Computing unambiguous Total Electron Content and ionosphere delays using only carrier phase data from NOAA's CORS network

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Topics of Discussion

- Geodetic need for ionosphere delays
- CORS network
- Assumptions and model
- Equations and implications
- Initial tests
- Full day solution
- Future directions

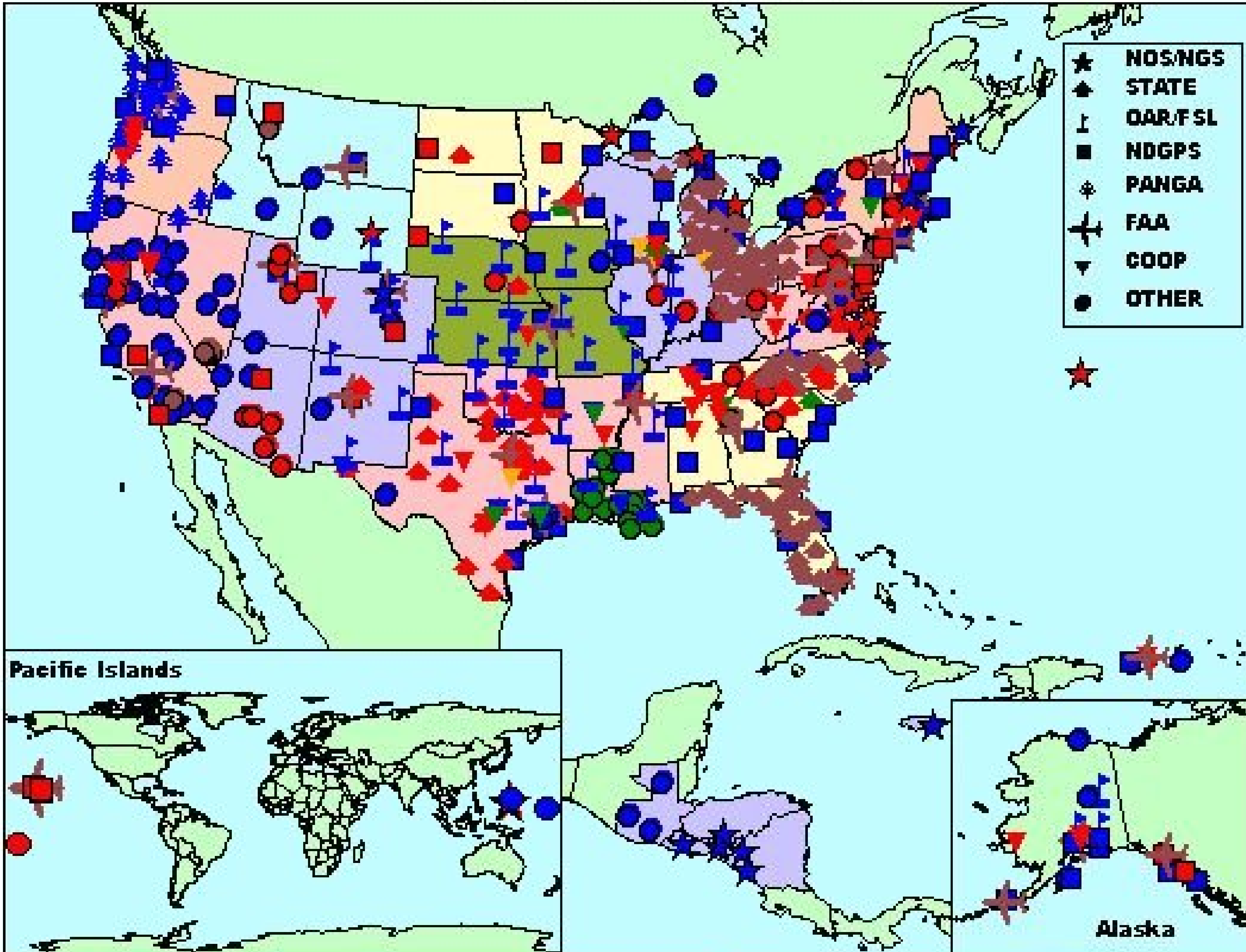
Geodetic need for ionosphere delays

- Dependent on
  - Absolute positioning versus single, double or triple differencing
  - 1, 2 (or 3) frequencies
- Solving for absolute (unambiguous) ionosphere delays for single frequency users solves all other geodetic positioning ionosphere needs

CORS Network

- ***Currently 400+ 24/7 receivers***
  - Dual frequency, carrier-phase
  - Multi-agency
  - Administered by NGS
  - All 50 states, Central America, others
  - *Ideally suited to serve as an ionosphere monitoring network for geodetic applications in the USA*

CORS Coverage - April 2004

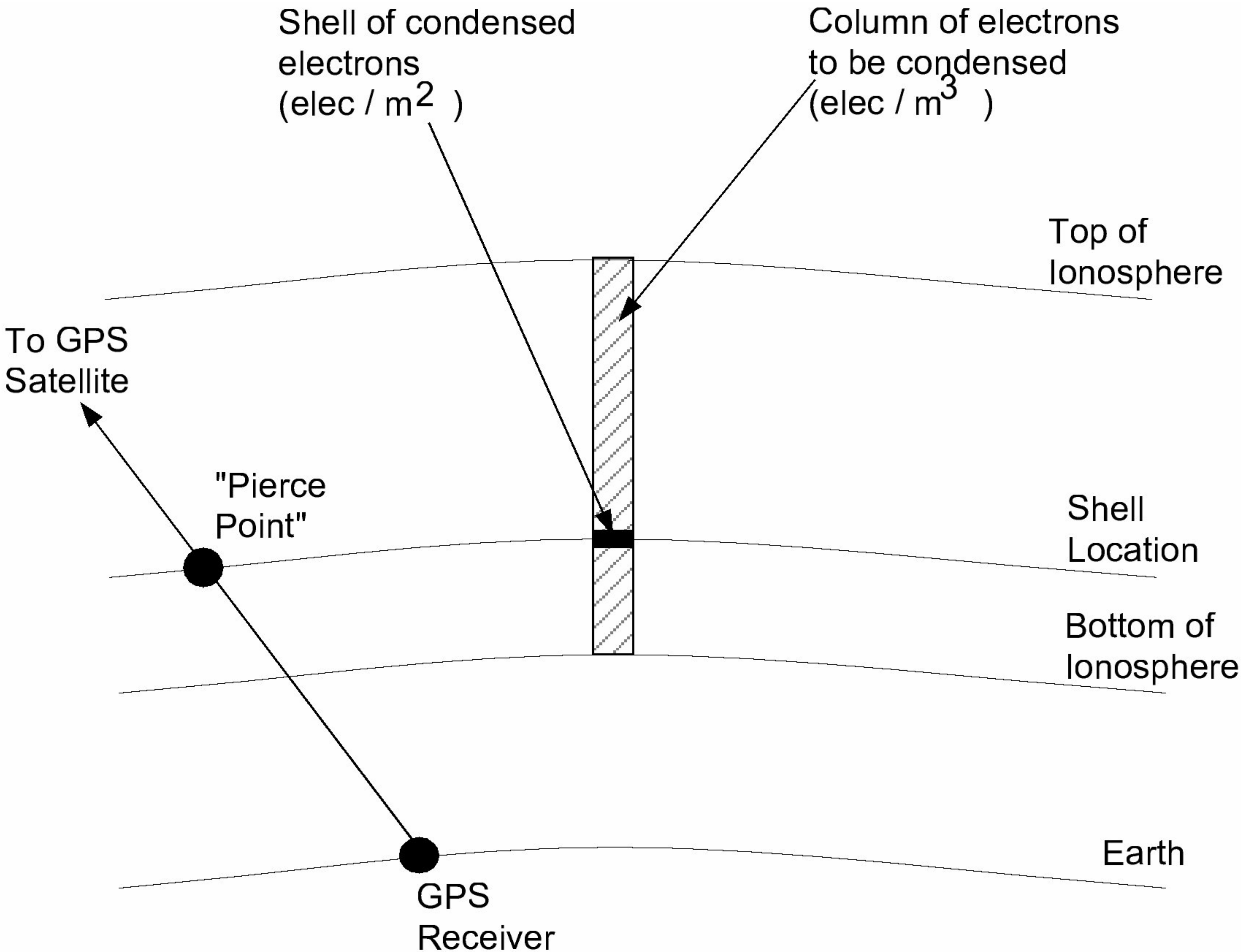


Symbol color denotes sampling rates: (1 sec) (5 sec) (10 sec) (15 sec) (30 sec)

