Secretary's Summit on Mathematics
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Mathematics in the 21st Century

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Thank you, Secretary Paige, for inviting me to speak at this summit on a topic that is very dear to my heart. I had the privilege of reading some of the papers that will be presented today, and it seems to me that the speakers are approaching important issues of mathematics education with admirable pragmatism and resolve. This morning I would like to share some personal perspectives on mathematics, which been an important part of my professional life.

Mathematics is beautiful

I must disclose at the outset that I love mathematics as many other people love music or poetry or fine art. To me mathematics is a language rich with metaphor, and deep with insight beyond any other form of communication. As others might treasure a favorite play of Shakespeare, or a Bach cantata, I have read and re-read the works of great scientists who were also great mathematical stylists -- especially physicists like Albert Einstein, Enrico Fermi, and Paul Dirac. I do also enjoy Bach and Shakespeare, and much else besides mathematics, but none of these more than math.

Mathematics is the language of nature

This thought -- that math is beautiful -- was the first that came to mind when I was asked to speak today about *Mathematics in the 21st Century*. But two more immediate reactions seemed equally important. The second is that the world is inherently mathematical. As Galileo first remarked, "The book of nature lies open for all to read, and it is written in the language of mathematics." Galileo was one of a line of thinkers that begins with Pythagoras who believed that the structure apparent in the natural world is described most deeply and most accurately by mathematics. In the generation following Galileo's, Isaac Newton attempted to account for natural phenomena in terms of a few fundamental principles in the same way that Euclid had deduced all of geometry from a few definitions and axioms. Today it is mathematics that links the steps in the impressive chain of explanation that stretches from quarks and leptons to the cosmos.

Mathematics is embedded in the entire universe of human and natural phenomena -- not only in the motions of elementary particles, or the intricate dance of molecules in the very cells of life, nor only in the realm of natural science, but also in the practical affairs of human society, in demographics, economics, and environmental issues. Even the dizzy swirl of chaos is captured in the metaphors of mathematics.

Mathematics is power

The third thought that came immediately to mind is that mathematics is power. I cannot recall when this dawned on me, but it was already part of my personal creed by my senior year in high school. I now realize how fortunate I was to appreciate at an early age the connection between the abstract games of mathematics and the behavior of real things around me. Even the simple formulas fascinated me, like that for distance traveled at fixed speed in a certain time. Or the law of the lever, or the relation between the lifting ability of pulleys and the number of ropes connecting them. These "laws" gave me the power to do things, to answer questions, to create ways to repair something, to accomplish some task. I became a physicist because I saw that mathematics was not only a language, but also a powerful tool.

I am not a mathematician

Despite my love of mathematics, and my life-long study of it, and its importance to my work, I am not a mathematician. I use math, but, except in a very limited way, I do not create it. I am a scientist, and my first love is for the tangible and complex reality in which we lead our lives. The aim of mathematics, for those who pursue it as mathematicians, is not primarily to search for truths about the natural world, it is to create beautiful and interesting conceptual structures. It is true that many mathematical inventions have been motivated by studying nature. But mathematics is not tied down to only natural things. You do not have to be a poet to use language, or to love poetry. But you do have to know the language. One of the aims of this conference is to stimulate the improvement of instruction in this beautiful, insightful, powerful language of mathematics.

Early learning experiences

Arithmetic, the art of counting and combining numbers, is indispensable to mathematics, and I acquired it with difficulty. I had trouble remembering the tables of addition and multiplication and their inverses. To help me keep up in school, my mother drilled me with "flash-cards." My father played card games with me in which we kept score with mental math. With their help I mastered the feat, and went rather smoothly, but without distinction, through the grade school material. My interest picked up with 'word problems,' and it was through them that I began to think of algebra as an abstraction of relationships among familiar things. I rebelled, however, at the dull repetition of the homework. Pages of equations to solve, identities to simplify, polynomials to factor. When my grades began to slip, not for lack of understanding, but for not doing the assignments, my father stepped in and imposed restrictions on my freedom until I caught up again. I did not actively dislike the algebra teacher who piled on all this work, but I did not appreciate her at the time, and I thought it unjust that I should be forced to do the work when I could easily pass all the exams without it.

