State Readjustments at the National Geodetic Survey

Kathryn O. Milbert and Dennis G. Milbert

ABSTRACT. *In the program of placing high accuracy global positioning satellite (GPS) surveys throughout the states, it is desirable to readjust existing control points by a state readjustment. The GPS work has detected sizable (0.5 m-plus) long-wavelength errors in the NAD 83 (86) control. The NAD 83 (86) errors are well within the accuracy classifications (1:100,000). However, the errors cause problems along the borders of readjusted states, where the coordinate differences must be accommodated. In this paper, we analyze a number of approaches to state readjustments. We find that readjusted states controlled by 1:1,000,000 GPS data may still absorb a surprising amount of error, and we develop a strategy and recommend procedures for future state readjustments.*

Introduction

Vectors from the Global Positioning Satellite (GPS) system have provided a windfall for geodetic control networks. The National Geodetic Survey (NGS) routinely uses GPS technology in its horizontal control field parties. The ability to obtain 1:1,000,000 relative accuracy, while freed from requirements of station intervisibility, has lead to unprecedented levels of quality and productivity.

Since the completion of the NAD 83 (86) readjustment of the United States, the decreasing cost of geodetic GPS receivers has made this technology available to a significant portion of the surveying and mapping community. Initially, NGS attempted to fit lower order (first through third) GPS projects into the existing NAD 83 (86) control network. It was soon realized, however, that surveys at 1:1,000,000 were now possible. The increasing use of GPS technology has brought about an interesting conflict. Suppose one surveys at 1:1,000,000, but ties into a NAD 83 (86) control network accurate to about 1:300,000. The dilemma of "degrading" GPS through a constrained adjustment, or "upsetting" network control points through a free adjustment is highlighted in the paper "The Trouble with Constrained Adjustments" (Schwarz 1994). We advise readers to refer to this paper for the mathematical basis of this dilemma.

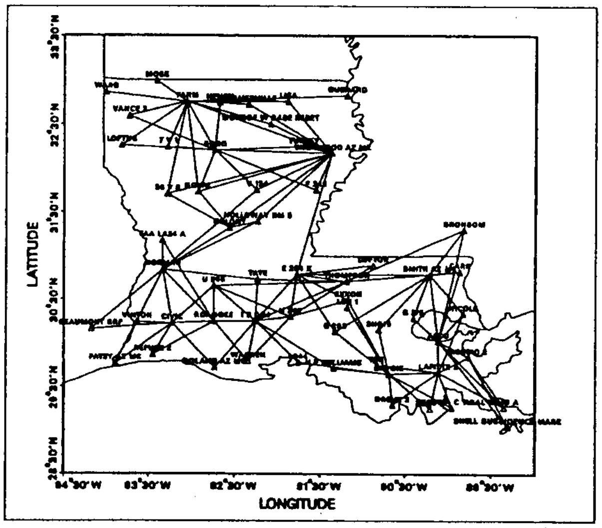
NGS is mitigating this dilemma by carrying out a program of individual state readjustments, which are performed in conjunction with the new high accuracy GPS surveys. Stations involved in these surveys are categorized as Federal Base Network (FBN) or Cooperative Base Network (CBN). FBN stations are established at a nominal 1° × 1° spacing, are surveyed at B order (1:1,000,000) accuracy, and are maintained at the expense of NGS. A portion of the FBN is set at a nominal 3° × 3° spacing, and is surveyed to A order (1:10,000,000) accuracy. NGS encourages individual states to establish additional B order stations at about 15′ × 15′ spacing. These additional stations are designated CBN. The FBN and CBN stations are often observed in a single cooperative GPS survey, frequently known as a High Accuracy Reference Network (HARN).

After the FBN/CBN is established in a state, the remaining horizontal control network is readjusted and the set of improved coordinates are designated NAD 83 (xx) (xx being the year of the GPS survey). The fundamental NAD 83 adjustment is named NAD 83 (86). This program of high accuracy state network upgrades across the nation is described by Bodnar (1990). After the state readjustment, the classical control has been brought into agreement with the FBN/CBN. This will lessen the dilemma faced by the surveyor when relating new GPS work to the control network.

At a higher level, the dilemma is still faced by NGS. Due to the piecemeal (state-by-state) surveying of the FBN/CBN and the immediate demand for consistent coordinates of nearby classical control points, the statewide readjustments are performed in a piecemeal fashion, also. This gives rise to inconsistencies along the border when a given state is readjusted to the FBN/CBN, but the adjacent state has not yet been readjusted.

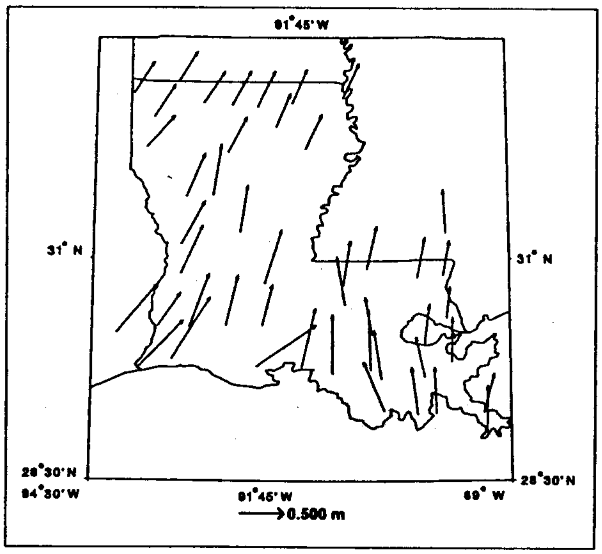
The differences between the NAD 83 (86) datum coordinates and the new FBN/CBN coordinates may be surprising at first glance. It is not unusual to see differences of 0.5 m or more. To illustrate, Figure 1 displays the B order survey conducted in the state of Louisiana. Figure 2 shows the horizontal shifts from the NAD 83 (86) coordinates to the FBN/CBN coordinates adjusted relative to A order control. Although the position shifts are sizable, the relative shifts are small. A 0.5-m position shift is entirely

