NOAA Manual NOS NGS 5

State Plane Coordinate System of 1983

James E. Stem

Rockville, MD
January 1989

Reprinted with minor corrections
March 1990

U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
National Ocean Service
Charting and Geodetic Services

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Reprinted February 1991

Reprinted July 1992

Reprinted January 1993

Reprinted August 1993

Reprinted April 1994

Reprinted January 1995

Reprinted September 1995

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For sale by the National Geodetic Information Center, NOAA, Rockville, MD 20852

PREFACE

This manual explains how to perform computations on the State Plane Coordinate System of 1983 (SPCS 83). It supplements Coast and Geodetic Survey Special Publication No. 235, "The State coordinate systems," and replaces Coast and Geodetic Survey Publication 62-4, "State plane coordinates by automatic data processing." These two widely distributed publications provided the surveying and mapping profession with information on deriving 1927 State plane coordinates from geodetic coordinates based on the North American Datum of 1927 (NAD 27) plus information for traverse and other computations with these coordinates. This manual serves the similar purpose for users of SPCS 83 derived from the North American Datum of 1983 (NAD 83). Emphasis is placed on computations that have changed as a result of SPCS 83.

This publication is neither a textbook on the theory, development, or applications of general map projections nor a manual on the use of coordinates in survey computations. Instead it provides the practitioner with the necessary information to work with three conformal map projections: the Lambert conformal conic, the transverse Mercator, and the oblique Mercator. Derivatives of these three map projections produce the system which the National Geodetic Survey (NGS) has named the State Plane Coordinate System (SPCS). Referred to NAD 83 or NAD 27, this system of plane coordinates is identified as SPCS 83 or SPCS 27, respectively.

The equations in chapter 3, Conversion Methodology, form a significant portion of the manual. Chapter 3 is required reading for programmers writing software, but practitioners with software available may skip this chapter. Although a modification of terminology and notation was suggested by some reviewers, consistency with NGS software was deemed more important. Hence, chapter 3 documents the SPCS 83 software available from the National Geodetic Survey.

ACKNOWLEDGMENT

The mathematics given in this manual were compiled or developed by T. Vincenty prior to his retirement from the National Geodetic Survey (NGS). His consultation was invaluable to the author.

Principal reviewer was Joseph F. Dracup, NGS, retired. His many excellent suggestions were incorporated into the manual. Once again, Joe gave generously of his time to assist in the education of the surveying profession.

The author appreciates the review and contributions made by Earl F. Burkholder, Oregon Institute of Technology. Earl spent a summer at NGS researching the subject of map projections and maintains a continuing interest in the subject.

In addition, the manual was reviewed by Charles A. Whitten, B. K. Meade, and Charles N. Claire, all retired employees of the former Coast and Geodetic Survey (now NGS). The author was very fortunate to have such experts donate their services.

Finally, the author appreciates the helpful guidance of John G. Gergen and Edward J. McKay, present NGS employees.

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THE STATE PLANE COORDINATE SYSTEM OF 1983

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ABSTRACT. This manual provides information and equations necessary to perform survey computations on the State Plane Coordinate System of 1983 (SPCS 83), a map projection system based on the North American Datum of 1983 (NAD 83). Given the geodetic coordinates on NAD 83 (latitude and longitude), the manual provides the necessary equations to compute State plane coordinates (northing, easting) using the "forward" mapping equation ($\phi,\lambda \rightarrow N,E$). "Inverse" mapping equations are given to compute the geodetic position of a point defined by State plane coordinates ($N,E \rightarrow \phi,\lambda$). The manual addresses corrections to angles, azimuths, and distances that are required to relate these geodetic quantities between the ellipsoid and the grid. The following map projections are defined within SPCS 83: Lambert conformal conic, transverse Mercator, and oblique Mercator. A section on the Universal Transverse Mercator (UTM) projection is included. UTM is a derivative of the general transverse Mercator projection as well as another projection, in addition to SPCS 83, on which NAD 83 is published by NGS.

1. INTRODUCTION

1.1 Requirement for SPCS 83

The necessity for SPCS 83 arose from the establishment of NAD 83. When NAD 27 was readjusted and redefined by the National Geodetic Survey, a project which began in 1975 and finished in 1986, SPCS 27 became obsolete. NAD 83 produced new geodetic coordinates for all horizontal control points in the National Geodetic Reference System (NGRS). The project was undertaken because NAD 27 values could no longer provide the quality of horizontal control required by surveyors and engineers without regional recomputations (least squares adjustments) to repair the existing network. NAD 83 supplied the following improvements:

- One hundred and fifty years of geodetic observations (approximately 1.8 million) were adjusted simultaneously, eliminating error propagation which occurs when projects must be mathematically assembled on a "piecemeal" basis.

- The precise transcontinental traverse, satellite triangulation, Doppler positions, baselines established by electronic distance measurements (EDM), and baselines established by very long baseline interferometry (VLBI), improved the internal consistency of the network.

- A new figure of the Earth, the Geodetic Reference System of 1980 (GRS 80), which approximates the Earth's true size and shape, supplied a better fit than the Clarke 1866 spheroid, the reference surface used with NAD 27.

- The origin of the datum was moved from station MEADES RANCH in Kansas to the Earth's center of mass, for compatibility with satellite systems.

Not only will the published geodetic position of each control point change, but the State plane coordinates will change for the following reasons:

- The plane coordinates are mathematically derived (using "mapping equations") from geodetic coordinates.

- The new figure of the Earth, the GRS 80 ellipsoid, has different values for the semimajor axis "a" and flattening "f" (and eccentricity "e" and semiminor axis "b"). These ellipsoidal parameters are often embedded in the mapping equations and their change produces different plane coordinates.

- The mapping equations given in chapter 3 are accurate to the millimeter, whereas previous equations promulgated by NGS were derivatives of logarithmic calculations with generally accepted approximations.

- The defining constants of several zones have been redefined by the States.

- The numeric grid value of the origin of each zone has been significantly changed to make the coordinates appear clearly different.

- The State plane coordinates for all points published on NAD 83 by NGS will be in metric units.

- The SPCS 83 uses the Gauss-Kruger form of the transverse Mercator projection, whereas the SPCS 27 used the Gauss-Schreiber form of the equations.

1.2 SPCS 27 Background

The State Plane Coordinate System of 1927 was designed in the 1930s by the U.S. Coast and Geodetic Survey (predecessor of the National Ocean Service) to enable surveyors, mappers, and engineers to connect their land or engineering surveys to a common reference system, the North American Datum of 1927. The following criteria were applied in the design of the State Plane Coordinate System of 1927:

- Use of conformal mapping projections.

- Restricting the maximum scale distortion (sec. 2.6) to less than one part in 10,000.

- Covering an entire State with as few zones of a projection as possible.

- Defining boundaries of projection zones as an aggregation of counties.

It is impossible to map a curved Earth on a flat map using plane coordinates without distorting angles, azimuths, distances, or area. It is possible to design a map such that some of the four remain undistorted by selecting an appropriate "map projection." A map projection in which angles on the curved Earth are preserved after being projected to a plane is called a "conformal" projection. (See sec. 2.3.) Three conformal map projections were used in designing the original State plane coordinate systems, the Lambert conformal conic projection, the transverse Mercator projection, and the oblique Mercator projection. The Lambert projection was used for States that are long in the east-west direction (e.g., Kentucky, Tennessee, North Carolina), or for States that prefer to be divided into several zones of east-west extent. The transverse Mercator projection was used for States (or zones within States) that are long in the north-south direction (e.g., Vermont and Indiana), and the oblique Mercator was used in one zone of Alaska when neither of these two was appropriate. These same map projections are also often custom designed to provide a coordinate system for a local or regional project. For example, the equations of the oblique Mercator projection produced project coordinates for the Northeast Corridor Rail Improvement project when a narrow coordinate system from Washington, DC, to Boston, MA, was required.

Land survey distance measurements in the 1930s were typically made with a steel tape, or something less precise. Accuracy rarely exceeded one part in 10,000. Therefore, the designers of the SPCS 27 concluded that a maximum systematic distance scale distortion (see sec. 2.6, "Grid scale factor") attributed to the projection of 1:10,000 could be absorbed in the computations without adverse impact on the survey. If distances were more accurate than 1:10,000, or if the systematic scale distortion could not be tolerated, the effect of scale distortion could be eliminated by computing and applying an appropriate grid scale factor correction. Admittedly, the one in 10,000 limit was set at an arbitrary level, but it worked well for its intended purpose and was not restrictive on the quality of the survey when grid scale factor was computed and applied.

To keep the scale distortion at less than one part in 10,000 when designing the SPCS 27, some States required multiple projection "zones." Thus some States have only one State plane coordinate zone, some have two or three zones, and the State of Alaska has 10 zones that incorporate all three projections. With the exception of Alaska, the zone boundaries in each State followed county boundaries. There was usually sufficient overlap from one zone to another to accommodate projects or surveys that crossed zone boundaries and still limit the scale distortion to 1:10,000. In more recent years, survey accuracy usually exceeded 1:10,000. More surveyors became accustomed to correcting distance observations for projection scale distortion by applying the grid scale factor correction. When the correction is used, zone boundaries become less important, as projects may extend farther into adjacent zones.

1.3 SPCS 83 Design

In the mid-1970s NGS considered several alternatives to SPCS 83. Some geodesists advocated retaining the design of the existing State plane coordinate system (projection type, boundaries, and defining constants) and others believed that a system based on a single projection type should be adopted. The single projection proponents contended that the present SPCS was cumbersome, since three projections involving 127 zones were employed.

A study was instituted to decide whether a single system would meet the principal requirements better than SPCS 27. These requirements included ease of understanding, computation, and implementation. Initially, it appeared that adoption of the Universal Transverse Mercator (UTM) system (sec. 2.7) would be the best solution because the grid had long been established, to some extent was being used, and the basic formulas were identical in all situations. However, on further examination, it was found that the UTM 6-degree zone widths presented several problems that might impede its overall acceptance by the surveying profession. For example, to accommodate the wider zone width, a grid scale factor of 1:2,500 exists on the central meridian while a grid scale factor of 1:1,250 exists at zone boundaries. As already discussed, similar grid scale factors on the SPCS rarely exceeded 1:10,000. In addition, the "arc-to-chord" correction term (sec. 2.5) that converts observed geodetic angles to grid angles is larger, requiring application more frequently. And finally, the UTM zone definitions did not coincide with State or county boundaries. These problems were not viewed as critical, but most surveyors and engineers considered the existing SPCS 27 the simpler system and the UTM as unacceptable because of rapidly changing grid scale factors.

The study then turned to the transverse Mercator projection with zones of 2° in width. This grid met the primary conditions of a single national system. By reducing zone width, the scale factor and the arc-to-chord correction would be no worse than in the SPCS 27. The major disadvantage of the 2° transverse Mercator grid was that the zones, being defined by meridians, rarely fell along State and county boundaries. A more detailed review showed that while many States would require two or more zones, the 2° grid could be defined to accommodate those who wanted the zones to follow county lines. Furthermore, seldom did this cause larger scale factor or arc-to-chord corrections than in the existing SPCS 27, although several of the larger counties would require two zones. However, the average number of zones per State was increased by this approach.

Throughout this study, three dominant factors for retaining the SPCS 27 design were evident: SPCS had been accepted by legislative action in 37 States. The grids had been in use for more than 40 years and most surveyors and engineers were familiar with the definition and procedures involved in using them. Except for academic and puristic considerations the philosophy of SPCS 27 was fundamentally sound. With availability of electronic calculators and computers, little merit was found in reducing the number of zones or projection types. There was merit in minimizing the number of changes to SPCS legislation. For these reasons a decision was made to retain the basic design philosophy of SPCS 27 in SPCS 83.

The above decision was expanded to enable NGS to also publish UTM coordinates for those users who preferred that system. Both grids are now fully supported by NGS for surveying and mapping purposes. It is recognized that requirements will arise when additional projections may be required, and there is no reason to limit use to only the SPCS 83 and UTM systems.

1.4 SPCS 83 Local Selection

The policy decision that NGS would publish NAD 83 coordinates in SPCS 83, a system designed similar to SPCS 27, was first announced in the Federal Register on March 24, 1977. From April 1978 through January 1979, NGS solicited comments on this published policy by canvassing member boards of the National Council of Engineering Examiners, all individual land surveyor members of each board, the secretary of each section and affiliate of the American Congress on Surveying and Mapping (ACSM), and State and local public agencies familiar to NGS. As of August 1988, the 1978-79 solicitations and responses to subsequently published articles had produced committees or liaison contacts in 43 States. Through these people NGS presented the options to be considered in delineation of SPCS 83 zones and options in the adoption of the defining mathematical constants for each zone.

Although most States left unchanged the list of counties that comprised a zone, three States (South Carolina, Montana, and Nebraska) elected to have a single zone cover the entire State, replacing what was several zones on the SPCS 27. In these States the grid scale factor correction to distances now exceeds 1:10,000, and the arc-to-chord correction to azimuths and angles may become significant. (See secs. 2.5 and 4.3.) A zone definition change also occurred in New Mexico due to creation of a new county, and in California where zone 7 of the SPCS 27 was incorporated into zone 5 of the SPCS 83. Figure 1.4 depicts the zone identification numbers and boundaries of the SPCS 83.

In 1982, NGS printed a map titled "Index of State Plane Coordinate (SPC) Zone Codes," which depicts the boundaries and identification numbers of the SPCS 27 zones. Figure 1.4 differs from that map in the following States and Possessions.

CALIFORNIA: CA 7 No. 0407 was eliminated, and its area, the County of Los Angeles, included in CA 5, No. 0405.

MICHIGAN: MI E No. 2101, MI C No. 2102, and MI W No. 2103, were eliminated in favor of the Lambert zones.

MONTANA: MT N No. 2501, MT C No. 2502, and MT S No. 2503 were eliminated in favor of a single State zone MT No. 2500.

NEBRASKA: NE N No. 2601 and NE S No. 2602 were eliminated in favor of a single State zone NE No. 2600.

SOUTH CAROLINA: SC N No. 3901 and SC S 3902 were eliminated in favor of a single State zone SC No. 3900.

PUERTO RICO AND VIRGIN ISLANDS: PR 5201, VI 5201, and VI SX 5202 were eliminated in favor of a single zone PR 5200.

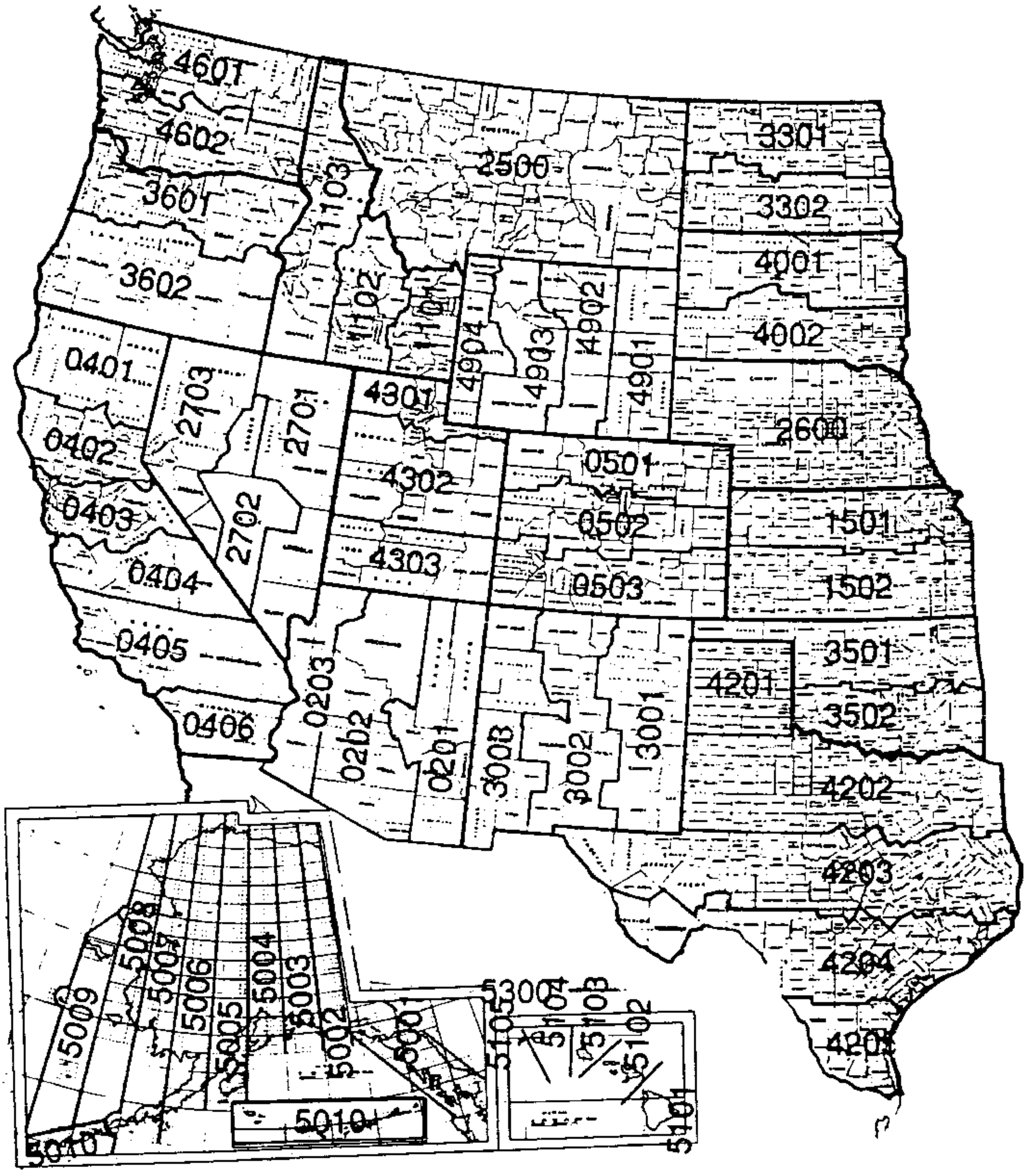

4601
4602
3601
3602
1103
2500
3301
3302
4001
4002
1102
4904
4903
4902
4901
0401
2703
2701
4301
2600
0402
4302
0501
0403
2702
0502
1501
0404
4303
0503
1502
0405
3501
4201
0203
3502
0406
0202
0201
3002
3001
4202
4203
4204
5008
5007
5009
5006
5004
5003
5005
5002
5300
5102
5101
5010

Figure 1.4.--State Plane Coordinate System of 1983 zones.

Several States chose to modify one or more of the defining constants of their zones. Appendix A contains the defining constants for all zones of the SPCS 83. Where the constant differs from the SPCS 27 definition, it is flagged with an asterisk. Some of these changes increase the magnitude of the grid scale factor and arc-to-chord correction terms. In addition to the flagged changes, all "grid origins" are different because, within the SPCS 83, origins that were redefined are defined in meters. This new grid origin was selected by liaison with the States based on the following criteria:

- o Keeping the number of digits in the coordinate to the minimum.
- o Creating a new range for easting and/or northing in meters on the 1983 datum that would not overlap the range of x and/or y in feet on the existing 1927 datum. If an overlap could not be avoided, the location of the band of overlap (i.e., where the range of x and/or y on the 1927 datum intersects with the range on the 1983 datum) could be positioned anywhere through selection of appropriate grid origin.
- o Selecting different grid origins (either in northing or easting) for each zone so the coordinate user could determine the zone from the magnitude of the coordinate. This usually requires the "false-easting" to be the smallest in the easternmost zone to avoid easting values close in magnitude for points near boundaries of adjacent transverse Mercator zones. It requires the "false-northing" of the northernmost zone to be the smallest for adjacent Lambert zones for the same reason.
- o Creating different orders of magnitude for northing and easting to reduce the possibility of transition errors.

The grid origin selection influenced only the appearance of the coordinate system, not its accuracy or usefulness.

1.5 SPCS 83 State Legislation

Before the NAD 83 project began, 37 States had passed a State Plane Coordinate System Act, the first in 1935. As of August 1988, 42 States had legislated a "1927 State Plane Coordinate System." The most recent additions during the NAD 83 project included Illinois, New Hampshire, North Dakota, South Carolina, and West Virginia. Of these five, only Illinois did not simultaneously include the definition of the SPCS 83 within its initial SPCS 27 legislation.

As of August 1988, 26 States had enacted 1983 State Plane Coordinate System legislation (table 1.5). For SPCS 83, as for SPCS 27, NGS prepared a model act to implement SPCS legislation by the States. The act was generally followed by the States except for minor changes, some of which are discussed below. The model SPCS 83 act may be found in appendix B. In addition to providing mathematical definitions of SPCS 83, enacted and proposed legislation contains other sections that warrant discussion.

In the old model a section stated that no coordinates "purporting to define the position of a point on a land boundary, shall be presented to be recorded in any public land records or deed records unless such point is within one-half mile" of

Table 1.5.--Status of SPCS 27 and SPCS 83 legislation
(as of August 1, 1988)

NO SPCS LEGISLATION (8 States)		EXISTING NAD 27 SPCS LEGISLATION (16 States)			ENACTED NAD 83 SPCS LEGISLATION (26 States)
No correspondence with NGS (4 States)	NAD 83 legislation drafted (4 States)	No correspondence with NGS (3 States)	Correspondence with recommendations (7 States)	NAD 83 legislation drafted (6 States)	
Hawaii Kansas Kentucky Oklahoma	Iowa Mississippi *Nebraska**(S) Wyoming**(S)	Arkansas Pennsylvania Tennessee	Alabama Delaware Florida Idaho Illinois North Dakota Wisconsin	Colorado(S) Massachusetts New Jersey New Mexico New York Washington	Alaska Arizona(I) *California(S) Connecticut(S) Georgia Indiana(S) Louisiana Maine Maryland(S) Michigan(I) Minnesota Missouri *Montana(I) Nevada New Hampshire North Carolina(S) Ohio Oregon(I) Rhode Island *South Carolina(I) South Dakota Texas(S) Utah(I) Vermont Virginia West Virginia

*NOTE: The only States that authorized changes in zone boundaries are: California, Montana, Nebraska, and South Carolina.

**NOTE: These States have not written legislation, but have corresponded definite new SPCS parameters to NGS.

UNITS: S = U.S. Survey feet and meters
I = International feet and meters
All others only meters

a first- or second-order control point. The new model changes only the "one-half mile" to "1 kilometer," and references the Federal Geodetic Control Committee (FGCC) as the source of the classifications of first- and second-order geodetic control points. The intent of this section has not been well understood.

To determine a boundary coordinate, the act explicitly states that at least a second-order monumented point must exist not more than 1 km away. It does not say that the second-order point must already exist. Adding that an "existing or newly established" control point needs to be within 1 km may clarify this confusion. The intent was that a property surveyor would either recover an existing point or use any survey methodology to establish a permanently monumented point of at least second-order, class II accuracy in an accessible but protected location within 1 km of the property to be surveyed. Then, using this point, coordinates of the "temporarily" monumented (essentially unmonumented) property corners would be determined. These corners, if determined from a second-order, class II point, are of third-order accuracy (1:10,000), following the usual practice of establishing the point to the next lower accuracy standard. Another approach would have been to legislate that property coordinates would be determined using FGCC third-order (1:10,000) positional standards but eliminate the monumentation standards. This approach may serve well with Global Positioning System methods, but it eliminates the nearby control point needed for retracement surveys by conventional means. The 1-km limit from monumented control is perhaps appropriate only for urban or suburban conditions. Of importance is not the distance, but the existence of monumented control. Land values may also affect the specifications for a State or county.

The following examples illustrate how some States have addressed the above requirement in their 1983 SPCS laws. South Carolina's law states that no point can be recorded unless "...such point is established in accordance with the Federal Geodetic Control Committee specification for second-order, class II ...." Virginia's law reads that no point can be recorded unless "...such point is within 2 km of a public or private monumented horizontal control station..." established in conformity with first- or second-order FGCC specifications. Minnesota is to be applauded for writing the most unambiguous section. The law reads that no point would be recorded unless "... coordinates have been established in conformity with the national prescribed standards for third-order, class II horizontal control surveys, and provided that these surveys have been tied to or originated off monumented first or second-order horizontal control stations, which are adjusted to and published in the national network of geodetic control and are within 3 km of the said boundary points or land corners." The statement continues by defining the national standards to be those of the FGCC.

Another debated portion of the model SPCS legislation involves the role of coordinates within legal land descriptions. The section that states, "It shall be considered a complete, legal, and satisfactory description of such location to give the position of the survey station or land boundary corner on the system of plane coordinates..." has been passed by many States. This section, in conjunction with the previously discussed section dealing with the accuracy and recording of such points, should be sufficient to permit, but not to require, the use of the SPCS 83 to describe real property or supplement parcel descriptions.

Often found in the 1927 SPCS laws, especially in Public Land Survey System (PLSS) States, and carried over to the SPCS 83 legislation, is a section that specifies coordinates are supplemental to any other means of land description, and in case of conflict the conventional description shall prevail over the description by coordinates. This legislates an unconditional priority to the order of evidence and could prevent the best surveyor from submitting sound boundary evidence based on coordinates. There are many who believe that the intent should be to evaluate each situation on its own merit and not to impose an invariable rule. Language such as, "In case of conflict between elements of a description, cast out doubtful data and adhere to the most certain" may suffice. Perhaps this is currently being accomplished by common law.

Several States have addressed the above situation with the statement, "...the conventional description shall prevail over the description by coordinates unless said coordinates are upheld by adjudication, at which time the coordinate description will prevail." This, at least, provides the opportunity for the competent land/property line surveyor to defend the use of coordinates in retracement surveys.

Pertaining to the use of coordinates on plats, States have inserted additional individual sections. Georgia defines, "Grid North" and requires the convergence term on "...maps of survey that are purported oriented to a Georgia Coordinate System Zone." Illinois states that plats of survey referencing the SPCS must indicate the zone and "...geodetic stations, azimuth, angles, and distances used for establishing the survey connection." Virginia added that "Nothing contained in this chapter shall be interpreted as preventing the use of the Virginia Coordinate System in any unrecorded deeds, maps, or computations."

The last section of most SPCS acts assigns responsibility for the act to a specific State agency. Our model law stated that sections of the law "could be modified by a State agency to meet local conditions." Many States specifically assign responsibility to a particular department. For example, the South Carolina Geodetic Survey "... shall maintain the South Carolina coordinate system, files, and such other maps and files as deemed necessary to make station information readily available...." Similarly, the Virginia Polytechnic Institute and State University is "...the authorized State agency to collect and distribute information," and it authorizes such modifications as are referred to elsewhere in the law.

NGS encourages the development of State level surveying and mapping offices. Responsibility for the States' geodetic networks would be one function of such an office. The need for SPCS 83 legislation provides an opportunity to designate this lead agency.

1.6 SPCS 83 Unit of Length

A Federal Register notice published jointly on July 1, 1959 (24 Fed. Reg. 5348) by the directors of the National Bureau of Standards (NBS) (now National Institute of Standards and Technology) and the U.S. Coast and Geodetic Survey refined the definition of the yard in metric terms. The notice also pointed out the very slight difference between the new definition of the yard (0.9144 m) and the 1893 definition (3600/3937 m), from which the U.S. survey foot is derived. The "international foot" of 0.3048 meter is shorter than the U.S. survey foot by 2 parts per million. The 1959 notice stated that the U.S. survey foot would continue to be used "until such time as it becomes desirable and expedient to readjust the basic geodetic survey networks in the United States, after which the ratio of a yard, equal to 0.9144 m, shall apply."

Because the profession desired to retain the U.S. survey foot, and because it is incorporated in legal definitions in many States as well as in practical usage, a tentative decision was made by NBS not to adopt the international foot of 0.3048 m for surveying and mapping activities in the United States. However, before reaching a final decision in this matter, it was deemed appropriate and necessary to solicit the comments of land surveyors; Federal, State, and local officials; and any others from among the public at large who are engaged in surveying and mapping or are interested in or affected by surveying and mapping operations.

NBS and NGS published their preliminary decision (Federal Register Doc. 88-16174) and as of this writing are awaiting comments. A final decision will be reached after careful consideration of all the comments received. The final decision will be published in the Federal Register and will be publicly announced in the communications media as deemed appropriate. Even if the final decision affirms the preliminary decision not to adopt the international definition of the foot in surveying and mapping services, it should be noted that the Office of Charting and Geodetic Services, National Ocean Service, NOAA, in a 1977 Federal Register notice (42 Fed. Reg. 15943), uses the meter exclusively and is providing the new SPCS 83 coordinates in meters.

In the 1927 SPCS legislation, the "foot" was the defining unit of measure, the conversion factor defined by the "U.S. survey foot" being implicit. In the States that have prepared 1983 SPCS legislation, the "U.S. survey foot" was explicitly stated as the unit of measure when using SPCS 27. If a foot unit has been selected for SPCS 83, it is explicitly written into the SPCS 83 legislation.

Most States define a metric SPCS 83. When the NAD 83-SPCS 83 publication policy was developed, and published in the Federal Register on March 24, 1977, the Department of Commerce had established a policy that the agency would use metric units exclusively. NGS concurs with the advantages of the metric system, and except for SPCS 27 applications, has always worked totally in metric units. Accordingly, a metric SPCS 83 was recommended to the States. Except for Arizona, States that have enacted legislation defined a metric system. In addition, 10 States defined which "foot" unit to use when converting from meters. (See table 1.5.)