Craig 4/15/2004

Assumptions and model

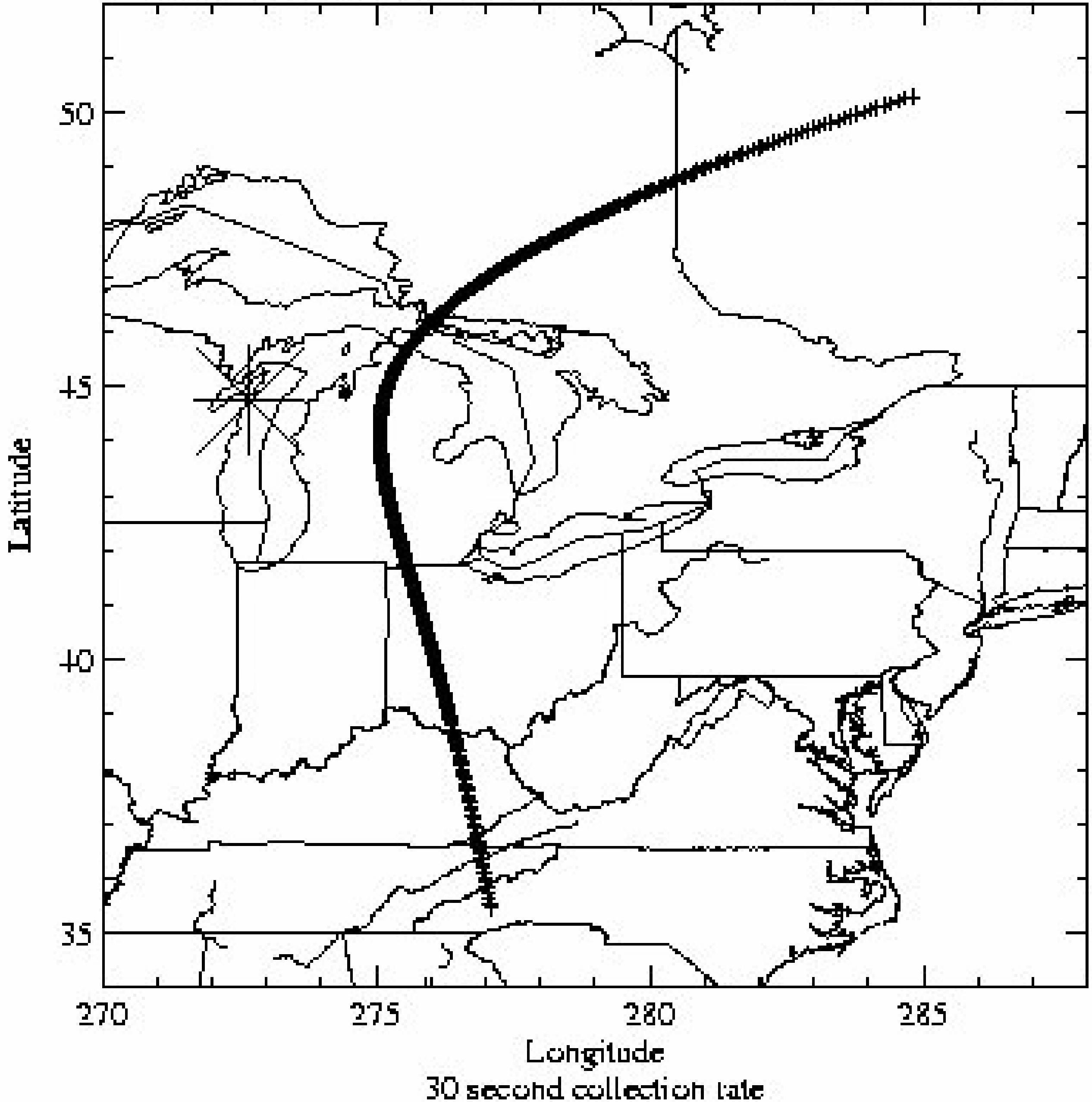
- 2-D “condensed” TEC shell
- Pros
  - Simplified equations, well known model
- Cons
  - Overly simple, far from true 3-D ionosphere
- Seen as a reasonable *first* step in a long range *geodetic* research strategy


Shell of condensed electrons (elec / m2)
Column of electrons to be condensed (elec / m3)
Top of Ionosphere
To GPS Satellite
"Pierce Point"
Shell Location
Bottom of Ionosphere
Earth
GPS Receiver

Pierce Points and Tracks

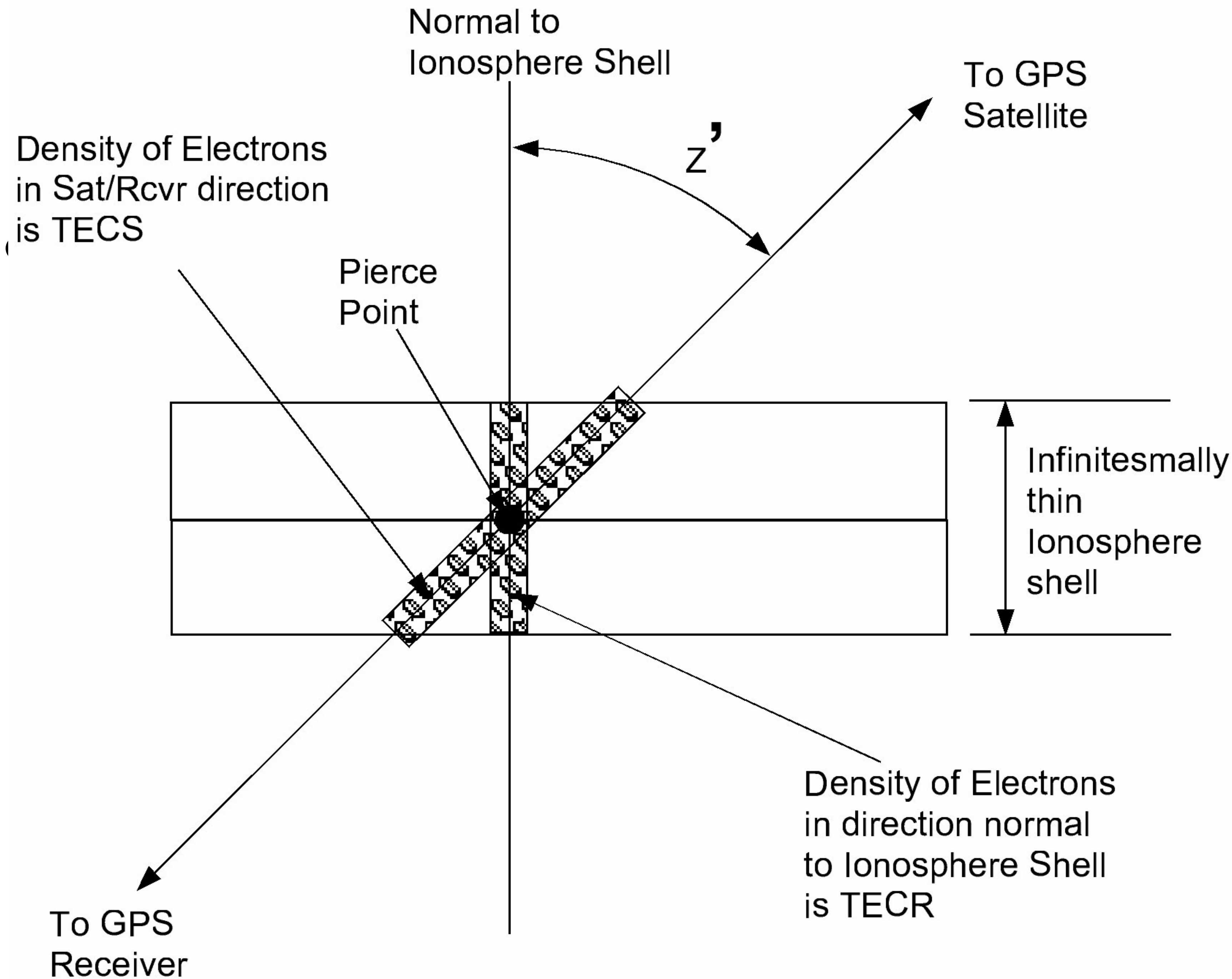
- A pierce point occurs at ionosphere shell for each data epoch
- Mapping pierce points without loss of lock yields a track
- CORS yields about 20,000 tracks every day

Track 10610 (CORS Station STB1 and GPS SV3)



TECS and TECR

- TECS is the TEC value seen in the satellite-receiver direction
- TECR is the vertical TEC value at the shell
  - TECS = TECR / cos z’
    - Questionable usefulness at low elevation angles



Normal to
Ionosphere Shell
To GPS
Satellite
Z'
Density of Electrons
in Sat/Rcvr direction
is TECS
Pierce
Point
Infinitesmally
thin
Ionosphere
shell
Density of Electrons
in direction normal
to Ionosphere Shell
is TECR
To GPS
Receiver

Equations

$$^{i}R_k = b_k + {}^{i}r + c({}^{i}\delta t) + {}^{i}T + {}^{i}I_k (+{}^{i}m_k) = \lambda_k {}^{\mathrm{i}}\Phi_k^{\mathrm{RINEX}} \qquad \text{(biased range, m, epoch "i", freq "k")}$$

$$I_k = -\frac{40.3}{f_k^2} TECS \qquad \text{(m)}$$

$$\therefore \quad \lambda_1 {}^{\mathrm{i}}\Phi_1^{\mathrm{RINEX}} - \lambda_2 {}^{\mathrm{i}}\Phi_2^{\mathrm{RINEX}} = (b_1 - b_2) + ({}^{i}I_1 - {}^{i}I_2)$$

$$\therefore \quad {}^{i}TECS = \left(\frac{1}{40.3}\right)\left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right)^{-1}\left[\lambda_1 {}^{\mathrm{i}}\Phi_1^{\mathrm{RINEX}} - \lambda_2 {}^{\mathrm{i}}\Phi_2^{\mathrm{RINEX}}\right]$$

$$-\left(\frac{1}{40.3}\right)\left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right)^{-1}(b_1 - b_2)$$

