FIG
June 1990

Establishing Vertical Control Using GPS Satellite Surveys

By David B. Zilkoski[1]

ABSTRACT: Analysis of Global Positioning System (GPS) survey data has shown that GPS can be used to establish precise relative positioning in a three-dimensional system. The results of many tests and operational projects have clearly shown that GPS survey methods can replace classical horizontal terrestrial survey methods. The problem of converting the ellipsoid height differences from GPS surveys to accurate orthometric height differences remains to be resolved. Can the accuracies achieved for these orthometric height differences provide a viable alternative to classical geodetic leveling techniques?

Some results of analyses performed at the National Geodetic Survey (NGS) in computing orthometric heights from ellipsoid heights obtained by GPS surveys indicate that with appropriate planning, consideration of GPS survey specifications for connection to bench marks, proper field observing procedures, and proper strategy for estimating geoid heights and final orthometric height values, it is possible to use GPS survey methods to compute orthometric heights that meet a wide range of engineering requirements for vertical control. It is clear that GPS-derived orthometric heights will have a major impact on the surveying community in the future. However, if horizontal and vertical distortions in existing geodetic control networks are not properly handled, their influences on GPS data can cause large errors in adjusted GPS-derived heights. There are several factors which must be investigated and documented before GPS-derived orthometric heights can be used routinely by the surveying community.

INTRODUCTION

Since early 1983, the National Geodetic Survey (NGS) has performed control survey projects in the United States using satellites of the Global Positioning System (GPS). Analysis of GPS survey data has shown that GPS can be used to establish precise relative positions in a three-dimensional Earth-centered coordinate system. GPS carrier phase measurements are used to determine vector base lines in space where the components of the base line are expressed in terms of cartesian coordinate differences (Remondi 1984). These vector base lines can be converted to distance, azimuth, and ellipsoidal height differences (dh) relative to a defined reference ellipsoid.

---

[1]Vertical Network Branch, National Geodetic Survey, Charting and Geodetic Services, NOS, NOAA, Rockville, MD 20852.

Orthometric height differences (dH) can then be obtained from ellipsoid height differences by subtracting the geoid height differences (dN):

$$dH = dh - dN.$$

(Note: this is an approximate equation; the error is always small and considered to be insignificant.)

The results of many tests and operational projects have clearly shown that GPS survey methods can replace classical horizontal terrestrial survey methods. However, there remains the problem of obtaining sufficiently accurate geoid height differences for converting these ellipsoid height differences to accurate orthometric height differences. Can the accuracies achieved for these orthometric height differences provide a viable alternative to classical geodetic leveling techniques?

It is clear that GPS-derived orthometric heights will have a major impact on the surveying community in the future. There are several factors which must be investigated and documented before GPS-derived orthometric heights can be used routinely by the surveying community. This report gives some results of the analyses performed by the author in comparing orthometric heights determined by differential leveling techniques with orthometric height differences from GPS surveys and predicted geoid height differences. Factors which need to be considered when estimating GPS-derived orthometric heights are discussed.

HEIGHTS AND HEIGHT DIFFERENCES

Orthometric heights (H) are referenced to an equipotential surface, e.g., the geoid. The orthometric height of a point on the Earth's surface is the distance from the reference surface to the point, measured along the plumb line normal to the geoid. Ellipsoid heights (h) are referenced to a reference ellipsoid. The ellipsoid height of a point is the distance from the reference ellipsoid to the point, measured along the line which is normal to the ellipsoid. The difference between an ellipsoid height and an orthometric height is defined as the geoid height (N) (to a sufficient approximation).

Several error sources that affect the accuracy of orthometric, ellipsoid, and geoid height values are generally common to nearby points. Because these error sources are common, the uncertainty of height differences between nearby points is significantly smaller than the "absolute" heights at a point.

Ellipsoidal height differences (dh) can be determined from GPS phase measurements with 1-sigma uncertainties that are typically +/- (0.5 cm + 1-2 ppm) (DMA and NGS 1986, NOAA 1985), although larger uncertainties have been seen recently caused by large disturbances in the ionosphere due to high solar radiation activity. With improved orbit determination techniques, dual frequency carrier phase data, and improved antenna designs, uncertainties approaching +/- (0.2 cm + 0.01-0.1 ppm) may be achieved for dh values in the future. Geoid height differences (dN) in the United States can be determined from gravity data and Stokes' integral method, or from astrogravimetric data and least squares collocation methods with uncertainties that are typically 1-10 cm for distances of as much as 20 km and 5-20 cm for distances from 20 to 50 km (Fury 1986; R. Fury, NGS, personal communication 1990). The smaller value for the uncertainties has been demonstrated in tests in several regions of the United States. Larger uncertainties can be expected in other areas, depending on the density of the gravity network and uncertainties in the determination of gravity anomalies.

When high-accuracy field procedures are used, orthometric height differences can be computed from measurements of precise geodetic leveling with an uncertainty of less than 1 cm over a 50-kilometer distance. Less accurate results are achieved when third-order leveling methods are employed. Depending on the accuracy requirements, GPS surveys and present geoid prediction models can be employed as an alternative to classical leveling methods. The primary limiting factor is the accuracy of estimating geoid height differences. Spherical harmonic models which are commonly used to estimate geoid heights are too generalized to accurately represent the local relief of the geoid. However, in many regions of the United States, over very small areas, i.e., 10 km by 10 km, the slope of the geoid can usually be assumed to be flat. It has been shown that when proper field procedures are followed and a significant number of vertical control points are occupied by GPS, it is possible to estimate GPS-derived orthometric heights which will meet a wide range of vertical control requirements for engineering projects (Zilkoski and Hothem 1989, Hajela 1990).

DATA EVALUATION

An important aspect of any geodetic positioning technique is to ensure that all data outliers have been removed from the data. The design of the network can be very helpful when analyzing the data.