When the metric grid origin of an SPCS 83 zone is other than a rounded number, it was derived from a rounded foot value using one of the definitions:

0.3048 m exactly = international foot

1200/3937 m = U.S. survey foot

1.7 The New GRS 80 Ellipsoid

The mathematics of map projection systems convert point and line data from the ellipsoid of a datum to a plane. Accordingly, the dimensions of the ellipsoid are an inherent part of the conversion process. As discussed above, NAD 83 adopted a new ellipsoid, the Geodetic Reference System of 1980 (GRS 80). Therefore, the dimensions of this ellipsoid must be incorporated within any map projection equations, SPCS 83 or otherwise, when the requirement is to project NAD 83 geodetic data into a plane system. Whereas the ellipsoid constants were imbedded within the map projection equations promulgated by NGS (and its predecessors) for the SPCS 27, and hence they were invisible to the user, the formulation given in chapter 3 requires entry of ellipsoid constants.

An ellipsoid is formed by rotating an ellipse about its minor axis. For geodetic purposes this regular mathematical surface is designed to approximate the irregular surface of the Earth or portion thereof. The SPCS 27 incorporated the defining parameters of an ellipsoid identified as the Clarke spheroid of 1866, the ellipsoid of NAD 27. The parameters defining the Clarke spheroid of 1866 were the semimajor axis "a" of 6,378,206.4 m and the semiminor axis "b" of 6,356,583.8 m.

The ellipsoid that forms the basis of NAD 83, and consequently the SPCS 83, is identified as the Geodetic Reference System of 1980 (GRS 80). GRS 80 provides an excellent global approximation of the Earth's surface. The Clarke spheroid of 1866, as used for NAD 27 approximated only the conterminous United States. Because the geoid separation at point MEADES RANCH was assumed equal to zero, a translation exists between ellipsoids. The ellipsoid change is the major contributor of the coordinate shift from NAD 27 to NAD 83.

The parameters of GRS 80 were adopted by the XVII General Assembly of the International Union of Geodesy and Geophysics meeting in 1979 at Canberra, Australia. Since only one of the four GRS 80 defining parameters (semimajor axis "a") is an element of the geometric ellipsoid, a second geometric constant ("b", "1/f," or "e^2") must be derived from the three GRS 80 parameters of physical geodesy. Accordingly, the geometric definition of the GRS 80 is:

$$a = 6,378,137.\ \text{m (exact by definition)}$$

$$1/f = 298.25722210088\ \text{(to 14 significant digits by computation)}$$

From these two numbers, any other desired constants of geometric geodesy may be derived. For example, to 14 significant digits:

$$b = 6,356,752.3141403$$

$$e^2 = 0.0066943800229034.$$

2. MAP PROJECTIONS

A "projection" is a function relating points on one surface to points on another surface so that for every point on the first surface there corresponds exactly one point on the second surface. A "map projection" is a function relating coordinates of points on a curved surface to coordinates of points on a surface of different curvature. The mathematical function defines a relationship of coordinates between the ellipsoid and a sphere, between that sphere and a plane, or directly between the ellipsoid and a plane.

In horizontal surveying operations, field observations are collected on the surface of the irregular nonmathematical Earth. For most applications, it is more convenient to represent the spatial relationship between surveyed points by a set of coordinates, the basis of which is a regular mathematical surface. Part of the process of reducing field survey observations consists of computing equivalent values for the survey observations from the measured value on the Earth's surface to their reduced value on the regular surface on which one wishes to compute coordinates. Because a selected regular surface can only approximate the physical surface on which the survey points are actually located, the degree of approximation, and hence selection of the regular surface, is usually a function of the ultimate accuracy requirements of the points. It would not show good judgment to perform difficult reductions of survey observations and place them on a complex mathematical surface if the final required accuracy of the points was such as could be achieved with more simple reducing procedures.

One mathematical surface traditionally used by surveyors is the local tangent plane with few, if any, reductions made to the field observations. Resulting plane coordinates from computations of this nature are sufficient for independent projects of a small extent. On the other end of the spectrum, survey observations are often reduced to an ellipsoid of revolution and its associated datum, with subsequent computations performed using geodetic coordinates and ellipsoidal geometry. To the majority of surveyors and engineers, the use of "map projections" provides a compromise solution to either of these two extremes.

2.1 Fundamentals

In the study of map projections the ultimate surface on which survey observations are reduced and on which coordinates are computed is a plane. Usually it is a plane that has been "developed" from another regular mathematical surface, as a cone in Lambert's conic projections or the cylinder in Mercator's projections. Survey observations are "projected" or reduced to a predefined cylinder or cone, as is the "graticule" of latitude and longitude. The regular mathematical surface of the cylinder or cone is then cut open, or "developed," and laid flat into a plane. The "grid" of northings and eastings is then overlaid. Map projection systems provide a compromise solution between a limited-in-extent and approximate local plane system, and performing ellipsoidal computations on the geodetic datum. Theoretically, field observations are first reduced to the ellipsoid and then to the map projection surface. But in practice this is often accomplished as one step. The conversion of angles, azimuths, distances, and coordinates between an ellipsoid (GRS 80 for NAD 83) and developable surfaces is one role of the science of map projections. Figure 2.1a illustrates the three basic projection surfaces.

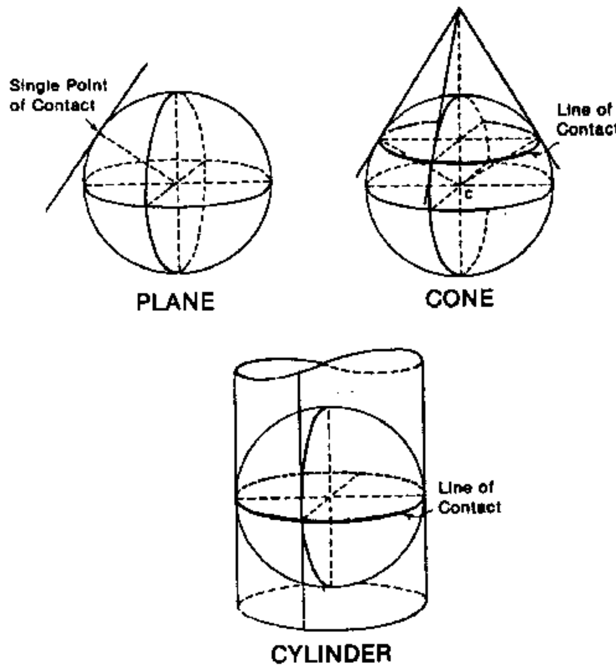


Figure 2.1a.--The three basic projection surfaces.

The plane, cone, or cylinder can be defined such that instead of being tangent to the datum surface, as illustrated in figure 2.1a, they intersect the datum surface as in figure 2.1b. This "secant" type of projection, a secant cone in Lambert's projections and secant cylinder in Mercator's projections, has been used for SPCS 27 and SPCS 83. In the Mercator projection, the secant cylinder has been rotated 90° so the axis of the cylinder is perpendicular to the axis of rotation of the datum surface, hence becoming a "transverse" Mercator projection. Occasionally the cylinder is rotated into a predefined azimuth, creating an "oblique" Mercator projection. Conceptually this is how one SPCS zone in Alaska was designed.

The secant cone intersects the surface of the ellipsoid along two parallels of latitude called "standard parallels" or "standard lines." Specifying these two parallels defines the cone. Specifying a "central meridian" orients the cone with respect to the ellipsoid. The transverse secant cylinder intersects the surface of the ellipsoid along two small ellipses equidistant from the meridian through the center of the zone. The secant cylinder is defined by specifying this central meridian, plus the desired grid scale factor on the central meridian. The ellipses of intersection are standard lines. Their location is a function of the selected central meridian grid scale factor. The specification of the latitude-longitude of the grid origin and the linear grid values assigned to that origin are all that remain to uniquely define a zone of either the Lambert or transverse Mercator projection. The above minimum specified values are the "defining constants" for a single zone of a projection.

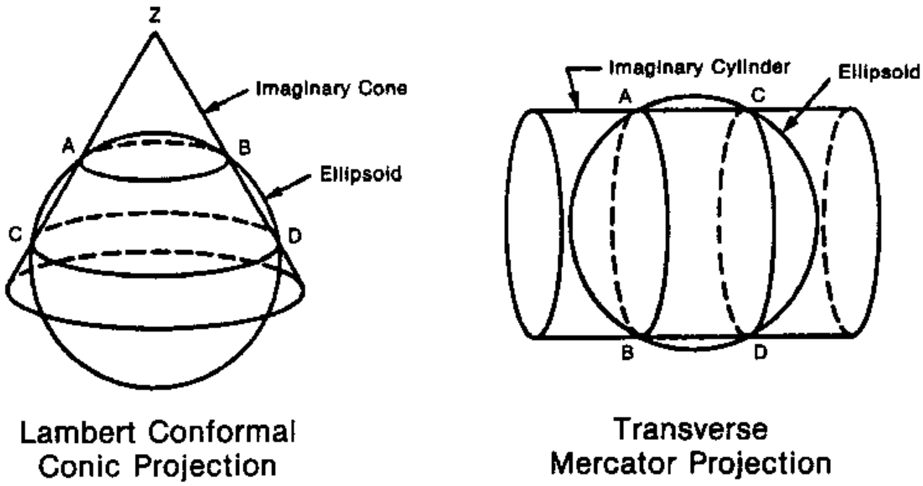


Figure 2.1b.--Surfaces used in State Plane Coordinate Systems.

Figure 2.1b illustrates the Lambert projection provides the closest approximation to the datum surface for a rectangular zone greatest in east-west extent, whereas the transverse Mercator projection provides the closest fit for an area north-south in extent. The narrower the strip of Earth's surface which it is desired to portray on a plane, the smaller will be the scale distortion designed in the projection. At a maximum width of 254 km, a maximum grid scale factor of 1:10,000 will exist at the zone boundaries. This maximum grid scale factor was designed into the SPCS 27 system, but in the SPCS 83 this maximum has been exceeded in the States of Montana, Nebraska, and South Carolina.

2.2 The SPCS 83 Grid

As with SPCS 27 or any plane-rectangular coordinate system, the SPCS 83 is represented on a map by two sets of uniformally spaced straight lines intersecting at right angles. The network thus formed is termed a "grid." One set of these lines is parallel to the plane of a meridian passing through the center of the grid, and the grid line corresponding to that meridian is the "northing axis" of the grid. It is also termed the "central meridian" of the grid. Forming right angles with the northing axis and to the south of the area covered by the grid is the easting axis. The point of intersection of these axes is the "grid origin" of the plane coordinate system. The grid origin differs from the "projection origin" by a constant. Knowledge of the origin is not required to use SPCS 83. The latitude and longitude of the grid origin are required defining-constants of a zone. The position of any point represented on the grid can be defined by stating two distances, termed "coordinates." One of these distances, known as the "northing coordinate," gives the position in a northern direction from the easting axis. The other distance, known as the "easting coordinate," gives the position in an east or west direction relative to the northing axis.

The northing coordinates increase numerically from south to north, the easting coordinates increase from west to east. Within the area covered by the grid, all northing coordinates are assured to be positive by placement of the grid origin south of the intended grid coverage. Easting coordinates are made positive by assigning the grid origin of the easting coordinates a large constant. For any point, the easting equals this value adopted for the grid origin, often identified as the "false easting," plus or minus the distance (E') of the point east or west from the central meridian (northing axis). Some zones have also assigned a "false northing" value at the grid origin. Accordingly, the northing equals this adopted value plus the distance of the point north of the easting axis.

The linear distance between two points on the SPCS 83, as obtained by computation or scaled from the grid, is termed the grid length of the line connecting those points. The angle between a line on the grid and the northing axis, reckoned clockwise from north through 360°, is the grid azimuth of the line. The computations involved in obtaining a grid length and a grid azimuth from grid coordinates are by means of the formulas of plane trigonometry.

2.3 Conformality

The commonly used examples of a developed cone for the Lambert grid and a developed cylinder for the transverse Mercator grid serve as excellent illustrations of the principles of map projections. Although some projections are truly "perspective," for SPCS 83 the mathematical equations of the map projection define the orderly system whereby the meridians and parallels of the ellipsoid are represented on the grid. Through the equations, controlled and computable distortion is placed into the map, the unavoidable result of representing a spherical surface on a flat plane. If correct relative depiction of an area is important, then "equal-area" mapping equations are selected. If correct depiction of select distances or azimuths is important, then other sets of mapping equations are selected.

In surveying and engineering, correct depiction of shapes is important. This is accomplished by mathematically constraining the grid scale factor (sec. 2.6) at a point, whatever it may be, such that it is the same in all directions from that point. This characteristic of a projection preserves angles between infinitesimal lines. That is, all lines on the grid cut each other at the same angles as do the corresponding lines on the ellipsoid for very short lines. Hence, for a small area, there is no local distortion of shape. But since the scale must change from point to point, distortion of shape can exist over large areas. Furthermore, for long lines the angle on the ellipsoid may not exactly equal the angle on the grid. This angular relationship is the property of conformality that has been mathematically imposed on SPCS 27, SPCS 83, the universal transverse Mercator projection, and most projections used in surveying and engineering. Although angles converted from the datum surface to the grid are preserved unchanged only for angles between lines of infinitesimal length, the angular difference of a single direction between the infinitesimal length and a finite length is a computable quantity identified as the "arc-to-chord" or (t-T) correction. (See secs. 2.5 and 4.3.)

By numerical example, the reader may verify that the distortion injected into the map projection is an exactly defined and computable quantity. All too often the concept of distortion in a map projection system is interpreted as an error of the system. However, map projection systems provide for a rigorous mathematical conversion of quantities between surfaces, and, as such, any inexactness that enters the conversion is caused by computational approximations.

2.4 Convergence Angle

The construction of all SPCS grids is such that geodetic north and grid north do not coincide at any point in a zone except along the central meridian. This condition is caused by the fact that the meridians converge toward the poles while the north-south grid lines are parallel to the central meridian. since geodetic azimuths are referred to meridians and grid azimuths are referred to north-south grid lines, it is evident that geodetic azimuths and grid azimuths must differ by a certain amount that depends on the position of the point of origin of the azimuth in relation to the central meridian of the SPCS zone. The "convergence angle," often also identified as the "mapping angle," is this angular difference between grid north and geodetic north. Defined another way, the convergence angle is the difference between a geodetic azimuth and the projection of that azimuth on the grid. Convergence is not the difference between geodetic and grid azimuths. (See sec. 2.5.) The "projected geodetic" azimuth is not the grid azimuth. Geodetic azimuths are symbolized as "α" and the convergence angle is symbolized as "γ". Note the change from SPCS 27, where the symbol "θ" was used for convergence within Lambert projections and "$\Delta\alpha$" for convergence within transverse Mercator systems, to SPCS 83 where "γ" represents convergence regardless of the projection type.

2.5 Grid Azimuth "t" and Projected Geodetic Azimuth "T"

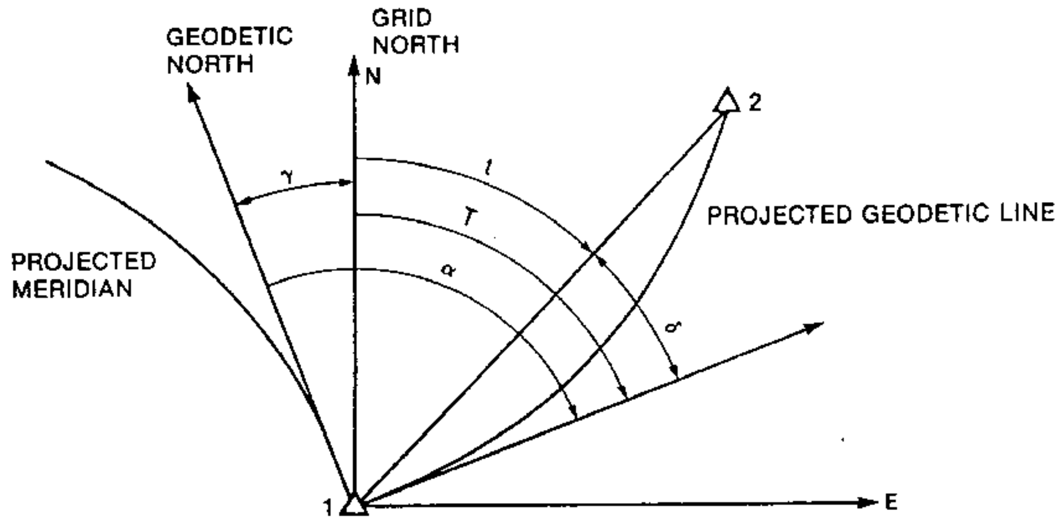
The projection of the geodetic azimuth "α" onto the grid is not the grid azimuth, but the "projected geodetic azimuth" symbolized as "T". Convergence "γ" is defined as the difference between geodetic and projected geodetic azimuths. Hence by definition, $\alpha = T+\gamma$, and the sign of "γ" should be applied accordingly. The angle obtained from two projected geodetic azimuths is a true representation of an observed angle.

When an azimuth is computed from two plane coordinate pairs, the resulting quantity is the grid azimuth symbolized as "t", or sometimes "α_g". The relationship between projected geodetic azimuth "T" and grid azimuth "t" is subtle and may be more clearly understood in figure 2.5. The difference between these azimuths is a computable quantity symbolized as "δ", or more often as (t-T). For the purpose of sign convention it is defined as $\delta = t-T$. For reasons apparent in figure 2.5, this term is also identified as the "arc-to-chord" correction. Given the above definition of α and δ, we obtain $t = \alpha-\gamma+\delta$.

Sometimes the convergence/mapping angle is incorrectly defined as the difference between the geodetic azimuth and the grid azimuth. This incorrect definition assumes the magnitude of (t-T) to be insignificant. While for many applications that assumption may be correct, (t-T) is often considered a "second-term" correction to the convergence term. Whether identified as δ, (t-T), arc-to-chord or second-term, the correction should be understood and always considered.

2.6 Grid Scale Factor at a Point

The grid scale factor is the measure of the linear distortion that has been mathematically imposed on ellipsoid distances so they may be projected onto a plane. At a given point, the ratio of the length of a linear increment on the grid to the length of the corresponding increment on the ellipsoid is identified



$$t = \alpha - \gamma + \delta$$

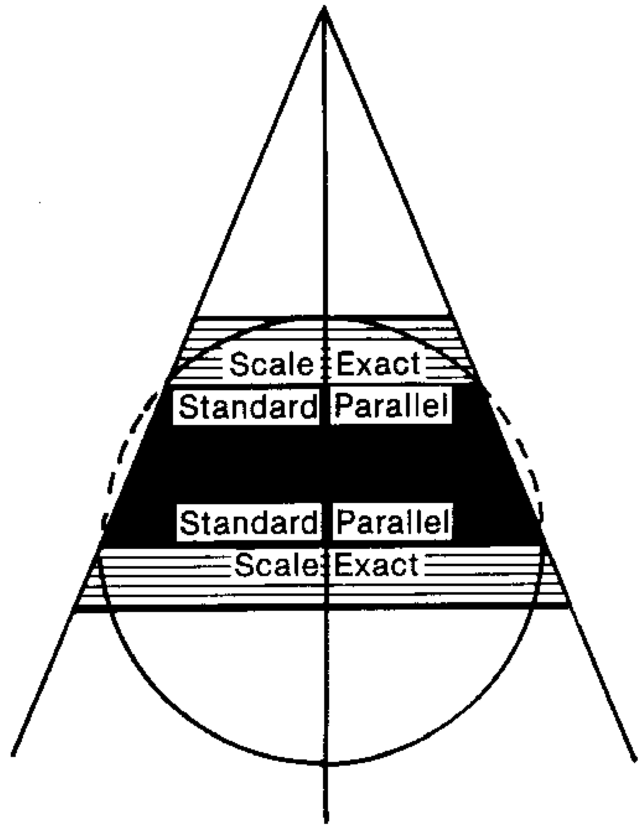
α = GEODETIC AZIMUTH RECKONED FROM NORTH
T = PROJECTED GEODETIC AZIMUTH
t = GRID AZIMUTH RECKONED FROM NORTH
γ = MAPPING ANGLE = CONVERGENCE ANGLE
δ = t-T = SECOND-TERM CORRECTION = ARC-TO-CHORD CORRECTION

Figure 2.5.--Azimuths.

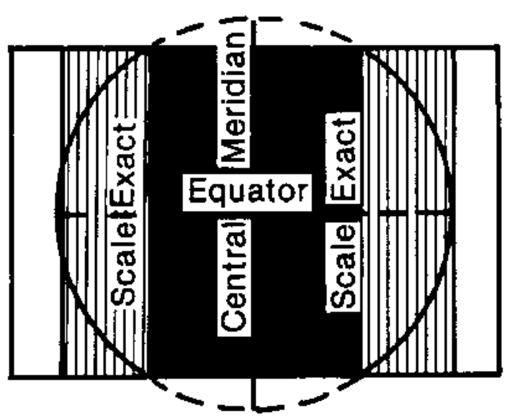
as the grid scale factor at that point, and symbolized by the letter "k". The grid scale factor is constant at a point, regardless of the azimuth, when conformal map projections are used as in the SPCS 83 and UTM systems. (See sec. 2.3, Conformality.) The grid scale factor is variable from point to point; mathematics refers to it as applying only to infinitesimal distances at a point.

The grid scale factor is equal to 1.0 along the "standard lines" (sec. 2.1) of the projection. Since the SPCS and UTM grids are secant type projections grid scale factors are less than 1.0 for the portion of the grid within the standard lines and greater than 1.0 for the remainder of the grid. In Lambert zones, the grid scale factor is less than 1.0 between the two standard parallels that define the zone. In transverse Mercator zones the scale factor is less than 1.0 between two north-south lines--the projection of the "ellipse of intersection" (sec. 2.1), their distance from the central meridian being a function of the scale factor assigned to that central meridian as part of the zone definition. Figure 2.6, although exaggerated, illustrates this concept.

Sometimes grid scale factor is defined as "scale distortion." On a map, map scale is correct only along the standard lines of the projection on which the map was cast. Everywhere else on the map, scale distortion exists and is defined as the ratio of the map scale at a given point to the map scale along a standard line. The scale distortion is identical to the grid scale factor. In small scale mapping, scale distortion is often expressed as "scale error" in percent, where scale error (%) = (scale distortion minus 1.0)*100.



Lambert Projection — Cone Secant to Sphere
Defined by Two Standard Parallels and the Origin



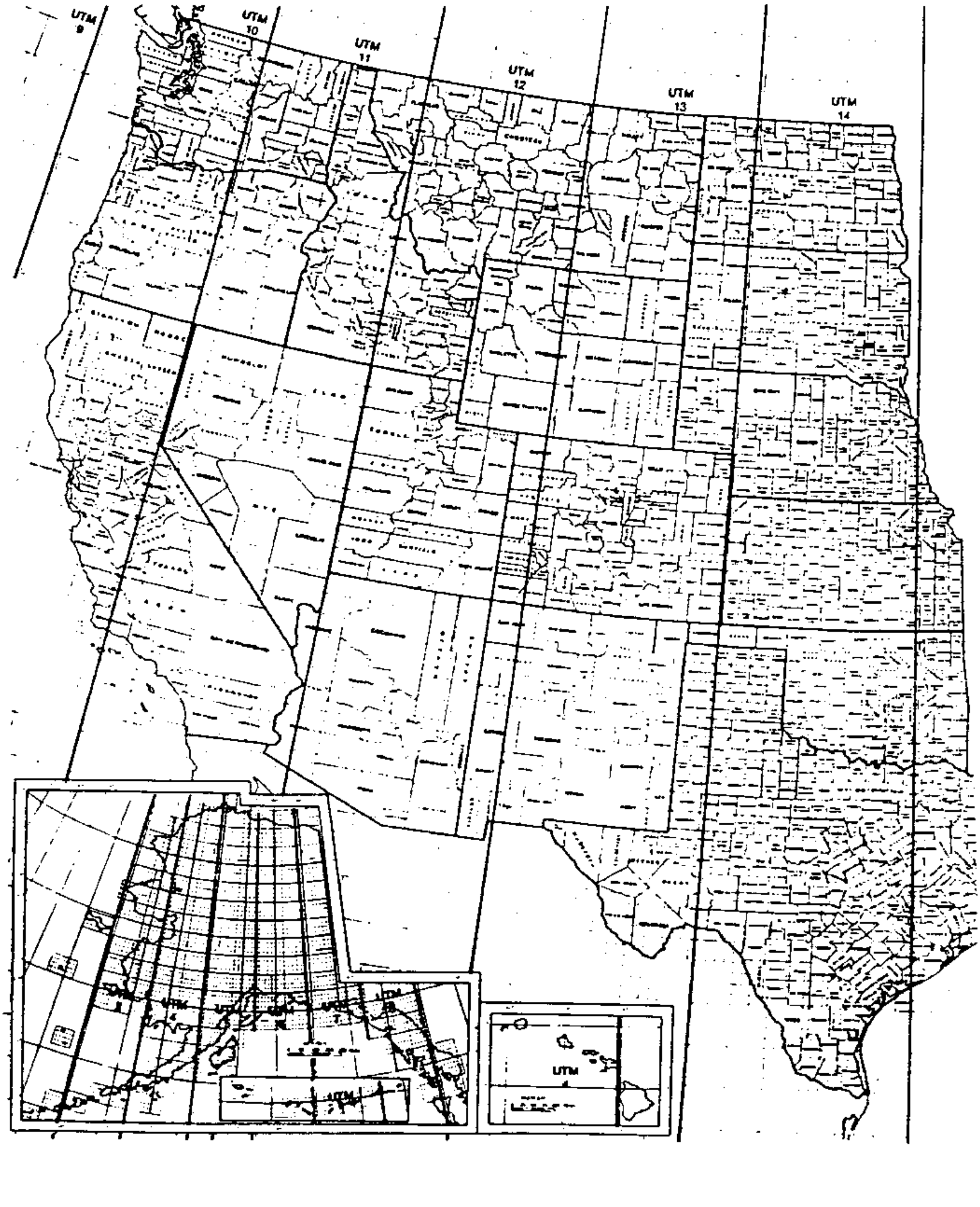
Transverse Mercator Projection —
Cylinder Secant to Sphere
Defined by Central Meridian and Its Scale
Factor, and the Origin

Figure 2.6.--Scale factor.

2.7 Universal Transverse Mercator Projection

The zones of the UTM projection system differ from the zones of other transverse Mercator projections by only the zone-defining constants. The basic mapping equations given in this manual for the transverse Mercator zones of the SPCS 83 may be used to obtain NAD 83 UTM coordinates upon substitution of UTM zone constants. The UTM zone constants have not changed, but when the constants are referenced to the GRS 80 ellipsoid of NAD 83, then 1983 UTM coordinates will be obtained. The UTM specifications, i.e., defining constants, on NAD 27 appear in many manuals of the Department of Army, originator of the system (e.g., Department of the Army 1958). To update the Department of Army specifications for NAD 83, only the ellipsoid ("spheroid" in the Army specifications) requires changing. The 1983 UTM specifications for the northern hemisphere are as follows.

Projection: Transverse Mercator (Gauss-Kruger type) in 6° wide zones
Ellipsoid: GRS 80 in North America
Longitude of origin: Central meridian of each zone
Latitude of origin: 0° (the equator)
Unit: Meter
False northing: 0
False easting: 500,000
Scale factor at central meridian: 0.9996 (exactly)
Zone numbering: Starting with No. 1 on the zone from 180° west to 174° west, and increasing eastward to No. 60 on the zone from 174° east to 180° east. (See fig. 2.7.)
Latitude limits of system: 0° to 80° north
Limits of zones: The zones are bounded by meridians whose longitudes are multiples of 6° west or east of Greenwich.



UTM 9
UTM 10
UTM 11
UTM 12
UTM 13
UTM 14

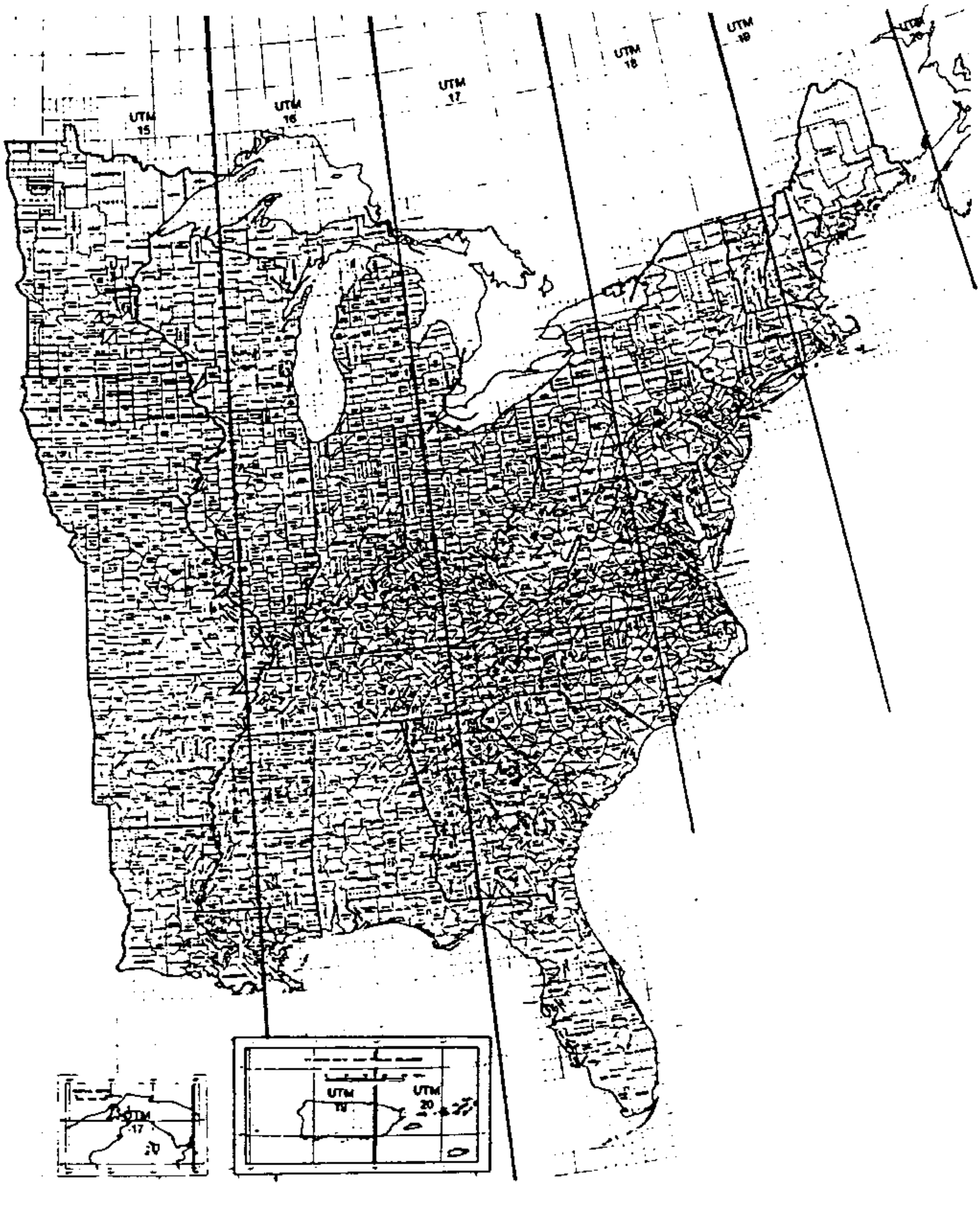


Figure 2.7.--Universal Transverse Mercator zones.