Kathryn O. Milbert and Dennis G. Milbert are geodesists at the National Geodetic Survey, Coast and Geodetic Survey, 1315 East-West Highway, Silver Spring, MD 20910.



Lousiana B Order

Figure 1: Sketch of Louisiana High Accuracy Reference Network (FBN/CBN) of B order (1:100,000) accuracy.



Shifts Louisiana B Order Versus NAD83(86)

Figure 2: Louisiana FBN/CBN minus published NAD 83 (86) coordinate differences. Control points are located at the base of arrows. Scale indicated at base of map.

consistent with first order NAD 83 (86) accuracy when considered across the entire United States. Further information on NAD 83 (86) accuracy may be found in Snay (1990).

The resolution of the dilemma posed above is achieved by minimizing coordinate inconsistencies along the border in a state readjustment. In order to make the positions consistent, observations from an area larger than the state being readjusted are retrieved from the data base and included in the adjustment. Positions in adjacent states are readjusted (as necessary) in order to maintain the relative shifts across the boundary below specified tolerances. This fine tuning, and readjustment of stations in adjacent states, is called "feathering."

Table 1. Summary of test adjustment notation.

Code	Description
[N]	NAD 83 (86) North American Datum of 1983
[L]	Published Louisiana state readjustment, NAD 83 (92)
[0]	Initial adjustment in feathering procedure
[1]	Second adjustment in feathering procedure
[2],..,[n]	Subsequent adjustments in feathering procedure
[A]	Only A and B order points held in Louisiana
[B]	A and B order held, plus points 15 arc-minutes beyond Louisiana border (Figure 3)
[C]	A and B order held, plus perimeter points of retrieval area (Figure 13)
[L0]	A file containing [L] coordinates and [A] coordinates outside Louisiana
[L1],..,[Ln]	Subsequent files containing different mixtures of [L] and [A] coordinates
Note: [L0], [L1],..., [Ln] are coordinate files simulating adjustments. They were not from a simultaneous least squares adjustment.	

In this paper we describe a procedure used to feather coordinate discontinuities along state readjustment borders. We then extend the analysis to encompass points internal to the state. Simulations are run to identify readjustment requirements, and an alternate feathering strategy is developed. The simulations and the alternate strategy are then verified through a set of constrained adjustments. Based on this research, procedures are recommended which we have found to be superior to previously used methods in accommodating coordinate discontinuities.

Notation

In a study of various adjustment strategies, one compares many different sets of coordinates. Confusion can be significantly reduced by adopting a notation. We use brackets to denote the coordinates of an adjustment expressed by a number or letter. Thus, [N] = coordinates of adjustment "N" (NAD 83 (86)).

Position shifts, dS, between two adjustments [A][B] are computed as

$$dS[A][B] = \sqrt{\left(dN^2 + dE^2\right)}$$

where

dN = position shift in latitude (meters)
dE = position shift in longitude (meters)

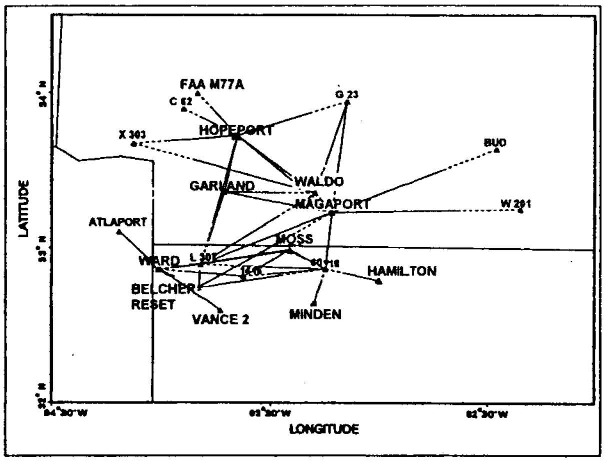
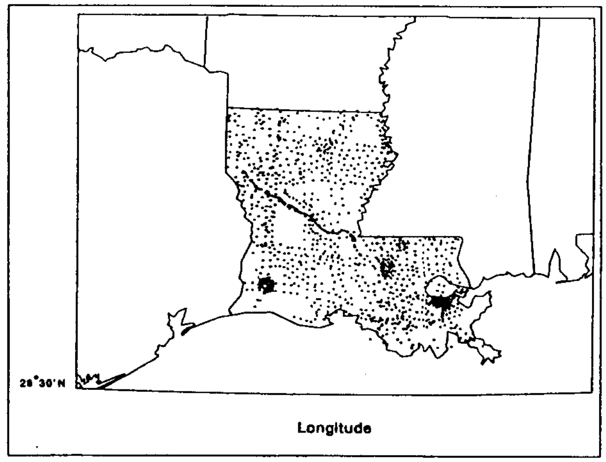


Figure 16: First order GPS network in the Louisiana/Texas/Arkansas area. Note the relationship of WARD, L 307, and HOPEPORT.