GPS results can be evaluated by analyzing network loop misclosures, repeat base line differences, and least squares adjustment results. The design of the network should be such that there are enough redundant observations to detect data outliers. As stated earlier, the largest contribution to the error budget is uncertainty in geoid height difference estimates. Therefore, it is important to evaluate these estimates. Since the slope of the geoid can change significantly between "widely" spaced monuments, it is necessary to perform a detailed study of the density and distribution of observed gravity values (or free-air anomalies) to determine the slope and changes in slope. The distribution of known orthometric heights is extremely important in verifying geoid height differences. Vertical control stations (bench marks) should be strategically located throughout the network in order to determine the geoid's slope and its changes in slope (flatness).

Four major items must be considered when using orthometric heights of bench marks obtained from leveling data to evaluate the results of GPS-derived orthometric heights. First, and most important, is that all orthometric heights must be referenced to the same datum, e.g., the National Geodetic Vertical Datum of 1929 (NGVD 29). Second, the network must be designed in such a manner that bench marks which have been disturbed or influenced by vertical crustal motion will be detected during the analyses. Third, all leveling data used to establish the heights should be corrected for known systematic effects. Last, the latest and/or most accurate data available should be used to estimate the orthometric height differences between monuments. Influences on GPS data due to horizontal and vertical distortions can cause significantly large errors in adjusted GPS-derived orthometric heights. It is, therefore, important that the survey is designed in such a manner that each item mentioned above can be evaluated.

INFLUENCES ON ADJUSTED GPS-DERIVED ORTHOMETRIC HEIGHTS DUE TO HORIZONTAL AND VERTICAL DISTORTIONS

If proper procedures are not followed, existing horizontal and vertical distortions, or apparent distortions, in geodetic control networks can cause large errors in estimated GPS-derived heights, both ellipsoidal and orthometric. Factors which cause horizontal and vertical distortions include the following: incorrect, inconsistent, or "less-accurate" coordinates, both horizontally and vertically, used to constrain GPS data; control station moved since coordinates were last checked, i.e., bench marks uplifted due to frost heave or subsided due to fluid withdrawal; station misidentified (e.g., reference mark 2 occupied instead of reference mark 1); antenna not centered over monument; height of antenna measured incorrectly; and an incorrect or inadequate model used to describe slope and/or change in slope of geoid.

It is difficult to accurately determine the effect each distortion will have on adjusted GPS-derived heights, because the influence on GPS data depends on many factors. Some of these factors include: the size and type of error, network design, and type of adjustment performed to estimate heights, e.g., one-dimensional, or three-dimensional, minimally constrained or partially constrained least squares adjustment.

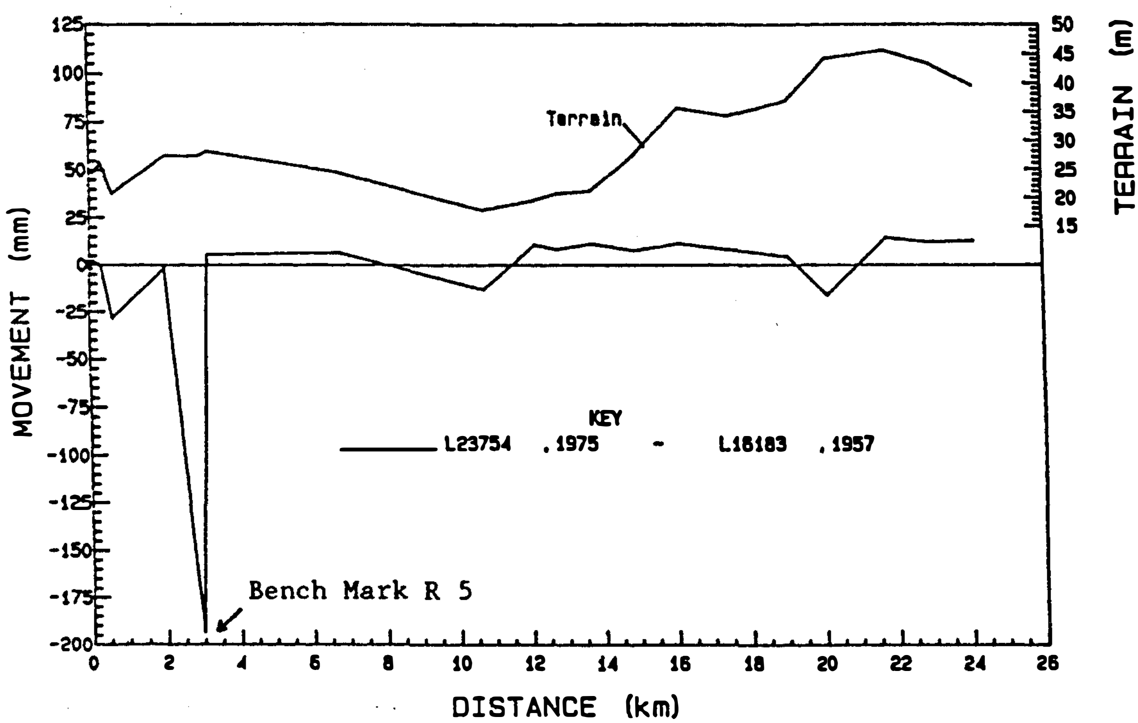

Figure 1.--Profile depicting differences in heights of bench marks computed from two different epochs (1957 and 1975) of leveling data near the GA/FL state line in the Folkston, GA, area

Bench mark movement is an error source that many analysts ignore, or do not have enough information in their survey project to evaluate properly. The profile in figure 1 depicts the differences in heights of bench marks estimated by two different epochs of leveling data. It is obvious from figure 1 that bench mark R 5 was disturbed between epochs 1957 and 1975. The latest estimate of the height for R 5 differs from the previous value by approximately 20 cm. If the incorrect height were held fixed, the 20 cm would be forced into the adjustment of GPS data and would distort other GPS-derived ellipsoid and orthometric heights in the network. Another possible problem, and one which cannot be determined from the available data, is whether R 5 has moved since epoch 1975.