3. CONVERSION METHODOLOGY

This chapter addresses both "manual" and "automated" methods for performing "conversions" on any Lambert conformal conic, transverse Mercator, or oblique Mercator projections. Included is conversion from NAD 83 latitude/longitude to SPCS 83 northing/easting, plus the reverse process. For these processes this manual uses the term "conversion," leaving the term "transformation" for the process of computing coordinate values between datums, for example, transforming from NAD 27 to NAD 83 or transforming from SPCS 27 to SPCS 83. In addition to converting point coordinates, methods for conversion of distances, azimuths, and angles are also given.

The "automated" methods for conversions given in sections 3.1 through 3.3 are equations that have been sequenced and structured to facilitate programming. "Manual" methods are generally a combination of simple equations, tables, and intermediate numerical input, requiring only a calculator capable of basic arithmetic operations. Section 3.4 provides such a manual method for the Lambert projection where the intermediate numerical input is polynomial coefficients. Table 3.0 summarizes the conversion computational methods that were used for SPCS 27 and the methods discussed in this manual for SPCS 83.

Table 3.0.--Summary of conversion methods

Datum	Mode	Projection	Method
SPCS 27	Manual	Lambert and Transverse Mercator	Projection tables
		Oblique Mercator	Intersection tables
	Automated	Lambert, transverse Mercator, and oblique Mercator	Equations and constants described in C&GS Publication 62-4 (Claire 1968)
SPCS 83	Manual	Lambert	Polynomial coefficients (sec. 3.4)
		Transverse Mercator	New projection tables (future)
		Oblique Mercator	Automated only
	Automated	Lambert	Polynomial coefficients or new mapping equations (sec. 3.1)
		Transverse Mercator	New mapping equations (sec. 3.2)
		Oblique Mercator	New mapping equations (sec. 3.3)

The mapping equations given in sections 3.1 through 3.3 are not really "new" and may differ little from equations found in geodetic literature. However, they are new in the sense that they are not in the same form as the equations published or programmed by NGS or its predecessors in connection with SPCS 27. Whereas the SPCS 27 equations given in C&GS Publication 62-4 were designed to reproduce exactly the numerical results of an earlier manual method using logarithmic computations and projection tables, the equations here were designed for accuracy and computational efficiency. The Gauss-Kruger form of the transverse Mercator equations was used in SPCS 83 and the Gauss-Schreiber form in SPCS 27 equations.

Because the mapping equations of the automated approach apply equally to main-frame computers and programmable hand-held calculators, the availability of sufficient significant digits warrants consideration. The equations of transverse and oblique projections as given here will produce millimeter accuracy on any machine handling 10 significant digits. For the Lambert projection, the method of polynomial coefficients (sec 3.4) was developed for machines with only 10 significant digits. With less than 12 digits, the general mapping equations could not guarantee millimeter accuracy in all Lambert zones, particularly in Florida, Louisiana, Texas, South Carolina, Nebraska, and Montana. However, the polynomial coefficient method may also prove to be the most efficient for any machine. The general mapping equations will produce submillimeter accuracy when adequate significant digits are available for the computation.

Since the equations are not difficult, the polynomial coefficient method also fills the requirement for a manual method for the Lambert projection. A manual method for the SPCS 83 transverse Mercator projections has not been fully developed by NGS pending the demonstrated requirement for such a method.

While it is easy to visualize map projections by considering them a perspective projection of the meridians and parallels of the datum surface onto a surface that develops into a plane, in this age of coordinate plotters a graticule is generally not constructed by these means. Although a set of mechanical procedures can sometimes be defined by which meridians and parallels can be geometrically constructed on the grid using a ruler, compass, and scale, a pair of functions, $N = f_1(\phi,\lambda)$ and $E = f_2(\phi,\lambda)$, always exist. That is, for a point of given latitude (ϕ) and longitude (λ), there exist equations to yield the northing coordinate and equations to yield the easting coordinate when ϕ and λ are substituted into the equations. Likewise, equations must exist to compute the convergence angle, $\gamma = f_3(\phi,\lambda)$, and grid scale factor, $k = f_4(\phi,\lambda)$. These four functions, or equations, comprise the direct conversion process.

Furthermore, it must be possible to perform the inverse computation, requiring another pair of formulas, latitude $(\phi) = f_5(N,E)$ and longitude $(\lambda) = f_6(N,E)$. Similarly needed are convergence and grid scale factor as a function of the plane coordinates, $\gamma = f_7(N,E)$ and $k = f_8(N,E)$. Because these are one-to-one mappings, the inverse computation must reproduce the original values.

This chapter provides these eight "mapping equations" for each of these projections: Lambert conformal conic (sec. 3.1), transverse Mercator (sec. 3.2), and oblique Mercator (sec. 3.3). For each projection, the definition of the adopted symbols will be given first. Two sets of symbols are listed, the conventional set which incorporates the Greek alphabet and a set available on standard keyboards. The equations in this chapter will use the conventional notation. The entries in the notation section flagged with an asterisk are the constants required to uniquely define one specific zone of that general type of map projection. The values of those zone-defining constants as adopted and legislated by the States are listed in appendix A.

Included within the notation section are the symbols and definition of ellipsoid constants. Although several geometric ellipsoid constants are used within the mapping equations, only two geometric constants are required to define an ellipsoid. The SPCS 83 uses the GRS 80 ellipsoid. Those constants are discussed in section 1.7. All other geometric ellipsoid constants are then derived from the two defining constants, usually for the purpose of eliminating repeated computations.

A section on computation of zone constants follows each section on notation and definitions. Within this section are equations to compute intermediate quantities derived from the zone-defining constants of appendix A. These need only to be derived once. The derived "intermediate computing constants" of this section that need to be saved for future computations are flagged with an asterisk. The advantage of segmenting the general mapping equations is to eliminate repeated computations.

Subsequent sections under each projection type list the equations of the direct and inverse coordinate conversion process. The equations for the (t-T) line correction term (see also sec. 4.3) are provided in a final section. The solution of the ultimate mapping equations will require the values of the asterisked terms of the first two sections (defining constants plus intermediate constants).

3.1 Lambert Conformal Conic Mapping Equations

3.11 Notation and Definitions

For some terms an optional symbol appears in parentheses. This optional symbol available on all keyboards is used exclusively in section 3.4 and appendix C. Asterisked terms define the projection. Their values are listed in appendix A. These terms are the "zone defining constants" included within State SPCS legislation where enacted.

	ϕ (B)	Parallel of geodetic latitude, positive north
*	ϕ_s (B_s)	Southern standard parallel
*	ϕ_n (B_n)	Northern standard parallel
	ϕ_0 (B_0)	Central parallel, the latitude of the true projection origin
*	ϕ_b (B_b)	Latitude of the grid origin
	λ (L)	Meridian of geodetic longitude, positive west
*	λ_0 (L_0)	Central meridian, longitude of the true and grid origin
	k	Grid scale factor at a general point
	k_{12}	Grid scale factor of a line (between point 1 and point 2)
	k_0	Grid scale factor at the central parallel ϕ_0
	γ (C)	Convergence angle
	δ (t-T)	Arc-to-chord or second-term correction
	N	Northing coordinate (formerly y)
*	N_b	The northing value for ϕ_b at the central meridian (the grid origin). Sometimes identified as the false northing.
	N_0	Northing value at the intersection of the central meridian with the central parallel (the true projection origin)

	E	Easting coordinate (formerly x)
*	E_0	The easting value at the central meridian λ_0. Sometimes identified as the false easting
	R	Mapping radius at latitude ϕ
	R_b	Mapping radius at latitude ϕ_b
	R_0	Mapping radius at latitude ϕ_0
	K	Mapping radius at the equator
	Q	Isometric latitude
*	a	Semimajor axis of the geodetic ellipsoid
	b	Semiminor axis of the geodetic ellipsoid
*	f	Flattening of the geodetic ellipsoid = $(a-b)/a$
	e	First eccentricity of the ellipsoid = $(2f-f^2)^{1/2}$

3.12 Computation of Zone Constants

In this section the zone defining constants, ellipsoid constants, and parts of the Lambert mapping equations are combined to form several intermediate computing constants that are zone specific. These intermediate constants, flagged with an asterisk, will be required within the working equations of sections 3.13 through 3.15. All angles are in radian measure where 1 radian equals $180/\pi$ degrees. Linear units are identical to the units of the ellipsoid (a and b) and grid origin (N_b and E_0).

$$Q_s = \frac{1}{2}\left[\ell n\,\frac{1 + \sin\,\phi_s}{1 - \sin\,\phi_s} - e\;\ell n\,\frac{1 + e\,\sin\,\phi_s}{1 - e\,\sin\,\phi_s}\right]$$

$$W_s = (1 - e^2\sin^2\phi_s)^{1/2}.$$

Similarly for Q_n, W_n, Q_b, Q_0, and W_0 upon substitution of the appropriate latitude

$$*\quad \sin\,\phi_0 = \frac{\ell n\,[W_n\cos\,\phi_s/(W_s\cos\,\phi_n)]}{Q_n - Q_s}$$

$$*\;K = \frac{a\,\cos\,\phi_s\exp(Q_s\sin\phi_0)}{W_s\sin\,\phi_0} = \frac{a\,\cos\,\phi_n\exp(Q_n\sin\,\phi_0)}{W_n\sin\,\phi_0}$$

NOTE: $\exp(x) = \varepsilon^x$

where $\varepsilon = 2.718281828\ldots$ (the base of natural logarithms)

$$*\quad R_b = K/\exp(Q_b\sin\,\phi_0)$$

* $R_0 = K/\exp(Q_0 \sin \phi_0)$ (R_0 used in δ computation)

* $k_0 = (W_0 \tan \phi_0 R_0)/a$

* $N_0 = R_b + N_b - R_0$.

3.13 Direct Conversion Computation

This computation starts with the geodetic coordinates of a point (ϕ,λ) from which the Lambert grid coordinates (N,E) are to be computed, with convergence angle (γ), and grid scale factor (k).

$$Q = \frac{1}{2}\left[\ln\frac{1+\sin\phi}{1-\sin\phi} - e\ \ln\frac{1+e\sin\phi}{1-e\sin\phi}\right]$$

$$R = K/\exp(Q \sin \phi_0)$$

$$\gamma = (\lambda_0 - \lambda)\sin\phi_0$$

$$N = R_b + N_b - R\cos\gamma$$

$$E = E_0 + R\sin\gamma$$

$$k = (1 - e^2\sin^2\phi)^{1/2}(R\sin\phi_0)/(a\cos\phi).$$

3.14 Inverse Conversion Computation

In this computation the Lambert grid coordinates of a point (N,E) are given and the geodetic coordinates (ϕ,λ), convergence (γ), and grid scale factor (k) are to be computed.

$$R' = R_b - N + N_b$$

$$E' = E - E_0$$

$$\gamma = \tan^{-1}(E'/R')$$

$$\lambda = \lambda_0 - \gamma/\sin\phi_0$$

$$R = (R'^2 + E'^2)^{1/2}$$

$$Q = [\ln(K/R)]/\sin\phi_0.$$

Computation of latitude is iterative. Starting with the approximation

$$\sin\phi = \frac{\exp(2Q) - 1}{\exp(2Q) + 1}$$

solve for sin ϕ three times, as follows:

$$f_1 = \frac{1}{2}\left[\ln\frac{1 + \sin\phi}{1 - \sin\phi} - e\ \ln\frac{1 + e\sin\phi}{1 - e\sin\phi}\right] - Q$$

$$f_2 = \frac{1}{1 - \sin^2\phi} - \frac{e^2}{1 - e^2\sin^2\phi}\ .$$

Add a correction of $(-f_1/f_2)$ to sin ϕ and iterate two times before obtaining ϕ.

$$k = (1 - e^2\sin^2\phi)^{1/2}\ (R\sin\phi_0)/(a\cos\phi).$$

Latitude can also be obtained without iteration as shown in computations for the oblique Mercator projection. Four additional ellipsoid constants required for this alternative, F_0, F_2, F_4, F_6, are computed in section 3.32.

If only k is desired from the grid coordinates, an approximate ϕ will suffice and its computation shortened. After computing Q, compute

$$\sin\theta = \frac{\exp(2Q) - 1}{\exp(2Q) + 1}$$

$$\phi = \theta + (A\sin\theta\cos\theta)(1 + B\cos^2\theta)$$

in which $A = e^2(1 - e^2/6)$ and $B = 7e^2/6$. For the GRS 80 ellipsoid A = 0.0066869 and B = 0.0078.

The grid scale factor may be approximated by the equation

$$k = k_0 + (N-N_0)^2/2r_0^2 + (N-N_0)^3(\tan\phi_0)/6r_0^3.$$

The quantity r_0 is defined in section 3.15. Values of r_0, k_0 and N_0 for each zone are given in appendix C.

A further approximation is given by the equation:

$$k = k_0 + (N-N_0)^2(1.231 \times 10^{-14}) + (N-N_0)^3(\tan\phi_0)(6.94 \times 10^{-22}).$$

These approximations may not be sufficiently accurate in the States with a single Lambert zone.

To derive the grid scale factor at a point directly from the grid coordinates, the method given in section 3.4, the method of polynomial coefficients, also warrants consideration.

3.15 Arc-to-Chord Correction "δ" (alias "t-T") (see also sec. 4.3.)

The relationship among grid azimuth (t), geodetic azimuth (α), convergence angle (γ), and arc-to-chord correction (δ) at any given point is

$$t = \alpha - \gamma + \delta.$$

To compute δ requires knowledge of the coordinates of both ends of the line to which δ is to be applied. If geodetic coordinates of the endpoints are available (ϕ_1, λ_1 and ϕ_2, λ_2), the δ from point 1 to point 2 can be computed from

$$\delta_{12} = (\sin \phi_3 - \sin \phi_0)\ (\lambda_1 - \lambda_2)/2$$

where $\phi_3 = (2\phi_1 + \phi_2)/3$ and ϕ_0 is the computed constant for the zone. In normal practice, however, δ is desired as a function of the grid coordinates. To that end the following sequence of equations will produce the best possible determination of δ_{12}, given points N_1, E_1 and N_2, E_2:

$$\begin{aligned} p_1 &= N_1 - N_0 & \qquad p_2 &= N_2 - N_0 \\ q_1 &= E_1 - E_0 & q_2 &= E_2 - E_0 \\ R'_1 &= R_0 - p_1 & R'_2 &= R_0 - p_2 \\ \Delta N &= N_2 - N_1 & & \\ M_0 &= k_0 a\ (1 - e^2)/(1 - e^2 \sin^2 \phi_0)^{3/2} & & \end{aligned}$$

NOTE: M_0 is the scaled radius of curvature in the meridian at ϕ_0 scaled to the grid. The value of M_0 for each zone appears in appendix 3 as a "computed constant."

$$u_1 = p_1 - q_1{}^2/2R'_1$$

$$\phi_3 = \phi_0 + (u_1 + \Delta N/3)/M_0$$

$$\delta_{12} = (\sin \phi_3/\sin \phi_0 - 1)(q_2/R'_2 - q_1/R'_1)/2. \qquad (1)$$

For most applications a less accurate determination of ϕ will suffice. For example, the original Coast and Geodetic Survey formula (Adams and Claire 1948: p. 13) should be adequate for all applications except the most precise surveys in the largest Lambert zones.

$$\delta_{12} = (p_1 + \Delta N/3)\Delta E/2r_0{}^2 \qquad (2)$$

where

$$p_1 = N_1 - N_0$$

$$\Delta N = N_2 - N_1 \qquad \Delta E = E_2 - E_1$$

$$r_0 = k_0 a(1 - e^2)^{1/2}/(1 - e^2 \sin^2 \phi_0).$$

The quantity r_0 is the geometric mean radius of curvature at ϕ_0 scaled to the grid and is constant for any one zone. The value of r_0 for each zone has been included with the computed constants in appendix C. A single value of $1/(2\ r_0{}^2)$ is often used and combined with the constant to convert radians to seconds (1 radian = $648000/\pi$ seconds).

Hence, $\delta_{12} = 25.4\ (p_1 + \Delta N/3)(\Delta E)10^{-10}$ seconds, where the coordinates are in meters. Sometimes the notation (ΔN) replaces $(p_1 + \Delta N/3)$. Then the above equation is analogous to the expression often used in connection with NAD 27:

$$\delta_{12} = 2.36\ \Delta x\ \Delta y\ 10^{-10} \quad \text{seconds}$$

where the coordinates are in feet. This expression also serves for NAD 83 coordinates that have been converted to feet.

For the SPCS 27 Lambert systems NGS suggested two other appropriate methods that provided more accurate (t-T) corrections at the zone extremities. One was similar to equation 3.15 (1) and gave essentially the same results. Since the computing effort was somewhat greater than for 3.15 (1) it is not given here. The second, while not as accurate as 3.15(1), may be simpler for manual calculations because it uses the SPCS 83 zone constants and readily understood rotation and translation formulas.

$$\delta_{12} = (e_2 - e_1)(2n_1 + n_2)/6r_0^2 \quad \text{(in radian measure)}$$

where

$n = D + E' \sin\gamma + N' \cos\gamma$
$e = E' \cos\gamma - N' \sin\gamma$
"γ" is the average convergence angle for the survey area and is considered positive. γ to minutes is sufficient.
$D = 2R_0 \sin^2\gamma/2$
$N' = N - N_0$
$E' = E - E_0$

The size of δ varies linearly with the length of the ΔE ($\Delta\lambda$) component of the line and with the distance of the standpoint from the central parallel. It does not vary with distance of standpoint from the central meridian. Hence the size of δ depends on the direction of the line, varying from a zero value between points on the same meridian to maximum values over east-west lines.

Table 3.1 gives an overview of the true numeric value of the arc-to-chord correction (δ) and of the computational errors expected from equations (1) and (2). The examples were computed for a hypothetical zone with central parallel of approximately 42° (standard parallels 41° and 43°), on the GRS 80 ellipsoid. Two cases, 1° and 2°, are illustrated for the distance of the standpoint from the central parallel ϕ_0. Two cases, 5° and 10°, are given for the distance of the standpoint from the central meridian. Although the magnitude of δ is not a function of the distance of the standpoint from the central meridian, equation (2) becomes less accurate as this distance increases. Table 3.1 also gives three cases for the orientation of the line, in azimuths of 90°, 135°, and 180°. Again note that although the true δ equals zero in an azimuth of 180°, the equations only approximate zero.

The final assumption in table 3.1 is that the length of the line for which δ is being computed is 20 km. Dividing the line into several traverse legs results in a proportional decrease in the required correction to a direction. It does nothing to diminish the closure error in azimuth because errors due to omission of δ are cumulative.

From data given in table 3.1 the persons performing the computing must decide which reduction formula is appropriate for their needs, remembering that the accuracy of the formula should exceed the expected accuracy of the field work by one order of magnitude and that an error of 1" in direction corresponds to a linear error of about 1:200,000, or 5 ppm.

Table 3.1.--True values of (t-T) and computational errors in their determination (in seconds of arc)

$\phi_1 - \phi_0$	1°	2°	1°	2°
$\lambda_1 - \lambda_0$	5°	5°	10°	10°
Azimuth	90°	90°	90°	90°
True δ	5.67	11.44	5.67	11.44
Error by (1)	0.00	0.02	0.03	0.11
Error by (2)	0.53	0.38	2.28	2.07
Azimuth	135°	135°	135°	135°
True δ	3.83	7.91	3.83	7.91
Error by (1)	0.00	0.01	0.02	0.08
Error by (2)	0.14	-0.20	0.99	0.38
Azimuth	180°	180°	180°	180°
True δ	0.00	0.00	0.00	0.00
Error by (1)	0.00	0.00	0.00	0.00
Error by (2)	-0.34	-0.67	-0.90	-1.55

3.2 Transverse Mercator Mapping Equations

3.21 Notation and Definitions

Asterisked terms define the projection for which values are given in appendix A. These zone specific "defining constants" are included within State SPCS legislation where enacted.

	ϕ	Parallel of geodetic latitude, positive north
	λ	Meridian of geodetic longitude, positive west
	ω	Rectifying latitude
	N	Northing coordinate on the projection (formerly y)
	E	Easting coordinate on the projection (formerly x)
*	λ_0	Central meridian
*	E_0	False easting (value assigned to the central meridian)
	S	Meridional distance
*	ϕ_0	Latitude of grid origin
*	N_0	False northing (value assigned to the latitude of grid origin)
	S_0	Meridional distance from the equator to ϕ_0, multiplied by the central meridian scale factor
*	k_0	Grid scale factor assigned to the central meridian
	k	Grid scale factor at a point
	k_{12}	Grid scale factor for a line (between points 1 and 2)
	ΔN	$N_2 - N_1$
	ΔE	$E_2 - E_1$
	E'	$E - E_0$
	γ	Meridian convergence
	δ_{12}	Arc-to-chord correction (t-T) (from point 1 to point 2)
	a	Semimajor axis of the ellipsoid
	b	Semiminor axis of the ellipsoid
	f	Flattening of the ellipsoid = (a - b)/a
	e^2	First eccentricity squared = $(a^2 - b^2)/a^2 = 2f - f^2$

e'^2 Second eccentricity squared = $(a^2 - b^2)/b^2 = e^2/(1 - e^2)$
n $(a - b)/(a + b) = f/(2 - f)$
R Radius of curvature in the prime vertical = $a/(1 - e^2\sin^2\phi)^{1/2}$
r_0 Geometric mean radius of curvature scaled to the grid
r Radius of the rectifying sphere
t $\tan\phi$ (secs. 3.23 and 3.24)
t grid azimuth (sec. 3.25 and others)
η^2 $e'^2\cos^2\phi$

3.22 Constants for Meridional Distance

In this section nine ellipsoid specific constants and one zone specific intermediate computing constant are derived. These intermediate constants, flagged with an asterisk, will be required within the working equations of sections 3.23 through 3.26.

The following ellipsoid specific constants may be directly entered into software. The equations are given for those with requirements for other ellipsoids.

$$* \; r = a(1 - n)(1 - n^2)(1 + 9n^2/4 + 225n^4/64) = 6367449.14577 \text{ m (GRS 80)}$$
$$u_2 = -3n/2 + 9n^3/16$$
$$u_4 = 15n^2/16 - 15n^4/32$$
$$u_6 = -35n^3/48$$
$$u_8 = 315n^4/512$$

$$* \; U_0 = 2(u_2 - 2u_4 + 3u_6 - 4u_8) = -0.00504\;82507\;76 \text{ (GRS 80)}$$
$$* \; U_2 = 8(u_4 - 4u_6 + 10u_8) = 0.00002\;12592\;04 \text{ (GRS 80)}$$
$$* \; U_4 = 32(u_6 - 6u_8) = -0.00000\;01114\;23 \text{ (GRS 80)}$$
$$* \; U_6 = 128\,u_8 = 0.00000\;00006\;26 \text{ (GRS 80)}$$
$$v_2 = 3n/2 - 27n^3/32$$
$$v_4 = 21n^2/16 - 55n^4/32$$
$$v_6 = 151n^3/96$$
$$v_8 = 1097n^4/512$$

$$* \; V_0 = 2(v_2 - 2v_4 + 3v_6 - 4v_8) = 0.00502\;28939\;48 \text{ (GRS 80)}$$
$$* \; V_2 = 8(v_4 - 4v_6 + 10\,v_8) = 0.00002\;93706\;25 \text{ (GRS 80)}$$
$$* \; V_4 = 32(v_6 - 6v_8) = 0.00000\;02350\;59 \text{ (GRS 80)}$$
$$* \; V_6 = 128v_8 = 0.00000\;00021\;81 \text{ (GRS 80)}$$

The following meridional constant is a zone specific constant computed once for a zone. Table 3.22 contains S_0 for each SPCS 83 transverse Mercator zone.

$$\omega_0 = \phi_0 + \sin\phi_0 \cos\phi_0(U_0 + U_2\cos^2\phi_0 + U_4\cos^4\phi_0 + U_6\cos^6\phi_0)$$
$$* \; S_0 = k_0\omega_0 r$$

3.23 Direct Conversion Computation

The following computation starts with the geodetic coordinates of a point (ϕ, λ) from which the transverse Mercator grid coordinates (N,E), convergence angle (γ), and the grid scale factor (k) are computed. All angles are in radian measure where one radian equals $180/\pi$ degrees. Linear units match the units of the ellipsoid and false origin.

Table 3.22.--Intermediate constants for transverse Mercator projections

State-zone-code	S_0 (m)	$1/(2\ r_0^2)$* (10^{14})	State-zone-code	S_0 (m)	$1/(2\ r_0^2)$* (10^{14})
AL-E-0101	3,375,406.7112	1.23256	IL-W-1202	4,056,280.6721	1.23068
AL-W-0102	3,319,892.0570	1.23262	IN-E-1301	4,151,863.7425	1.23062
AK-2-5002	5,985,317.4367	1.22473	IN-W-1302	4,151,863.7425	1.23062
AK-3-5003	5,985,317.4367	1.22473	ME-E-1801	4,836,302.3615	1.22906
AK-4-5004	5,985,317.4367	1.22473	ME-W-1802	4,744,046.5583	1.22918
AK-5-5005	5,985,317.4367	1.22473	MS-E-2301	3,264,526.0416	1.23258
AK-6-5006	5,985,317.4367	1.22473	MS-W-2302	3,264,526.0416	1.23258
AK-7-5007	5,985,317.4367	1.22473	MO-E-2401	3,966,785.2908	1.23126
AK-8-5008	5,985,317.4367	1.22473	MO-C-2402	3,966,785.2908	1.23126
AK-9-5009	5,985,317.4367	1.22473	MO-W-2403	4,003,800.5632	1.23124
AZ-E-0201	3,430,631.2260	1.23244	NV-E-2701	3,846,473.6437	1.23106
AZ-C-0202	3,430,631.2260	1.23244	NV-C-2702	3,846,473.6437	1.23106
AZ-W-0203	3,430,745.5918	1.23236	NV-W-2703	3,846,473.6437	1.23106
DE---0700	4,207,476.9816	1.23083	NH---2800	4,707,019.0442	1.22976
FL-E-0901	2,692,050.5001	1.23387	NJ---2900	4,299,571.6693	1.23078
FL-W-0902	2,692,050.5001	1.23387	NM-E-3001	3,430,662.4167	1.23242
GA-E-1001	3,319,781.3865	1.23271	NM-C-3002	3,430,631.2260	1.23244
GA-W-1002	3,319,781.3865	1.23271	NM-W-3003	3,430,688.4089	1.23240
HI-1-5101	2,083,150.1655	1.23570	NY-E-3101	4,299,571.6693	1.22992
HI-2-5102	2,249,193.4045	1.23532	NY-C-3102	4,429,252.1847	1.22983
HI-3-5103	2,341,506.4725	1.23527	NY-W-3103	4,429,252.1847	1.22983
HI-4-5104	2,415,321.4658	1.23507	RI---3800	4,549,799.4141	1.22998
HI-5-5105	2,396,891.1333	1.23505	VT---4400	4,707,007.8366	1.22948
ID-E-1101	4,614,370.6555	1.22980	WY-E-4901	4,484,768.4357	1.22983
ID-C-1102	4,614,370.6555	1.22952	WY-EC-4902	4,484,768.4357	1.22983
ID-W-1103	4,614,305.8890	1.22926	WY-WC-4903	4,484,768.4357	1.22983
IL-E-1201	4,059,417.9793	1.23060	WY-W-4904	4,484,768.4357	1.22983

$$L = (\lambda - \lambda_0) \cos \phi$$

Note: The sign convention used in SPCS 27 was $(\lambda_0 - \lambda)$.

$$\omega = \phi + \sin \phi \cos \phi(U_0 + U_2\cos^2\phi + U_4\cos^4\phi + U_6\cos^6\phi).$$

Suggestion: Use nested form.

$$\omega = \phi + (\sin \phi \cos \phi)[U_0 + \cos^2\phi\{U_2 + \cos^2\phi(U_4 + U_6\cos^2\phi)\}]$$

$$S = k_0\omega\ r$$

$$R = k_0a/(1 - e^2\sin^2\phi)^{1/2}$$

$$A_2 = \frac{1}{2} R\ t$$

$$A_4 = \frac{1}{12}[5 - t^2 + \eta^2(9 + 4\eta^2)]$$

$$A_6 = \frac{1}{360}[61 - 58t^2 + t^4 + \eta^2(270 - 330t^2)]$$

$$N = S - S_0 + N_0 + A_2L^2[1 + L^2(A_4 + A_6L^2)]$$

$$A_1 = -R$$

$$A_3 = \frac{1}{6}(1 - t^2 + \eta^2)$$

$$A_5 = \frac{1}{120}[5 - 18t^2 + t^4 + \eta^2(14 - 58t^2)]$$

$$A_7 = \frac{1}{5040}(61 - 479t^2 + 179t^4 - t^6)$$

$$E = E_0 + A_1L[1 + L^2(A_3 + L^2(A_5 + A_7L^2))]$$

$$L = (\lambda - \lambda_0)\cos\phi$$

$$C_1 = -t$$

$$C_3 = \frac{1}{3}(1 + 3\eta^2 + 2\eta^4)$$

$$C_5 = \frac{1}{15}(2 - t^2)$$

$$\gamma = C_1L[1 + L^2(C_3 + C_5L^2)]$$

$$F_2 = \frac{1}{2}(1 + \eta^2)$$

$$F_4 = \frac{1}{12}[5 - 4t^2 + \eta^2(9 - 24t^2)]$$

$$k = k_0[1 + F_2L^2(1 + F_4L^2)].$$

The A_6, A_7, C_5, and F_4 terms are negligible when computing within the approximate boundaries of the SPCS 83 zones. To use the SPCS 83 beyond the defined SPCS 83 boundaries and to compute UTM coordinates, the significance of these terms should be evaluated.