First Order Control Points, Louisiana

Figure 17: Distribution of first order control in Louisiana.

If our goal is merely to provide a consistent set of coordinates for the surveyor's use, then we have a great deal of flexibility in our approach to state readjustments. The Louisiana State Readjustment [L] produced coordinates that met the accuracy requirements of the NAD 83 (86) with only a minimum amount of readjustment of positions in the adjacent states.

We feel that, in general, it is far preferable to disperse coordinate shifts into adjacent states, rather than to let the NAD 83 (86) coordinate shifts in those adjacent states affect the updated coordinates in the state being readjusted. This implies that stations in adjacent states will have to be readjusted much more freely than they were for the Louisiana adjustment [L].

This preference is based in large measure on the fact that FBN/CBN surveys are proceeding at a rapid pace. The adjacent state networks will soon receive their state upgrade readjustments, we don't want those coordinates to contain residual distortions from NAD 83 (86) constraints, as occurred in Louisiana [L].

We recognize that it is an inconvenience to the states without FBN/CBN networks in place to have updated coordinates thrust upon them, when the previously published NAD 83 (86) coordinates may have been adequately meeting their needs. However, we have to weigh this inconvenience against the needs of the state being readjusted.

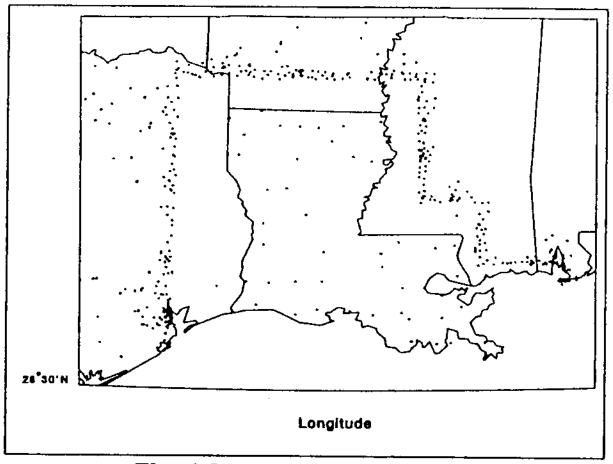
The analysis of Louisiana indicates that at least some readjustment of positions in Louisiana will be necessary when Mississippi, Texas and Arkansas are adjusted to the FBN/CBN. Again, it might be argued that the readjustment need not be as extensive as our readjustment analysis suggests. If we are willing to settle for the minimum requirement of maintaining accuracies to the NAD 83 (86), then it would be possible to minimize the number of readjusted positions. In doing this, we would maintain a set of consistent coordinates but they would be a cross between the NAD 83 (86) and the FBN/CBN near the state boundaries. When the entire FBN/CBN is in place, we would find a nationwide network with very FBN/CBN-consistent coordinates in the interior of states, degrading towards the NAD 83 (86) at the various state boundaries. We must avoid this situation, and we must minimize future readjustments of upgraded states.

Conclusions and Recommendations

The coordinate shifts between the various readjustments are all caused by the discrepancies between highly accurate GPS data, expressed by the FBN/CBN, and less accurate triangulation surveys, expressed by NAD 83 (86). The results in this paper show that coordinate shifts display a surprising amount of variation, based on one's selection of constrained points. And, as seen in Figure 16, behavior can be complicated when observation lines are very long. For these reasons it is mandatory that graphical analysis of all points, inside and outside a state, be a routine part of the feathering procedure.

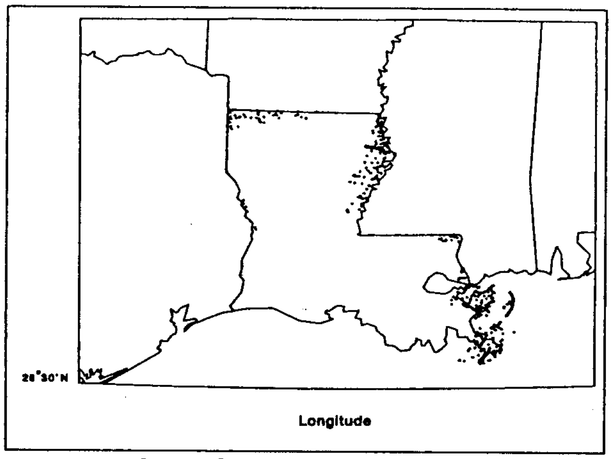
When FBN/CBN versus NAD 83 (86) coordinate differences are large, it will be necessary to retrieve an area large enough to effectively spread these shifts through adjacent states. To assist in obtaining satisfactory area retrievals, we can propose a rule of thumb.

- Examine the first order control in a state (see Figure 17).
- Compute the dS[FBN/CBN][NAD 83 (86)] position shifts within that state (see Figure 2).
- At the boundaries, divide the average position shifts by first order accuracy.



Fixed Control, Perimeter Held

Figure 13: Constrained points used in test adjustment [C]. These points are at the perimeter of the retrieval area and include Louisiana FBN/CBN constraints.