The recommended procedure to check for movement of bench marks is to perform check leveling between two or more bench marks and compare the results with published values. Another method that can be used, which may be less expensive during a GPS survey, is to occupy two bench marks with GPS that are only 2-3 km apart. With new GPS techniques becoming operational, e.g., kinematic and pseudo-static GPS (Remondi 1988), these additional "GPS-leveling" ties should not require much in the way of additional resources. The geoid height differences over small areas in most regions of the United States should be small enough that ellipsoid height differences can be compared with published orthometric height differences to check the stability of bench marks to the 5-to-10 centimeter level.

The analyst must also ensure that all bench mark values are referenced to the same vertical reference system, e.g., NGVD 29, and that published values do not contain inconsistencies due to previous adjustment constraints. This is not a major problem for bench marks published by NGS, but there are a few inconsistencies in NGVD 29. This is one of the reasons NGS is performing the new adjustment of the North American Vertical Datum of 1988 (Zilkoski and Young 1985, Zilkoski et al. 1989). An example of one inconsistency in NGVD 29 is near Oak Hill, Florida. Here a 10 cm difference exists between published NGVD 29 height differences and adjusted height differences computed in a minimally constrained adjustment of the Florida primary leveling network between bench marks D 227 and JLR 370, which are 0.85 km apart (See table 1.)

Table. 1 -- An inconsistency in NGVD 29 near Oak Hill, Florida

Bench Mark	Special Adjusted Height (m)	Published NGVD 29 Height (m)	Difference Between Special Adj. and Published Height (cm)	Second Difference (cm)
D 227	3.624	3.624	0.0	
JLR 370	3.296	3.192	10.4	-10.4
J 211	3.502	3.405	9.7	0.7
JLR 371	3.358	3.263	9.5	0.2
Hale RM 2	2.882	2.884	-0.2	9.7

In some portions of NGVD 29, influences from distortions of past adjustment constraints and/or crustal movement made it impossible to fit new leveling data into NGVD 29 without performing a major readjustment of the region. The NAVD 88 readjustment project will remove most of these inconsistencies.

An error source in published NGVD 29 heights which will be eliminated by NAVD 88 is the differences in adjusted orthometric heights due to using "true" geopotential differences based on observed gravity, instead of the presently published orthometric height differences based on normal (theoretical) gravity values. At this time, the difference is insignificant compared with the large uncertainty of geoid height differences. However, as GPS is used to estimate GPS-derived orthometric heights over longer base lines and estimates of geoid height differences become more accurate, this difference could become significant. For example, the difference between "normal" orthometric heights and "true" orthometric heights for leveling from Seattle, Washington (approximate elevation of 2 m), to the Mt. Rainier area (approximate elevation of 1650 m), approaches 17 cm. It should be noted that the uncertainties in the geoid height differences in this region are much higher than this error.

The point that needs to be understood here is that without additional information, these inconsistencies and bench mark movement could incorrectly appear to be problems in the geoid height differences to many analysts.