3.24 Inverse Conversion Computation

This computation starts with the transverse Mercator grid coordinates of a point (N,E) from which the geodetic coordinates (ϕ,λ), convergence angle (γ), and grid scale factor (k) are computed.

$$\omega = (N - N_0 + S_0)/(k_0r)$$
$$\phi_f = \omega + (\sin\omega\cos\omega)(V_0 + V_2\cos^2\omega + V_4\cos^4\omega + V_6\cos^6\omega).$$

(This is sometimes referred to as the "footpoint latitude.")

Suggestion: Use nested form.

$\phi_f = \omega + (\sin\omega\cos\omega)[V_0 + \cos^2\omega\,\{V_2 + \cos^2\omega\,(V_4 + V_6\cos^2\omega)\}]$

$R_f = k_0 a/(1 - e^2\sin^2\phi_f)^{1/2}$

$Q = E'/R_f$

$B_2 = -\frac{1}{2}\,t_f(1 + \eta_f^2)$

$B_4 = -\frac{1}{12}\,[5 + 3t_f^2 + \eta_f^2(1 - 9t_f^2) - 4\eta_f^4]$

$B_6 = \frac{1}{360}\,[61 + 90t_f^2 + 45t_f^4 + \eta_f^2(46 - 252t_f^2 - 90t_f^4)]$

$\phi = \phi_f + B_2Q^2[1 + Q^2(B_4 + B_6Q^2)]$

$B_3 = -\frac{1}{6}\,(1 + 2t_f^2 + \eta_f^2)$

$B_5 = \frac{1}{120}\,[5 + 28t_f^2 + 24t_f^4 + \eta_f^2(6 + 8t_f^2)]$

$B_7 = -\frac{1}{5040}\,(61 + 662t_f^2 + 1320t_f^4 + 720t_f^6)$

$L = Q[1 + Q^2\{B_3 + Q^2(B_5 + B_7Q^2)\}]$

$\lambda = \lambda_0 - L/\cos\phi_f$

$D_1 = t_f$

$D_3 = -\frac{1}{3}\,(1 + t_f^2 - \eta_f^2 - 2\eta_f^4)$

$D_5 = \frac{1}{15}\,(2 + 5t_f^2 + 3t_f^4)$

$\gamma = D_1Q[1 + Q^2(D_3 + D_5Q^2)]$

$G_2 = \frac{1}{2}\,(1 + \eta_f^2)$

$G_4 = \frac{1}{12}\,(1 + 5\eta_f^2)$

$k = k_0[1 + G_2Q^2(1 + G_4Q^2)]$

The B_6, B_7, D_5, and G_4 terms are negligible when using the SPCS 83 within the approximate boundaries of the SPCS 83 zones. To compute beyond the defined SPCS 83 boundaries and to compute UTM coordinates, the use of these terms should be evaluated.

For most requirements the point grid scale factor k may be determined from the approximation:

$$k = k_0 + (E')^2/2r_0^2$$

where r_0 is the geometric mean radius of curvature scaled to the grid defined in section 3.15, and evaluated at the mean latitude of the zone. Table 3.22 contains $(1/2r_0^2)$ for each of the transverse Mercator zones.

3.25 Arc-to-Chord Correction (t-T)

The relationship among grid azimuth (t), geodetic azimuth (α), convergence angle (γ), and arc-to-chord correction (δ) at any given point is

$$t = \alpha - \gamma + \delta. \quad \text{(Remember that } \delta \text{ is defined as t-T).}$$

To compute δ requires knowledge of the coordinates of both ends of the line to which δ is to be applied. The following equations will compute δ_{12}, the δ from point (N_1, E_1) to (N_2, E_2):

$$N_m = \frac{1}{2}(N_1 + N_2)$$

$$\omega = (N_m - N_0 + S_0)/(k_0 r)$$

$$\phi_f = \omega + V_0 \sin\omega \cos\omega$$

$$F = (1 - e^2 \sin^2\phi_f)(1 + \eta_f^2)/(k_0 a)^2$$

$$E_3 = 2E'_1 + E'_2$$

$$\delta_{12} = -\frac{1}{6}\Delta N\, E_3 F\left(1 - \frac{1}{27} E_3^2 F\right).$$

When computing within the approximate boundaries of the SPCS 83 zones, the term "$\frac{1}{27} E_3^2 F$" is negligible and a single value of F can be precomputed for a mean latitude.

Often a single value of (F/2) is combined with the constant to convert radians to seconds (1 radian = $648000/\pi$ seconds) yielding the expression:

$$\delta_{12} = -25.4\, \Delta N (E_3/3)\, 10^{-10} \text{ seconds}$$

where the coordinates are in meters. Sometimes the notation ΔE replaces $(E_3/3)$. Then this equation is analogous to the expression often used in connection with the SPCS 27:

$$\delta_{12} = 2.36\, \Delta x\, \Delta y\, 10^{-10} \text{ seconds}$$

where the coordinates are in feet. This expression would serve for SPCS 83 coordinates that have been converted to feet upon insertion of the negative sign to conform to the sign convention. This expression with SPCS 27 coordinates derived the correct sign graphically or from a table.

3.26 Grid Scale Factor of a Line

As covered in section 4.2, the grid scale factor is different at each end of a line, but a single value is required to reduce a measured line. Given the grid scale factor of endpoints of a line (k_1 and k_2), a grid scale factor of the line (k_{12}) is required. Below is an alternative to the methods stated in section 4.2. This equation computes k_{12} using the function "F" derived in section 3.25 for the δ_{12} correction.

$$G = F(E'^2_1 + E'_1E'_2 + E'^2_2)/6$$

$$k_{12} = k_0[1 + G(1 + G/6)].$$

As above, the term "G/6" is often negligible within the bounds of the SPCS 83 and a single value of F will usually give results within $\pm(3)(10^{-7})$ at zone extremes.

3.3 Oblique Mercator Mapping Equations

3.31 Notation and Definition

Asterisked terms define the projection. Their value can be found in appendix A for the one zone in SPCS 83 that uses this projection.

	ϕ	Parallel of geodetic latitude, positive north
	λ	Meridian of geodetic longitude, positive west
	Q	Isometric latitude
	χ	Conformal latitude
	N	Northing coordinate
	E	Easting coordinate
*	N_0	False northing
*	E_0	False easting
	k	Point grid scale factor
	γ	Convergence
*	ϕ_c	Latitude of local origin
*	λ_c	Longitude of local origin
*	α_c	Azimuth of positive skew axis (u-axis) at local origin
*	k_c	Grid scale factor at the local origin
	α_0	Azimuth of positive skew axis at equator
	λ_0	Longitude of the true origin
	k_{12}	Line scale factor (between points 1 and 2)
	δ_{12}	Arc-to-chord correction (t-T) (from point 1 to point 2)
	a	Equatorial radius of the ellipsoid
	f	Flattening of the ellipsoid

3.32 Computation of GRS 80 Ellipsoid Constants

This section lists the equations for the ellipsoid-specific constants and the constants derived for the GRS 80 ellipsoid. The asterisked terms are required in section 3.34 through 3.36.

$$e^2 = 2f - f^2$$

$$e'^2 = e^2/(1 - e^2)$$

$$c_2 = e^2/2 + 5e^4/24 + e^6/12 + 13e^8/360$$

$$c_4 = 7e^4/48 + 29e^6/240 + 811e^8/11520$$

$$c_6 = 7e^6/120 + 81e^8/1120$$

$$c_8 = 4279e^8/161280$$

* $F_0 = 2(c_2 - 2c_4 + 3c_6 - 4c_8)$ = 0.00668 69209 27 (GRS 80)

* $F_2 = 8(c_4 - 4c_6 + 10c_8)$ = 0.00005 20145 84 (GRS 80)

* $F_4 = 32(c_6 - 6c_8)$ = 0.00000 05544 30 (GRS 80)

* $F_6 = 128\ c_8$ = 0.00000 00068 20 (GRS 80)

3.33 Computation of Zone Constants

In this section the zone defining constants, ellipsoid constants, and expressions within the oblique Mercator mapping equations are combined to form several intermediate computing constants that are zone and ellipsoid specific. These intermediate constants, flagged with an asterisk, will be required within the working equations of section 3.34 through 3.36.

All angles are in radian measure, where 1 radian equals $180/\pi$ degrees. Linear units are in meters.

* $$B = (1 + e'^2\cos^4\phi_c)^{1/2}$$

$$W_c = (1 - e^2\sin^2\phi_c)^{1/2}$$

$$A = aB(1 - e^2)^{1/2}/W_c^{\,2}$$

$$Q_c = (1/2)\left[\ln \frac{1 + \sin \phi_c}{1 - \sin \phi_c} - e \ln \frac{1 + e \sin \phi_c}{1 - e \sin \phi_c}\right]$$

* $$C = \cosh^{-1} \frac{B\ (1 - e^2)^{1/2}}{W_c \cos \phi_c} - BQ_c$$

Note: $\cosh^{-1}x = \ln[x+(x^2-1)^{1/2}]$

* $D = k_c A/B$

$\sin \alpha_0 = (a \sin \alpha_c \cos \phi_c)/(AW_c)$

For zone 5001:

$\tan \alpha_c = -0.75$
$\sin \alpha_c = -0.6$
$\cos \alpha_c = +0.8$

* $\lambda_0 = \lambda_c + \{\sin^{-1} [\sin \alpha_0 \sinh(BQ_c + C)/\cos \alpha_0]\}/B$

Note: $\sinh x = (e^x - e^{-x})/2$ (e = base of natural logarithms)

* $F = \sin \alpha_0$

* $G = \cos \alpha_0$

* $I = k_c A/a$

For Alaska zone 1, these constants are:

$B = 1.00029\ 64614\ 04$
$C = 0.00442\ 68339\ 26$
$D = 6\ 386\ 186.73253$
$F = -0.32701\ 29554\ 38$
$G = 0.94501\ 98553\ 34$
$I = 1.00155\ 89176\ 62$
$\lambda_0 = 101.51383\ 9560$ degrees

3.34 Direct Conversion Computation

This computation starts with the geodetic coordinates of a point (ϕ,λ), and computes the oblique Mercator grid coordinates (N,E), convergence angle (γ), and the grid scale factor (k).

$$L = (\lambda - \lambda_0)B$$

$$Q = (1/2)\left[\ln \frac{1 + \sin \phi}{1 - \sin \phi} - e \ln \frac{1 + e \sin \phi}{1 - e \sin \phi}\right]$$

$$J = \sinh(BQ + C)$$

$$K = \cosh(BQ + C)$$

Note: $\cosh x = (e^{x} + e^{-x})/2$ (e = base of natural logarithms)

$$u = D \tan^{-1} [(JG - F \sin L)/\cos L]$$

$$v = \frac{D}{2} \ln \frac{K - FJ - G \sin L}{K + FJ + G \sin L}$$

$$N = u \cos \alpha_c - v \sin \alpha_c + N_0$$

$$E = u \sin \alpha_c + v \cos \alpha_c + E_0$$

For zone 5001:

$$N = 0.8\ u + 0.6\ v - 5{,}000{,}000.$$
$$E = -0.6\ u + 0.8\ v + 5{,}000{,}000.$$

$$\gamma = \tan^{-1} \frac{F - JG \sin L}{KG \cos L} - \alpha_c$$

$$k = \frac{I(1 - e^2 \sin^2 \phi)^{1/2} \cos(u/D)}{\cos \phi \cos L}\ .$$

3.35 Inverse Conversion Computation

This computation starts with the oblique Mercator grid coordinates (N,E) and computes the geodetic coordinates (ϕ,λ). To compute the convergence angle (γ) and the grid scale factor (k), the computed (ϕ,λ) is then used in the equations of the direct conversion computation.

$$u = (E - E_0) \sin \alpha_c + (N - N_0) \cos \alpha_c$$

$$v = (E - E_0) \cos \alpha_c - (N - N_0) \sin \alpha_c$$

For zone 5001:

$$u = -0.6E + 0.8N + 7{,}000{,}000.$$
$$v = 0.8E + 0.6N - 1{,}000{,}000.$$

$$R = \sinh(v/D)$$

$$S = \cosh(v/D)$$

Note: $\cosh x = (e^{x} + e^{-x})/2$

$$T = \sin(u/D)$$

$$Q = \left[(1/2) \ln \frac{S - RF + GT}{S + RF - GT} - C \right] / B$$

$$\chi = 2 \tan^{-1} \frac{\exp(Q) - 1}{\exp(Q) + 1}$$

where $\exp(Q) = e^Q$ and $e = 2.718281828...$(base of natural logarithms)

$$\phi = \chi + (\sin\chi \cos\chi)(F_0 + F_2\cos^2\chi + F_4\cos^4\chi + F_6\cos^6\chi)$$

$$\lambda = \lambda_0 - \frac{1}{B} \tan^{-1} \frac{RG + TF}{\cos(u/D)} .$$

3.36 Arc-to-chord Correction (t-T) and Grid Scale Factor of a Line

Having first obtained coordinates (u_1,v_1) and (u_2,v_2) from either the direct or inverse conversion computation, the (t-T) correction for the line from point 1 to point 2 (δ_{12}) and line correction k_{12} may be computed.

$$\phi = (\phi_1 + \phi_2)/2$$

$$Q = \frac{1}{2} \left[\ln \frac{1 + \sin\phi}{1 - \sin\phi} - e \ln \frac{1 + e \sin\phi}{1 - e \sin\phi} \right]$$

$$\delta_{12} = (u_1 - u_2)\ (2v_1 + v_2)/(6D^2)$$

$$k_{12} = \frac{k_c\ [1 + (v_1{}^2 + v_1v_2 + v_2{}^2)/(6D^2)](1 - e^2\sin^2\phi)^{1/2}}{\cos\phi \cosh\ (BQ + C)}$$

3.4 Polynomial Coefficients for the Lambert projection

Conversion of coordinates from NAD 83 geodetic positions to SPCS 83 plane coordinate positions, and vice versa, can be greatly simplified for the Lambert projection using precomputed zone constants obtained by polynomial curve fitting. NGS developed the Lambert "polynomial coefficient" approach as an alternative to the rigorous mapping equations given section 3.1. For many zones the solution of the textbook mapping equations for the Lambert projection requires the use of more than 10 significant digits to obtain millimeter accuracy, and in light of the programmable calculators generally in use by surveyors/engineers, an alternative approach was warranted. The mapping equations of the transverse Mercator projection do not present the same numerical problem as does the Lambert projection. Therefore, 10 significant digits are adequate. For the polynomial coefficient method of the Lambert projection, 10 significant digits will produce millimeter accuracy in all zones.

Given the precomputed polynomial coefficients, the conversion process by this method reduces to the solution of simple algebraic equations, requiring no exponential or logarithmic functions. It is therefore very efficient for hand calculators and small computers. In addition, the conversion is not too difficult to apply manually without the aid of programming. For this reason, the polynomial coefficient approach has also been listed as a manual approach in table 3.0. When programmed, this approach may be more efficient than the mapping equations of section 3.1.

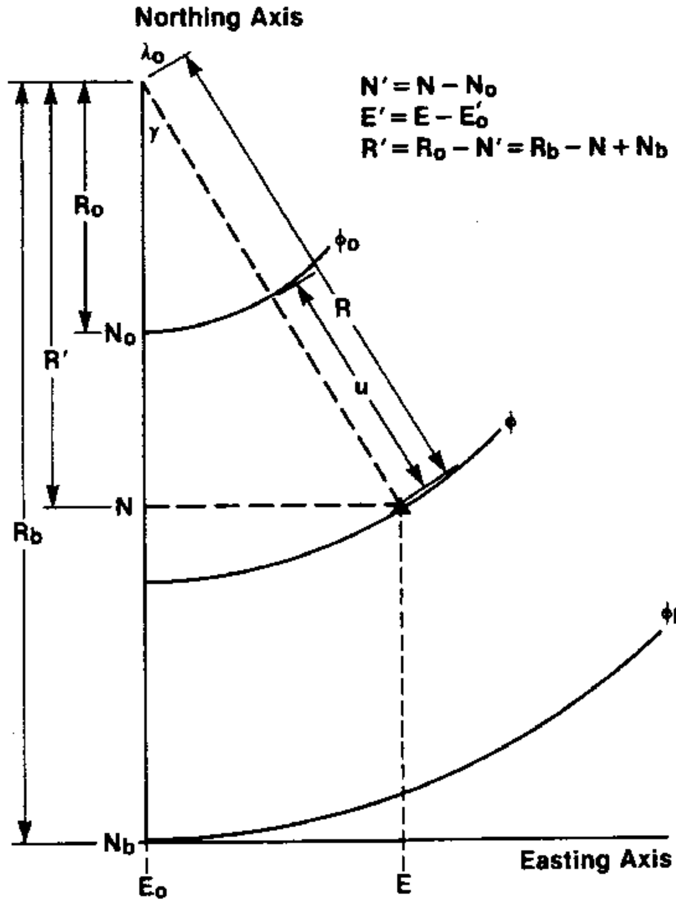


Figure 3.4.--The Lambert grid.

The equations in this section are similar to those in section 3.1, with the symbols representing the same quantities. Four new symbols are introduced, three of which are for polynomial coefficients--L's, G's, and F's--and the fourth is the symbol "u". From the equations and figure 3.4, it will be discovered that "u" is a distance on the mapping radius "R" between the central parallel and a given point. The "L" coefficients (L_1, L_2, L_3, etc.) are used in the forward conversion process (sec. 3.41), the "G" coefficients (G_1, G_2, G_3, etc.) are used in the inverse conversion process (sec. 3.42), and the "F" coefficients are used in the computation of grid scale factor. For the computation of (t-T), the methods in section 3.15 are applicable.

The fundamental polynomial equations of this method are

$$u = L_1\Delta\phi + L_2\Delta\phi^2 + L_3\Delta\phi^3 + L_4\Delta\phi^4 + L_5\Delta\phi^5 \text{ (forward conversion)}$$

$$\Delta\phi = \phi - \phi_0 = G_1u + G_2u^2 + G_3u^3 + G_4u^4 + G_5u^5 \text{ (inverse conversion).}$$

The determination of "u" in meters on a plane by a polynomial, given point (ϕ,λ) in the forward conversion, and the determination by a polynomial of $\Delta\phi$ in radians on the ellipsoid given point (N,E) in the inverse conversion, is the unique aspect of this method. The L-coefficients perform the functions: (1) computing the length of the meridian arc between ϕ and ϕ_0, and (2) converting that length to (R_0-R) which is its equivalent on the mapping radius. The G-coefficients serve the same two stage process, but in reverse. The polynomial coefficients of these equations, L's and G's, were separately determined by a least squares curve fitting program that also provided information as to the accuracy of the fit. Ten data points were used for each Lambert zone and the model solved for the fewest number of coefficients possible that provided 0.5 mm coordinate accuracy in the conversion. Consequently, some small zones required only three coefficients, three L's and three G's, whereas a few large zones required five coefficients for each the forward and inverse conversion.

Appendix C discusses the computed constants and coefficients required for this method, which are defined as follows:

The Defining Constants of a Zone:

ϕ_s or B_s	Southern standard parallel
ϕ_n or B_n	Northern standard parallel
ϕ_b or B_b	Latitude of grid origin
λ_0 or L_0	Central meridian - longitude of true and grid origin
N_b	Northing value at grid origin (B_b)
E_0	Easting value at grid and projection origin (L_0)

The Derived Constants:

ϕ_0 or B_0	Central parallel - Latitude of the projection origin
N_0	Northing value at projection origin (B_0)
k_0	Grid scale factor at the central parallel
R_0	Mapping radius at (B_0)
R_b	Mapping radius at (B_b)
M_0	Scaled radius of curvature in the meridian at B_0 used in section 3.15.

The Polynomial Coefficients:

L_1 through L_5 used in the forward conversion
G_1 through G_5 used in the inverse conversion
F_1 through F_3 used in the grid scale-factor computation.

3.41 Direct Conversion Computation

The computation starts with the geodetic position of a point (ϕ,λ), and computes the Lambert grid coordinates (N,E), convergence angle (γ), and grid scale factor (k).

$$\Delta\phi = \phi - B_0 \quad (\Delta\phi \text{ in decimal degrees})$$

$$u = L_1\Delta\phi + L_2\Delta\phi^2 + L_3\Delta\phi^3 + L_4\Delta\phi^4 + L_5\Delta\phi^5.$$

Note: The only required terms are those for which polynomial coefficients are provided in appendix C. Either three, four, or five L's are required depending on the size of the zone.

Suggestion: Use nested form.

$$u = \Delta\phi[L_1 + \Delta\phi\{L_2 + \Delta\phi(L_3 + \Delta\phi(L_4 + L_5\Delta\phi))\}]$$

$R = R_0 - u$

$\gamma = (L_0 - \lambda)\sin(B_0)$ convergence angle

$E' = R\sin\gamma$
$N' = u + E'\tan(\gamma/2)$
$E = E' + E_0$ easting

$N = N' + N_0$ northing

$k = F_1 + F_2u^2 + F_3u^3$ grid scale factor

3.42 Inverse Conversion Computation

This computation starts with the Lambert grid coordinates (N,E) from which are computed the geodetic coordinates (ϕ,λ), convergence angle (γ), and grid scale factor (k):

$N' = N - N_0$

$E' = E - E_0$

$R' = R_0 - N'$

$\gamma = \tan^{-1}(E'/R')$ convergence angle

$\lambda = L_0 - \gamma/\sin(B_0)$ longitude

$u = N' - E'\tan(\gamma/2)$

$\Delta\phi = \phi - B_0 = G_1u + G_2u^2 + G_3u^3 + G_4u^4 + G_5u^5$ ($\Delta\phi$ in decimal degrees)

Note: The only required terms are those for which polynomial coefficients are provided in appendix C. Either three, four, or five G's are required depending on the size of the zone.

Suggestion: Use factored form.

$$\Delta\phi = u[G_1 + u\{G_2 + u(G_3 + u(G_4 + G^5u))\}]$$

$\phi = B_0 + \Delta\phi$ latitude

$k = F_1 + F_2u^2 + F_3u^3$ grid scale factor

4. LINE CONVERSION METHODS REQUIRED TO PLACE A SURVEY ON SPCS 83

State plane coordinates are derived from latitudes and longitudes. Latitudes and longitudes are based on an ellipsoid of reference and a horizontal datum that approximates the surface of the Earth. Accordingly, field observations measured on the ground must first be reduced to the surface of the horizontal datum before they are further reduced to the map projection surface--the grid. The mathematical process of reducing field observations does not necessarily imply that the numbers are reduced in magnitude although often that is the case.

Section 4.1 addresses the reduction of measured distances to the datum surface, not a subject of map projections, but included here for convenience. Only the geometric aspect of reduction is discussed. Reductions relating to the influence of the atmosphere are not included. Section 4.2 contains the further reduction of measured distances to the grid, expanding on section 2.6 and applying the concept of point grid scale factors to an entire measured line. Section 4.3 discusses the reduction of azimuths and angles from the ellipsoid to the grid, applying the concepts stated in section 2.5. Reduction of angles and azimuths to the ellipsoid is beyond the scope of this manual. The reader is referred to texts on higher geodesy.

4.1 Reduction of Observed Distances to the Ellipsoid

Before a measured distance can be reduced to a grid distance in a zone of the SPCS 83, it must first be reduced to a geodetic distance. Classically, observed distances have been reduced to one of two surfaces, either the geoid (sea level) or the ellipsoid. (See fig. 4.1a.) To which surface distances were reduced depended on available information. Generally, in conjunction with NAD 27, distances were reduced only to sea level, although subsequent computations using those distances were performed on the ellipsoid. This incomplete reduction was adequate for NAD 27, as the ellipsoid of NAD 27 (Clarke Spheroid of 1866) closely approximated sea level. For NAD 83, due to availability of information on geoid-ellipsoid separation, distances may be reduced to the ellipsoid. Furthermore,

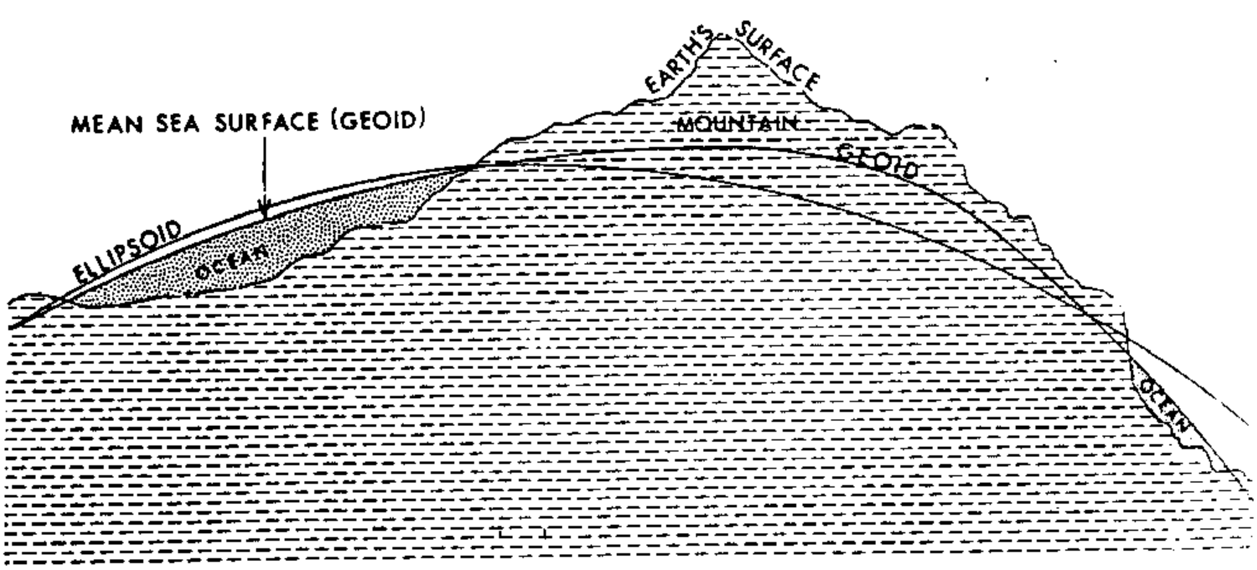


Figure 4.1a.--Geoid-ellipsoid-surface relationships.

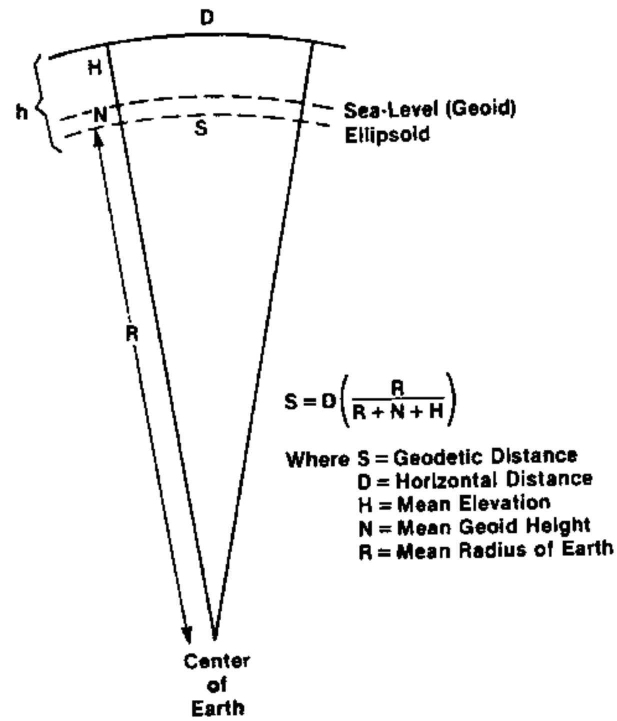


Figure 4.1b.--Reduction to the ellipsoid.

the worldwide datum of NAD 83 does not fit the North American continent as well as the previous NAD 27. The impact of this on surveyors may be the requirement to use this geoidal separation information in connection with the reduction of observed distances to the ellipsoid. The approximation of using sea level may not always be adequate, but those occurrences should be few and affect only surveys of highest order.