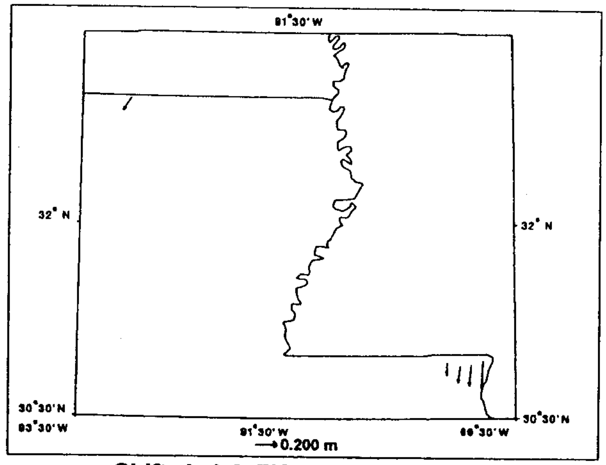
Large Shifts, Perimeted Held

Figure 14: Locations of coordinate shifts, *dS*[C][A], in excess of 0.1 m, between test adjustment [C] and test adjustment [A] (only A and B order points held).

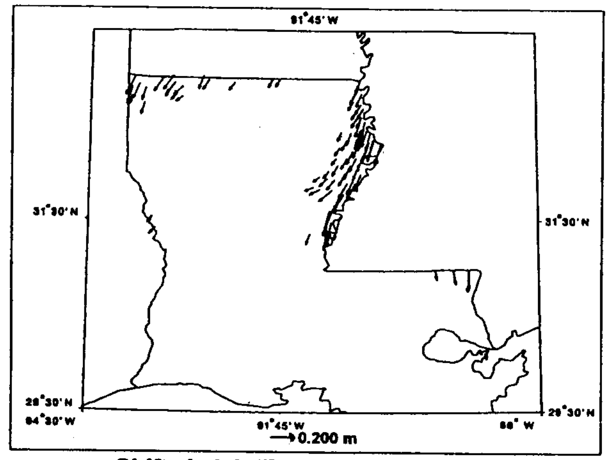
- to minimize readjustment effects in the adjacent states.

The results of this study show that both conditions cannot be met at the same time in Louisiana, due to the magnitude of the coordinate discrepancies in Figure 2. We must choose between the two elements of our initial objective. We cannot avoid the dilemma of constrained adjustments.

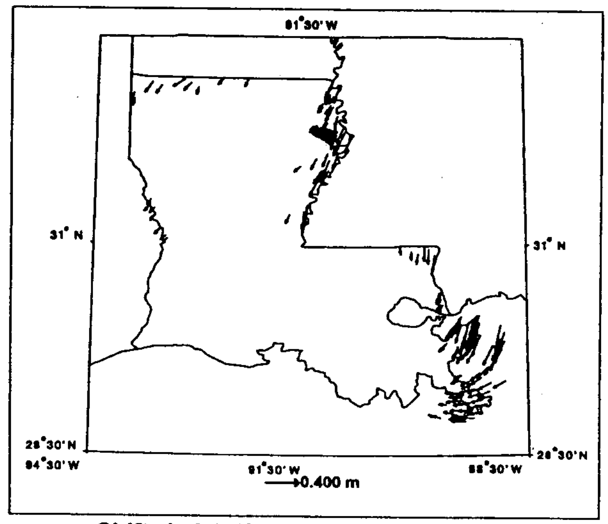
If our goal is to provide a set of the most FBN/CBN consistent coordinates in the state being readjusted, then it becomes apparent that we cannot constrain the NAD 83 (86) coordinates along the borders of the adjacent states which do not yet have a FBN/CBN network in place. To do so, forces discontinuities into the state being readjusted.



Shifts in LA (TCT) Perimeter Held



Shifts in LA (First) Perimeter Held



Shifts in LA (Second) Perimeter Held

Figure 15: Locations of coordinate shifts, *dS*[C][A], in excess of 0.1 m:

(top) Transcontinental Traverse (TCT) control points
(middle) First order control points
(bottom) Second order control points

Along the northern boundary of Louisiana and Arkansas there are quite a few B order stations (see Figure 5). When compared to the NAD 83 (86), the shifts dS[L][N] are all about 0.500 m (see Figure 2). We retrieved about 30 minutes into the state of Arkansas. A large area would have been necessary to disperse this shift.

To illustrate this point, there is a station in Louisiana named L 307 close to the Louisiana border (see Figure 16). It is directly tied to the B order stations, WARD and MOSS. Unfortunately, L 307 is also tied directly to a perimeter point named HOPEPORT through a first order GPS survey. This is about a 100,000 m line or 60 arc-minutes! We would like the position of L 307 to be determined only from the B order points. But, with the restrictions of retrieval size, the fixed position for HOPEPORT still contributes to its determination. The shift dS[C][A] at this station is 0.157 m.

Figure 15b shows an area near Mississippi along the northeast boundary of Louisiana which has many first order stations with large dS[C][A] shifts. Although there are three B order stations (TURKEY, WESTWOOD AZ MK, and F 248) (see Figure 1), only TURKEY is strongly tied to the NAD 83(86) network. The station F 248 was newly determined in the B order project and WESTWOOD AZ MK has only one directional tie from WESTWOOD in the state readjustment. Without a strong tie to the FBN/CBN, the stations in this area are being pulled toward the perimeter points which were held in this adjustment. When Mississippi is readjusted, the location of its B order control will be critical in determining how much of Louisiana should be readjusted.