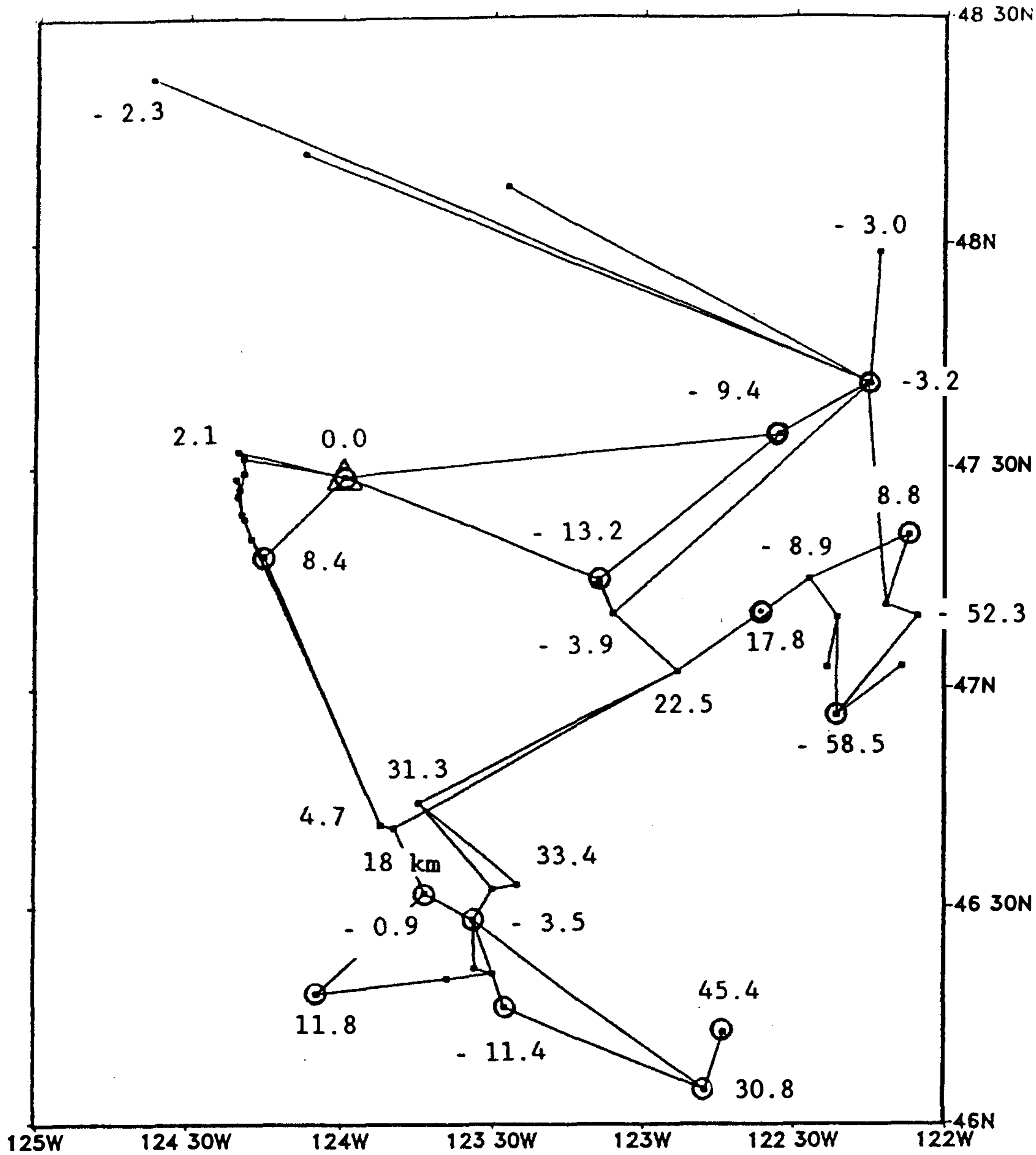

Figure 2.--Ellipsoid height differences between a 3-dimensional, minimally constrained (mc) least squares adjustment and a 3-dimensional, partially constrained (pc) least squares adjustment; h(pc) minus h(mc); units=cm; © NAD 83 published values of latitude and longitude held fixed in partially constrained adjustment only; △ NAD 83 published ellipsoid height value held fixed in both adjustments

GPS provides a determination of a three-dimensional position for all stations in the network. Integrating very precise three-dimensional results into two separate, existing geodetic reference systems, i.e., a system of latitudes and longitudes: NAD 83, and a system of heights: NGVD 29, can cause large distortions to be forced into GPS data if proper procedures are not followed. These distortions would degrade the adjusted GPS-derived coordinates.

Horizontal coordinates used as constraints in 3-dimensional least squares adjustments can distort adjusted GPS-derived orthometric heights if the coordinates are less accurate than the GPS data. Figure 2 depicts the differences in adjusted ellipsoid heights obtained from two adjustments of the same data using different constraints. The first adjustment was minimally constrained by fixing the latitude, longitude, and height values of one station, while the second adjustment was partially constrained by fixing the height value of one station and the latitudes and longitudes of several known horizontal control stations. It is easy to see from figure 2 that the horizontal constraints had an adverse influence on the ellipsoid heights. In fact, one station's value changed by more than 50 cm. This is probably an extreme case, but depending on the accuracy of the horizontal coordinates, GPS vectors, and lengths of lines some effect will be noticed. GPS-derived orthometric heights should not be estimated by constraining horizontal coordinates if they are less accurate than the GPS results. It should be noted that a GPS-derived coordinate, i.e., latitude, longitude, and ellipsoid height, estimated using high-precision GPS survey data could help control errors in lower-order GPS surveys. The geoid still needs to be properly handled when estimating GPS-derived orthometric heights.

Using incorrect or inadequate models to describe the slope and/or change in slope of the geoid can cause large errors in GPS-derived orthometric heights. In small areas, e.g., 10 km by 10 km, the slope of the geoid can usually be assumed to be flat. However, the project still requires enough bench marks strategically located throughout the project to properly evaluate the slope and to determine if there are any changes in slope of the geoid.

Figure 3 is a plot of geoid heights that were estimated using four techniques for five bench marks which were in a Boulder County, Colorado, GPS survey (Zilkoski and Hothem 1989). The following four techniques were used: (1) computation using the Earth's gravity field represented by spherical harmonic coefficients to order and degree 180, derived by Rapp (1981), (2) spherical harmonics to order and degree 360, derived by Rapp and Cruz (1986), (3) geoid heights estimated using GPS and leveling data ($N = h - N$), and (4) gravimetric geoid heights using Stokes' integration procedure. Figure 3 indicates that the spherical harmonic models provide only the long wavelength of the geoid and that Stokes' integration method using gravity data improves the estimates of geoid heights. It is apparent from figure 3 that bench marks are required throughout the surveying project.

Figure 3 also shows that the geoid over the extent of the project is not flat (see line labeled 3 on figure 3), even though two models indicate that the geoid is flat. (See lines labeled 1 and 2 on figure 3.) There is an obvious change in slope of the geoid at bench mark "NOAA." If the geoid were assumed to be flat over the entire area, an error of more than 1 meter would be introduced into the final GPS-derived orthometric heights. (See line labeled 5 on figure 3.) Once again, this is an extreme case, but depending on the change in slope in the geoid and the locations of known vertical control, there will be some effect.