To reduce measurements to the ellipsoid instead of sea level requires the addition of the geoid height (sea level/ellipsoid separation) to the station elevations prior to reduction. In the conterminous United States the ellipsoid is above the geoid. In Alaska the ellipsoid is below the geoid. Since the geoid height of a station is defined as the height above the ellipsoid minus the height above the geoid, except in Alaska it is a negative value. The geoid height is published by NGS together with NAD 83 coordinate information. The geodetic height of a control station (height above ellipsoid) "h" is the sum of elevation above mean sea level "H" and geoid height "N". The failure to use geoid height will introduce an error in reduced distance of 0.16 ppm for each meter of geoid height. A geoid height of -30 m systematically affects all reduced distances by -4.8 ppm (1:208,000). Clearly a single geoid height may be applied for a region or project, and even ignored for many types of surveys.

The application of geoid height and the precise determination of a radius of curvature on the ellipsoid (below) are the only occurrences where NAD 83 may affect changes to distance reduction procedures.

Knowledge of radii of curvature on ellipsoids is paramount to the reduction of distances measured on the surface of the Earth to either sea level or the ellipsoid surface. Ellipsoid radii are a function of both latitude and azimuth. Fortunately, for many uses any mean radius of curvature is often a satisfactory approximation. Distance reductions are often performed on a sphere having a radius equal to the mean radius of curvature of the ellipsoid at an average latitude of the conterminous United States. Figure 4.1b illustrates the quantities involved in the reduction assuming this sphere. From the proportion:

$$S/D = R/(R+h),$$

we solve: $S = D*R/(R+h)$

$$h = N + H \text{ by definition.}$$

Therefore,

$$S = D*R/(R+N+H).$$

The ratio R/(R+N+H) is similar to the familiar sea level factor except that the average elevation of line "D" above the ellipsoid, usually denoted as "h" and called sea level in most NAD 27 literature, is replaced by "H" as the height above sea level (the geoid) and "N" the ellipsoidal separation. To emphasize the difference, the ratio has been designated as an elevation factor. On NAD 83, "h" remains the height above the ellipsoid, but is obtained by adding together the geoid-ellipsoid separation "N" and the height of the station above the geoid "H". In line reductions by this method, a mean geoid height "N" and mean elevation "H" are used to obtain a mean height of the line "h". Figure 4.1c depicts the situation of a negative N, as is the case in the conterminous United States.

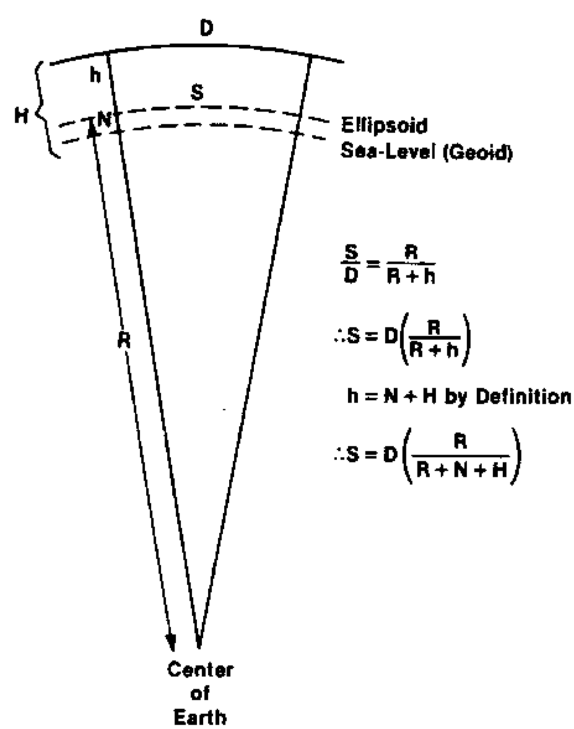


Figure 4.1c.--Reduction to the ellipsoid (shown with a negative geoid height).

The mean radius "R" used in connection with NAD 27 was 20,906,000 ft, or 6,372,000 m. This approximate radius serves equally well for NAD 83.

The elevation factor is often combined with the grid scale factor of a line (see 4.2) to form a single multiplier that reduces an observed horizontal distance at an average elevation directly to the SPCS grid. These two factors are quickly combined by obtaining the product of the factors. This product is approximated by subtracting "1" from the sum of the two factors. Identified as the "combined factor," when multiplied by the horizontal distance it has the same effect as each factor multiplied separately, yielding the grid distance. If the area of a parcel of land at ground elevation is desired, the area obtained from using SPCS 83 coordinates should be divided by the square of the combined factor (the same factor that was used to reduce the measured distances) to obtain the area at ground elevation.

Although the above approximate method serves most surveyors and engineers well, sometimes a more rigorous reduction procedure will be required. Such a procedure is found in NOAA Technical Memorandum NOS NGS-10, Use of calibration base lines, appendix I: "The geometrical transformation of electronically measured distances" (1977).

4.2 Grid scale factor k_{12} of a Line

As discussed in section 2.6 on grid scale factor, an incremental length on the ellipsoid must be multiplied by a grid scale factor to obtain the length of that increment on the grid. However, measured survey lines are not infinitesimal increments, and grid scale factor ratios change from point to point. Therefore, we are faced with the problem of deriving a single grid scale factor that can be applied to an entire measured length (that has first been reduced to the ellipsoid), when in fact the value of the grid scale factor is changing from point to point. Required is a grid scale factor ratio that when multiplied by the measured ellipsoid-reduced distance will yield the grid distance. (See fig. 4.2.) This grid scale factor which applies to a line between points 1 and 2 is symbolized as k_{12}.

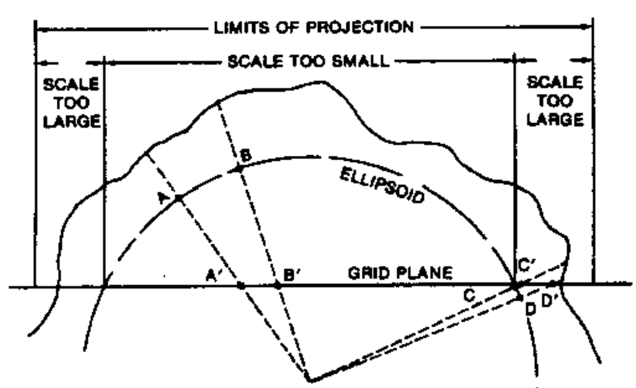


Figure 4.2.--Geodetic vs. grid distances.

Recalling we are computing on a conformal projection, the grid scale factor is the same in any direction, but increases in magnitude with distance of the point from the central meridian in a transverse Mercator projection, or central parallel in a Lambert projection. This means that the grid scale factor is different at each end of a measured line, and that this difference is greatest for an east-west line in the transverse Mercator projection, or north-south line in the Lambert projection. There are several solutions to the problem of deriving a grid scale factor for a line. The solution depends on the required accuracy of the reduction, the lengths of lines involved, and, to a lesser degree, the areal extent of the zone and location of the line within the zone.

The application of the grid scale factor to measured lengths for each project should begin with an analysis of the magnitude of the correction within that project--a function of the average length of measured lines and location of the project within the zone--compared with the desired project accuracy. For surveys of third-order accuracy or less (as classified by the Federal Geodetic Control Committee) a single scale factor for all lines in the project may suffice. This grid scale factor would be computed at the center of the project. It may be determined that in the reduction of measured geodetic lengths to the grid, the grid scale factor could be ignored.

To determine an appropriate method for computing the line grid scale factor for any project, it is suggested that one first determine point grid scale factors for the worst case situation in the project--ends of the longest line that run in a direction perpendicular to the central axis of the projection, and at the greatest distance from the central axis. The central axis is the central meridian in the transverse Mercator or central parallel in the Lambert projection. The appropriateness of approximations for each line or for a project, i.e., a single project grid scale factor, is dependent on the computing error that can be tolerated.

When a single grid scale factor for a project is acceptable, it may also be an acceptable approximation to use a single elevation reduction factor, a similar looking multiplier that reduces a measured horizontal line on the Earth's surface to its equivalent ellipsoid length. (See sec. 4.1.) Sometimes a combined project factor is used to reduce all measured horizontal distances from the average elevation of the project directly to the grid. The appropriateness of a single project elevation reduction factor requires the similar analysis as a project grid scale factor.

When a line grid scale factor must be determined for each measured line of the survey, there are several approaches for handling the fact that point grid scale factors are different at each end of a line. Each approach requires computing one or more point grid scale factors. Approximate equal results are obtained from either using the point scale factor of the midpoint of the line or a mean scale factor computed from the point scale factor for each end of the line. The most accurate determination of the line grid scale factor (k_{12}) requires computing point scale factors at each end of the line (k_1 and k_2) plus the midpoint (k_m) and combining according to:

$$k_{12} = (k_1 + 4\,k_m + k_2)/6.$$

4.3 Arc-to-Chord Correction (t-T)

As given in section 2.5, the representation (projection) of a geodetic azimuth "α" on any plane grid does not produce the grid azimuth "t" but the projected geodetic azimuth "T". Figure 2.5 illustrates the small difference, "t-T". "t-T" (alias "arc-to-chord" and "second difference") is an angular correction to the line of sight between two points, whether that "direction" is an azimuth or part of an angle measure. The "t-T" is the difference between the "pointing" observed on the ellipsoid (generally the same as on the ground) and the pointing on the grid. The difference is often insignificant.

From a purely theoretical perspective, grid azimuths should be used with grid angles and directions, while geodetic azimuths should be used with observed angles and directions. To perform survey computations on a plane, observed directions should be corrected for the arc-to-chord (t-T) correction to derive an equivalent value for the observed direction on the grid; otherwise observed angles should be used with geodetic azimuths with survey computations performed on the ellipsoid. For example, in a traverse computation on the ellipsoid, the azimuth would need to be carried forward by geodetic methods where forward and backward azimuths differ by approximately $\Delta\lambda''\sin\phi_m \pm 180°$.

From a practical perspective in many survey operations the (t-T) correction is negligible, observed angles are used with grid azimuths, and survey computations are done on a plane. In a precise survey it is necessary to evaluate the magnitude of (t-T). Table 4.3a provides an approximation.

Table 4.3a.--Approximate size of (t-T) in seconds of arc for Lambert or transverse Mercator projection (see note 1)

ΔE or ΔN (See note 2) (km)	Perpendicular distance from central axis to midpoint of the line (see note 3) (km)				
	50	100	150	200	250
2	0.3	0.5	0.8	1.0	1.3
5	0.6	1.3	1.9	2.5	3.2
10	1.3	2.5	3.8	5.1	6.4
20	2.5	5.1	7.6	10.2	12.7

(1) (t-T) is also a function of latitude, but often is estimated by $(t-T) = 25.4\,(\Delta N)(\Delta E)10^{-10}$ seconds, where ΔN and ΔE are in meters.

(2) The length of the line to which the correction is to be applied is in a direction parallel to central axis.

(3) A better approximation is obtained by taking the distance from the central axis to a point one-third of the distance from point 1 to point 2 when estimating (t-T) at point 1.

Note on the use of Table 4.3a

* IN A STRAIGHT TRAVERSE OF EQUAL LINE LENGTHS, THE *
* CORRECTION TO AN ANGLE WILL BE DOUBLE THE ABOVE *
* CORRECTION TO EACH DIRECTION, AND THESE ANGULAR *
* CORRECTIONS SYSTEMATICALLY ACCUMULATE ALONG THE *
* TRAVERSE. BECAUSE OF THE DOUBLING ACTION, IF *
* THIS TABLE IS USED TO ESTIMATE THE PORTION OF AZIMUTH *
* MISCLOSURE OF AN ENTIRE TRAVERSE SURVEY ATTRIBUTED *
* TO IGNORING THIS CORRECTION, THE CONTRIBUTION WOULD *
* BE TWICE THE TABLE VALUE. *

Use of the table requires knowledge of two items. First, the approximate perpendicular distance from the central axis (central parallel in Lambert projection or the central meridian in Mercator projection) to the midpoint of your line is required. The midpoint is derived from differencing mean coordinates of the line (northings for Lambert and eastings for Mercator) from the northing of the central parallel or the easting of the central meridian.

Also needed to use table 4.3a is the length of the line in a direction parallel to the central parallel (Lambert) or parallel to the central meridian (Mercator). Again ΔE or ΔN is derived from the point coordinates. From studying table 4.3a it is apparent that the (t-T) correction will be its largest on lines parallel, and the greatest distance, from the central axis of the projection zone.

Table 4.3b.--Sign of (t-T) correction

Map projection		Azimuth of the line from north	
Lambert:	Sign of $N-N_0$	0 to 180°	180 to 360°
	Positive	+	-
	Negative	-	+
		270-90°	90-270°
Transverse Mercator:	Sign of $E-E_0$ (or E_3)		
	Positive	-	+
	Negative	+	-

Figure 4.3 illustrates the relative orientation of projected geodetic lines (T) and grid lines (t) for traverses located on either side of the central axis. It should be observed that the projected geodetic line is always concave towards the central meridian or parallel. This fact provides a visual check on the correct sign of the (t-T) correction. For a conventional nearly straight traverse, the signs of the (t-T) correction on each direction of an observed angle are opposite, thus the corrections accumulate. Therefore, for a straight traverse of

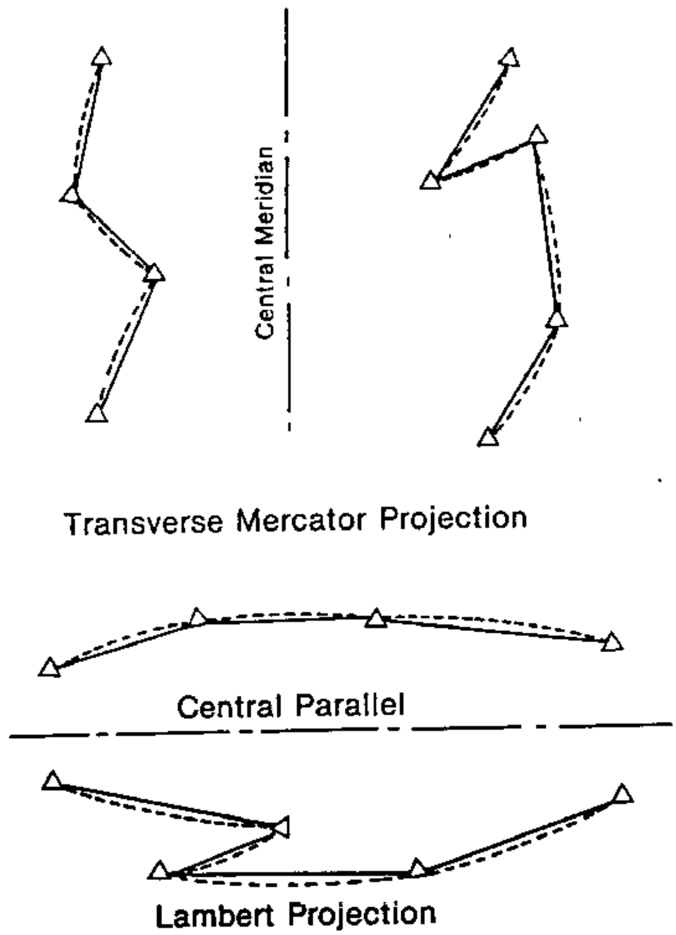


Figure 4.3.--Projected geodetic vs. grid angles.

approximately equal line lengths, the (t-T) correction for the observed angle will be twice the computed correction to a single direction.

Although the formulas in chapter 3 will provide the proper sign of (t-T), table 4.3b may also be used as a guide.

4.4 Traverse Example

This section illustrates the solution to a traverse computation. Given the terrestrial survey observations for a closed connecting traverse between points of known position, final adjusted coordinates are computed for new points. The emphasis in the example is the procedure to reduce angles and distances measured on the surface of the Earth to an equivalent value on the grid of the SPCS 83. These reduction procedures are applicable regardless of the positioning method or network geometry. After the field data are reduced to the SPCS 83 grid, the data are made consistent with the SPCS 83 plane coordinates of the fixed control station using the method of the compass rule adjustment. Mathematical rigor plus data encoding and programming consideration often make adjustment by least squares the preferred method, but the compass rule adjustment is widely used and serves well for this example where computations on a plane are being illustrated. Furthermore, the example in this section was a sample NAD 27 problem used at more than 30 workshops instructed by NGS, so adjusting identical field data to NAD 83 control illustrates datum differences.

Figure 4.4a is a sketch of the sample traverse. For instructional consideration, it purposely violates Federal Geodetic Control Committee specifications for network design. Four new points are to be positioned between two points of known position. The starting azimuth (azimuth at station number 1) is derived from published angles while the closing azimuth (azimuth at station number 6) is derived from published coordinates. The solid lines depict grid ines; dotted lines show projected geodetic lines. The difference is (t-T). The illustrated northing of the central axis together with the approximate point coordinates are used in the computation of (t-T). The computation of (t-T) requires the distance of the traverse line from the central axis. Similarly, a traverse line in a transverse Mercator zone requires E_0, the easting of the central axis.

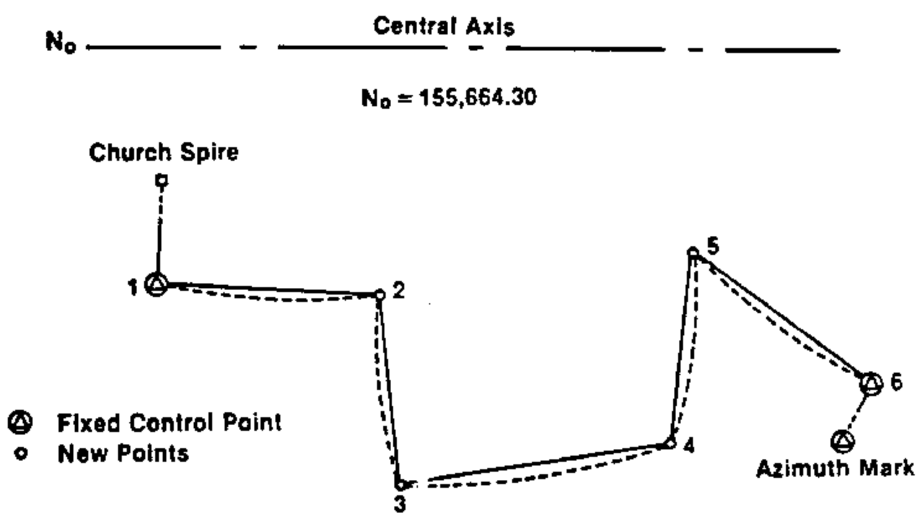
The following steps are required to compute any traverse:

1. Obtain starting and closing azimuth.

2. Analyze the grid scale factor for the project. A mean of the published point grid scale factors of the control points may be adequate for all lines in the project, or a grid scale factor for each line may be required.

3. Analyze the elevation factor for the project. A mean of the published elevations of the control points corrected for the geoid height (N) may be adequate to compute the elevation factor. Otherwise each line may need to be reduced individually.

4. If a project grid scale factor and project elevation factor are applicable, compute a project combined factor.



Approximate Coordinates

Point	N	E
1	61,400 meters	660,300 meters
2	61,300	665,100
3	57,300	665,400
4	58,200	670,300
5	61,800	670,500
6	58,900	674,000

Figure 4.4a.--Sample traverse. (N specifies northing component and E specifies easting component.)

5. Reduce the horizontal distances to the grid.
6. Using preliminary azimuths derived from unreduced angles and grid distances, compute approximate coordinates.
7. Analyze magnitude of (t-T) corrections, and if their application is required, compute the (t-T) corrections for each line using approximate coordinates for each point.
8. Apply (t-T) corrections to the measured angles to obtain grid angles.
9. Adjust the traverse.
10. Compute the final adjusted 1983 State Plane Coordinates for the new points, adjusted azimuths and distances between the points, and if required ground level distances.

Step 1

To obtain the starting azimuth, use the published azimuth and angle information (Fig. 4.4b) and compute the grid azimuth from point 1 to the church spire.

Plane azimuth point 1 to azimuth mark	30°30'12.6"
Plus observed clockwise angle from azimuth mark to church spire	329 50 18.6
Starting azimuth	0 °20'31.2"

Recalling that plane angles are used with plane azimuths and observed spherical angles are used with geodetic azimuths, in theory the above observed angle should have been corrected for the (t-T) correction. However, because each direction of the angle was short, the (t-T) correction is zero.

To obtain the closing azimuth, use the published coordinate information in figure 4.4b and compute the grid azimuth from Point 6 to Point 6 Azimuth Mark using a plane coordinate inverse:

$$\tan \text{azimuth}_{12} = (E_1 - E_2)/(N_1 - N_2)$$

$$\text{azimuth}_{12} = \text{arc tan } (121.457/485.047) = 194°03'28.5"$$

Steps 2 through 4

Grid scale factor point 1	=	1.0000420
Grid scale factor point 6	=	1.0000480
Mean grid scale factor	=	1.0000450

Elevation point 1	=	830.0 ft
Elevation point 6	=	900.0 ft
Mean H	=	865.0 ft
Mean N	=	-30.5 m = -100 ft

$$\begin{aligned}\text{Elevation factor} &= (20{,}906{,}000)/(20{,}906{,}000 + H + N)\\ &= (20{,}906{,}000)/(20{,}906{,}765)\\ &= 0.9999634\end{aligned}$$

NORTH AMERICAN DATUM 1983
ADJUSTED HORIZONTAL CONTROL DATA

NAME OF STATION: Point 1

STATE: Wisconsin YEAR: 1980 Second ORDER

SOURCE: G-17289 Geoid Height = -30.5 meters

	° ′ ″	ELEVATION:
GEODETIC LATITUDE:	42 33 00.01150	830.5 METERS
GEODETIC LONGITUDE:	89 15 56.24590	FEET

STATE COORDINATES				
STATE & ZONE	CODE	Northing	Easting	Mapping angle ° ′ ″
Wisconsin	4803	61,367.006	660,318.626	+ 0 30 16.5

Scale Factor = 1.0000420

TO STATION OR OBJECT	GEODETIC AZIMUTH ° ′ ″	PLANE AZIMUTH* ° ′ ″	CODE
Point 1 Azimuth Mark	31 00 29.1	30 30 12.6	4803

DESCRIPTION OF TRAVERSE STATION

NAME OF STATION: Point 1 STATE: Wisconsin COUNTY: LEO

CHIEF OF PARTY: E.J. McKay YEAR: 1980 DESCRIBED BY: JES

NOTE.* HEIGHT OF TELESCOPE ABOVE STATION MARK METERS.† HEIGHT OF LIGHT ABOVE STATION MARK METERS.

SURFACE-STATION MARK, UNDERGROUND-STATION MARK

DISTANCES AND DIRECTIONS TO AZIMUTH MARK, REFERENCE MARKS AND PROMINENT OBJECTS WHICH CAN BE SEEN FROM THE GROUND AT THE STATION

OBJECT	BEARING	DISTANCE FEET	DISTANCE METERS	DIRECTION† ° ′ ″
Point 6	E		13.886.795	0 00 00.0
RM1	ESE		30.287	33 46 28.1
RM2	WSW		22.984	123 33 29.2
Church Spire (unidentified)	N		(2.3 miles)	260 19 01.5
Azimuth Mark	NNE		(0.2 miles)	290 28 42.9

Figure 4.4b.--Fixed station control information.

NORTH AMERICAN DATUM 1983
ADJUSTED HORIZONTAL CONTROL DATA

NAME OF STATION: Point 6

STATE: Wisconsin YEAR: 1980 Second -ORDER

SOURCE: G-17289

Geoid Height = -30.5 meters

	°	′	″	ELEVATION:
GEODETIC LATITUDE:	42	31	37.32888	METERS
GEODETIC LONGITUDE:	89	05	58.04271	900.0 FEET

1983 STATE COORDINATES (meters)				
STATE & ZONE	CODE	Northing	Easting	Mapping Angle ° ′ ″
Wisconsin, S	4803	58,949.532	673,994.015	+ 0 37 07.5

Scale Factor = 1.0000480

ADJUSTED HORIZONTAL CONTROL DATA

NAME OF STATION: Point 6 Azimuth Mark

STATE: Wisconsin YEAR: 1980 Second -ORDER

SOURCE: G-14402

	°	′	″	ELEVATION:
GEODETIC LATITUDE:	42	31	21.65360	METERS
GEODETIC LONGITUDE:	89	06	03.59289	750.0 FEET

STATE COORDINATES				
STATE & ZONE	CODE	Northing	Easting	Mapping angle ° ′ ″
Wisconsin, S	4803	58,464.485	673,872.558	+ 0 37 03.7

Scale Factor = 1.0000491

Figure 4.4b.--Fixed station control information (continued).

Combined factor = (1.0000450)(0.9999634)
= 1.0000084

Step 5

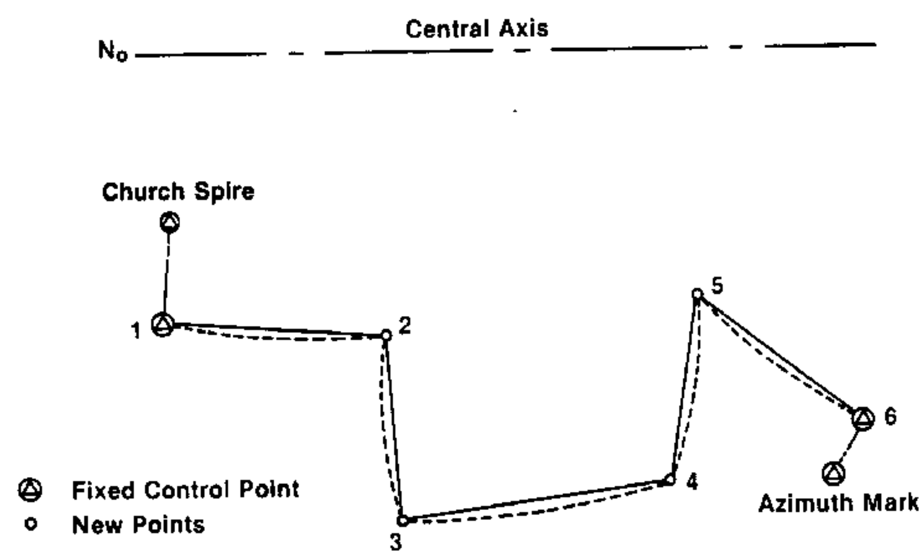
From	To	Measured horizontal lengths	Grid lengths
1	2	4,805.468	4,805.508
2	3	3,963.694	3,963.727
3	4	4,966.083	4,966.125
4	5	3,501.223	3,501.252
5	6	4,466.935	4,466.973

Step 6

The computation of preliminary coordinates is not illustrated. The procedure used to obtain preliminary coordinates for the adjustment is generally used.

Step 7

Figure 4.4c illustrates computation of (t-T) using the abbreviated formula (t-T) = (25.4)(ΔN)(ΔE)(10^{-10}) seconds. The (ΔN) is the distance of the midpoint of the line from the central axis. For example:



From	To	ΔN	ΔE	25.4 (ΔN) (ΔE)
1	2	−0.943	0.048	−1.1
2	3	−0.964	0.003	−0.1
3	4	−0.979	0.049	−1.2
4	5	−0.957	0.002	−0.1
5	6	−0.953	0.035	−0.8

Figure 4.4c.--(t-T) correction (Δ northing and Δ easting expressed in meters).

$$\Delta N_{12} = [(61,400 + 61,300)/2 - 155,664]10^{-5} = -0.943$$

(ΔE) is the difference of eastings of the endpoints of the line. For example:

$$\Delta E_{12} = (E_2 - E_1)10^{-5} = 0.048$$

Note that (ΔN) and (ΔE) are each scaled by (10^{-5}) to account for the (10^{-10}) in the equation for (t-T).

Step 8

The corrections computed in step 7 are applied to an observed pointing in one direction. Using this approximate equation for (t-T), the correction from the other end of the line is identical but with opposite sign. Figure 4.4d lists the observed traverse angles, (t-T) corrections to each direction, angle correction, and the grid angle.

Point	Observed Angle			Backsight Correction	Foresight Correction	Angle Correction	Grid Angle
	°	'	"	"	"	"	"
1	90	44	18.3	0	-1.1	-1.1	17.2
2	265	15	55.2	+1.1	-0.1	-1.2	54.0
3	82	48	26.9	+0.1	-1.2	-1.3	25.6
4	105	03	08.6	+1.2	-0.1	-1.3	07.3
5	304	33	46.2	+0.1	-0.8	-0.9	45.3
6	245	17	39.5	+0.8	0	-0.8	38.7

STATION FROM	STATION TO	PRELIMINARY AZIMUTH °	'	"	CORRECTION FOR CLOSURE "	CORRECTED AZIMUTH °	'	"
1	Azimuth Mk.	00	20	31.2*		00	20	31.2*
∠		90	44	17.2	-1.8	90	44	15.4
1	2	91	04	48.4		91	04	46.6
2	1	271	04	48.4		271	04	46.6
∠		265	15	54.0	-1.8	265	15	52.2
2	3	176	20	42.4		176	20	38.8
3	2	356	20	42.4		356	20	38.8
∠		82	48	25.6	-1.8	82	48	23.8
3	4	79	09	08.0		79	09	02.6
4	3	259	09	08.0		259	09	02.6
∠		105	03	07.3	-1.8	105	03	05.5
4	5	4	12	15.3		4	12	08.1
5	4	184	12	15.3		184	12	08.1
∠		304	33	45.3	-1.8	304	33	43.5
5	6	128	46	00.6		128	45	51.6
6	5	308	46	00.6		308	45	51.6
∠		245	17	38.7	-1.8	245	17	36.9
6	Azimuth Mark	194	03	39.3		194	03	28.5*
		Closure =			-10.8			
∠ *Fixed Grid Azimuths								

Figure 4.4d.--Azimuth adjustment.