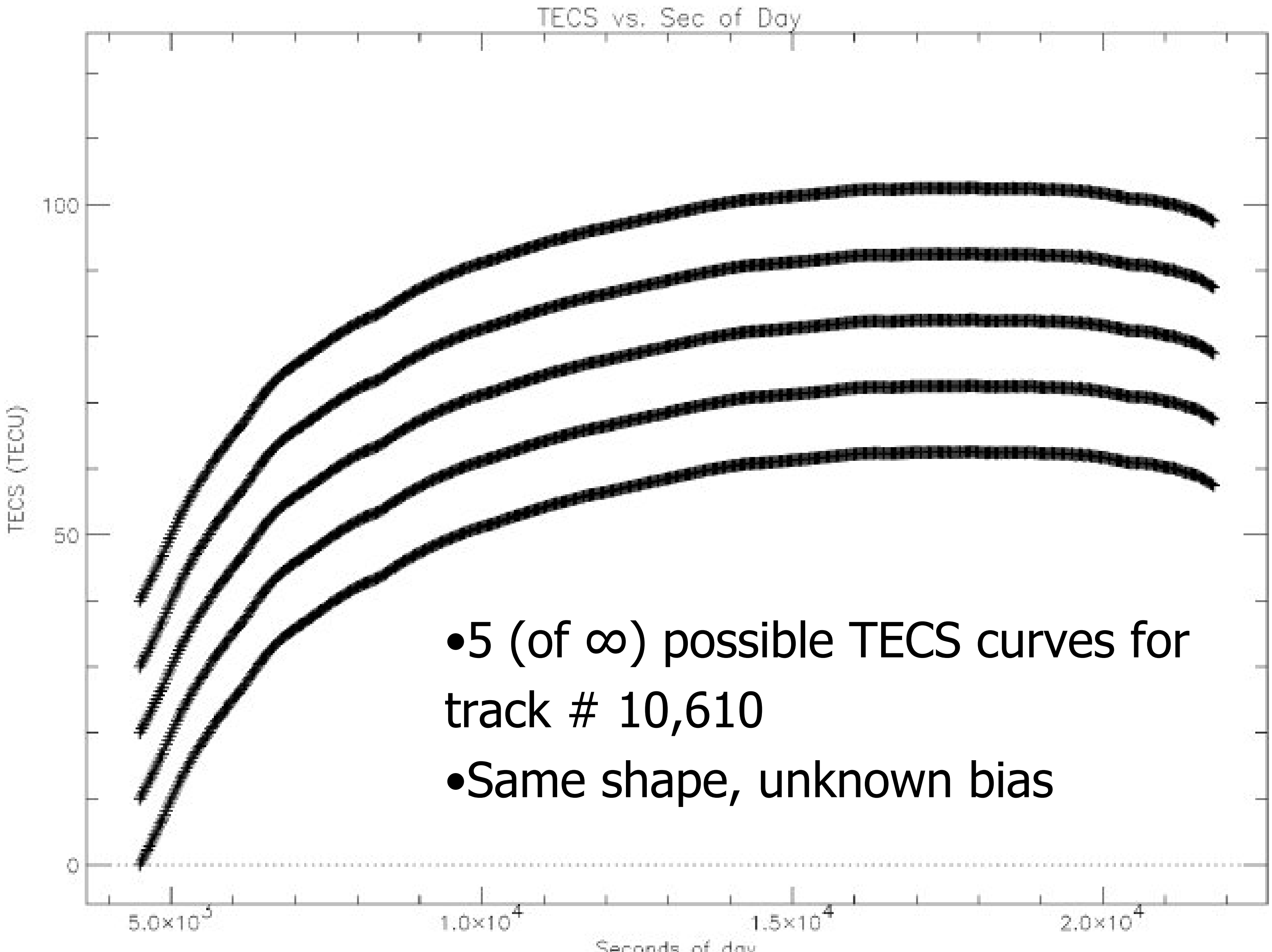
$$\therefore \quad {}^{i,j}\Delta TECS = {}^{j}TECS - {}^{i}TECS$$

$$= \left(\frac{1}{40.3}\right)\left(\frac{1}{f_1^2} - \frac{1}{f_2^2}\right)^{-1}\left(\lambda_1 {}^{i,j}\Delta\Phi_1^{\mathrm{RINEX}} - \lambda_2 {}^{i,j}\Delta\Phi_2^{\mathrm{RINEX}}\right)$$

Smith, D.A., Ionosphere from CORS, PLANS 2004

Implications of Equations

- Knowing ΔTECS:
  - Shape of "TECS vs time" curve known
  - Absolute level unknown
- Single, unknown bias per "track"



- 5 (of ∞) possible TECS curves for track # 10,610
- Same shape, unknown bias

$\Delta TECR$ vs $\Delta TECS$

$$ {}^{i}TECS = {}^{i}TECR / \cos {}^{i}z' $$

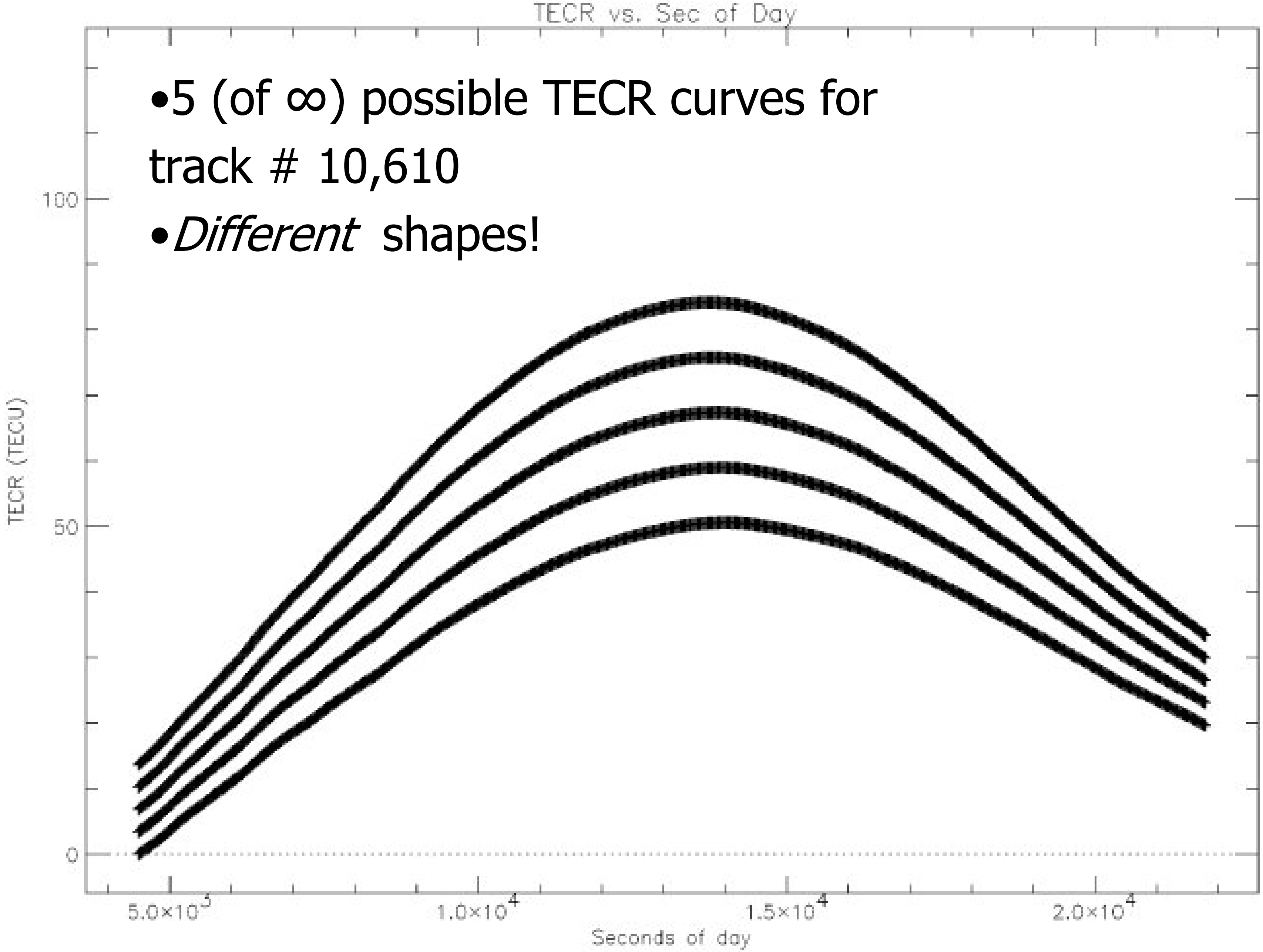
$$ {}^{i,j}\Delta TECR = {}^{j}TECR - {}^{i}TECR $$

$$ \therefore \quad {}^{i,j}\Delta TECR = {}^{i,j}\Delta TECS \cos {}^{j}z' + {}^{i}TECS \left(\cos {}^{j}z' - \cos {}^{i}z' \right) $$

Smith, D.A., Ionosphere from CORS, PLANS 2004

Implications

- Epoch-dependent cos z’ in TECR:
  - Shape of “TECR vs time” curve is *unknown*
  - Absolute level unknown



- 5 (of ∞) possible TECR curves for track # 10,610
- *Different* shapes!

Solving for biases

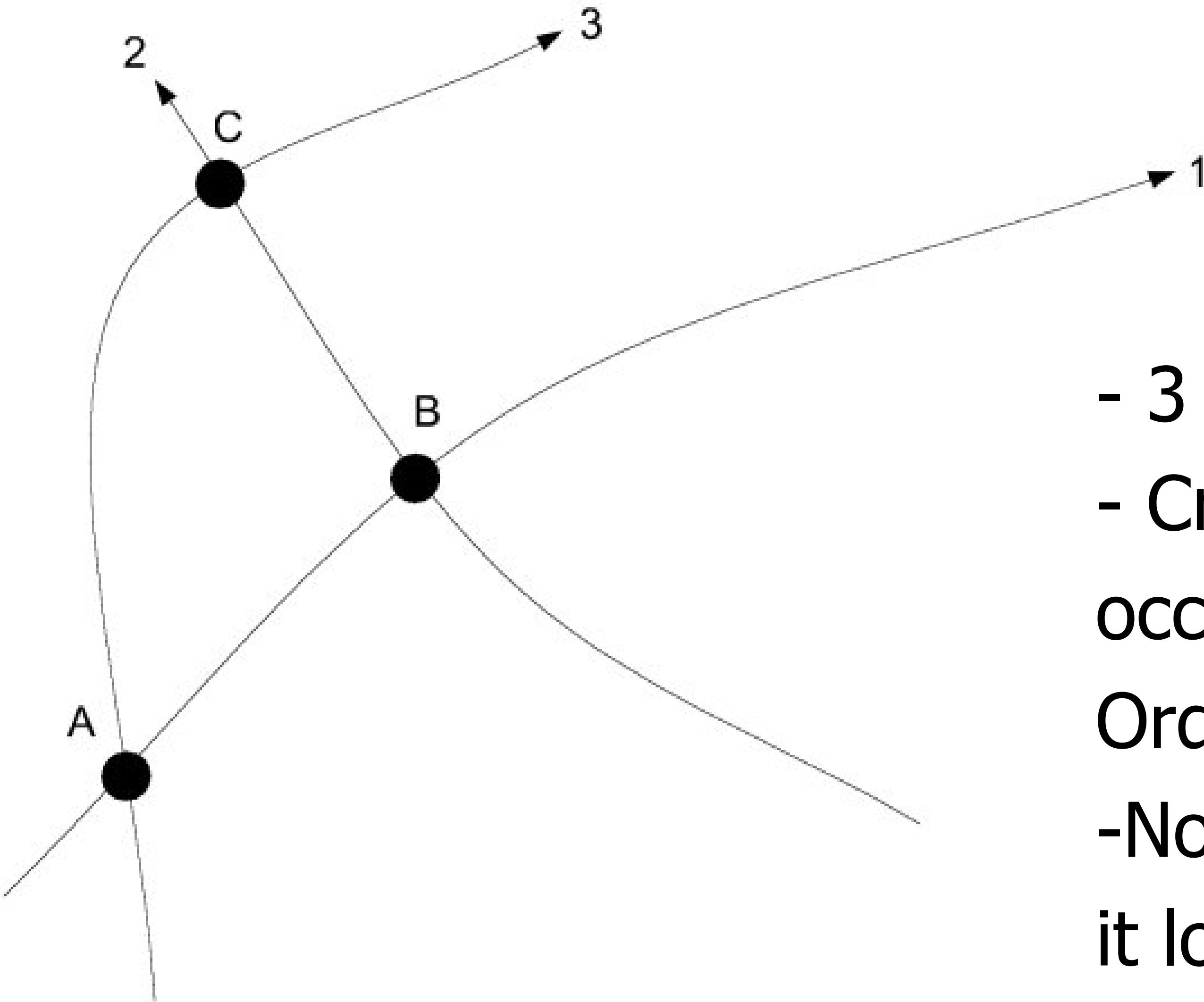
- Solve:
  - 1 TECS bias per track
- Yields:
  - Known TECS values
    - And thus known TECR values
- How? Modifying an old lesson from altimetry…