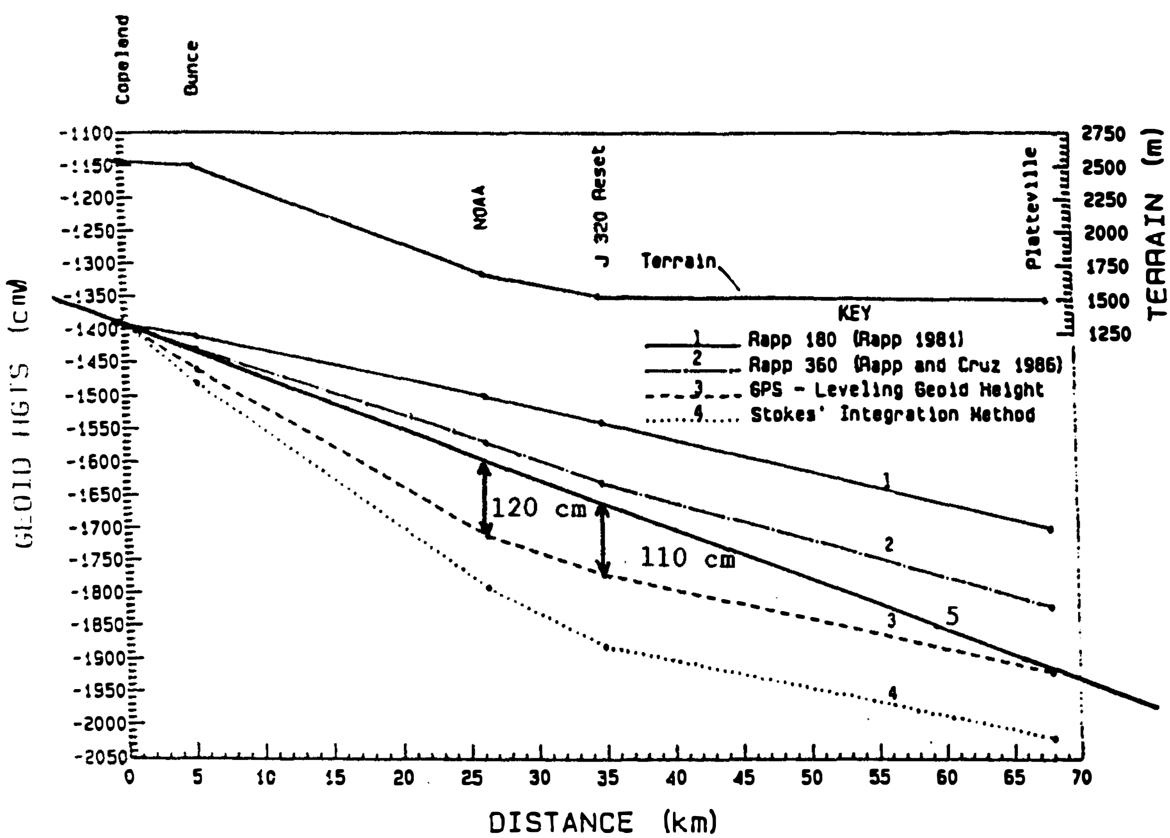

Figure 3.--Plot of geoid heights that were estimated using four techniques for five bench marks in a Boulder County, CO, GPS survey (modified version of Zilkoski and Hothem 1989)

RESULTS

Zilkoski and Hothem (1989) documented results of some analyses performed in estimating GPS-derived orthometric heights. In the study, precise geodetic leveling data were used to determine orthometric height differences between monuments. These height differences provided the standards used for the comparisons. GPS-derived orthometric height differences estimated from ellipsoid height differences and geoid height differences were then subtracted from the differential leveling results. The study showed that it is possible to estimate GPS-derived orthometric heights with uncertainties of 5-15 cm, but it also showed how GPS-derived orthometric height difference estimates can vary considerably in accuracy within the same survey. The main problem with using GPS-derived orthometric heights is estimating the accuracy of the value where there is no known orthometric height, which is exactly what the user needs. A better estimate of the shape of the geoid, as well as changes in the slope, must be obtained before GPS-derived orthometric heights can be routinely used by the surveying community. In certain areas of the country, GPS-derived orthometric heights can be estimated accurately enough to meet the needs of many users (Zilkoski and Hothem 1989, Hajela 1990). The estimates, however, must be used with caution because of the uncertainty in the estimates of geoid height differences.

Even the best estimates of geoid heights usually have systematic errors which are local in nature. These errors are in absolute magnitude as well as in tilt. Vincenty (1987a, 1987b) describes the mathematical models required to solve for these parameters. The geoidal slope is absorbed by two rotations (one around the north axis and the other around the east axis in the horizon system) and the geoidal heights are absorbed by the scale correction.