The dS[C][A] shifts along the southern tip of Louisiana (near Mississippi) are, very likely, due to the retrieval area being too small. Figure 13 shows how close to the state boundaries the perimeter points are located. Fortunately, most of this control is second order (see Figure 15c) and the relative shift analysis showed that, although there were sizable dS[L][A] shifts in this area, it should not be necessary to readjust many points (see Figure 9c).

Discussion

The closeness of the NAD 83 (86) fixed control to the Louisiana state boundary suggests the possibility that the coordinate discrepancies between the NAD 83 (86) and the FBN/CBN could not have been adequately dispersed into the adjacent states. Unfortunately, at the time of the Louisiana State Readjustment, a sketch of the fixed control (Figure 4) was not available.

Instead, we assumed that the relative shift accuracy analysis between fixed and readjusted stations (Phase 2) would be sufficient to detect any problems with the fixed control. This assumption was flawed. It is relatively easy for lower order stations to meet the required accuracy specifications (e.g., third order = 1:10,000). The shift analysis (Phase 1), inadvertently allowed stations close to the boundary to be fixed, and the Phase 2 accuracy analysis tolerances were not stringent enough to detect the problem.

In addition to the accuracy analysis, we assumed that large observation residuals between stations along the boundary would occur in the adjustments that held NAD 83 (86) control too close to the Louisiana state boundary. Although the residuals did grow larger as the perimeter of fixed control contracted, they did not exceed the tolerances set.

To illustrate this, the statistics of several adjustments with different constraints can be compared.

Adjustment	Variance of unit weight	Degrees of freedom
[A]	1.51	38,089
[B]	2.05	42,598
[C]	2.06	39,293
[L]	2.10	44,218
[A]	(fixed control = Figure 5)	
[B]	(fixed control = Figure 3)	
[C]	(fixed control = Figure 13)	
[L]	(fixed control = Figure 4)	

The higher variance of unit weight for [B], [C] and [L] was largely due to large observation residuals involving the A order station BRONSOM, located in Mississippi (Figure 1). Since the retrieval area around this station was not large enough, large residuals involving this station occur even when only the perimeter is held ([C]). The uniformity of the variance of unit weight for [B], [C], and [L] demonstrate the ability of the terrestrial network to absorb varying amounts of coordinate discontinuity without displaying significant problems.

Despite the lack of adequate feathering in [L], it should be noted that the relative accuracy analysis (Phase 2) did ensure that the positions from [L] were good enough in a relative sense to maintain their published accuracy classification. In addition, the number of readjusted positions of stations in adjacent states was minimized. However, the procedures did not produce the most FBN/CBN consistent coordinates along the border. This forced us to consider our objective.

Our initial objective was twofold:

- to compute positions in the Louisiana state readjustment consistent with the FBN/CBN, and

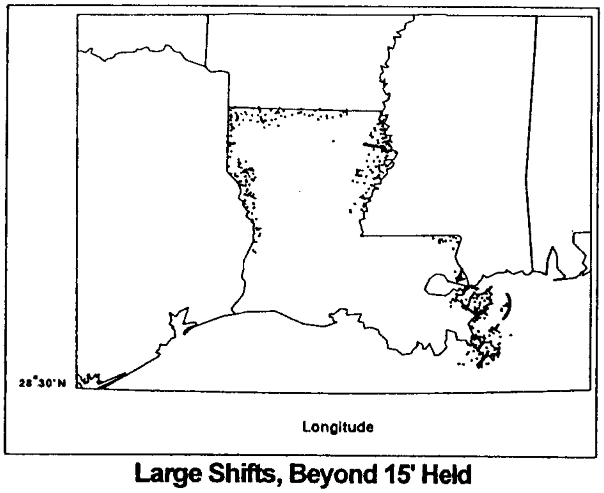


Figure 11: Locations of coordinate shifts dS[B][A], in excess of 0.1 m, between test adjustment [B] and test adjustment [A] (only A and B order points held).

hundred and eighty-two (2,382) positions of stations were held fixed in this adjustment and 1,270 positions of points in Mississippi, Arkansas, and Texas were readjusted in the adjacent states. Figures 11, and 12a, 12b, and 12c show where large shifts dS[B][A] in Louisiana remain. Nine hundred and seventy-seven (977) stations still have shifts greater than one decimeter (784 stations, not counting intersections). By comparing Figure 6 with Figure 11, one can see that by feathering further into adjacent states, the stations with large shifts within Louisiana are fewer and are located closer to the boundary.

Test Adjustment [C]

For this test, the readjustment zone is extended even further. The test readjustment, [C], is computed by holding the A and B order coordinates, and holding the points only at the perimeter of the data retrieval area. Seven hundred and thirty (730) positions of stations were held fixed in this adjustment. Two thousand, nine hundred and twenty-two (2,922) positions of points in Mississippi, Arkansas and Texas were readjusted in the adjacent states. Figure 13 shows the fixed control in this adjustment. Note that the retrieval area went much further into Texas, than Mississippi or Arkansas. As a result of this, most of the large shifts dS[C][A] in position disappear along the Texas / Louisiana border (see Figure 14). Figures 14, 15a, 15b, and 15c show where large shifts in Louisiana remain. Seven hundred and fourteen (714) stations still have shifts greater than one decimeter (565 stations, not counting intersections). The coordinate shifts dS[C][A] in these figures seem to be caused by the failure to retrieve a large enough area or the lack of B order control in the area.