STATION	PLANE AZIMUTH ° ′ ″	GRID DISTANCE Meters	SIN AZIMUTH / COS AZIMUTH	LATITUDE Meters	DEPARTURE Meters	GRID COORDINATES Northing	Easting
1					Fixed	61,367.006	660,318.626
			+0.99982248		+4804.655		
	91 04 46.6	4805.508				61,276.462	665,123.281 (1)
			-0.01884165	-90.544		- 0.223	+ 0.232 (2)
2						61,276.239	665,123.513 (3)
			+0.06376401		+252.743		
	176 20 38.8	3963.727				57,320.801	665,376.024
			-0.99796501	-3955.661		- 0.407	+ 0.423
3						57,320.394	665,376.447
			+0.98212573		+4877.359		
	79 09 02.6	4966.125				58,255.555	670,253.383
			+0.18822607	+934.754		- 0.637	+ 0.662
4						58,254.918	670,254.045
			+0.07327736		+256.563		
	4 12 08.1	3501.252				61,747.394	670,509.946
			+0.99731160	+3491.839		- 0.799	+ 0.831
5						61,746.595	670,510.777
			+0.77972788		+3483.023		
	128 45 51.6	4466.973				58,950.539	673,992.969
			-0.62611855	-2796.855		- 1.007	+ 1.046
6					Fixed	58,949.532	673,994.015

(1) Preliminary Coordinate
(2) Accumulative Correction
(3) Adjusted Coordinate

Figure 4.4e.--Traverse computation by latitudes and departures.

FINAL ADJUSTED RESULTS

FROM	TO	LATITUDE	DEPARTURE	GRID LENGTH	TAN (OR COT*)	GROUND LENGTH	AZIMUTH
1	2	- 90.767	+4804.887	4805.744	-0.01889056*	4805.704	91 04 56.0
2	3	-3955.845	+ 252.934	3963.923	-0.06393931	3963.890	176 20 29.5
3	4	+ 934.524	+4877.598	4966.316	+0.19159513*	4966.274	79 09 13.9
4	5	+3491.677	+ 256.732	3501.103	+0.07352685	3501.074	4 12 18.8
5	6	-2797.063	+3483.238	4467.271	-0.80300657*	4467.233	128 45 52.9

Figure 4.4f.--Adjusted traverse data.

Step 9

Using the starting grid azimuth and grid angles, the closing azimuth is computed. (See fig. 4.4d.) The misclosure of (-10.8) seconds is prorated among the grid angles and final corrected azimuths computed. The adjusted azimuths and grid distances are transferred to figure 4.4e where the coordinate misclosures are determined and misclosures prorated according to the compass rule adjustment method.

Step 10

Plane coordinate inverses between adjusted coordinates provide adjusted grid azimuths and distances. If ground level distances are required, the adjusted grid distance is divided by the combined factor that was previously used to reduce the observed distances. Figure 4.4f shows the adjusted data.

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APPENDIX A.--DEFINING CONSTANTS FOR THE 1983 STATE PLANE COORDINATE SYSTEM

Transverse Mercator (T.M.), Oblique Mercator (O.M.), and Lambert (L.) Projections

State/Zone/Code			Projection	Central Meridian and Scale Factor (T.M.) or Standard Parallels (L.)	Grid Origin Longitude Latitude	Easting Northing (meters)
Alabama	AL					
East	E	0101	T.M.	85 50 1:25,000	85 50 30 30	200,000. 0
West	W	0102	T.M.	87 30 1:15,000	87 30 30 00	600,000. 0
Alaska	AK					
Zone 1		5001	O.M.	Axis Azimuth = arc tan -3/4 1:10,000	133 40 57 00	5,000,000. -5,000,000.
Zone 2		5002	T.M.	142 00 1:10,000	142 00 54 00	500,000. 0
Zone 3		5003	T.M.	146 00 1:10,000	146 00 54 00	500,000. 0
Zone 4		5004	T.M.	150 00 1:10,000	150 00 54 00	500,000. 0
Zone 5		5005	T.M.	154 00 1:10,000	154 00 54 00	500,000. 0
Zone 6		5006	T.M.	158 00 1:10,000	158 00 54 00	500,000. 0
Zone 7		5007	T.M.	162 00 1:10,000	162 00 54 00	500,000. 0
Zone 8		5008	T.M.	166 00 1:10,000	166 00 54 00	500,000. 0
Zone 9		5009	T.M.	170 00 1:10,000	170 00 54 00	500,000. 0
Zone 10		5010	L	51 50 53 50	176 00 51 00	1,000,000. 0

State/Zone/Code			Projection	Central Meridian and Scale Factor (T.M.) or Standard Parallels (L.)	Grid Origin Longitude Latitude	Grid Origin Easting Northing
Arizona	AZ					
East	E	0201	T.M.	110 10 1:10,000	110 10 31 00	213,360. 0
Central	C	0202	T.M.	111 55 1:10,000	111 55 31 00	213,360. 0
West	W	0203	T.M.	113 45 1:15,000	113 45 31 00	213,360. 0

(State law defines the origin in International Feet)
(213,360M. = 700,000 International Feet)

State/Zone/Code			Projection	Central Meridian and Scale Factor (T.M.) or Standard Parallels (L.)	Grid Origin Longitude Latitude	Grid Origin Easting Northing
Arkansas	AR					
North	N	0301	L	34 56 36 14	92 00 34 20	400,000. 0
South	S	0302	L	33 18 34 46	92 00 32 40	400,000. 400,000.
California	CA					
Zone 1		0401	L	40 00 41 40	122 00 39 20	2,000,000. 500,000.
Zone 2		0402	L	38 20 39 50	122 00 37 40	2,000,000. 500,000.
Zone 3		0403	L	37 04 38 26	120 30 36 30	2,000,000. 500,000.
Zone 4		0404	L	36 00 37 15	119 00 35 20	2,000,000. 500,000.
Zone 5		0405	L	34 02 35 28	118 00 33 30	2,000,000. 500,000.
Zone 6		0406	L	32 47 33 53	116 15 32 10	2,000,000. 500,000.
Colorado	CO					
North	N	0501	L	39 43 40 47	105 30 39 20	914,401.8289 304,800.6096
Central	C	0502	L	38 27 39 45	105 30 37 50	914,401.8289 304 800.6096
South	S	0503	L	37 14 38 26	105 30 36 40	914 401.8289 304 800.6096

State/Zone/Code			Projection	Central Meridian and Scale Factor (T.M.) or Standard Parallels (L.)	Grid Origin Longitude Latitude	Grid Origin Easting Northing
Connecticut	CT					
		0600	L	41 12 41 52	72 45 40 50	304,800.6096 152,400.3048
Delaware	DE					
		0700	T.M.	75 25 1:200,000	75 25 38 00	200,000. 0
Florida	FL					
East	E	0901	T.M.	81 00 1:17,000	81 00 24 20	200,000. 0
West	W	0902	T.M.	82 00 1:17,000	82 00 24 20	200,000. 0
North	N	0903	L	29 35 30 45	84 30 29 00	600,000. 0
Georgia	GA					
East	E	1001	T.M.	82 10 1:10,000	82 10 30 00	200,000. 0
West	W	1002	T.M.	84 10 1:10,000	84 10 30 00	700,000. 0
Hawaii	HI					
Zone 1		5101	T.M.	155 30 1:30,000	155 30 18 50	500,000. 0
Zone 2		5102	T.M.	156 40 1:30,000	156 40 20 20	500,000. 0
Zone 3		5103	T.M.	158 00 1:100,000	158 00 21 10	500,000. 0
Zone 4		5104	T.M.	159 30 1:100,000	159 30 21 50	500,000. 0
Zone 5		5105	T.M.	160 10 0	160 10 21 40	500,000. 0

State/Zone/Code			Projection	Central Meridian and Scale Factor (T.M.) or Standard Parallels (L.)	Grid Origin Longitude Latitude	Easting Northing
Idaho	ID					
East	E	1101	T.M.	112 10 1:19,000	112 10 41 40	200,000. 0
Central	C	1102	T.M.	114 00 1:19,000	114 00 41 40	500,000. 0
West	W	1103	T.M.	115 45 1:15,000	115 45 41 40	800,000. 0
Illinois	IL					
East	E	1201	T.M.	88 20 1:40,000	88 20 36 40	300,000. 0
West	W	1202	T.M.	90 10 1:17,000	90 10 36 40	700,000. 0
Indiana	IN					
East	E	1301	T.M.	85 40 1:30,000	85 40 37 30	100,000. 250,000.
West	W	1302	T.M.	87 05 1:30,000	87 05 37 30	900,000. 250,000.
Iowa	IA					
North	N	1401	L	42 04 43 16	93 30 41 30	1,500,000. 1,000,000.
South	S	1402	L	40 37 41 47	93 30 40 00	500,000. 0
Kansas	KS					
North	N	1501	L	38 43 39 47	98 00 38 20	400,000. 0
South	S	1502	L	37 16 38 34	98 30 36 40	400,000. 400,000.
Kentucky	KY					
North	N	1601	L	37 58 38 58	84 15 37 30	500,000. 0
South	S	1602	L	36 44 37 56	85 45 36 20	500,000. 500,000.

State/Zone/Code			Projection	Central Meridian and Scale Factor (T.M.) or Standard Parallels (L.)	Grid Origin Longitude Latitude	Easting Northing
Louisiana	LA					
North	N	1701	L	31 10 32 40	92 30 30 30*	1,000,000. 0
South	S	1702	L	29 18 30 42	91 20 28 30*	1,000,000. 0
Offshore	SH	1703	L	26 10 27 50	91 20 25 30*	1,000,000. 0
Maine	ME					
East	E	1801	T.M.	68 30 1:10,000	68 30 43 40*	300,000. 0
West	W	1802	T.M.	70 10 1:30,000	70 10 42 50	900,000. 0
Maryland	MD					
		1900	L	38 18 39 27	77 00 37 40*	400,000. 0
Massachusetts	MA					
Mainland	M	2001	L	41 43 42 41	71 30 41 00	200,000. 750,000.
Island	I	2002	L	41 17 41 29	70 30 41 00	500,000. 0
Michigan	MI					
North	N	2111	L	45 29 47 05	87 00 44 47	8,000,000. 0
Central	C	2112	L	44 11 45 42	84 22* 43 19	6,000,000. 0
South	S	2113	L	42 06 43 40	84 22* 41 30	4,000,000. 0
Minnesota	MN					
North	N	2201	L	47 02 48 38	93 06 46 30	800,000. 100,000.
Central	C	2202	L	45 37 47 03	94 15 45 00	800,000. 100,000.
South	S	2203	L	43 47 45 13	94 00 43 00	800,000. 100,000.

State/Zone/Code			Projection	Central Meridian and Scale Factor (T.M.) or Standard Parallels (L.)	Grid Origin Longitude Latitude	Easting Northing
Mississippi	MS					
East	E	2301	T.M.	88 50 1:20,000*	88 50 29 30*	300,000. 0
West	W	2302	T.M.	90 20 1:20,000*	90 20 29 30*	700,000. 0
Missouri	MO					
East	E	2401	T.M.	90 30 1:15,000	90 30 35 50	250,000. 0
Central	C	2402	T.M.	92 30 1:15,000	92 30 35 50	500,000. 0
West	W	2403	T.M.	94 30 1:17,000	94 30 36 10	850,000. 0
Montana	MT	2500	L	45 00* 49 00*	109 30 44 15*	600,000. 0
Nebraska	NE	2600	L	40 00* 43 00*	100 00* 39 50*	500,000. 0
Nevada	NV					
East	E	2701	T.M.	115 35 1:10,000	115 35 34 45	200,000. 8,000,000.
Central	C	2702	T.M.	116 40 1:10,000	116 40 34 45	500,000. 6,000,000.
West	W	2703	T.M.	118 35 1:10,000	118 35 34 45	800,000. 4,000,000.
New Hampshire	NH	2800	T.M.	71 40 1:30,000	71 40 42 30	300,000. 0
New Jersey (New York East)	NJ	2900	T.M.	74 30* 1:10,000*	74 30* 38 50	150,000. 0

State/Zone/Code			Projection	Central Meridian and Scale Factor (T.M.) or Standard Parallels (L.)	Grid Origin Longitude Latitude	Easting Northing
New Mexico	NM					
East	E	3001	T.M.	104 20 1:11,000	104 20 31 00	165,000. 0
Central	C	3002	T.M.	106 15 1:10,000	106 15 31 00	500,000. 0
West	W	3003	T.M.	107 50 1:12,000	107 50 31 00	830,000. 0
New York	NY					
East (New Jersey)	E	3101	T.M.	74 30* 1:10,000*	74 30* 38 50*	150,000. 0
Central	C	3102	T.M.	76 35 1:16,000	76 35 40 00	250,000. 0
West	W	3103	T.M.	78 35 1:16,000	78 35 40 00	350,000. 0
Long Island	L	3104	L	40 40 41 02	74 00 40 10*	300,000. 0
North Carolina	NC					
		3200	L	34 20 36 10	79 00 33 45	609,601.22 0
North Dakota	ND					
North	N	3301	L	47 26 48 44	100 30 47 00	600,000. 0
South	S	3302	L	46 11 47 29	100 30 45 40	600,000. 0
Ohio	OH					
North	N	3401	L	40 26 41 42	82 30 39 40	600,000. 0
South	S	3402	L	38 44 40 02	82 30 38 00	600,000. 0
Oklahoma	OK					
North	N	3501	L	35 34 36 46	98 00 35 00	600,000. 0
South	S	3502	L	33 56 35 14	98 00 33 20	600,000. 0

State/Zone/Code			Projection	Central Meridian and Scale Factor (T.M.) or Standard Parallels (L.)	Grid Origin Longitude Latitude	Easting Northing
Oregon	OR					
North	N	3601	L	44 20 46 00	120 30 43 40	2,500,000. 0
South	S	3602	L	42 20 44 00	120 30 41 40	1,500,000. 0
Pennsylvania	PA					
North	N	3701	L	40 53 41 57	77 45 40 10	600,000. 0
South	S	3702	L	39 56 40 58	77 45 39 20	600,000. 0
Rhode Island	RI					
		3800	T.M.	71 30 1:160,000	71 30 41 05	100,000. 0
South Carolina	SC					
		3900	L	32 30* 34 50*	81 00* 31 50*	609,600. 0
South Dakota	SD					
North	N	4001	L	44 25 45 41	100 00 43 50	600,000. 0
South	S	4002	L	42 50 44 24	100 20 42 20	600,000. 0
Tennessee	TN					
		4100	L	35 15 36 25	86 00 34 20*	600,000. 0

State/Zone/Code			Projection	Central Meridian and Scale Factor (T.M.) or Standard Parallels (L.)	Grid Origin Longitude Latitude	Grid Origin Easting Northing
Texas	TX					
North	N	4201	L	34 39 36 11	101 30 34 00	200,000. 1,000,000.
North Central	NC	4202	L	32 08 33 58	98 30* 31 40	600,000. 2,000,000.
Central	C	4203	L	30 07 31 53	100 20 29 40	700,000. 3,000,000.
South Central	SC	4204	L	28 23 30 17	99 00 27 50	600,000. 4,000,000.
South	S	4205	L	26 10 27 50	98 30 25 40	300,000. 5,000,000.
Utah	UT					
North	N	4301	L	40 43 41 47	111 30 40 20	500,000. 1,000,000.
Central	C	4302	L	39 01 40 39	111 30 38 20	500,000. 2,000,000.
South	S	4303	L	37 13 38 21	111 30 36 40	500,000. 3,000,000.
Vermont	VT	4400	T.M.	72 30 1:28,000	72 30 42 30	500,000. 0
Virginia	VA					
North	N	4501	L	38 02 39 12	78 30 37 40	3,500,000. 2,000,000.
South	S	4502	L	36 46 37 58	78 30 36 20	3,500,000. 1,000,000.
Washington	WA					
North	N	4601	L	47 30 48 44	120 50 47 00	500,000. 0
South	S	4602	L	45 50 47 20	120 30 45 20	500,000. 0

State/Zone/Code			Projection	Central Meridian and Scale Factor (T.M.) or Standard Parallels (L.)	Grid Origin Longitude Latitude	Grid Origin Easting Northing
West Virginia	WV					
North	N	4701	L	39 00 40 15	79 30 38 30	600,000. 0
South	S	4702	L	37 29 38 53	81 00 37 00	600,000. 0
Wisconsin	WI					
North	N	4801	L	45 34 46 46	90 00 45 10	600,000. 0
Central	C	4802	L	44 15 45 30	90 00 43 50	600,000. 0
South	S	4803	L	42 44 44 04	90 00 42 00	600,000. 0
Wyoming	WY					
East	E	4901	T.M.	105 10 1:16,000*	105 10 40 30*	200,000. 0
East Central	EC	4902	T.M.	107 20 1:16,000*	107 20 40 30*	400,000. 100,000
West Central	WC	4903	T.M.	108 45 1:16,000*	108 45 40 30*	600,000. 0
West	W	4904	T.M.	110 05 1:16,000*	110 05 40 30*	800,000. 100,000.
Puerto Rico and Virgin Islands	PR	5200	L	18 02 18 26	66 26 17 50	200,000. 200,000.

* This represents a change from the defining constant used for the 1927 State Plane Coordinate System. All metric values assigned to the origins also are changes.

APPENDIX B. MODEL ACT FOR STATE PLANE COORDINATE SYSTEMS

An act to describe, define, and officially adopt a system of coordinates for designating the geographic position of points on the surface of the Earth within the State of ....

BE IT ENACTED BY THE LEGISLATURE OF THE STATE OF ....

Section 1. The systems of plane coordinates which have been established by the National Ocean Service/National Geodetic Survey (formerly the United States Coast and Geodetic Survey) or its successors for defining and stating the geographic positions or locations of points on the surface of the Earth within the State of .... are hereafter to be known and designated as the .... (name of State) Coordinate System of 1927 and the .... (name of State) Coordinate System of 1983.

For the purpose of the use of these systems, the State is divided into a ..... Zone and a ..... Zone (or as many zone identifications as now defined by the National Ocean Service.

The area now included in the following counties shall constitute the ..... Zone: (here enumerate the name of the counties included).

The area now included ..... (likewise for all zones).

Section 2. As established for use in the ..... Zone, the .... (name of State) Coordinate System of 1927 or the .... (name of State) Coordinate System of 1983 shall be named; and in any land description in which it is used, it shall be designated the "......... (name of State) Coordinate System 1927 ..... Zone" or .... (name of State) Coordinate System of 1983 ..... Zone."

As established for use ..... (likewise for all zones).

Section 3. The plane coordinate values for a point on the Earth's surface, used to express the geographic position or location of such point in the appropriate zone of this system, shall consist of two distances expressed in U.S. Survey Feet and decimals of a foot when using the .... (name of State) Coordinate System of 1927 and expressed in meters and decimals of a meter when using the .... (name of State) Coordinate System of 1983. For SPCS 27, one of these distances, to be known as the "x-coordinate," shall give the position in an east-and-west direction; the other, to be known as the "y-coordinate," shall give the position in a north-and-south direction. For SPCS 83, one of the distances, to be known as the "northing" or "N", shall give the position in a north-and-south direction; the other, to be known as the "easting" or "E" shall give the position in an east-and-west direction. These coordinates shall be made to depend upon and conform to plane rectangular coordinate values for the monumented points of the North American National Geodetic Horizontal Network as published by the National Ocean Service/National Geodetic Survey (formerly the United States Coast and Geodetic Survey), or its successors, and whose plane coordinates have been computed on the systems defined in this chapter. Any such station may be used for establishing a survey connection to either .... (name of State) Coordinate System.

Section 4. For purposes of describing the location of any survey station or land boundary corner in the State of ...., it shall be considered a complete, legal, and satisfactory description of such location to give the position of said survey station or land boundary corner on the system of plane coordinates defined in this act.

Nothing contained in this act shall require a purchaser or mortgagee of real property to rely wholly on a land description, any part of which depends exclusively upon either .... (name of State) coordinate system.

Section 5. When any tract of land to be defined by a single description extends from one into the other of the above coordinate zones, the position of all points on its boundaries may be referred to either of the two zones, the zone which is used being specifically named in the description.

Section 6. (a) For purposes of more precisely defining the .... (name of State) Coordinate System of 1927, the following definition by the United States Coast and Geodetic Survey (now National Ocean Service/National Geodetic Survey) is adopted:

(For Lambert zones)

The ".......... (name of State) Coordinate System of 1927 ..... (Zone ID) Zone," is a Lambert conformal conic projection of the Clarke spheroid of 1866, having standard parallels at north latitudes ..... degrees ..... minutes and ..... degrees ..... minutes along which parallels the scale shall be exact. The origin of coordinates is at the intersection of the meridian ..... degrees ..... minutes west of Greenwich and the parallel ..... degrees ..... minutes north latitude. This origin is given the coordinates: x = ... feet and y = ... feet (as now defined).

(Use similar paragraphs for other Lambert zones on the 1927 Datum.)

(For transverse Mercator zones)

The ".......... (name of State) Coordinate System of 1927 ..... (Zone ID) zone," is a transverse Mercator projection of the Clarke spheroid of 1866, having a central meridian ..... degrees ..... minutes west of Greenwich, on which meridian the scale is set one part in ..... too small. The origin of coordinates is at the intersection of the meridian ..... degrees ..... minutes west of Greenwich and the parallel ..... degrees ..... minutes north latitude. This origin is given the coordinates: x = ... feet and y = ... feet (as now defined).

(Use similar paragraphs for other transverse Mercator zones on the 1927 Datum).

(b) For purposes of more precisely defining the .... (name of State) Coordinate System of 1983, the following definition by the National Ocean Service/National Geodetic Survey is adopted:

(For Lambert zones)

The ".......... (name of State) Coordinate System of 1983 ..... (Zone ID) Zone" is a Lambert conformal conic projection of the North American Datum of 1983, having standard parallels at north latitudes ..... degrees ..... minutes and

..... degrees ..... minutes along which parallels the scale shall be exact. The origin of coordinates is at the intersection of the meridian ..... degrees ..... minutes west of Greenwich and the parallel ..... degrees ..... minutes north latitude. This origin is given the coordinates: N = ... meters and E = ... meters.

(Use similar paragraphs for other Lambert zones on the 1983 Datum).

(For transverse Mercator zones)

The ".......... (name of State) Coordinate System of 1983 ..... (Zone ID) Zone," is a transverse Mercator projection of the North American Datum of 1983, having a central meridian ..... degrees ..... minutes west of Greenwich, on which meridian the scale is set one part in ..... too small. The origin of coordinates is at the intersection of the meridian ..... degrees ..... minutes west of Greenwich and the parallel ..... degrees ..... minutes north latitude. This origin is given the coordinates: N = ... meters and E = ... meters.

(Use similar paragraphs for other transverse Mercator zones on the 1983 Datum.)

Section 7. No coordinates based on either .... (name of State) coordinate system, purporting to define the position of a point on a land boundary, shall be presented to be recorded in any public land records or deed records unless such point is within 1 kilometer of a monumented horizontal control station established in conformity with the standards of accuracy and specifications for first- or second-order geodetic surveying as prepared and published by the Federal Geodetic Control Committee (FGCC) of the United States Department of Commerce. Standards and specifications of the FGCC or its successor in force on date of said survey shall apply. Publishing existing control stations, or the acceptance with intent to publish the newly established stations, by the Natinal Ocean Service/National Geodetic Survey will constitute evidence of adherence to the FGCC specifications. Above limitations may be modified by a duly authorized State agency to meet local conditions.

Section 8. The use of the term ".......... (name of State) Coordinate System of 1927 .... Zone" or ".......... (name of State) Coordinate System of 1983 .... Zone" on any map, report of survey, or other document shall be limited to coordinates based on the .... (name of State) coordinate system as defined in this act.

Section 9. If any provision of this act shall be declared invalid, such invalidity shall not affect any other portion of this act which can be given effect without the invalid provision; and to this end, the provisions of this act are declared severable.

Section 10. The .... (name of State) Coordinate System of 1927 shall not be used after .... (date); the .... (name of State) Coordinate System of 1983 will be the sole system after this date.

(Note: This model act was prepared in 1977. In light of GPS technology, the 1 kilometer limitation of Section 7 should be reevaluated.)

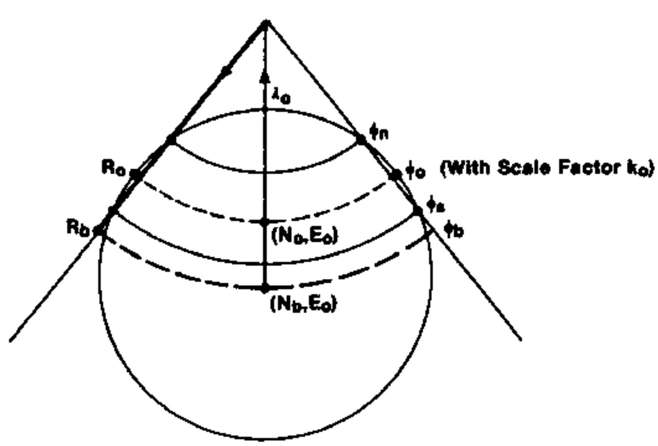
APPENDIX C.--CONSTANTS FOR THE LAMBERT PROJECTION BY THE POLYNOMIAL COEFFICIENT METHOD

Constants	Description
Bs =	Southern standard parallel
Bn =	Northern standard parallel
Bb =	Latitude of grid origin
Lo =	Longitude of the true and grid origin, the "central meridian"
Nb =	Northing value at grid orgin "Bb"
Eo =	Easting value at the origin "Lo"
Bo =	Latitude of the true projection origin, the "central parallel"
SinBo=	Sine of Bo
Rb =	Mapping radius at Bb
Ro =	Mapping radius at Bo
K =	Mapping radius at the equator
No =	Northing value at the true projection origin "Bo"
ko =	Central parallel grid scale factor
Mo =	Scaled radius of curvature in the meridian at "Bo"
ro =	Geometric mean radius of curvature at Bo scaled to the grid

Bs, Bn, Bb, and Lo in degrees: minutes
Bo in decimal degrees
Linear units in meters

(See page 44 for equivalent notation of defining and derived constants used in the figure below.)

PARAMETERS OF A LAMBERT PROJECTION



AK 10 ALASKA 10

ZONE # 5010

Defining Constants

Bs	=	51:50
Bn	=	53:50
Bb	=	51:00
Lo	=	176:00
Nb	=	0.0000
Eo	=	1000000.0000

Computed Constants

Bo	=	52.8372090915
SinBo	=	0.796922389486
Rb	=	5048740.3829
Ro	=	4844318.3515
No	=	204422.0314
K	=	11499355.8664
ko	=	0.999848059991
Mo	=	6375089.0366
ro	=	6382923.

Coefficients for GP to PC

L(1)	=	111266.2938
L(2)	=	9.42762
L(3)	=	5.60521
L(4)	=	0.032566
L(5)	=	0.0008745

Coefficients for PC to GP

G(1)	=	8.987447734E-06
G(2)	=	-6.84405E-15
G(3)	=	-3.65605E-20
G(4)	=	-1.7676E-27
G(5)	=	-9.143E-36

Coefficients for Grid Scale Factor

F(1)	=	0.999848059991
F(2)	=	1.22755E-14
F(3)	=	8.34E-22

AR N ARKANSAS NORTH ZONE # 0301

Defining Constants

Bs	=	34:56
Bn	=	36:14
Bb	=	34:20
Lo	=	92:00
Nb	=	0.0000
Eo	=	400000.0000

Computed Constants

Bo	=	35.5842283444
SinBo	=	0.581899128039
Rb	=	9062395.1981
Ro	=	8923619.0696
No	=	138776.1285
K	=	13112784.4998
ko	=	0.999935935348
Mo	=	6356634.6561
ro	=	6370786.

Coefficients for GP to PC

L(1)	=	110944.2037
L(2)	=	9.22246
L(3)	=	5.64616
L(4)	=	0.017597

Coefficients for PC to GP

G(1)	=	9.013539789E-06
G(2)	=	-6.75356E-15
G(3)	=	-3.72463E-20
G(4)	=	-9.0676E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999935935348
F(2)	=	1.23195E-14
F(3)	=	4.52E-22

AR S ARKANSAS SOUTH ZONE # 0302

Defining Constants

Bs	=	33:18
Bn	=	34:46
Bb	=	32:40
Lo	=	92:00
Nb	=	400000.0000
Eo	=	400000.0000

Computed Constants

Bo	=	34.0344093756
SinBo	=	0.559690686832
Rb	=	9604584.2290
Ro	=	9452884.9686
No	=	551699.2604
K	=	13438989.7695
ko	=	0.999918469533
Mo	=	6354902.0291
ro	=	6369591.

Coefficients for GP to PC

L(1)	=	110913.9635
L(2)	=	9.03498
L(3)	=	5.64949
L(4)	=	0.016534

Coefficients for PC to GP

G(1)	=	9.015997271E-06
G(2)	=	-6.62159E-15
G(3)	=	-3.73079E-20
G(4)	=	-8.5429E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999918469533
F(2)	=	1.23240E-14
F(3)	=	4.26E-22

CA 01 CALIFORNIA 1

ZONE # 0401

Defining Constants

Bs	=	40:00
Bn	=	41:40
Bb	=	39:20
Lo	=	122:00
Nb	=	500000.0000
Eo	=	2000000.0000

Computed Constants

Bo	=	40.8351061249
SinBo	=	0.653884305400
Rb	=	7556554.6408
Ro	=	7389802.0597
No	=	666752.5811
K	=	12287826.3052
ko	=	0.999894636561
Mo	=	6362067.2798
ro	=	6374328.

