1 is now reduced: Using the results of Section 4, we have for the difference between the central pressure p_o and p_1:

$$p_o - p_1 = \frac{p_o \gamma \Delta V}{V_o} \approx \frac{\gamma p_o u_1}{c(t) V_o} = \rho_o c(t) u_1 \tag{58}$$

and therefore

$$u_1 \quad \frac{p_o}{\rho_o c(t) + (p_1/u_1)} = \frac{p_o}{\rho_o c(t) + \rho_B \ell \alpha} \tag{59}$$

Comparing this with (7) and Table 1, we get for the ratio of u_1

to the velocity u_o at points where the reflector is complete:

$$\frac{u_1}{u_o} = \frac{\rho_U \, c_U \, F}{\rho_o \, c(t) + \rho_B \, l \mathcal{L}} \tag{60}$$

where F is the factor calculated in Table 1. With $\rho_B = 1.8$, $l = 3.5$, $\mathcal{L} = 5 \times 10^4$, $c(t) = c_U = 2.2 \times 10^5$, $F = 0.61$:

$$\frac{u_1}{u_o} = 2.2 \tag{61}$$

Since the interface in general moves by about 1 mm, the interface at the place of missing reflector rods will move about 2 mm. The danger of increased moderation is therefore small. For the less serious case $\Delta k = 0.01$, the ratio u_1/u_o is somewhat greater. However, the approximations of Section 4 are no longer well justified in our case so that the actual ratio u_1/u_o may be somewhat larger.

7. Subsequent Developments

At the time when neutron multiplication stops, a certain amount of energy has been developed which has been calculated in Section 3. This energy is located, at that time, almost entirely inside the core. Subsequently, the gaseous fuel in the core will expand and do work on the reflector, blanket, etc. This means that the compression waves moving through these components will not stop when the nuclear reaction stops, but will continue until all the energy in the fuel has been transformed into work. Of course, in cooling, the fuel will condense again and the heat of condensation is made available as work; it was, therefore, included in Section 3. Similarly, electrons and ions will recombine and make the ionization energy available.

The compression in the compression wave in the blanket is appreciable (about 4% for $\Delta k = 0.02$) and the pressure will exceed the yield stress. The strain caused by the wave consists of a radial compression and a tangential expansion, giving rise to a hoop tension. Most of the deformation is undoubtedly plastic; i.e., irreversible, and a certain amount of work, necessary to overcome the yield stress, will be transformed into heat. This amount is, per cm^3:

$$W = S \left(\frac{y}{r} - \frac{\partial y}{\partial r} \right) \qquad (62)$$

where S is the yield stress, y the final radial displacement of a material element originally at r, y/r the tangential and $\partial y/\partial r$ the radial strain. In an outgoing spherical wave, the behavior of y is between 1/r and $1/r^2$ (Eq. 49); therefore

$$W = \beta S\, y/r \qquad (63)$$

where β is between 2 and 3. The 1/r behavior, and hence $\beta = 2$, is likely to be a closer approximation and will be used in the following.

The yield stress S is known to be much greater for rapid deformations than for slow ones; a value S of about 10 kilobars $\approx$ 140,000 psi seems appropriate for U, and somewhat less for the concrete shield. If y_o is the displacement of the core-reflector interface, $y = y_o r_1/r$ and the total energy dissipation by plastic deformation is

$$W_{tot} = 4\pi \int_{r_1}^{r_3} r^2 \, dr \, W = 8\pi S \, y_o \, r_1 (r_3 - r_1) \quad (64)$$

where the upper limit r_3 is either the outer radius of the shield, or the point where the wave becomes too weak to do plastic work. The different values of S in blanket and shield should be taken into account. Now

$$4\pi y_o \, r_1^2 \approx \Delta\Omega \quad (65)$$

is the expansion of the core volume; thus we get

$$W_{tot} = 2 S \Delta\Omega (r_3/r_1 - 1) \quad (66)$$

The energy originally developed is

$$E_{tot} = \frac{V_o}{\gamma - 1} p_o \Omega \approx p_o \Omega \quad (67)$$

Now the expansion $\Delta\Omega$ is of the order of magnitude Ω, or rather greater, the limitation being given by the pressure wave becoming too weak to do further plastic work. Further, if we take the outer radius of the shield for r_3, we have r_3/r_1 about 10. Therefore

$$\frac{W_{tot}}{E_{tot}} \gtrsim 20 \, S/p_o \quad (68)$$

But S/p_o, even for the larger accident contemplated, is about 1/4 if we take for S the yield stress for U, and perhaps 1/8 for concrete. This means that the plastic work can almost certainly dissipate essentially all the energy developed in the nuclear reaction. Clearly, the thick concrete shield is very important in this respect.

It may be questioned whether r_3 should really be taken equal to the shield radius, or whether at a smaller radius the wave gets too weak to do plastic work. However, if the latter were true, this would automatically mean that the wave getting to the outside of the shield involves a purely elastic deformation and is therefore extremely weak.

We believe, therefore, that only a very small fraction of the energy developed will appear ultimately as kinetic energy of fragments of the shield or the reactor. Even if <u>all</u> of it were transformed into kinetic energy, then, for a shield weight of 200 tons = 2×10^8 grams, the average velocity would be according to (12):

$$v_{Av} = 2 E_{tot}/M = 3 \times 10^5 \quad k \qquad (69)$$

i.e., 30 meters per second (70 miles per hour) for $k = 0.01$. This makes the event look like a car accident, rather than an explosion. $k = 0.02$ gives, of course, twice this velocity.

On the other hand, if all the energy is transformed into heat, the average temperature rise, even for the larger accident, is only $1^{\circ}C$.

Of course, it must be assumed that the concrete shatters, that all pipes, containers and working parts break, and that liquids and gases from the reactor can freely escape after the accident. There will almost surely be a sodium fire, and precautions must be taken to minimize its size. The fission product gases will escape into the reactor building, but the plutonium formed in the blanket will remain contained in the still-solid uranium blocks. There is very good hope that the reactor building will remain intact, and that the fission gases can remain confined in it for a considerable time, allowing a regulated slow escape later on. The only plutonium that escapes into the reactor building will be that which has been formed in the 238 contained in the fuel itself, but even this is likely to condense almost quantitatively before it can escape into the atmosphere. The accident, therefore, while destroying the reactor, is not likely to cause major health hazards to the surrounding country.

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