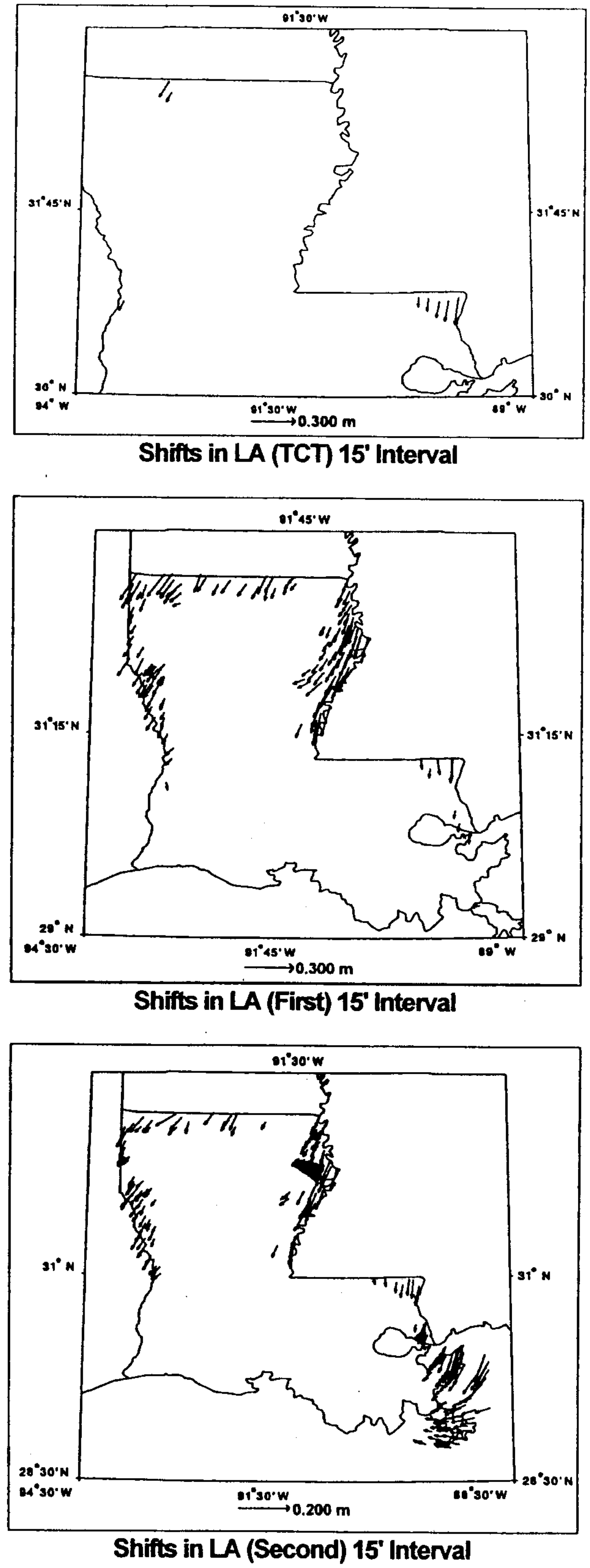
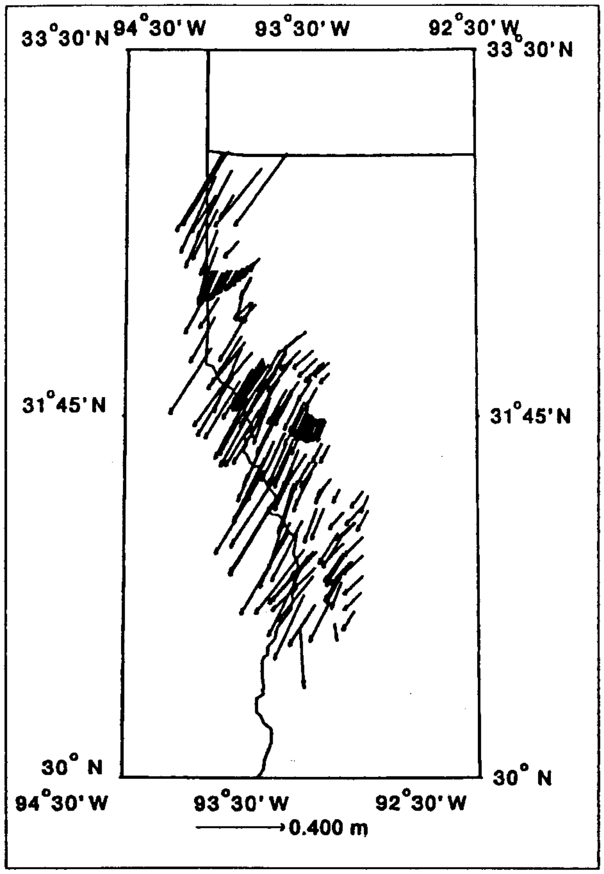
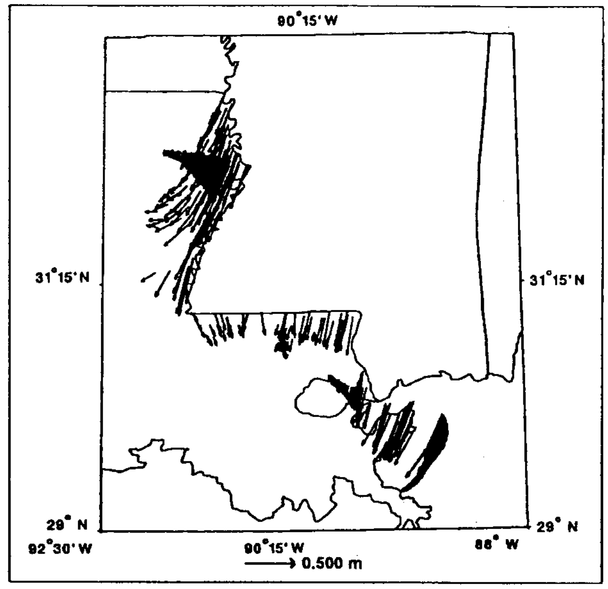