Coefficients for GP to PC

L(1)	=	111039.0203
L(2)	=	9.65524
L(3)	=	5.63491
L(4)	=	0.021275

Coefficients for PC to GP

G(1)	=	9.005843038E-06
G(2)	=	-7.05240E-15
G(3)	=	-3.70393E-20
G(4)	=	-1.1142E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999894636561
F(2)	=	1.23062E-14
F(3)	=	5.47E-22

CA 02 CALIFORNIA 2

ZONE # 0402

Defining Constants

Bs	=	38:20
Bn	=	39:50
Bb	=	37:40
Lo	=	122:00
Nb	=	500000.0000
Eo	=	2000000.0000

Computed Constants

Bo	=	39.0846839219
SinBo	=	0.630468335285
Rb	=	8019788.9307
Ro	=	7862381.4027
No	=	657407.5280
K	=	12520351.6538
ko	=	0.999914672977
Mo	=	6360268.3937
ro	=	6373169.

Coefficients for GP to PC

L(1)	=	111007.6240
L(2)	=	9.54628
L(3)	=	5.63874
L(4)	=	0.019988

Coefficients for PC to GP

G(1)	=	9.008390180E-06
G(2)	=	-6.97872E-15
G(3)	=	-3.71084E-20
G(4)	=	-1.0411E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999914672977
F(2)	=	1.23106E-14
F(3)	=	5.14E-22

CA 03 CALIFORNIA 3 ZONE # 0403

Defining Constants

Bs	=	37:04
Bn	=	38:26
Bb	=	36:30
Lo	=	120:30
Nb	=	500000.0000
Eo	=	2000000.0000

Computed Constants

Bo	=	37.7510694363
SinBo	=	0.612232038295
Rb	=	8385775.1723
Ro	=	8246930.3684
No	=	638844.8039
K	=	12724574.9735
ko	=	0.999929178853
Mo	=	6358909.6841
ro	=	6372292.

Coefficients for GP to PC

L(1)	=	110983.9104
L(2)	=	9.43943
L(3)	=	5.64142
L(4)	=	0.019048

Coefficients for PC to GP

G(1)	=	9.010315015E-06
G(2)	=	-6.90503E-15
G(3)	=	-3.71614E-20
G(4)	=	-9.8819E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999929178853
F(2)	=	1.23137E-14
F(3)	=	4.89E-22

CA 04 CALIFORNIA 4 ZONE # 0404

Defining Constants

Bs	=	36:00
Bn	=	37:15
Bb	=	35:20
Lo	=	119:00
Nb	=	500000.0000
Eo	=	2000000.0000

Computed Constants

Bo	=	36.6258593071
SinBo	=	0.596587149880
Rb	=	8733227.3793
Ro	=	8589806.8935
No	=	643420.4858
K	=	12916986.0281
ko	=	0.999940761703
Mo	=	6357772.8978
ro	=	6371557.

Coefficients for GP to PC

L(1)	=	110964.0696
L(2)	=	9.33334
L(3)	=	5.64410
L(4)	=	0.018382

Coefficients for PC to GP

G(1)	=	9.011926076E-06
G(2)	=	-6.83121E-15
G(3)	=	-3.72043E-20
G(4)	=	-9.4223E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999940761703
F(2)	=	1.23168E-14
F(3)	=	4.70E-22

CA 05 CALIFORNIA 5

ZONE # 0405

Defining Constants

Bs	=	34:02
Bn	=	35:28
Bb	=	33:30
Lo	=	118:00
Nb	=	500000.0000
Eo	=	2000000.0000

Computed Constants

Bo	=	34.7510553142
SinBo	=	0.570011896174
Rb	=	9341756.1389
Ro	=	9202983.1099
No	=	638773.0290
K	=	13282624.8345
ko	=	0.999922127209
Mo	=	6355670.9697
ro	=	6370113.

Coefficients for GP to PC

L(1)	=	110927.3840
L(2)	=	9.12439
L(3)	=	5.64805
L(4)	=	0.017445

Coefficients for PC to GP

G(1)	=	9.014906468E-06
G(2)	=	-6.68534E-15
G(3)	=	-3.72796E-20
G(4)	=	-8.6394E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999922127209
F(2)	=	1.23221E-14
F(3)	=	4.41E-22

CA 06 CALIFORNIA 6

ZONE # 0406

Defining Constants

Bs	=	32:47
Bn	=	33:53
Bb	=	32:10
Lo	=	116:15
Nb	=	500000.0000
Eo	=	2000000.0000

Computed Constants

Bo	=	33.3339229447
SinBo	=	0.549517575763
Rb	=	9836091.7896
Ro	=	9706640.0762
No	=	629451.7134
K	=	13602026.7133
ko	=	0.999954142490
Mo	=	6354407.2007
ro	=	6369336.

Coefficients for GP to PC

L(1)	=	110905.3274
L(2)	=	8.94188
L(3)	=	5.65087
L(4)	=	0.016171

Coefficients for PC to GP

G(1)	=	9.016699372E-06
G(2)	=	-6.55499E-15
G(3)	=	-3.73318E-20
G(4)	=	-8.2753E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999954142490
F(2)	=	1.23251E-14
F(3)	=	4.15E-22

CO N COLORADO NORTH ZONE # 0501

Defining Constants

```
Bs    =        39:43
Bn    =        40:47
Bb    =        39:20
Lo    =       105:30
Nb    =   304800.6096
Eo    =   914401.8289
```

Computed Constants

```
Bo    =  40.2507114537
SinBo= 0.646133456811
Rb    =   7646051.6244
Ro    =   7544194.6172
No    =    406657.6168
K     =  12361909.8309
ko    = 0.999956846063
Mo    =   6361817.5470
ro    =    6374293.
```

Coefficients for GP to PC

```
L(1)  = 111034.6624
L(2)  =       9.62324
L(3)  =       5.63555
L(4)  =       0.021040
```

Coefficients for PC to GP

```
G(1)   =  9.006196586E-06
G(2)   = -7.02998E-15
G(3)   = -3.70588E-20
G(4)   = -1.0841E-27
```

Coefficients for Grid Scale Factor

```
F(1)  = 0.999956846063
F(2)  = 1.23060E-14
F(3)  = 5.37E-22
```

CO C COLORADO CENTRAL ZONE # 0502

Defining Constants

```
Bs    =        38:27
Bn    =        39:45
Bb    =        37:50
Lo    =       105:30
Nb    =   304800.6096
Eo    =   914401.8289
```

Computed Constants

```
Bo    =  39.1010150117
SinBo= 0.630689555225
Rb    =   7998699.7391
Ro    =   7857977.9317
No    =    445522.4170
K     =  12518269.8410
ko    = 0.999935909777
Mo    =   6360421.3434
ro    =    6373316.
```

Coefficients for GP to PC

```
L(1)  = 111010.2938
L(2)  =       9.54770
L(3)  =       5.63848
L(4)  =       0.019957
```

Coefficients for PC to GP

```
G(1)   =  9.008173565E-06
G(2)   = -6.97922E-15
G(3)   = -3.71064E-20
G(4)   = -1.0428E-27
```

Coefficients for Grid Scale Factor

```
F(1)  = 0.999935909777
F(2)  = 1.23099E-14
F(3)  = 5.14E-22
```

CO S COLORADO SOUTH ZONE # 0503

Defining Constants

Bs	=	37:14
Bn	=	38:26
Bb	=	36:40
Lo	=	105:30
Nb	=	304800.6096
Eo	=	914401.8289

Computed Constants

Bo	=	37.8341602703
SinBo	=	0.613378042371
Rb	=	8352015.4059
Ro	=	8222442.4013
No	=	434373.6143
K	=	12711335.3256
ko	=	0.999945398499
Mo	=	6359102.7444
ro	=	6372455.

Coefficients for GP to PC

L(1)	=	110987.2800
L(2)	=	9.44685
L(3)	=	5.64118
L(4)	=	0.019105

Coefficients for PC to GP

G(1)	=	9.010041469E-06
G(2)	=	-6.90983E-15
G(3)	=	-3.71567E-20
G(4)	=	-9.9134E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999945398499
F(2)	=	1.23131E-14
F(3)	=	4.91E-22

CT CONNECTICUT ZONE # 0600

Defining Constants

```
Bs    =        41:12
Bn    =        41:52
Bb    =        40:50
Lo    =        72:45
Nb    =    152400.3048
Eo    =    304800.6096
```

Computed Constants

```
Bo    =  41.5336239347
SinBo= 0.663059457532
Rb    =   7288924.5189
Ro    =   7211151.4122
No    =    230173.4115
K     =  12206545.8602
ko    = 0.999983140478
Mo    =   6363404.7042
ro    =    6375409.
```

Coefficients for GP to PC

```
L(1)  = 111062.3637
L(2)  =      9.68962
L(3)  =      5.63247
L(4)  =      0.021924
```

Coefficients for PC to GP

```
G(1)   =  9.003950270E-06
G(2)   = -7.07309E-15
G(3)   = -3.70044E-20
G(4)   = -1.1414E-27
```

Coefficients for Grid Scale Factor

```
F(1)  = 0.999983140478
F(2)  = 1.23017E-14
F(3)  = 5.61E-22
```

FL N FLORIDA NORTH ZONE # 0903

Defining Constants

Bs	=	29:35
Bn	=	30:45
Bb	=	29:00
Lo	=	84:30
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	30.1672535540
SinBo	=	0.502525902671
Rb	=	11111265.2070
Ro	=	10981878.2256
No	=	129386.9814
K	=	14473086.8984
ko	=	0.999948432740
Mo	=	6351211.3497
ro	=	6367189.

Coefficients for GP to PC

L(1)	=	110849.5492
L(2)	=	8.45478
L(3)	=	5.65723
L(4)	=	0.014285

Coefficients for PC to GP

G(1)	=	9.021236462E-06
G(2)	=	-6.20727E-15
G(3)	=	-3.74501E-20
G(4)	=	-7.2421E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999948432740
F(2)	=	1.23332E-14
F(3)	=	3.67E-22

IA N IOWA NORTH — ZONE # 1401

Defining Constants

Bs	=	42:04
Bn	=	43:16
Bb	=	41:30
Lo	=	93:30
Nb	=	1000000.0000
Eo	=	1500000.0000

Computed Constants

Bo	=	42.6676459541
SinBo	=	0.677744566795
Rb	=	7059740.0263
Ro	=	6930042.0331
No	=	1129697.9931
K	=	12083972.0985
ko	=	0.999945367870
Mo	=	6364426.3661
ro	=	6376011.

Coefficients for GP to PC

L(1)	=	111080.1947
L(2)	=	9.73155
L(3)	=	5.63034
L(4)	=	0.022691

Coefficients for PC to GP

G(1)	=	9.002504885E-06
G(2)	=	-7.10023E-15
G(3)	=	-3.69607E-20
G(4)	=	-1.1953E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999945367870
F(2)	=	1.22997E-14
F(3)	=	5.83E-22

IA S IOWA SOUTH — ZONE # 1402

Defining Constants

Bs	=	40:37
Bn	=	41:47
Bb	=	40:00
Lo	=	93:30
Nb	=	0.0000
Eo	=	500000.0000

Computed Constants

Bo	=	41.2008797613
SinBo	=	0.658701013169
Rb	=	7429044.5139
Ro	=	7295688.5838
No	=	133355.9301
K	=	12244655.5752
ko	=	0.999948369709
Mo	=	6362814.2760
ro	=	6374941.

Coefficients for GP to PC

L(1)	=	111052.0582
L(2)	=	9.67367
L(3)	=	5.63393
L(4)	=	0.021895

Coefficients for PC to GP

G(1)	=	9.004785763E-06
G(2)	=	-7.06375E-15
G(3)	=	-3.70197E-20
G(4)	=	-1.1221E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999948369709
F(2)	=	1.23041E-14
F(3)	=	5.56E-22

KS N KANSAS NORTH ZONE # 1501

Defining Constants

```
Bs    =        38:43
Bn    =        39:47
Bb    =        38:20
Lo    =        98:00
Nb    =         0.0000
Eo    =    400000.0000
```

Computed Constants

```
Bo    =  39.2506869474
SinBo= 0.632714613092
Rb    =   7918239.4709
Ro    =   7816402.7262
No    =    101836.7447
K     =  12497179.1821
ko    = 0.999956851054
Mo    =   6360718.3963
ro    =    6373559.
```

Coefficients for GP to PC

```
L(1)  = 111015.4786
L(2)  =      9.55844
L(3)  =      5.63780
L(4)  =      0.020306
```

Coefficients for PC to GP

```
G(1)   =  9.007752883E-06
G(2)   = -6.98626E-15
G(3)   = -3.70994E-20
G(4)   = -1.0424E-27
```

Coefficients for Grid Scale Factor

```
F(1)  = 0.999956851054
F(2)  = 1.23088E-14
F(3)  = 5.18E-22
```

KS S KANSAS SOUTH ZONE # 1502

Defining Constants

```
Bs    =        37:16
Bn    =        38:34
Bb    =        36:40
Lo    =        98:30
Nb    =    400000.0000
Eo    =    400000.0000
```

Computed Constants

```
Bo    =  37.9176400609
SinBo= 0.614528111936
Rb    =   8336559.0467
Ro    =   8197720.0530
No    =    538838.9936
K     =  12697806.8013
ko    = 0.999935918480
Mo    =   6359132.8597
ro    =    6372455.
```

Coefficients for GP to PC

```
L(1)  = 110987.8057
L(2)  =      9.45414
L(3)  =      5.64091
L(4)  =      0.018964
```

Coefficients for PC to GP

```
G(1)   =  9.009998800E-06
G(2)   = -6.91489E-15
G(3)   = -3.71545E-20
G(4)   = -1.0003E-27
```

Coefficients for Grid Scale Factor

```
F(1)  = 0.999935918480
F(2)  = 1.23130E-14
F(3)  = 4.91E-22
```

KY N KENTUCKY NORTH ZONE # 1601

Defining Constants

Bs	=	37:58
Bn	=	38:58
Bb	=	37:30
Lo	=	84:15
Nb	=	0.0000
Eo	=	500000.0000

Computed Constants

Bo	=	38.4672539691
SinBo	=	0.622067254038
Rb	=	8145306.4712
Ro	=	8037943.9917
No	=	107362.4795
K	=	12612341.7840
ko	=	0.999962079530
Mo	=	6359896.1212
ro	=	6373021.

Coefficients for GP to PC

L(1)	=	111001.1272
L(2)	=	9.49969
L(3)	=	5.63960
L(4)	=	0.019624

Coefficients for PC to GP

G(1)	=	9.008917501E-06
G(2)	=	-6.94594E-15
G(3)	=	-3.71303E-20
G(4)	=	-1.0140E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999962079530
F(2)	=	1.23109E-14
F(3)	=	5.03E-22

KY S KENTUCKY SOUTH ZONE # 1602

Defining Constants

Bs	=	36:44
Bn	=	37:56
Bb	=	36:20
Lo	=	85:45
Nb	=	500000.0000
Eo	=	500000.0000

Computed Constants

Bo	=	37.3341456532
SinBo	=	0.606462358287
Rb	=	8483079.4552
Ro	=	8372015.2303
No	=	611064.2249
K	=	12793783.0812
ko	=	0.999945401603
Mo	=	6358562.7562
ro	=	6372094.

Coefficients for GP to PC

L(1)	=	110977.8556
L(2)	=	9.40195
L(3)	=	5.64201
L(4)	=	0.018759

Coefficients for PC to GP

G(1)	=	9.010806634E-06
G(2)	=	-6.87874E-15
G(3)	=	-3.71775E-20
G(4)	=	-9.7208E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999945401603
F(2)	=	1.23142E-14
F(3)	=	4.82E-22

LA N LOUISIANA NORTH ZONE # 1701

Defining Constants

Bs	=	31:10
Bn	=	32:40
Bb	=	30:30
Lo	=	92:30
Nb	=	0.0000
Eo	=	1000000.0000

Computed Constants

Bo	=	31.9177055892
SinBo	=	0.528700659421
Rb	=	10405759.0459
Ro	=	10248571.1515
No	=	157187.8944
K	=	13961752.4737
ko	=	0.999914740906
Mo	=	6352722.0540
ro	=	6368127.

Coefficients for GP to PC

L(1)	=	110875.9156
L(2)	=	8.73673
L(3)	=	5.65399
L(4)	=	0.015313

Coefficients for PC to GP

G(1)	=	9.019091156E-06
G(2)	=	-6.40970E-15
G(3)	=	-3.73877E-20
G(4)	=	-7.8031E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999914740906
F(2)	=	1.23296E-14
F(3)	=	3.93E-22

LA S LOUISIANA SOUTH ZONE # 1702

Defining Constants

Bs	=	29:18
Bn	=	30:42
Bb	=	28:30
Lo	=	91:20
Nb	=	0.0000
Eo	=	1000000.0000

Computed Constants

Bo	=	30.0008395428
SinBo	=	0.500012689631
Rb	=	11221678.1079
Ro	=	11055318.6368
No	=	166359.4711
K	=	14525497.0844
ko	=	0.999925744553
Mo	=	6350906.2899
ro	=	6366937.

Coefficients for GP to PC

L(1)	=	110844.2246
L(2)	=	8.42633
L(3)	=	5.65782
L(4)	=	0.014018

Coefficients for PC to GP

G(1)	=	9.021669771E-06
G(2)	=	-6.18701E-15
G(3)	=	-3.74568E-20
G(4)	=	-7.2616E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999925744553
F(2)	=	1.23343E-14
F(3)	=	3.64E-22

LA SH LOUISIANA OFFSHORE ZONE # 1703

Defining Constants

Bs	=	26:10
Bn	=	27:50
Bb	=	25:30
Lo	=	91:20
Nb	=	0.0000
Eo	=	1000000.0000

Computed Constants

Bo	=	27.0010512832
SinBo	=	0.454006848165
Rb	=	12690863.7281
Ro	=	12524558.0674
No	=	166305.6607
K	=	15621596.5270
ko	=	0.999894794114
Mo	=	6347907.1071
ro	=	6364866.

Coefficients for GP to PC

L(1)	=	110791.8786
L(2)	=	7.86506
L(3)	=	5.66365
L(4)	=	0.012775

Coefficients for PC to GP

G(1)	=	9.025932193E-06
G(2)	=	-5.78388E-15
G(3)	=	-3.75631E-20
G(4)	=	-6.1764E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999894794114
F(2)	=	1.23421E-14
F(3)	=	3.23E-22

MD MARYLAND ZONE # 1900

Defining Constants

Bs	=	38:18
Bn	=	39:27
Bb	=	37:40
Lo	=	77:00
Nb	=	0.0000
Eo	=	400000.0000

Computed Constants

Bo	=	38.8757880051
SinBo	=	0.627634132356
Rb	=	8055622.7373
Ro	=	7921405.1556
No	=	134217.5816
K	=	12551136.6396
ko	=	0.999949847842
Mo	=	6360263.7936
ro	=	6373240.

Coefficients for GP to PC

L(1)	=	111007.5442
L(2)	=	9.53130
L(3)	=	5.63889
L(4)	=	0.019736

Coefficients for PC to GP

G(1)	=	9.008396710E-06
G(2)	=	-6.96769E-15
G(3)	=	-3.71144E-20
G(4)	=	-1.0352E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999949847842
F(2)	=	1.23102E-14
F(3)	=	5.09E-22

MA M MASS MAINLAND

ZONE # 2001

Defining Constants

Bs	=	41:43
Bn	=	42:41
Bb	=	41:00
Lo	=	71:30
Nb	=	750000.0000
Eo	=	200000.0000

Computed Constants

Bo	=	42.2006252872
SinBo	=	0.671728673921
Rb	=	7177701.7404
Ro	=	7044348.7021
No	=	883353.0384
K	=	12132804.7336
ko	=	0.999964550086
Mo	=	6364028.0516
ro	=	6375786.

Coefficients for GP to PC

L(1)	=	111073.2431
L(2)	=	9.71650
L(3)	=	5.63098
L(4)	=	0.021759

Coefficients for PC to GP

G(1)	=	9.003068344E-06
G(2)	=	-7.09026E-15
G(3)	=	-3.69789E-20
G(4)	=	-1.1855E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999964550086
F(2)	=	1.23003E-14
F(3)	=	5.69E-22

MA I MASS ISLAND

ZONE # 2002

Defining Constants

Bs	=	41:17
Bn	=	41:29
Bb	=	41:00
Lo	=	70:30
Nb	=	0.0000
Eo	=	500000.0000

Computed Constants

Bo	=	41.3833593510
SinBo	=	0.661093979591
Rb	=	7291990.4498
Ro	=	7249415.2230
No	=	42575.2267
K	=	12223979.6222
ko	=	0.999998482670
Mo	=	6363335.5426
ro	=	6375395.

Coefficients for GP to PC

L(1)	=	111061.1569
L(2)	=	9.68480
L(3)	=	5.62745

Coefficients for PC to GP

G(1)	=	9.004048113E-06
G(2)	=	-7.06961E-15
G(3)	=	-3.69799E-20

Coefficients for Grid Scale Factor

F(1)	=	0.999998482670
F(2)	=	1.23015E-14
F(3)	=	5.56E-22

MI N MICHIGAN NORTH ZONE # 2111

Defining Constants

Bs	=	45:29
Bn	=	47:05
Bb	=	44:47
Lo	=	87:00
Nb	=	0.0000
Eo	=	8000000.0000

Computed Constants

Bo	=	46.2853056176
SinBo	=	0.722789934733
Rb	=	6275243.8434
Ro	=	6108308.6036
No	=	166935.2398
K	=	11779843.7720
ko	=	0.999902834466
Mo	=	6368201.9117
ro	=	6378442.

Coefficients for GP to PC

L(1)	=	111146.0908
L(2)	=	9.76397
L(3)	=	5.62053
L(4)	=	0.025777
L(5)	=	0.0007325

Coefficients for PC to GP

G(1)	=	8.997167538E-06
G(2)	=	-7.11123E-15
G(3)	=	-3.68190E-20
G(4)	=	-1.3725E-27
G(5)	=	8.019E-35

Coefficients for Grid Scale Factor

F(1)	=	0.999902834466
F(2)	=	1.22919E-14
F(3)	=	6.70E-22

MI C MICHIGAN CENTRAL ZONE # 2112

Defining Constants

Bs	=	44:11
Bn	=	45:42
Bb	=	43:19
Lo	=	84:22
Nb	=	0.0000
Eo	=	6000000.0000

Computed Constants

Bo	=	44.9433587575
SinBo	=	0.706407406862
Rb	=	6581660.2321
Ro	=	6400902.4399
No	=	180757.7922
K	=	11878338.0174
ko	=	0.999912706253
Mo	=	6366762.5687
ro	=	6377502.

Coefficients for GP to PC

L(1)	=	111120.9691
L(2)	=	9.77091
L(3)	=	5.62494
L(4)	=	0.023788

Coefficients for PC to GP

G(1)	=	8.999201531E-06
G(2)	=	-7.12032E-15
G(3)	=	-3.68711E-20
G(4)	=	-1.3161E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999912706253
F(2)	=	1.22939E-14
F(3)	=	6.25E-22

MI S MICHIGAN SOUTH ZONE # 2113

Defining Constants

Bs	=	42:06
Bn	=	43:40
Bb	=	41:30
Lo	=	84:22
Nb	=	0.0000
Eo	=	4000000.0000

Computed Constants

Bo	=	42.8850151357
SinBo	=	0.680529259912
Rb	=	7031167.2907
Ro	=	6877323.4058
No	=	153843.8848
K	=	12061671.8385
ko	=	0.999906878420
Mo	=	6364423.8607
ro	=	6375928.

Coefficients for GP to PC

L(1)	=	111080.1507
L(2)	=	9.73761
L(3)	=	5.63002
L(4)	=	0.022802

Coefficients for PC to GP

G(1)	=	9.002508421E-06
G(2)	=	-7.10459E-15
G(3)	=	-3.69552E-20
G(4)	=	-1.2067E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999906878420
F(2)	=	1.23000E-14
F(3)	=	5.87E-22

MN N MINNESOTA NORTH ZONE # 2201

Defining Constants

Bs	=	47:02
Bn	=	48:38
Bb	=	46:30
Lo	=	93:06
Nb	=	100000.0000
Eo	=	800000.0000

Computed Constants

Bo	=	47.8354141053
SinBo	=	0.741219640371
Rb	=	5934713.4739
Ro	=	5786251.1143
No	=	248462.3596
K	=	11685145.4281
ko	=	0.999902816593
Mo	=	6369933.6096
ro	=	6379598.

Coefficients for GP to PC

L(1)	=	111176.3136
L(2)	=	9.72967
L(3)	=	5.61897
L(4)	=	0.027729

Coefficients for PC to GP

G(1)	=	8.994721600E-06
G(2)	=	-7.08107E-15
G(3)	=	-3.67535E-20
G(4)	=	-1.4515E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999902816593
F(2)	=	1.22867E-14
F(3)	=	7.04E-22

MN C MINNESOTA CENTRAL ZONE # 2202

Defining Constants

Bs	=	45:37
Bn	=	47:03
Bb	=	45:00
Lo	=	94:15
Nb	=	100000.0000
Eo	=	800000.0000

Computed Constants

Bo	=	46.3349188114
SinBo	=	0.723388068681
Rb	=	6246233.9437
Ro	=	6097862.9029
No	=	248371.0408
K	=	11776732.4900
ko	=	0.999922022624
Mo	=	6368379.6277
ro	=	6378602.

Coefficients for GP to PC

L(1)	=	111149.1920
L(2)	=	9.76378
L(3)	=	5.62196
L(4)	=	0.025568

Coefficients for PC to GP

G(1)	=	8.996916454E-06
G(2)	=	-7.11028E-15
G(3)	=	-3.68130E-20
G(4)	=	-1.3780E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999922022624
F(2)	=	1.22899E-14
F(3)	=	6.61E-22

MN S MINNESOTA SOUTH ZONE # 2203

Defining Constants

Bs	=	43:47
Bn	=	45:13
Bb	=	43:00
Lo	=	94:00
Nb	=	100000.0000
Eo	=	800000.0000

Computed Constants

Bo	=	44.5014884140
SinBo	=	0.700927792688
Rb	=	6667126.8494
Ro	=	6500294.5043
No	=	266832.3451
K	=	11914387.7514
ko	=	0.999922039553
Mo	=	6366327.3480
ro	=	6377231.

Coefficients for GP to PC

L(1)	=	111113.3724
L(2)	=	9.76742
L(3)	=	5.62679
L(4)	=	0.024208

Coefficients for PC to GP

G(1)	=	8.999816728E-06
G(2)	=	-7.12002E-15
G(3)	=	-3.68868E-20
G(4)	=	-1.2821E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999922039553
F(2)	=	1.22957E-14
F(3)	=	6.22E-22

MT MONTANA ZONE # 2500

Defining Constants

Bs	=	45:00
Bn	=	49:00
Bb	=	44:15
Lo	=	109:30
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	47.0126454240
SinBo	=	0.731504203765
Rb	=	6259119.5655
Ro	=	5952137.2048
No	=	306982.3608
K	=	11726990.9793
ko	=	0.999392636277
Mo	=	6365765.4708
ro	=	6375730.

Coefficients for GP to PC

L(1)	=	111103.5668
L(2)	=	9.74667
L(3)	=	5.61611
L(4)	=	0.026479
L(5)	=	0.0007162

Coefficients for PC to GP

G(1)	=	9.000611125E-06
G(2)	=	-7.10687E-15
G(3)	=	-3.68456E-20
G(4)	=	-1.4141E-27
G(5)	=	7.257E-35

Coefficients for Grid Scale Factor

F(1)	=	0.999392636277
F(2)	=	1.23001E-14
F(3)	=	6.75E-22

NE NEBRASKA ZONE # 2600

Defining Constants

Bs	=	40:00
Bn	=	43:00
Bb	=	39:50
Lo	=	100:00
Nb	=	0.0000
Eo	=	500000.0000

Computed Constants

Bo	=	41.5058803333
SinBo	=	0.662696910933
Rb	=	7401530.8340
Ro	=	7215835.9104
No	=	185694.9237
K	=	12205748.1618
ko	=	0.999658595062
Mo	=	6361308.6623
ro	=	6373319.

Coefficients for GP to PC

L(1)	=	111025.7809
L(2)	=	9.68528
L(3)	=	5.63025
L(4)	=	0.021792
L(5)	=	0.0006372

Coefficients for PC to GP

G(1)	=	9.006917060E-06
G(2)	=	-7.07688E-15
G(3)	=	-3.70427E-20
G(4)	=	-1.1443E-27
G(5)	=	1.251E-34

Coefficients for Grid Scale Factor

F(1)	=	0.999658595062
F(2)	=	1.23079E-14
F(3)	=	5.62E-22

NY L NEW YORK LONG ISLAND ZONE # 3104

Defining Constants

```
Bs    =        40:40
Bn    =        41:02
Bb    =        40:10
Lo    =        74:00
Nb    =       0.0000
Eo    =  300000.0000
```

Computed Constants

```
Bo    =  40.8500858421
SinBo= 0.654082091204
Rb    =   7462536.3011
Ro    =   7386645.0143
No    =     75891.2868
K     =  12287232.6151
ko    = 0.999994900400
Mo    =   6362721.8083
ro    =     6374978.
```

Coefficients for GP to PC

```
L(1)  = 111050.4466
L(2)  =       9.66003
L(3)  =       5.62096
```

Coefficients for PC to GP

```
G(1)   =  9.004916524E-06
G(2)   = -7.05345E-15
G(3)   = -3.69553E-20
```

Coefficients for Grid Scale Factor

```
F(1)  = 0.999994900400
F(2)  = 1.23032E-14
F(3)  = 5.44E-22
```

NC NORTH CAROLINA ZONE # 3200

Defining Constants

Bs	=	34:20
Bn	=	36:10
Bb	=	33:45
Lo	=	79:00
Nb	=	0.0000
Eo	=	609601.2199

Computed Constants

Bo	=	35.2517586002
SinBo	=	0.577170255241
Rb	=	9199785.5932
Ro	=	9033195.6010
No	=	166589.9922
K	=	13178320.6222
ko	=	0.999872591882
Mo	=	6355881.3611
ro	=	6370148.