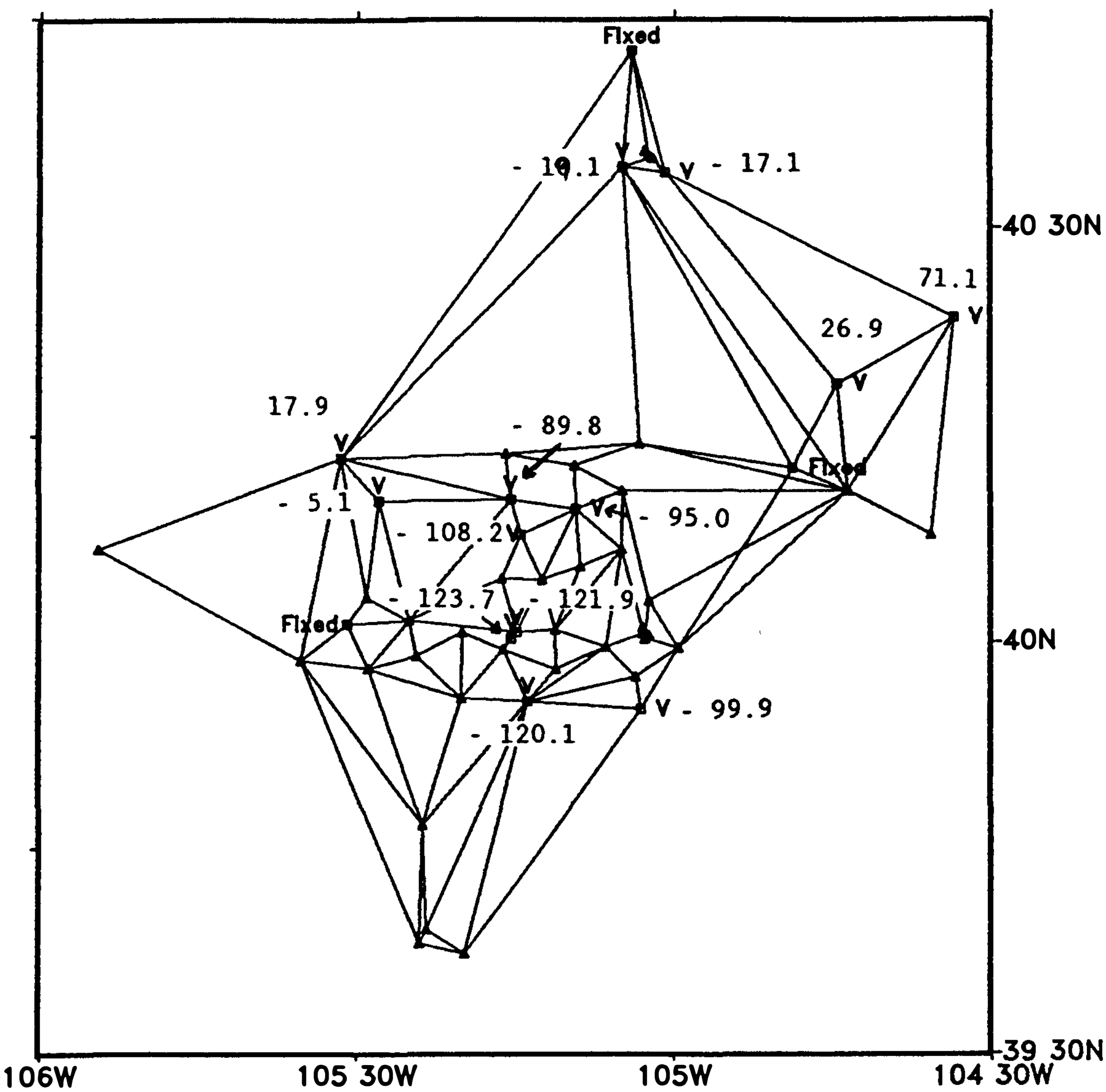

Figure 4.--Plot depicting the differences between adjusted GPS-derived orthometric heights and published NGVD 29 heights in the Boulder County, CO, GPS project, where height values of three bench marks which did not accurately represent the change in slope of the geoid were held fixed; units=cm

In order to evaluate the process, Zilkoski and Hothem (1989) solved for trend (bias) parameters in two networks: Boulder County, Colorado, and Summit County, Ohio. The Boulder County project was separated into two components. The boundaries of the components were based on the large change in geoid slope as indicated by using differential leveling orthometric heights and GPS-derived orthometric heights to estimate geoid heights. (See figures 3 and 4.) It is important that enough bench marks with known elevations are evenly distributed throughout the project to separate the network into trend (bias) groups which best represent the slope and changes in slope of the geoid (Vincenty 1987b). The points must be distributed in such a manner as to provide a strong geometric determination of a plane. Three fixed elevations are required to solve for each additional set of parameters. Therefore, for every set of parameters the network should have at least five marks with known elevations: three to solve for the parameters and two to check the results.