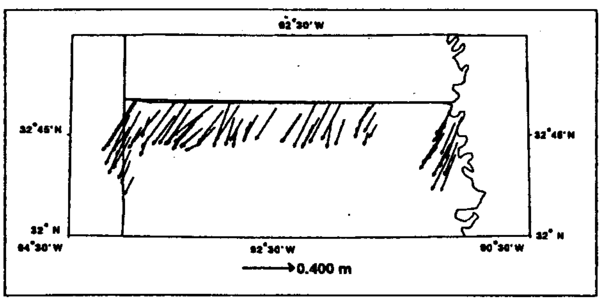
Figure 12: Subsets of coordinate shifts, dS[B][A], in excess of 0.1 m:
(top) Transcontinental Traverse (TCT) control points.
(middle) First order control points.
(bottom) Second order control points.



Potential Readjustment in LA (Near Texas)



Potential Readjustment in LA (Near MS)



Potential Readjustment in LA (Near AR)

Figure 10: Regional sketches of the position shifts, dS[A][L] for points with excessive relative shifts:
(top left) near Texas
(top right) near Mississippi
(above) near Arkansas

Figures 9a, 9b, and 9c plot the points in Figure 8 when grouped by accuracy classification. In Figure 9c one can note several second order traverse lines running from the boundary into the state. The short lines (1 to 3 km) along these traverses made them sensitive to the relative shift analysis. However, they should be readjusted to remove the coordinate discontinuity error which was distributed along their lengths.

To assist in the future readjustments of the adjacent states, we plot the position shifts dS[A][L] for the points identified in the relative shift analysis. These are shown in Figures 10a, 10b, and 10c. Their count breaks down as follows: 245 near Texas, 501 near Mississippi, and 85 near Arkansas.

These results illustrate the constrained adjustment dilemma. The coordinate discontinuities between NAD 83 (86) and the FBN/CBN are real (Figure 2). A state readjustment may absorb the discontinuity, as was done in [L], or it may spread the discontinuity into adjoining states.

A Study of Coordinate Discontinuity Spread

In this section, we examine the problem of minimizing coordinate discontinuity absorbed into a state being upgraded. As discussed above, this alternative involves readjustment of control points in adjoining states. These test adjustments are compared against the [A] set of coordinates, since these values most closely represent the best, undistorted coordinates in Louisiana.

Test Adjustment [B]

In this adjustment, we constrain the A and B order FBN/CBN coordinates plus NAD 83 (86) stations 15 arc-minutes beyond the state boundaries. This adjustment is essentially the same as adjustment [0], which initiated the shift analysis during the Louisiana State Readjustment. In the southern half of Texas, we held points a little further away, since the A order point TOWNSEND is located in Texas near the Louisiana border. Figure 3 displays the fixed control for this adjustment, which we denote as [B]. Two thousand, three

Traverse (TCT, nominal 1:1,000,000 classical), first order, and second order stations. The stations are located at the base of the arrows. One sees smaller shifts within the state, and larger shifts near the boundary. Shifts approaching 0.5 m may be found in all cases.

Inspection of the figures shows that the coordinate discrepancies (Figure 2) were forced into the state by the constraints (Figure 4). The discrepancies "flowed around" the fixed FBN/CBN control within Louisiana (Figures 4 and 5), and were accommodated by the lower order control within the state. This occurred without exceeding the tolerances in the feathering process.

Accuracy Analysis to Determine Positions To Be Readjusted in Louisiana

Due to the large differences in positions, an accuracy analysis was undertaken to determine just how much of Louisiana should be readjusted when adjacent states are readjusted.

In this analysis, two assumptions were followed:

1. When the state readjustment for Mississippi (for example) is performed, the positions of stations in the buffer area close to Louisiana should be approximately the same as those derived from adjustment [A] in Louisiana (only A and B order stations held).
2. Ideally, Louisiana's positions [L] at that time should be fixed, but this will not be possible, since too many NAD 83 (86) positions from Mississippi, Texas, and Arkansas were constrained in the Louisiana State Readjustment.

By making assumption (1), the disparity [A][L] between the coordinates from the Louisiana State Readjustment (which should be close to [A]) can be simulated, by forming a coordinate file [L0] with the following attributes:

1. The Louisiana positions are from the final constrained adjustment from the Louisiana State Readjustment, [L].
2. The other positions (from adjacent states) are from adjustment [A] (only A and B order positions held).