Crossovers

- Consider two tracks that pierce the ionosphere at the same place, at the same time
  - $TECS(\phi,\lambda,t,\text{track a}) \neq TECS(\phi,\lambda,t,\text{track b})$
  - $TECR(\phi,\lambda,t,\text{track a}) = TECR(\phi,\lambda,t,\text{track b})$
- Known:
  - ΔTECS on all tracks
  - TECR equality at crossovers

Using Crossovers

- By itself, one crossover has:
  - 1 condition (TECR equality)
  - 2 unknowns (TECS biases for 2 tracks)
  - Thus, unsolvable as is
- Need conditions ≥ unknowns
- Closed polygons is the solution



- 3 Tracks
- Crossovers A,B,C occur in sequential Order
-Not as rare as it looks

Closed Polygons

- Altimetry or Leveling (ΔH & H-equality):
  - # conditions = # vertices – 1
- Ionosphere (ΔTECS & TECR-equality)
  - # conditions = # vertices
- Any time that a closed polygon is formed on the ionosphere "shell" we have:
  - # Conditions = # Unknowns

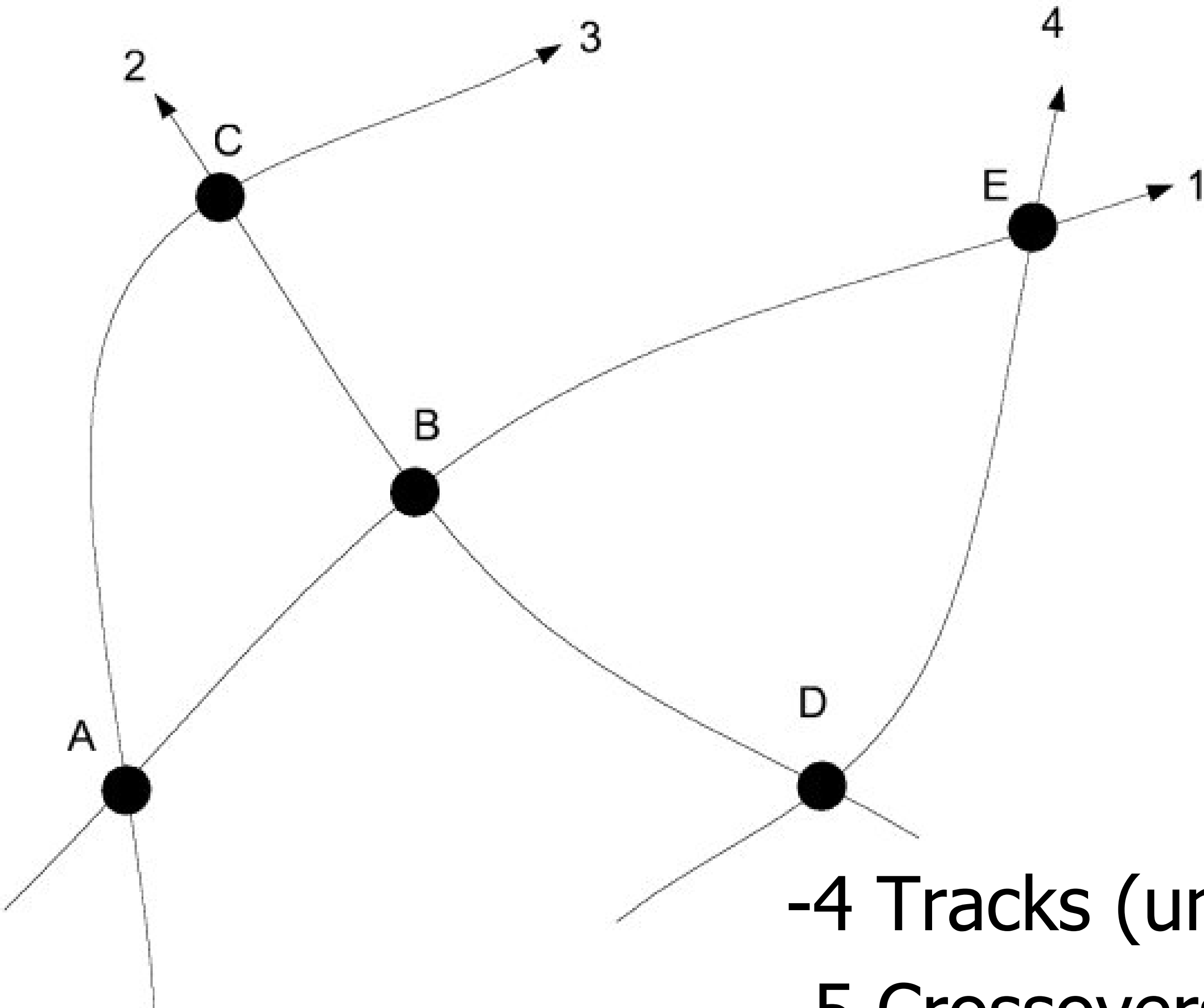
Polygon Crossover Equations

$$\begin{bmatrix} {}_1^A\Delta TECS \cos {}_1^A z' - {}_3^A\Delta TECS \cos {}_3^A z' \\ {}_1^B\Delta TECS \cos {}_1^B z' - {}_2^B\Delta TECS \cos {}_2^B z' \\ {}_2^C\Delta TECS \cos {}_2^C z' - {}_3^C\Delta TECS \cos {}_3^C z' \end{bmatrix} = \begin{bmatrix} -\cos {}_1^A z' & 0 & +\cos {}_3^A z' \\ -\cos {}_1^B z' & +\cos {}_2^B z' & 0 \\ 0 & -\cos {}_2^C z' & +\cos {}_3^C z' \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Smith, D.A., Ionosphere from CORS, PLANS 2004

Polygon Crossover Equations

- The existence of the cos z' values on the RHS allows for matrix inversion
  - (as opposed to +1,0 and -1 for altimetry)
- Solvability
- Can we have redundancy?
  - YES



-4 Tracks (unknowns)

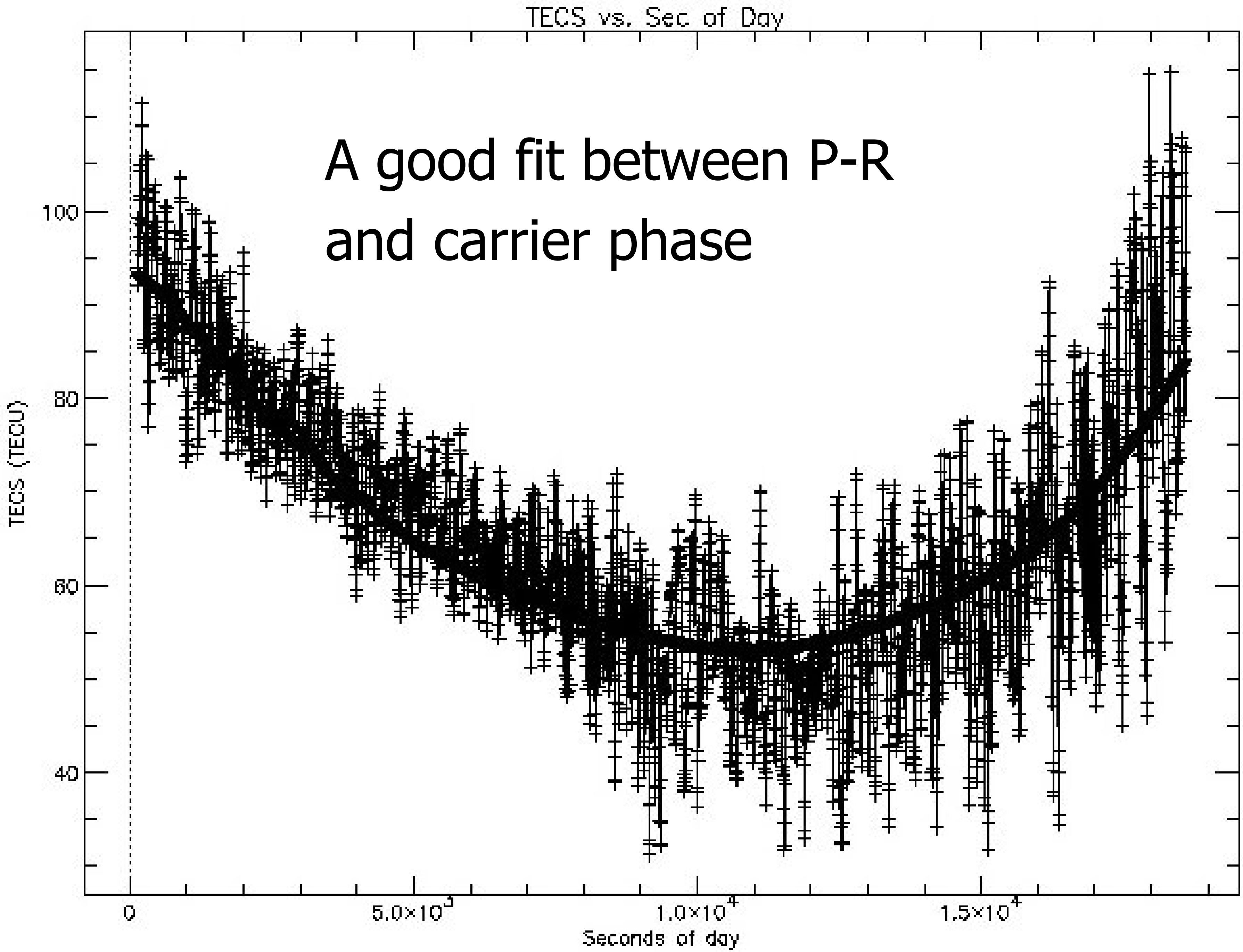
-5 Crossovers (conditions)