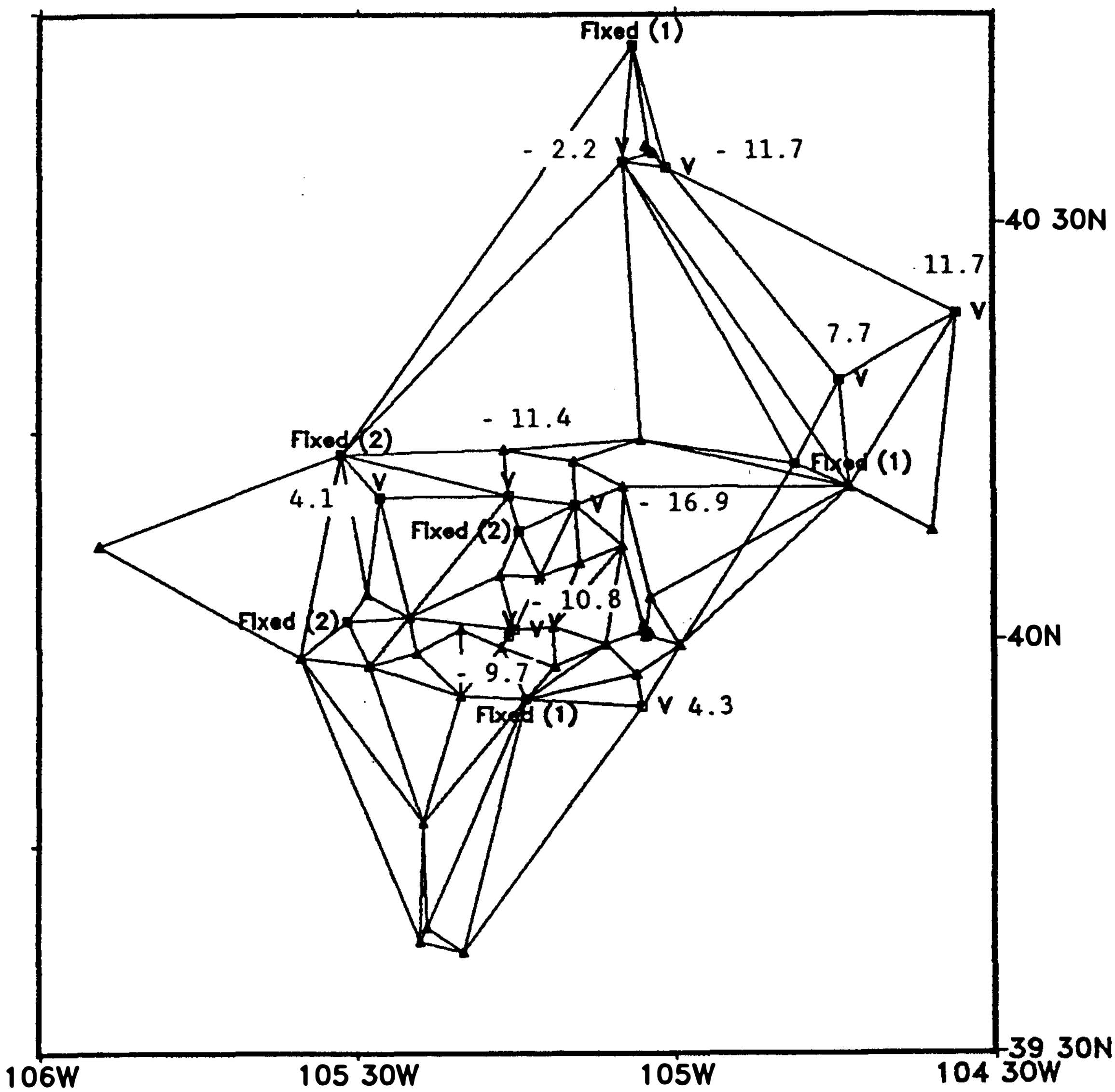

Figure 5.--Plot depicting the differences between adjusted GPS-derived orthometric heights in the Boulder County, CO, GPS project, where height values of six bench marks which accurately represent the change in slope of the geoid were held fixed; units=cm

It was mentioned above that using incorrect models to describe the slope and/or change in slope of the geoid can cause large errors in GPS-derived orthometric heights. The Boulder County GPS project illustrates what can happen if an analyst is not careful or if a project is not properly designed. Figure 4 depicts the differences between adjusted GPS-derived orthometric heights and published NGVD 29 heights in the Boulder County GPS project, where height values of three bench marks which did not accurately represent the slope and change in slope of the geoid were held fixed. From figure 4, it can be seen that some GPS-derived orthometric heights would be in error by more than 1 meter. This is exactly what was indicated in figure 3.

However, figure 5, also representing the Boulder County GPS project, shows that by properly solving for all parameters the overall estimates of GPS-derived orthometric heights were improved. All differences between GPS-derived orthometric heights and published heights decreased when two sets of parameters were solved for to account for the change in slope of the geoid. The Boulder County project represents an extreme case. It is not a typical "small" GPS project being performed in the United States, nor does it represent a typical "roughness" of the geoid. In the report by Zilkoski and Hothem (1989), GPS-derived orthometric heights for stations that were located close to one another showed good agreement compared with leveling data, indicating that over small areas, GPS can be used to replace lower-order surveys for some engineering projects. The results are encouraging and show that GPS-derived orthometric height determination deserves more attention in the future.

STEPS REQUIRED WHEN ESTIMATING GPS-DERIVED ORTHOMETRIC HEIGHTS

The minimum steps required for a project when analyzing GPS-derived orthometric heights are listed below.

1. During the planning stage, perform a detailed analysis of the geoid in the area of the survey in order to determine if additional gravity and/or leveling data are required to adequately estimate the geoid slope and changes in slope.
    a. Perform a detailed study of the density and distribution of observed gravity values.
2. During the planning stage, perform a detailed study of the leveling network in the area, i.e., plot all leveling lines, note the age of leveling data, determine if bench marks can be occupied by GPS equipment, and other considerations.
    a. Perform a history check on monuments to determine if they are stable bench marks.
3. Perform a 3-D minimum constraint least squares adjustment.
    a. Compare GPS-derived coordinates with results of higher-order surveys to determine if coordinates estimated from higher-order surveys can be used to control errors in lower-order survey.
4. Compare adjusted GPS-derived orthometric height differences obtained from step 3 with leveling-derived orthometric height differences.
5. Detect and remove all data outliers determined in steps 2 through 4.
6. Analyze the local geoid in detail.
    a. Plot the geoid heights in the area.
    b. Plot the estimated slope of the geoid using differences between GPS-derived ellipsoid height differences and leveling-derived orthometric height differences ($dN = dh - dH$).

7. Estimate local systematic errors in the geoid heights by solving for the geoidal slope and scale using the method described by Vincenty (1987a).

8. Compare adjusted GPS-derived orthometric height differences from step 7 with leveling-derived orthometric height differences.

a. Using the results of steps 6b and 8, determine if additional parameters are required to solve for changes in the slope of the geoid within the projects boundaries.

9. Estimate GPS-derived heights by performing a 3-dimensional least squares adjustment holding all appropriate height values of published bench marks (and appropriate GPS-derived coordinates estimated from higher-order surveys) and solving for appropriate scale and rotation parameters. Over small areas, the geoid may be flat, as well as level. The analyst must ensure that bench mark movement is not interpreted as slopes in the geoid or does not significantly influence the solution of rotation parameters. In addition, over large areas, if there are not enough bench marks strategically located throughout the project to detect changes in slope of the geoid, solving for rotation parameters may produce erroneous results. The project must have enough information to ensure the results can be evaluated properly or the results from the adjustment solving for rotational parameters could provide incorrect GPS-derived orthometric heights.

10. Use the results from steps 3 through 9 to document the estimated accuracy of the GPS-derived orthometric heights.

Of course, it must be understood that each project is different and, therefore, the procedures used to estimate GPS-derived orthometric heights will be slightly different for each project. At this time, there is not an economical standard method that works well all the time everywhere. The results of all steps and comparisons with known values must be considered before estimating final GPS-derived heights. NGS is working on algorithms and models to improve the computation of geoid heights and geoid height differences. A current project uses the Integrated Geodesy approach, where leveling data and GPS measurements are combined with gravity data to solve for an improved geoid (Milbert and Dewhurst 1990).

ESTIMATING VERTICAL CRUSTAL MOTION USING GPS

This report indicates the largest error source in estimating orthometric heights using GPS and gravity data is the inadequacy of the models to accurately represent the relief of the geoid. The main purpose of performing geodetic leveling is to estimate orthometric heights that are consistent with a particular datum, e.g., NGVD 29. Therefore knowing the uncertainties of the estimates of geoid height differences is critical.