College math and physics

When I arrived in college as a freshman physics major, I discovered to my surprise that I could do the homework faster than most of my classmates. All that drill in high school had paid off. College level physics homework is very difficult. It is the first time most students learn how mathematics is actually used in science. If math skills are weak, students bog down in computation and miss the physical principles key to understanding the material. Calculus, in particular, carries a heavy load of algebraic manipulation, and algebra requires absolute mastery of arithmetic. Of the more than one hundred of my classmates who thought they would be physicists, more eighty percent dropped out, most after the first course. This was in the late 1950's. Failure rates in freshman physics are often high, and deficiency in math skills of the simplest kind are the most common reason. This explanation does not account for gender differences in dropout rates, which is a curious phenomenon.

Some things never change

You might ask what all this has to do with mathematics in the twenty-first century. In the first place, the beauty of mathematics, its relation to reality, and the power it gives us in the business of life will never diminish. These are constants that will persist in future centuries as they have in centuries past. And these are the factors that make us want our children to study mathematics: to understand the deep beauty of the world they live in, and to be usefully equipped for life in an increasingly technical economy.

Some things had better change

At the same time, the factors that led to huge dropout rates among a highly achieving class of physics students fifty years in the past, will be present fifty years in the future if we don't discover how to prepare and motivate students as I was prepared and motivated by my teachers and my parents. My motivation came from understanding the link between mathematics and science. I needed the one to do the other. Loving math came only after I was proficient at it. It was as if I had made such a tremendous investment in learning it, that I had become attached to it, and curious about it.

Some things are changing

While the beauty, relevance, and power of mathematics are constant, some features of twenty-first century life will have an impact on mathematics and how we learn it. We have already seen changes in computational habits associated with the introduction of inexpensive computing power. I have four slide rules in a drawer at home. A nearby shelf holds a row books of tables of logarithms. I used both until around 1970. I did not use graph paper after the mid-1980's. I stopped writing my own computer programs to evaluate functions after the early 1990's. If I were to return to an active research career, I would probably work as a member of a team that included computer specialists who would provide me with programs, written by other teams, to handle needed computations. The everyday tools of computation have already changed, and there is no going back.

Computation is not the same as mathematics

But mathematics is mostly not about computation These changes in the paraphernalia of calculation, in my opinion, should have no effect at all on how we teach math in the early grades. Ease of computation does not obviate the need for basic mathematics skills. Deep relations among natural phenomena are not revealed through similarities in the graphs of their dependent and independent variables; they are found in the mathematical structures that underlie them. Understanding does not come from simulation and visualization, it comes from analysis. There is a crucial difference.

Yes, there are computer programs now that solve equations symbolically, and these are useful labor-savers. Using them successfully, however, still requires that you know what you are doing. No computer tells the significance of a symbolic expression. Mathematicians have found many other ways of using computers to simplify their work, and to expand the domain of problems they can attack. The proof not too many years ago of the four-color map theorem is a good example. This long standing problem yielded only to a systematic analysis of a very large number of special cases in which symbolic computation played a crucial role.

From the point of view of education, the most useful application of powerful computing is not to do arithmetic, but to display relationships visually. Computer graphics of a relatively simple kind can stimulate the imagination and permit the extraordinary acuity of the human eye and brain to pick out features that can then be investigated more closely with other means.

The greatest challenge

With respect to the future of mathematics education, however, the greatest challenge is not how to use the immense and growing power of information technology. The challenge is to find ways to prepare a much larger percentage of young people for success in mathematics. Throughout history some teachers have been more successful than others, some students have learned faster than others, some textbooks and curricula appear to have been more effective than others in motivating and preparing youngsters for the next steps. We have to find out what works and make it available to more people. This is the whole point of President Bush's education initiative *"No Child Left Behind."* The basic skills of reading, math, and science, are essential not only to prepare a technically competent workforce of the future. It is only through these skills that our society can fully understand and enjoy the wonders of the world around us.

Thank you again for inviting me to say these words. And thank you too, audience and panelists, for the time you are taking and the energy you are investing to improve instruction in mathematics, and to increase knowledge of the beauty, the relevance, and the power of mathematics among future generations.