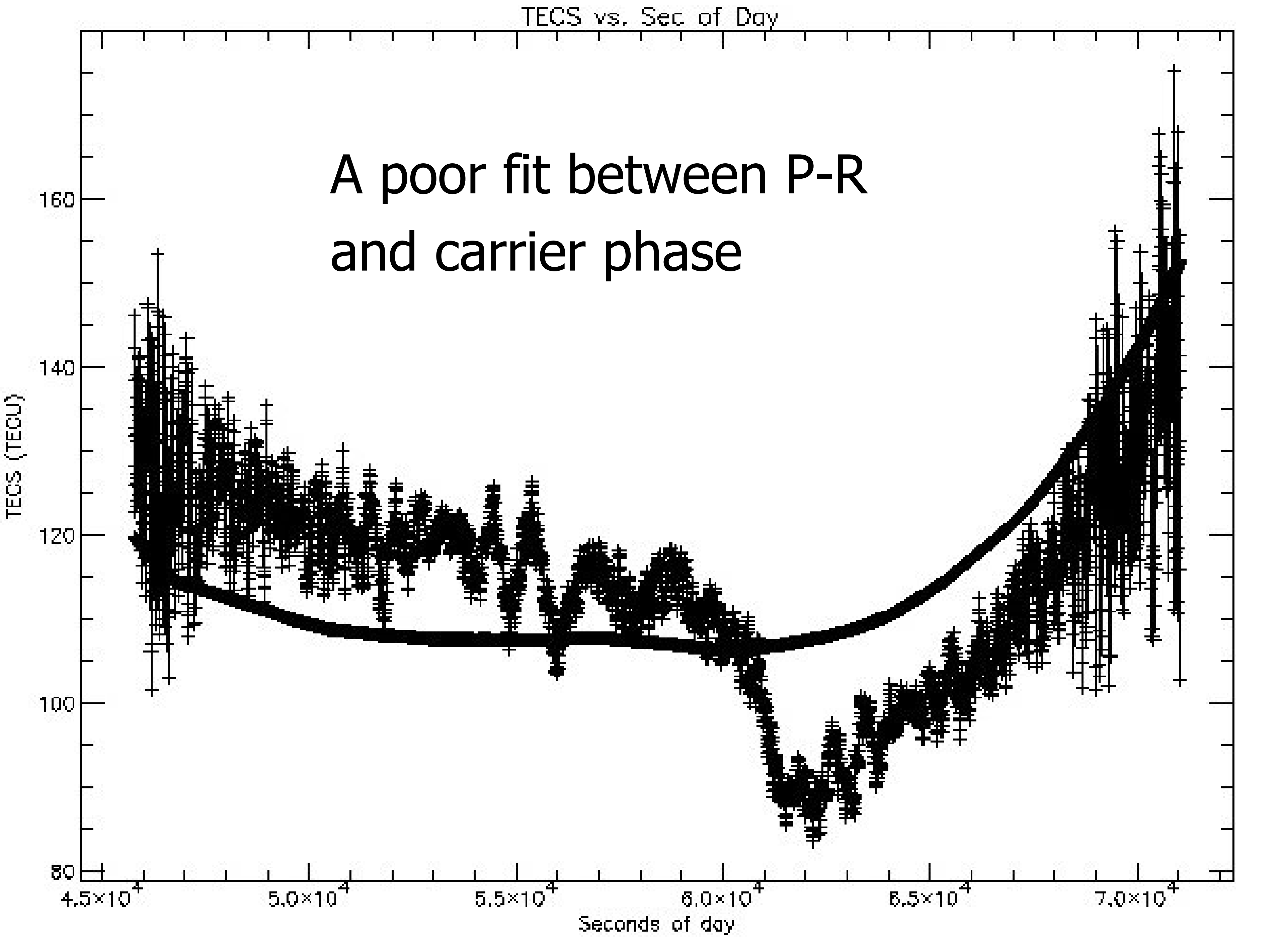
-Redundancy!

-(Yes, this really happens)

Initial Tests

- Form small (10-12 track) “tracknets”
  - Built around double-differencing
  - 2 CORS + 2 SV = 4 base tracks
- With/without closed polygons
  - If without polygons, use pseudo-range fitting
  - If with polygons, test with/without p-r fitting
- Test against NGS double-difference positioning software ionosphere estimates


TECS vs. Sec of Day
A good fit between P-R
and carrier phase
TECS (TECU)
100
80
60
40
0
5.0×10^3
1.0×10^4
1.5×10^4
Seconds of day


TECS vs. Sec of Day
A poor fit between P-R
and carrier phase
TECS (TECU)
80
100
120
140
160
4.5×10⁴
5.0×10⁴
5.5×10⁴
6.0×10⁴
6.5×10⁴
7.0×10⁴
Seconds of day

Initial Tests

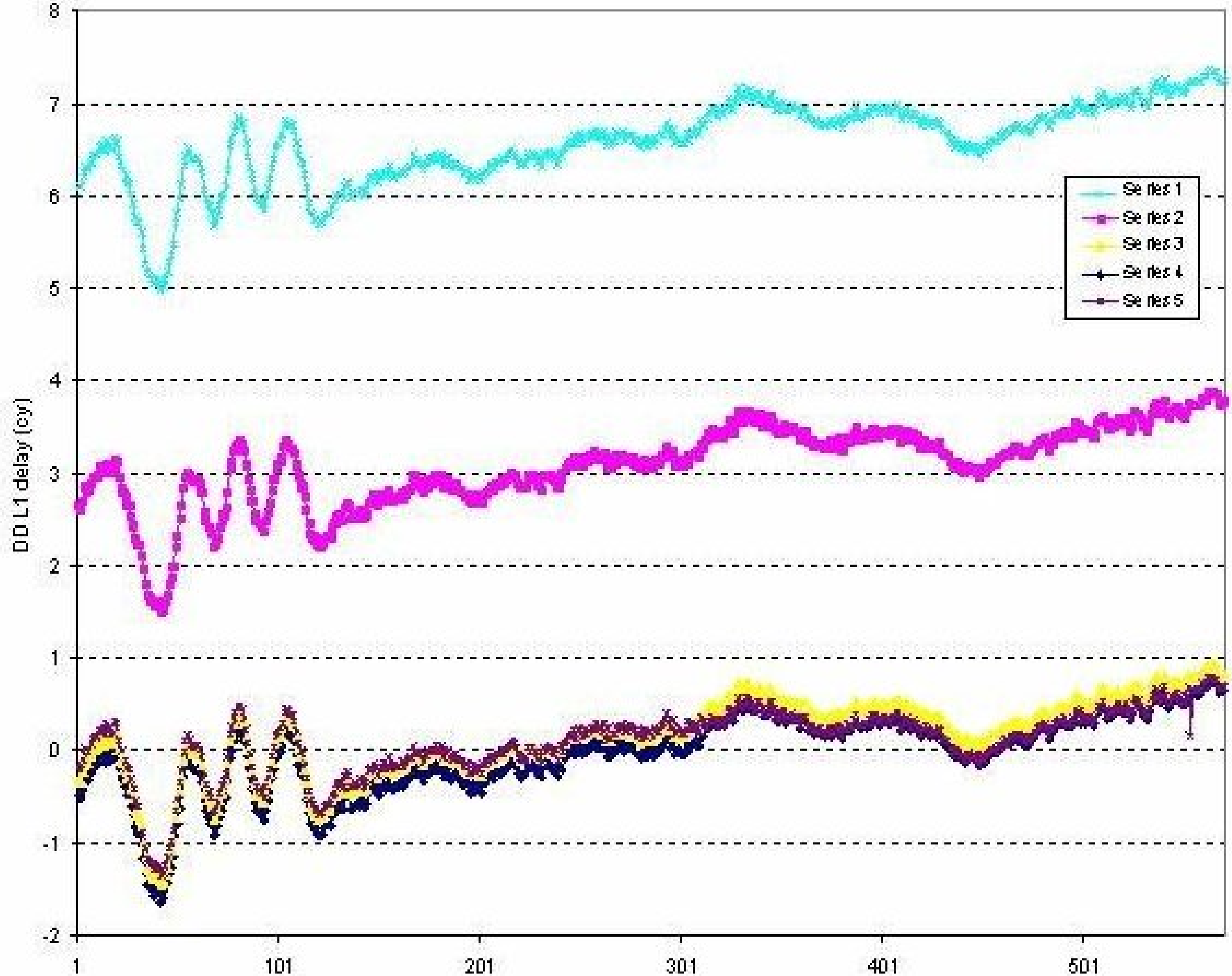
- Parameters:
  - Shell height = 300 km
  - Crossover definition: 0.1° x 0.1° x 1 min
  - Cut-off angle: 10° (for data and crossovers)

Initial Tests (all contain the 4 base tracks)