Leveling is also used to estimate vertical crustal motion when two or more leveling surveys have been performed over some of the same bench marks, enabling one to estimate changes in height differences between bench marks over time. GPS satellite survey data can also be used to estimate vertical crustal movement.

Changes in ellipsoid heights determined from repeat GPS surveys of the same survey marks can be evaluated independently of the geoid, i.e., the uncertainties associated with estimates for geoid height differences can be ignored. Thus, repeat GPS surveys can be used as an accurate alternative to repeat leveling surveys for the purpose of estimating vertical movement. Results obtained by Strange (1989) for a project southeast of Phoenix, Arizona, showed that it is possible to use multiple GPS occupations of the same point to estimate subsidence with uncertainties that are typically less than 2 cm over 20-kilometer distances.

CONCLUSION

Since early 1983, NGS has performed geodetic control survey projects in the United States using GPS satellites. These surveys have met the requirements of many users.

It is obvious that GPS-derived orthometric heights will have a major impact on the surveying community in the future. However, several factors need to be understood by users before GPS-derived orthometric heights can be routinely used by the surveying community. The user should perform a detailed analysis of the geoid and leveling data in the area of the survey to determine if additional gravity data and/or leveling are required to adequately estimate the geoid's slope and its changes in slope. NGS is working on algorithms and models to improve the computation of geoid heights and geoid height differences. NGS is actively pursuing the Integrated Geodesy approach of combining leveling data and GPS measurements with gravity data to solve for an improved geoid. Network design must include bench marks with known orthometric heights strategically located throughout the network to verify the estimates of geoid height differences. This may require obtaining additional leveling data or performing "GPS-leveling" ties in certain portions of the network where control is sparse or where orthometric heights are based on old surveys.

Results of studies performed by NGS show that orthometric heights in certain regions of the United States can be determined by GPS and geoid heights derived from gravity data with uncertainties between 5-15 cm. Analyses indicate that with appropriate planning, consideration of GPS survey specifications for connection to bench marks, proper field observing procedures, and a proper strategy for estimating geoid undulation differences and final orthometric height values, it is possible to use GPS survey methods to estimate orthometric heights to meet a wide range of engineering and land surveying requirements for vertical control.

Efforts to improve the accuracies of geoid undulation differences will depend on overall national accuracy needs for determining GPS-derived orthometric heights and on costs of differential leveling versus GPS and gravity survey methods. Therefore, another question needs to be addressed. What are the accuracy requirements of most engineering and land surveying applications, as well as mapping applications? This is best answered by the users, and will influence how much effort should be directed toward improving the models to estimate more accurate geoid values.

REFERENCES

Defense Mapping Agency and National Geodetic Survey, 1986: Proceedings of the Fourth International Geodetic Symposium on Satellite Positioning. University of Texas at Austin, Austin, Texas, April 28-May 2.

Federal Geodetic Control Committee, 1984: Standards and Specifications for Geodetic Control Networks. National Geodetic Information Center, NOAA, Rockville, MD 20852, 33 pp.

Fury, R. J., 1986: Geoid height estimation technology development to improve computed orthometric height as a complement to GPS, Internal report of the National Geodetic Survey, NOAA,Rockville, MD.

Hajela, D., 1990: Obtaining centimeter precision heights by GPS observations over small areas, GPS World, January/February, 55-59.

Milbert, D. G. and Dewhurst, W. T., 1990: The Yellowstone-Hebgen Lake geoid obtained through the integrated geodesy approach, Journal of Geophysical Research (submitted 1989).

NOAA, 1985: The Proceedings of the First International Symposium on Precise Positioning with the Global Positioning System. Rockville, MD, April 15-19, National Geodetic Information Center, NOAA, Rockville, MD 20852, 946 pp.

Rapp, R. H., 1981: The Earth's gravity field to degree and order 180 using Seasat altimeter data, terrestrial gravity data, and other data, Report No. 322. Department of Geodetic Science and Surveying, The Ohio State University, Columbus, Ohio.

Rapp, R. H. and Cruz, J. Y., 1986: Spherical harmonic expansions of the Earth's gravitational potential to degree 360 using 30' mean anomalies, Report No. 376. Department of Geodetic Science and Surveying, The Ohio State University, Columbus, Ohio.

Remondi, B. W., 1984: Using the Global Positioning System (GPS) phase observable for relative geodesy: modeling, processing, and results, Ph.D. dissertation, Center for Space Research, The University of Texas at Austin, Austin, Texas, 360 pp.

Remondi, B W., 1988: Kinematic and pseudo-kinematic GPS, Proceedings of the Satellite Division's International Technical Meeting, Colorado Springs, Colorado, September 19-23, 115-122.

Strange, W. E., 1989: GPS determination of groundwater withdrawal subsidence, Journal of Surveying Engineering, Vol. 115, No. 2, 198-206.

Vincenty, T., 1987a: On the use of GPS vectors in densification adjustment, Surveying and Mapping. Vol. 47, No. 2, 103-108.

Vincenty, T., 1987b: Geoid heights for GPS densifications, ACSM Bulletin, No. 111, December, 25-26.

Zilkoski, D. B., Balazs, E. I., and Bengston, J. M., 1989: NAVD 88 datum definition study, Draft NOAA Technical Report, (in preparation).

Zilkoski, D. B. and Hothem, L. D., 1989: GPS satellite surveys and vertical control, Journal of Surveying Engineering, Vol. 115, No. 2, May, 262-282.

Zilkoski, D. B. and Young, G. M., 1985: North American Vertical Datum (NAVD) Update, Proceedings of the U.S. Army Corps of Engineers 1985 Survey Conference, Jacksonville, Florida, February 4-8.