Coefficients for GP to PC

L(1)	=	110931.0558
L(2)	=	9.18403
L(3)	=	5.64691
L(4)	=	0.017289

Coefficients for PC to GP

G(1)	=	9.014608051E-06
G(2)	=	-6.72767E-15
G(3)	=	-3.72650E-20
G(4)	=	-8.9805E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999872591882
F(2)	=	1.23215E-14
F(3)	=	4.46E-22

ND N NORTH DAKOTA NORTH ZONE # 3301

Defining Constants

Bs	=	47:26
Bn	=	48:44
Bb	=	47:00
Lo	=	100:30
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	48.0847188415
SinBo	=	0.744133404458
Rb	=	5856720.4592
Ro	=	5736120.4804
No	=	120599.9788
K	=	11672088.5605
ko	=	0.999935842096
Mo	=	6370421.8763
ro	=	6379995.

Coefficients for GP to PC

L(1)	=	111184.8361
L(2)	=	9.72243
L(3)	=	5.61786
L(4)	=	0.027700

Coefficients for PC to GP

G(1)	=	8.994032200E-06
G(2)	=	-7.07375E-15
G(3)	=	-3.67405E-20
G(4)	=	-1.4677E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999935842096
F(2)	=	1.22846E-14
F(3)	=	7.08E-22

ND S NORTH DAKOTA SOUTH ZONE # 3302

Defining Constants

Bs	=	46:11
Bn	=	47:29
Bb	=	45:40
Lo	=	100:30
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	46.8346602257
SinBo	=	0.729382600558
Rb	=	6122339.5950
Ro	=	5992509.2670
No	=	129830.3280
K	=	11744429.2917
ko	=	0.999935851558
Mo	=	6369026.6161
ro	=	6379063.

Coefficients for GP to PC

L(1)	=	111160.4842
L(2)	=	9.75568
L(3)	=	5.62076
L(4)	=	0.026264

Coefficients for PC to GP

G(1)	=	8.996002517E-06
G(2)	=	-7.10242E-15
G(3)	=	-3.67913E-20
G(4)	=	-1.4014E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999935851558
F(2)	=	1.22881E-14
F(3)	=	6.75E-22

OH N OHIO NORTH — ZONE # 3401

Defining Constants

Bs	=	40:26
Bn	=	41:42
Bb	=	39:40
Lo	=	82:30
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	41.0676989228
SinBo	=	0.656950312341
Rb	=	7485451.5983
Ro	=	7329872.6916
No	=	155578.9068
K	=	12260321.3670
ko	=	0.999939140422
Mo	=	6362607.9595
ro	=	6374783.

Coefficients for GP to PC

L(1)	=	111048.4575
L(2)	=	9.66786
L(3)	=	5.63397
L(4)	=	0.021060

Coefficients for PC to GP

G(1)	=	9.005077760E-06
G(2)	=	-7.05943E-15
G(3)	=	-3.70266E-20
G(4)	=	-1.1329E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999939140422
F(2)	=	1.23043E-14
F(3)	=	5.48E-22

OH S OHIO SOUTH — ZONE # 3402

Defining Constants

Bs	=	38:44
Bn	=	40:02
Bb	=	38:00
Lo	=	82:30
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	39.3843585118
SinBo	=	0.634519536788
Rb	=	7932869.0374
Ro	=	7779186.9467
No	=	153682.0906
K	=	12478096.2534
ko	=	0.999935907680
Mo	=	6360731.6589
ro	=	6373523.

Coefficients for GP to PC

L(1)	=	111015.7097
L(2)	=	9.56783
L(3)	=	5.63800
L(4)	=	0.020081

Coefficients for PC to GP

G(1)	=	9.007734087E-06
G(2)	=	-6.99281E-15
G(3)	=	-3.70945E-20
G(4)	=	-1.0564E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999935907680
F(2)	=	1.23093E-14
F(3)	=	5.18E-22

OK N OKLAHOMA NORTH ZONE # 3501

Defining Constants

Bs	=	35:34
Bn	=	36:46
Bb	=	35:00
Lo	=	98:00
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	36.1674456022
SinBo	=	0.590147072450
Rb	=	8864259.9258
Ro	=	8734728.4814
No	=	129531.4444
K	=	13001040.2487
ko	=	0.999945408786
Mo	=	6357313.3855
ro	=	6371260.

Coefficients for GP to PC

L(1)	=	110956.0498
L(2)	=	9.28617
L(3)	=	5.64482
L(4)	=	0.017979

Coefficients for PC to GP

G(1)	=	9.012577476E-06
G(2)	=	-6.79804E-15
G(3)	=	-3.72229E-20
G(4)	=	-9.2812E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999945408786
F(2)	=	1.23177E-14
F(3)	=	4.62E-22

OK S OKLAHOMA SOUTH ZONE # 3502

Defining Constants

Bs	=	33:56
Bn	=	35:14
Bb	=	33:20
Lo	=	98:00
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	34.5841961094
SinBo	=	0.567616677812
Rb	=	9399243.5141
Ro	=	9260493.6985
No	=	138749.8157
K	=	13318364.4294
ko	=	0.999935942436
Mo	=	6355584.5004
ro	=	6370084.

Coefficients for GP to PC

L(1)	=	110925.8751
L(2)	=	9.10472
L(3)	=	5.64812
L(4)	=	0.016766

Coefficients for PC to GP

G(1)	=	9.015029132E-06
G(2)	=	-6.67047E-15
G(3)	=	-3.72859E-20
G(4)	=	-8.7733E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999935942436
F(2)	=	1.23220E-14
F(3)	=	4.34E-22

OR N OREGON NORTH ZONE # 3601

Defining Constants

Bs	=	44:20
Bn	=	46:00
Bb	=	43:40
Lo	=	120:30
Nb	=	0.0000
Eo	=	2500000.0000

Computed Constants

Bo	=	45.1687259619
SinBo	=	0.709186016884
Rb	=	6517624.6963
Ro	=	6350713.9300
No	=	166910.7663
K	=	11860484.1452
ko	=	0.999894582577
Mo	=	6366899.4862
ro	=	6377555.

Coefficients for GP to PC

L(1)	=	111123.3583
L(2)	=	9.77067
L(3)	=	5.62487
L(4)	=	0.024544

Coefficients for PC to GP

G(1)	=	8.999007999E-06
G(2)	=	-7.12020E-15
G(3)	=	-3.68630E-20
G(4)	=	-1.3188E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999894582577
F(2)	=	1.22939E-14
F(3)	=	6.35E-22

OR S OREGON SOUTH ZONE # 3602

Defining Constants

Bs	=	42:20
Bn	=	44:00
Bb	=	41:40
Lo	=	120:30
Nb	=	0.0000
Eo	=	1500000.0000

Computed Constants

Bo	=	43.1685887665
SinBo	=	0.684147361010
Rb	=	6976289.2382
Ro	=	6809452.2816
No	=	166836.9566
K	=	12033772.6984
ko	=	0.999894607592
Mo	=	6364662.2994
ro	=	6376061.

Coefficients for GP to PC

L(1)	=	111084.3129
L(2)	=	9.74486
L(3)	=	5.62774
L(4)	=	0.023107
L(5)	=	0.0006671

Coefficients for PC to GP

G(1)	=	9.002171179E-06
G(2)	=	-7.10916E-15
G(3)	=	-3.69482E-20
G(4)	=	-1.2185E-27
G(5)	=	1.111E-34

Coefficients for Grid Scale Factor

F(1)	=	0.999894607592
F(2)	=	1.23002E-14
F(3)	=	5.97E-22

PA N PENNSYLVANIA NORTH ZONE # 3701

Defining Constants

Bs	=	40:53
Bn	=	41:57
Bb	=	40:10
Lo	=	77:45
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	41.4174076242
SinBo	=	0.661539733811
Rb	=	7379348.3668
Ro	=	7240448.7701
No	=	138899.5967
K	=	12219540.4665
ko	=	0.999956840202
Mo	=	6363108.3386
ro	=	6375155.

Coefficients for GP to PC

L(1)	=	111057.1908
L(2)	=	9.68441
L(3)	=	5.63320
L(4)	=	0.021500

Coefficients for PC to GP

G(1)	=	9.004369625E-06
G(2)	=	-7.07004E-15
G(3)	=	-3.70106E-20
G(4)	=	-1.1439E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999956840202
F(2)	=	1.23030E-14
F(3)	=	5.56E-22

PA S PENNSYLVANIA SOUTH ZONE # 3702

Defining Constants

Bs	=	39:56
Bn	=	40:58
Bb	=	39:20
Lo	=	77:45
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	40.4506723597
SinBo	=	0.648793151619
Rb	=	7615193.7581
Ro	=	7491129.9649
No	=	124063.7931
K	=	12336392.1867
ko	=	0.999959500101
Mo	=	6362055.0747
ro	=	6374457.

Coefficients for GP to PC

L(1)	=	111038.8080
L(2)	=	9.63502
L(3)	=	5.63528
L(4)	=	0.020898

Coefficients for PC to GP

G(1)	=	9.005860337E-06
G(2)	=	-7.03760E-15
G(3)	=	-3.70500E-20
G(4)	=	-1.0995E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999959500101
F(2)	=	1.23055E-14
F(3)	=	5.38E-22

SC SOUTH CAROLINA ZONE # 3900

Defining Constants

Bs	=	32:30
Bn	=	34:50
Bb	=	31:50
Lo	=	81:00
Nb	=	0.0000
Eo	=	609600.0000

Computed Constants

Bo	=	33.6693534716
SinBo	=	0.554399350127
Rb	=	9786198.7935
Ro	=	9582591.5259
No	=	203607.2676
K	=	13520786.8598
ko	=	0.999793656965
Mo	=	6353731.8876
ro	=	6368544.

Coefficients for GP to PC

L(1)	=	110893.5412
L(2)	=	8.98578
L(3)	=	5.64832
L(4)	=	0.016390
L(5)	=	0.0005454

Coefficients for PC to GP

G(1)	=	9.017657737E-06
G(2)	=	-6.58928E-15
G(3)	=	-3.73407E-20
G(4)	=	-8.3932E-28
G(5)	=	1.748E-34

Coefficients for Grid Scale Factor

F(1)	=	0.999793656965
F(2)	=	1.23274E-14
F(3)	=	4.21E-22

SD N SOUTH DAKOTA NORTH ZONE # 4001

Defining Constants

Bs	=	44:25
Bn	=	45:41
Bb	=	43:50
Lo	=	100:00
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	45.0511846016
SinBo	=	0.707738185595
Rb	=	6512395.0582
Ro	=	6377064.4907
No	=	135330.5675
K	=	11870154.6246
ko	=	0.999939111894
Mo	=	6367051.4253
ro	=	6377751.

Coefficients for GP to PC

L(1)	=	111126.0105
L(2)	=	9.77054
L(3)	=	5.62503
L(4)	=	0.024765

Coefficients for PC to GP

G(1)	=	8.998793259E-06
G(2)	=	-7.11994E-15
G(3)	=	-3.68635E-20
G(4)	=	-1.3078E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999939111894
F(2)	=	1.22932E-14
F(3)	=	6.35E-22

SD S SOUTH DAKOTA SOUTH ZONE # 4002

Defining Constants

Bs	=	42:50
Bn	=	44:24
Bb	=	42:20
Lo	=	100:20
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	43.6183915831
SinBo	=	0.689851962794
Rb	=	6846221.9383
Ro	=	6703463.3332
No	=	142758.6051
K	=	11991572.8665
ko	=	0.999906870345
Mo	=	6365242.9133
ro	=	6376475.

Coefficients for GP to PC

L(1)	=	111094.4459
L(2)	=	9.75472
L(3)	=	5.62829
L(4)	=	0.023597

Coefficients for PC to GP

G(1)	=	9.001350018E-06
G(2)	=	-7.11454E-15
G(3)	=	-3.69253E-20
G(4)	=	-1.2373E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999906870345
F(2)	=	1.22979E-14
F(3)	=	6.04E-22

TN TENNESSEE ZONE # 4100

Defining Constants

Bs	=	35:15
Bn	=	36:25
Bb	=	34:20
Lo	=	86:00
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	35.8340607459
SinBo	=	0.585439726459
Rb	=	9008631.3113
Ro	=	8842127.1422
No	=	166504.1691
K	=	13064326.2967
ko	=	0.999948401424
Mo	=	6356978.3321
ro	=	6371042.

Coefficients for GP to PC

L(1)	=	110950.2019
L(2)	=	9.25072
L(3)	=	5.64572
L(4)	=	0.017374

Coefficients for PC to GP

G(1)	=	9.013052490E-06
G(2)	=	-6.77268E-15
G(3)	=	-3.72351E-20
G(4)	=	-9.2828E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999948401424
F(2)	=	1.23188E-14
F(3)	=	4.54E-22

TX N TEXAS NORTH ZONE # 4201

Defining Constants

Bs	=	34:39
Bn	=	36:11
Bb	=	34:00
Lo	=	101:30
Nb	=	1000000.0000
Eo	=	200000.0000

Computed Constants

Bo	=	35.4179042823
SinBo	=	0.579535862261
Rb	=	9135570.8896
Ro	=	8978273.3931
No	=	1157297.4965
K	=	13145417.7356
ko	=	0.999910875663
Mo	=	6356299.7601
ro	=	6370509.

Coefficients for GP to PC

L(1)	=	110938.3584
L(2)	=	9.20339
L(3)	=	5.64670
L(4)	=	0.017491

Coefficients for PC to GP

G(1)	=	9.014014675E-06
G(2)	=	-6.74066E-15
G(3)	=	-3.72545E-20
G(4)	=	-9.0079E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999910875663
F(2)	=	1.23205E-14
F(3)	=	4.49E-22

TX NC TEXAS NORTH CENTRAL ZONE # 4202

Defining Constants

Bs	=	32:08
Bn	=	33:58
Bb	=	31:40
Lo	=	98:30
Nb	=	2000000.0000
Eo	=	600000.0000

Computed Constants

Bo	=	33.0516205542
SinBo	=	0.545394412971
Rb	=	9964225.7538
Ro	=	9810648.6091
No	=	2153577.1446
K	=	13669256.3042
ko	=	0.999872622628
Mo	=	6353600.5552
ro	=	6368624.

Coefficients for GP to PC

L(1)	=	110891.2484
L(2)	=	8.90195
L(3)	=	5.65144
L(4)	=	0.016070

Coefficients for PC to GP

G(1)	=	9.017844103E-06
G(2)	=	-6.52831E-15
G(3)	=	-3.73499E-20
G(4)	=	-8.1560E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999872622628
F(2)	=	1.23272E-14
F(3)	=	4.11E-22

TX C TEXAS CENTRAL ZONE # 4203

Defining Constants

Bs	=	30:07
Bn	=	31:53
Bb	=	29:40
Lo	=	100:20
Nb	=	3000000.0000
Eo	=	700000.0000

Computed Constants

Bo	=	31.0013908377
SinBo	=	0.515058882235
Rb	=	10770561.1034
Ro	=	10622600.3250
No	=	3147960.7784
K	=	14219009.8813
ko	=	0.999881743629
Mo	=	6351602.5419
ro	=	6367308.

Coefficients for GP to PC

L(1)	=	110856.3764
L(2)	=	8.59215
L(3)	=	5.65568
L(4)	=	0.015131

Coefficients for PC to GP

G(1)	=	9.020680826E-06
G(2)	=	-6.30758E-15
G(3)	=	-3.74251E-20
G(4)	=	-7.3651E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999881743629
F(2)	=	1.23324E-14
F(3)	=	3.81E-22

TX SC TEXAS SOUTH CENTRAL ZONE # 4204

Defining Constants

Bs	=	28:23
Bn	=	30:17
Bb	=	27:50
Lo	=	99:00
Nb	=	4000000.0000
Eo	=	600000.0000

Computed Constants

Bo	=	29.3348388416
SinBo	=	0.489912625143
Rb	=	11523512.5584
Ro	=	11357106.1291
No	=	4166406.4293
K	=	14743501.7826
ko	=	0.999863243591
Mo	=	6349870.7242
ro	=	6366112.

Coefficients for GP to PC

L(1)	=	110826.1504
L(2)	=	8.30885
L(3)	=	5.65894
L(4)	=	0.013811

Coefficients for PC to GP

G(1)	=	9.023141055E-06
G(2)	=	-6.10399E-15
G(3)	=	-3.74868E-20
G(4)	=	-6.9867E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999863243591
F(2)	=	1.23368E-14
F(3)	=	3.55E-22

TX S TEXAS SOUTH ZONE # 4205

Defining Constants

Bs	=	26:10
Bn	=	27:50
Bb	=	25:40
Lo	=	98:30
Nb	=	5000000.0000
Eo	=	300000.0000

Computed Constants

Bo	=	27.0010512832
SinBo	=	0.454006848165
Rb	=	12672396.4573
Ro	=	12524558.0674
No	=	5147838.3899
K	=	15621596.5270
ko	=	0.999894794114
Mo	=	6347907.1071
ro	=	6364866.

Coefficients for GP to PC

L(1)	=	110791.8791
L(2)	=	7.86549
L(3)	=	5.66316
L(4)	=	0.012519

Coefficients for PC to GP

G(1)	=	9.025932226E-06
G(2)	=	-5.78365E-15
G(3)	=	-3.75655E-20
G(4)	=	-6.2913E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999894794114
F(2)	=	1.23417E-14
F(3)	=	3.21E-22

UT N UTAH NORTH ZONE # 4301

Defining Constants

Bs	=	40:43
Bn	=	41:47
Bb	=	40:20
Lo	=	111:30
Nb	=	1000000.0000
Eo	=	500000.0000

Computed Constants

Bo	=	41.2507366798
SinBo	=	0.659355481817
Rb	=	7384852.1452
Ro	=	7282974.6766
No	=	1101877.4686
K	=	12238904.9538
ko	=	0.999956841041
Mo	=	6362923.4572
ro	=	6375032.

Coefficients for GP to PC

L(1)	=	111053.9642
L(2)	=	9.67638
L(3)	=	5.63329
L(4)	=	0.021795

Coefficients for PC to GP

G(1)	=	9.004631262E-06
G(2)	=	-7.06511E-15
G(3)	=	-3.70181E-20
G(4)	=	-1.1272E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999956841041
F(2)	=	1.23032E-14
F(3)	=	5.56E-22

UT C UTAH CENTRAL ZONE # 4302

Defining Constants

Bs	=	39:01
Bn	=	40:39
Bb	=	38:20
Lo	=	111:30
Nb	=	2000000.0000
Eo	=	500000.0000

Computed Constants

Bo	=	39.8349774741
SinBo	=	0.640578595825
Rb	=	7822240.6085
Ro	=	7655530.3911
No	=	2166710.2174
K	=	12415886.8989
ko	=	0.999898820765
Mo	=	6360990.5575
ro	=	6373617.

Coefficients for GP to PC

L(1)	=	111020.2282
L(2)	=	9.59755
L(3)	=	5.63694
L(4)	=	0.020325

Coefficients for PC to GP

G(1)	=	9.007367459E-06
G(2)	=	-7.01354E-15
G(3)	=	-3.70800E-20
G(4)	=	-1.0771E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999898820765
F(2)	=	1.23087E-14
F(3)	=	5.26E-22

UT S UTAH SOUTH

ZONE # 4303

Defining Constants

Bs	=	37:13
Bn	=	38:21
Bb	=	36:40
Lo	=	111:30
Nb	=	3000000.0000
Eo	=	500000.0000

Computed Constants

Bo	=	37.7840696241
SinBo	=	0.612687337234
Rb	=	8361336.2313
Ro	=	8237322.9910
No	=	3124013.2403
K	=	12719504.1729
ko	=	0.999951297078
Mo	=	6359086.0437
ro	=	6372457.

Coefficients for GP to PC

L(1)	=	110986.9886
L(2)	=	9.44259
L(3)	=	5.64118
L(4)	=	0.018991

Coefficients for PC to GP

G(1)	=	9.010065135E-06
G(2)	=	-6.90671E-15
G(3)	=	-3.71585E-20
G(4)	=	-9.9163E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999951297078
F(2)	=	1.23130E-14
F(3)	=	4.89E-22

VA N VIRGINIA NORTH ZONE # 4501

Defining Constants

Bs	=	38:02
Bn	=	39:12
Bb	=	37:40
Lo	=	78:30
Nb	=	2000000.0000
Eo	=	3500000.0000

Computed Constants

Bo	=	38.6174703154
SinBo	=	0.624117864647
Rb	=	8100315.6826
Ro	=	7994777.9034
No	=	2105537.7792
K	=	12589455.5135
ko	=	0.999948385156
Mo	=	6359972.6472
ro	=	6373043.

Coefficients for GP to PC

L(1)	=	111002.4628
L(2)	=	9.51137
L(3)	=	5.63918
L(4)	=	0.019770

Coefficients for PC to GP

G(1)	=	9.008809102E-06
G(2)	=	-6.95425E-15
G(3)	=	-3.71258E-20
G(4)	=	-1.0190E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999948385156
F(2)	=	1.23106E-14
F(3)	=	5.06E-22

VA S VIRGINIA SOUTH ZONE # 4502

Defining Constants

Bs	=	36:46
Bn	=	37:58
Bb	=	36:20
Lo	=	78:30
Nb	=	1000000.0000
Eo	=	3500000.0000

Computed Constants

Bo	=	37.3674799550
SinBo	=	0.606924846589
Rb	=	8476701.8059
Ro	=	8361937.6230
No	=	1114764.1829
K	=	12788171.0476
ko	=	0.999945401397
Mo	=	6358598.6747
ro	=	6372118.

Coefficients for GP to PC

L(1)	=	110978.4824
L(2)	=	9.40495
L(3)	=	5.64206
L(4)	=	0.018900

Coefficients for PC to GP

G(1)	=	9.010755731E-06
G(2)	=	-6.88091E-15
G(3)	=	-3.71758E-20
G(4)	=	-9.6990E-28

Coefficients for Grid Scale Factor

F(1)	=	0.999945401397
F(2)	=	1.23143E-14
F(3)	=	4.83E-22

WA N WASHINGTON NORTH ZONE # 4601

Defining Constants

Bs	=	47:30
Bn	=	48:44
Bb	=	47:00
Lo	=	120:50
Nb	=	0.0000
Eo	=	500000.0000

Computed Constants

Bo	=	48.1179151437
SinBo	=	0.744520326553
Rb	=	5853778.6038
Ro	=	5729486.2170
No	=	124292.3869
K	=	11670409.5559
ko	=	0.999942253481
Mo	=	6370499.7054
ro	=	6380060.

Coefficients for GP to PC

L(1)	=	111186.1944
L(2)	=	9.72145
L(3)	=	5.61785
L(4)	=	0.027630

Coefficients for PC to GP

G(1)	=	8.993922319E-06
G(2)	=	-7.07270E-15
G(3)	=	-3.67384E-20
G(4)	=	-1.4705E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999942253481
F(2)	=	1.22844E-14
F(3)	=	7.08E-22

WA S WASHINGTON SOUTH ZONE # 4602

Defining Constants

Bs	=	45:50
Bn	=	47:20
Bb	=	45:20
Lo	=	120:30
Nb	=	0.0000
Eo	=	500000.0000

Computed Constants

Bo	=	46.5850847865
SinBo	=	0.726395784020
Rb	=	6183952.2755
Ro	=	6044820.3632
No	=	139131.9123
K	=	11760132.9643
ko	=	0.999914597644
Mo	=	6368612.1773
ro	=	6378741.

Coefficients for GP to PC

L(1)	=	111153.2505
L(2)	=	9.75921
L(3)	=	5.62165
L(4)	=	0.026539

Coefficients for PC to GP

G(1)	=	8.996587928E-06
G(2)	=	-7.10693E-15
G(3)	=	-3.68032E-20
G(4)	=	-1.3823E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999914597644
F(2)	=	1.22897E-14
F(3)	=	6.73E-22

WV N WEST VIRGINIA NORTH ZONE # 4701

Defining Constants

Bs	=	39:00
Bn	=	40:15
Bb	=	38:30
Lo	=	79:30
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	39.6259559060
SinBo	=	0.637772979172
Rb	=	7837787.7954
Ro	=	7712787.3235
No	=	125000.4720
K	=	12444726.9475
ko	=	0.999940741388
Mo	=	6361027.5180
ro	=	6373731.

Coefficients for GP to PC

L(1)	=	111020.8737
L(2)	=	9.58417
L(3)	=	5.63702
L(4)	=	0.020271

Coefficients for PC to GP

G(1)	=	9.007315138E-06
G(2)	=	-7.00383E-15
G(3)	=	-3.70855E-20
G(4)	=	-1.0658E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999940741388
F(2)	=	1.23081E-14
F(3)	=	5.23E-22

WV S WEST VIRGINIA SOUTH ZONE # 4702

Defining Constants

Bs	=	37:29
Bn	=	38:53
Bb	=	37:00
Lo	=	81:00
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	38.1844729967
SinBo	=	0.618195407531
Rb	=	8250940.5496
Ro	=	8119477.8143
No	=	131462.7353
K	=	12655491.0285
ko	=	0.999925678359
Mo	=	6359357.1532
ro	=	6372583.

Coefficients for GP to PC

L(1)	=	110991.7203
L(2)	=	9.47644
L(3)	=	5.64030
L(4)	=	0.019308

Coefficients for PC to GP

G(1)	=	9.009681018E-06
G(2)	=	-6.93061E-15
G(3)	=	-3.71449E-20
G(4)	=	-1.0063E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999925678359
F(2)	=	1.23124E-14
F(3)	=	4.97E-22

WI N WISCONSIN NORTH ZONE # 4801

Defining Constants

Bs	=	45:34
Bn	=	46:46
Bb	=	45:10
Lo	=	90:00
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	46.1677715519
SinBo	=	0.721370788570
Rb	=	6244929.5105
Ro	=	6133662.3561
No	=	111267.1544
K	=	11788334.3169
ko	=	0.999945345317
Mo	=	6368341.1351
ro	=	6378625.

Coefficients for GP to PC

L(1)	=	111148.5205
L(2)	=	9.76579
L(3)	=	5.62201
L(4)	=	0.025652

Coefficients for PC to GP

G(1)	=	8.996970839E-06
G(2)	=	-7.11207E-15
G(3)	=	-3.68179E-20
G(4)	=	-1.3661E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999945345317
F(2)	=	1.22894E-14
F(3)	=	6.59E-22

WI C WISCONSIN CENTRAL ZONE # 4802

Defining Constants

Bs	=	44:15
Bn	=	45:30
Bb	=	43:50
Lo	=	90:00
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	44.8761466967
SinBo	=	0.705576614409
Rb	=	6531967.9926
Ro	=	6416091.9604
No	=	115876.0322
K	=	11884020.9704
ko	=	0.999940704902
Mo	=	6366865.5955
ro	=	6377630.

Coefficients for GP to PC

L(1)	=	111122.7674
L(2)	=	9.76998
L(3)	=	5.62509
L(4)	=	0.024632

Coefficients for PC to GP

G(1)	=	8.999055911E-06
G(2)	=	-7.12016E-15
G(3)	=	-3.68710E-20
G(4)	=	-1.2987E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999940704902
F(2)	=	1.22933E-14
F(3)	=	6.31E-22

WI S WISCONSIN SOUTH ZONE # 4803

Defining Constants

Bs	=	42:44
Bn	=	44:04
Bb	=	42:00
Lo	=	90:00
Nb	=	0.0000
Eo	=	600000.0000

Computed Constants

Bo	=	43.4012400263
SinBo	=	0.687103235566
Rb	=	6910290.1546
Ro	=	6754625.8558
No	=	155664.2988
K	=	12012072.0457
ko	=	0.999932547079
Mo	=	6365163.6776
ro	=	6376476.

Coefficients for GP to PC

L(1)	=	111093.0630
L(2)	=	9.75085
L(3)	=	5.62892
L(4)	=	0.023110

Coefficients for PC to GP

G(1)	=	9.001462070E-06
G(2)	=	-7.11165E-15
G(3)	=	-3.69314E-20
G(4)	=	-1.2326E-27

Coefficients for Grid Scale Factor

F(1)	=	0.999932547079
F(2)	=	1.22981E-14
F(3)	=	5.97E-22

PR VI PUERTO RICO & VIRGIN I ZONE # 5200

Defining Constants

```
Bs     =        18:02
Bn     =        18:26
Bb     =        17:50
Lo     =        66:26
Nb     =    200000.0000
Eo     =    200000.0000
```

Computed Constants

```
Bo     =  18.2333725907
SinBo= 0.312888187729
Rb     =  19411706.1974
Ro     =  19367429.4540
No     =    244276.7435
K      =  21418025.2279
ko     = 0.999993944472
Mo     =   6341634.1470
ro     =    6360883.
```

Coefficients for GP to PC

```
L(1)   = 110682.3958
L(2)   =      5.76845
L(3)   =      5.67659
L(4)   =      0.008098
```

Coefficients for PC to GP

```
G(1)    =  9.034860445E-06
G(2)    = -4.25426E-15
G(3)    = -3.78192E-20
G(4)    = -3.9493E-28
```

Coefficients for Grid Scale Factor

```
F(1)   = 0.999993944472
F(2)   = 1.23576E-14
F(3)   = 2.08E-22
```