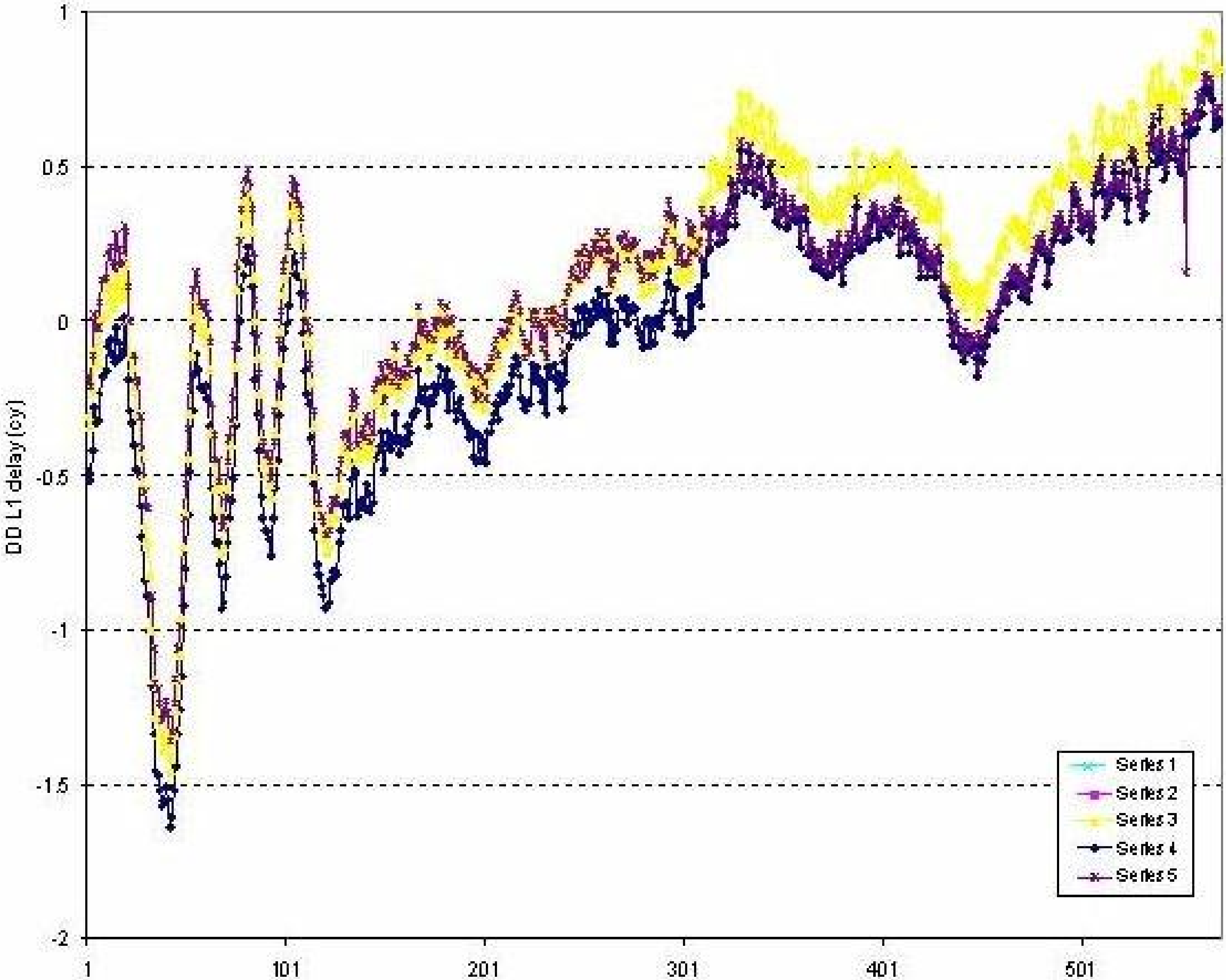
- Solution 1 (smallest tracknet possible containing the 4 base tracks)
  - 8 tracks, No polygons, PR-fit 6 of 8 tracks
- Solution 2
  - 10 tracks, 2 polygons, PR-fit 7 of 10 tracks
- Solution 3
  - 10 tracks, 2 polygons, no PR-fitting
- Solution 4
  - 10 tracks, 2 polygons, PR-fit 1 of 10 tracks

Formal σ_{bias} estimates for first tracknet tests (in TECU)

Track #	Soln 1 (PR fit to 6 of 8; no polygons)	Soln 2 (PR fit to 7 of 10; 2 polygons)	Soln 3 (No PR fit; 2 polygons)	Soln 4 (PR fit to 1 of 10; 2 polygons)
4300 (base)	3.5	2.9	0.1	1.2
4303 (base)	8.8	4.7	0.2	2.1
9484 (base)	9.3	4.6	0.2	2.0
9487 (base)	9.4	3.1	0.1	1.3
2253	13.6	5.9	0.3	2.5
10146	9.7	3.3	0.1	1.4
11416	6.5	4.9	0.2	2.0
12565	6.1	3.9	0.2	1.6
2224	-	4.3	0.2	1.7
11580	-	3.0	0.1	1.2



8
7
6
5
4
3
2
1
0
-1
-2
DD L1 delay (cy)
1
101
201
301
401
501
Series 1
Series 2
Series 3
Series 4
Series 5



DD L1 delay (cy)
1
0.5
0
-0.5
-1
-1.5
-2
1
101
201
301
401
501
Series 1
Series 2
Series 3
Series 4
Series 5

First tracknet tests

- Pseudo-range fitting tends to bias the tracknet
- Better fit to Double Difference estimated ionosphere by using just polygons and no P-R fitting

Full day solution

- After doing multiple small tracknets to prove the method, a full day was attempted
- Day 193 (July 12) of 2002
- 307 CORS stations
- 16,896 Crossovers (conditions)
- 8298 Tracks (unknowns)

Full day solution

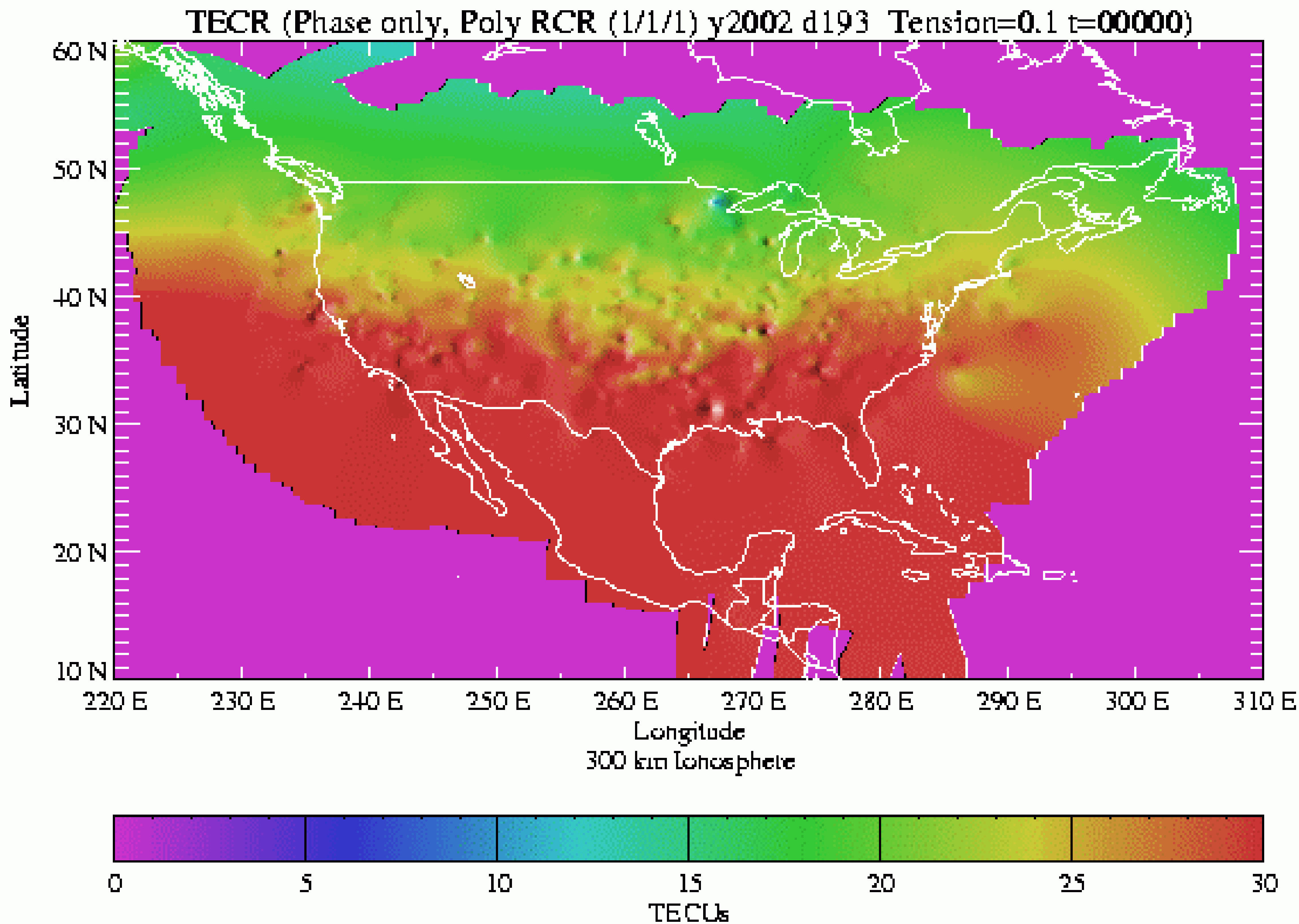
- Unzip/read hundreds of RINEX.gz files
  - 4 hours
- Clean 11 Million data pts (cycle slips, etc)
  - 1 hour
- Solve 8298 x 16896 sparse linear system
  - 30 seconds to get 8298 biases
  - 10 minutes to get σ_{bias}

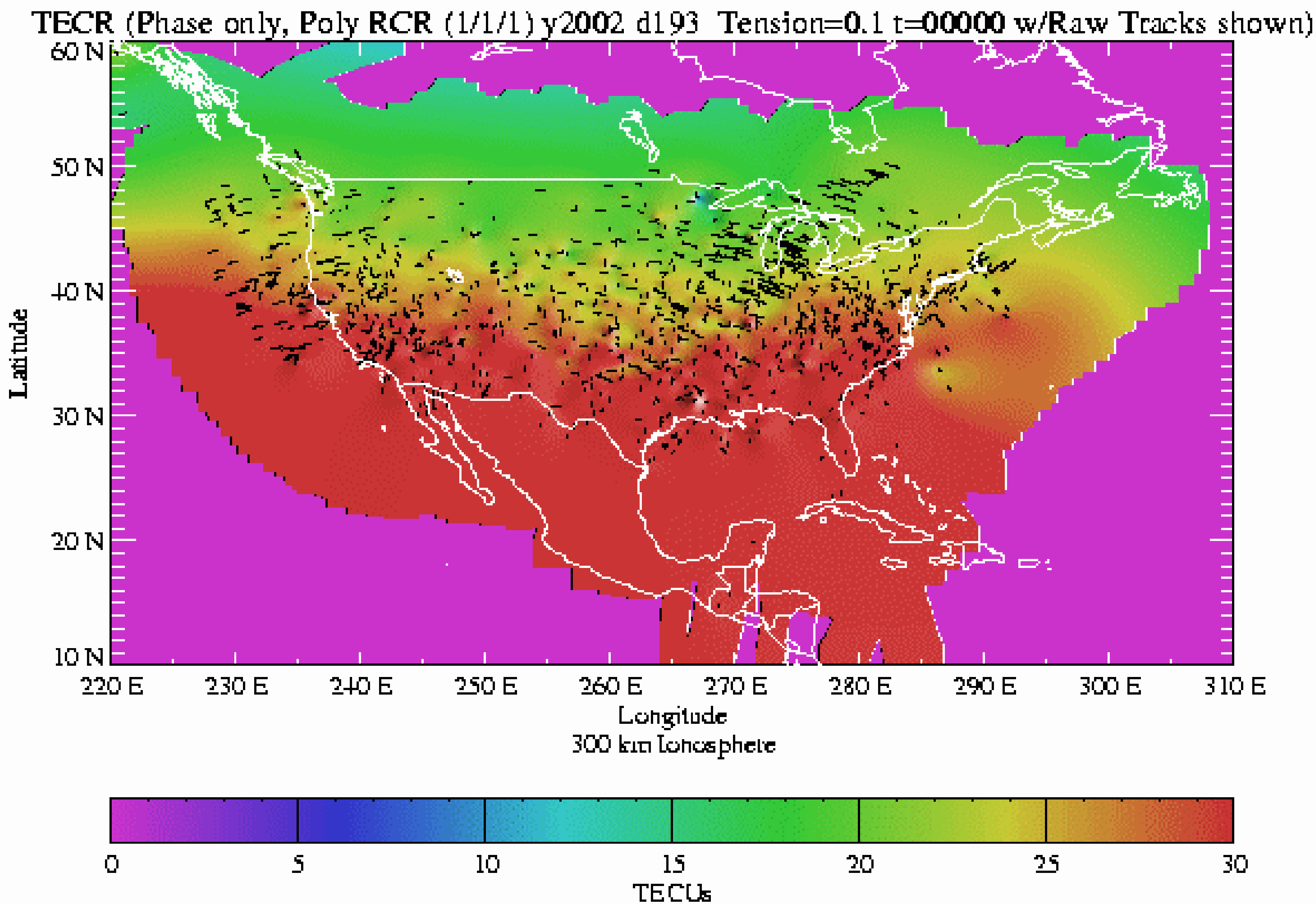
Full day solution (cont)

- Post Fit Crossover stats (TECUs)
  - -0.004 ± 0.51 (Min -3.7; Max +4.0)
- A-posteriori σ_{bias} estimates:
  - Ave(σ_{bias}) = $\pm$ 1.1 TECU (Min 0.22, Max 10.7)
- Tracks were gridded and animated

Full day solution Animations

- Animation without tracks
  - gif
  - avi
- Animation with tracks
  - gif
  - avi


TECR (Phase only, Poly RCR (1/1/1) y2002 d193 Tension=0.1 t=00000)
60 N
50 N
40 N
30 N
20 N
10 N
Latitude
220 E
230 E
240 E
250 E
260 E
270 E
280 E
290 E
300 E
310 E
Longitude
300 km Ionosphere
0
5
10
15
20
25
30
TECUs


TECR (Phase only, Poly RCR (1/1/1) y2002 d193 Tension=0.1 t=00000 w/Raw Tracks shown)
Latitude
60 N
50 N
40 N
30 N
20 N
10 N
220 E
230 E
240 E
250 E
260 E
270 E
280 E
290 E
300 E
310 E
Longitude
300 km Ionosphere
0
5
10
15
20
25
30
TECUs

Full day solution (cont)

- Average a-posteriori σ_{bias} of ± 1.1 TECU reasonable, but larger than hoped for
- Sub-TECU crossover residuals show tight "locking" or consistency of tracknet
- Overall noise in grids needs improvement
- General conclusion:
  - "Promising" but not by any means "done"

Summary and Conclusions

- With certain assumptions, a model for the ionosphere can be computed as an entire network
  - to ~1 TECU (absolute), *subject to biases*
  - to ~0.01 TECU agreement to Double Difference estimates, subject to cycle-slip fixing

Summary and Conclusions (cont)

- Further sensitivity studies:
  - Removing near-horizon crossovers
  - Shell height
  - CORS thinning
- Independent tests forthcoming:
  - Against other ionosphere models
  - In ambiguity resolving software

Contact Information

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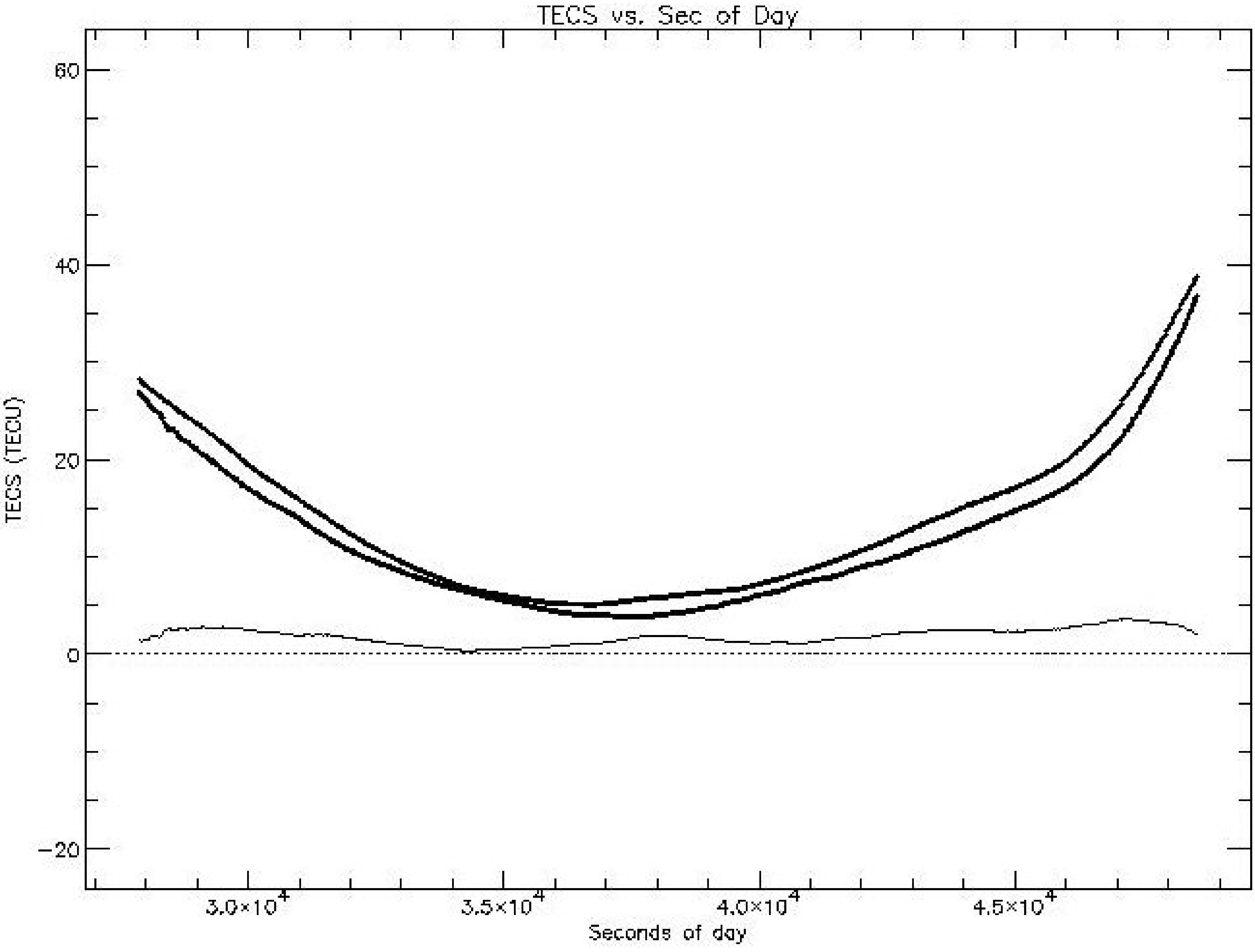
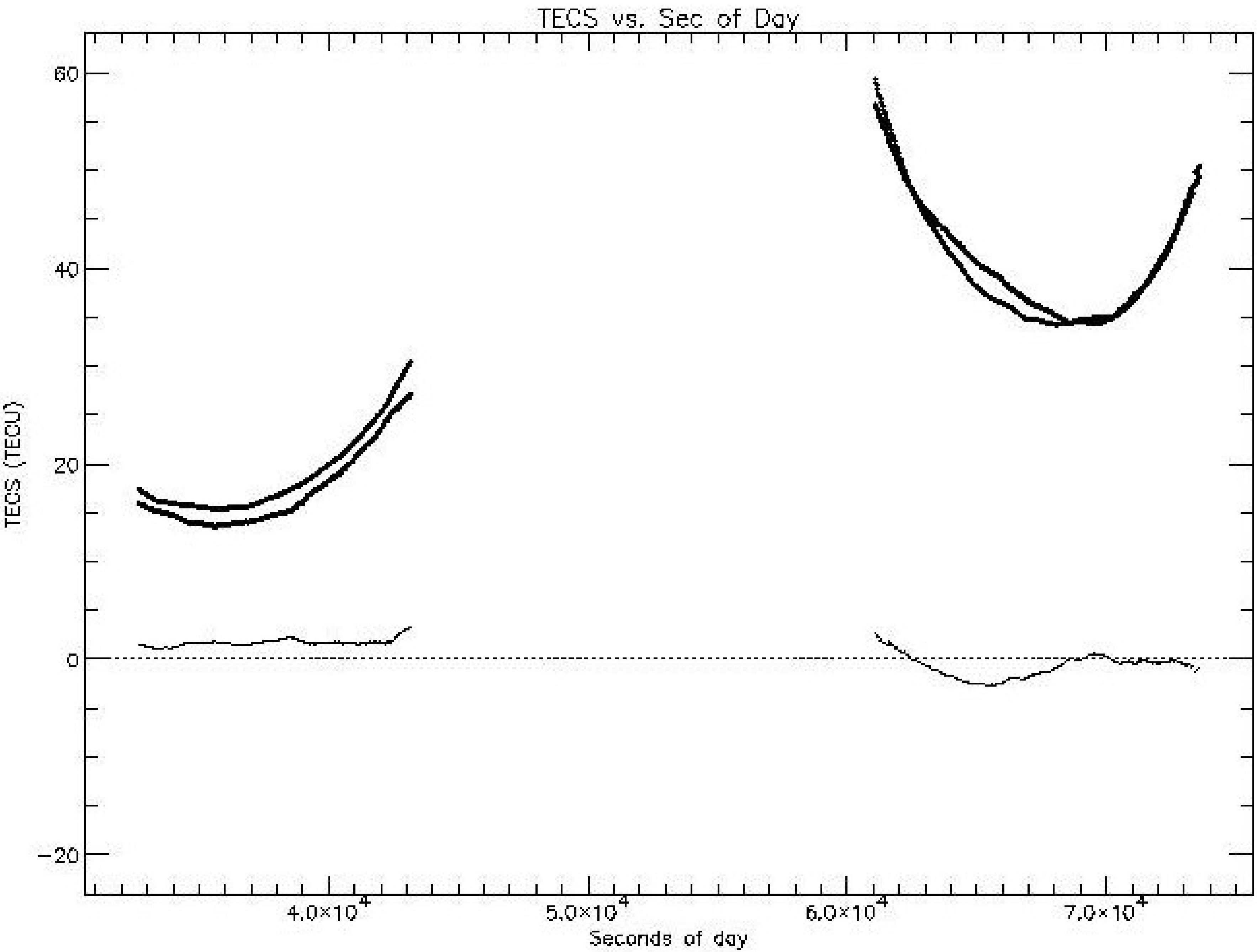
Questions?

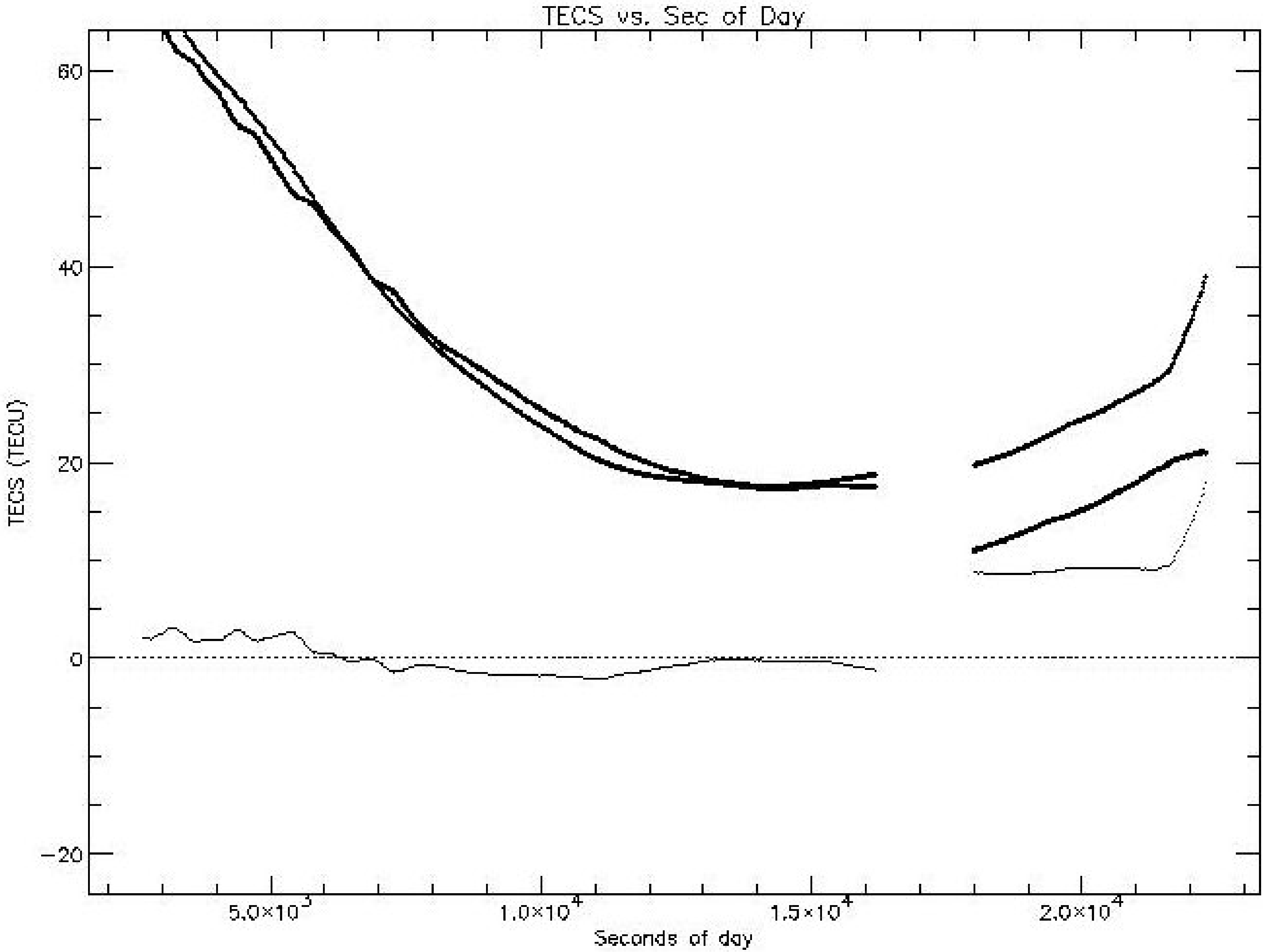
Extra Slides



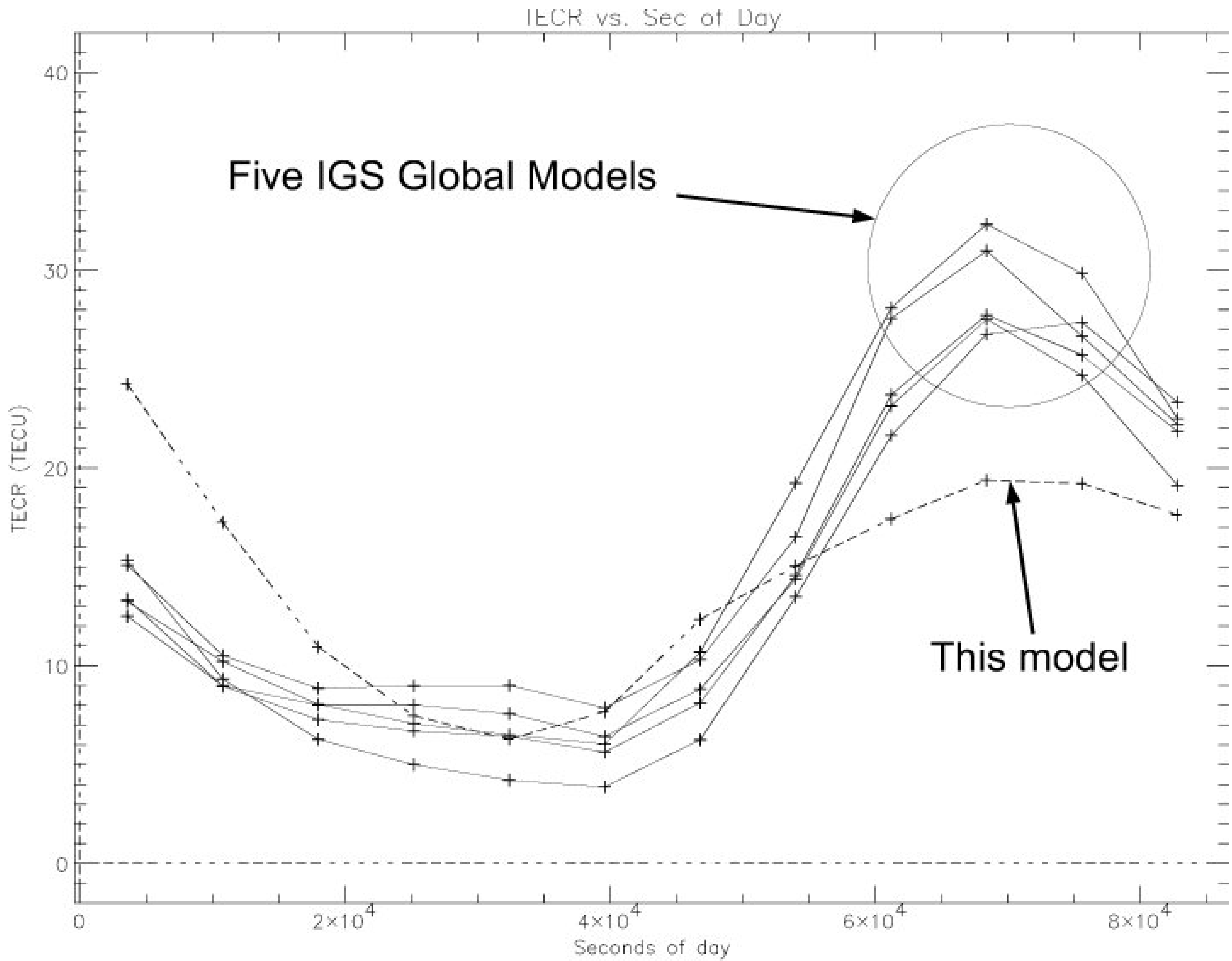
TECS vs. Sec of Day
60
40
20
0
-20
TECS (TECU)
3.0×10^4
3.5×10^4
4.0×10^4
4.5×10^4
Seconds of day



TECS vs. Sec of Day
60
40
20
0
−20
TECS (TECU)
4.0×10^4
5.0×10^4
6.0×10^4
7.0×10^4
Seconds of day



TECS vs. Sec of Day
60
40
20
0
−20
TECS (TECU)
5.0×10^3
1.0×10^4
1.5×10^4
2.0×10^4
Seconds of day



TECR vs. Sec of Day
Five IGS Global Models
This model
TECR (TECU)
Seconds of day
0
10
20
30
40
0
2×10^4
4×10^4
6×10^4
8×10^4