The coordinate file [L0] simulates the results from an adjustment where the surrounding states have been upgraded with FBN/CBN surveys. The [A] coordinates represent newly upgraded points in adjacent state FBN/CBN projects. The [L] coordinates represent the fixed points used to maintain the new Louisiana state readjustment.

Relative shifts, dR[A][L0] are computed over observed lines where one end of the line is taken from the [A] adjustment, and where the other end of the line is taken from the [L] adjustment part of the L0 file. These relative shifts are tested against a tolerance based on the accuracy classification of the observed line (determined by the accuracy classification of the stations involved). If any relative shift involving a given station (with an [L] coordinate) exceeds the tolerance, this station is upgraded with the corresponding [A] coordinate. No action is taken for relative shifts within the tolerance. The new coordinate file is denoted [L1]. Note that this file has the same number of positions as the [L0] file. The only difference is that some of the positions from within Louisiana have been changed from [L] values to [A] values.

One can see immediately that this testing procedure is similar to that of the second phase of the feathering process described earlier. One difference is that the testing procedure operates on a file of coordinates with no adjustment being performed, whereas the second phase feathering iteration used coordinates from constrained adjustments.

The relative shift analysis on the coordinate files is also iterated, producing files [L1],[L2],...,[Ln]. The number of points in these coordinate files is constant. All that changes is the mixture of points with [A] coordinates versus points with [L] coordinates. The [A] coordinates simulate control points to be readjusted when an FBN/CBN survey is carried out in a nearby state. The [L] coordinates simulate points in Louisiana that would remain fixed.

Based on this relative shift analysis, 831 positions in Louisiana may need to be readjusted in order to meet accuracy requirements. These points are displayed in Figure 8. One will notice close, but not identical, distribution to the large dS[A][L] position shifts of Figure 6. It should also be noted in Figure 8 that, even in cases where there is an abundance of FBN/CBN points (northern Louisiana along the Arkansas border), holding the FBN/CBN control was not sufficient to remove the influences of fixing NAD 83 (86) coordinates in the adjacent state.

It must be emphasized that Figure 8 is the result of a simulation. FBN/CBN surveys were not available in the adjacent states at the time of this investigation. FBN/CBN control is abundant near the Arkansas border, so the prediction of which points require readjustment is probably very good. On the other hand, where FBN/CBN control is sparse, such as in northeast Louisiana, the FBN/CBN survey in Mississippi will be needed in order to determine the full extent of readjustment required in that portion of Louisiana.

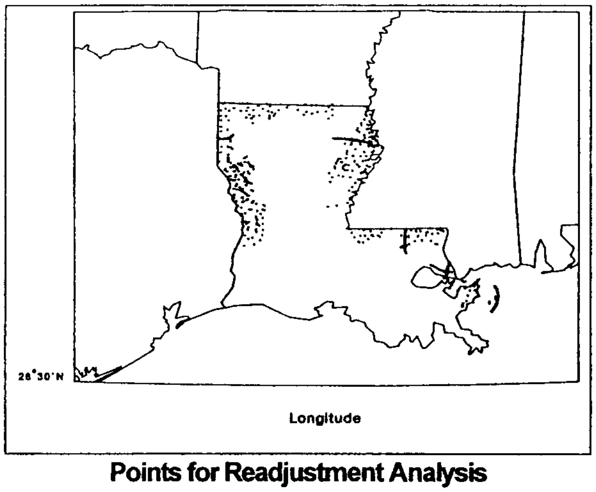


Figure 8: Control points in Louisiana which may need to be readjusted as a result of an accuracy analysis of relative shifts using a mixture of coordinates from adjustments [A] and [L].

adjustments [m+1], [m+2]....[n]. The process converged when all the fixed/readjusted pairs met their accuracy tolerances. The general effect was to free some first order points near the constrained perimeter.

This process of fine tuning the fixed control by examining shifts required a fair amount of time, and over 30 preliminary constrained adjustments. Figure 4 displays the fixed control used in the final constrained adjustment of Louisiana. Three thousand, one hundred and seventy-seven (3,177) positions of stations were held fixed in this adjustment. Only 475 positions of stations in Mississippi, Arkansas, and Texas were readjusted. This adjustment is denoted [L].

Effect of Fixing Positions Close to State Boundary

After the completion of the Louisiana State Readjustment, [L], the question of the effect of "feathering" on stations within the state being readjusted was raised. In order to respond to this question, a test adjustment was run. In this test adjustment only the A and B order FBN/CBN coordinates were held. This adjustment is denoted [A]. Figure 5 displays the fixed control used in this adjustment. The coordinates of adjustment [A] should be close to those obtainable when the entire United States FBN/CBN is completed, and all control has been simultaneously readjusted.

Position shifts, *dS*[A][L], were computed for stations within Louisiana. The expectation was that the shifts would be small (a few centimeters) due to the Louisiana FBN/CBN constraints in both [A] (Figure 5) and in [L] (Figure 4). It was discovered that 1,527 stations in Louisiana have shifts of 0.1 m or greater. Those stations with large shifts are plotted in Figure 6. Even if one does not count intersection stations, 1,224 position shifts exceed 0.1 m. Figures 7a, 7b, and 7c display the position shift vectors *dS*[A][L] for Transcontinental

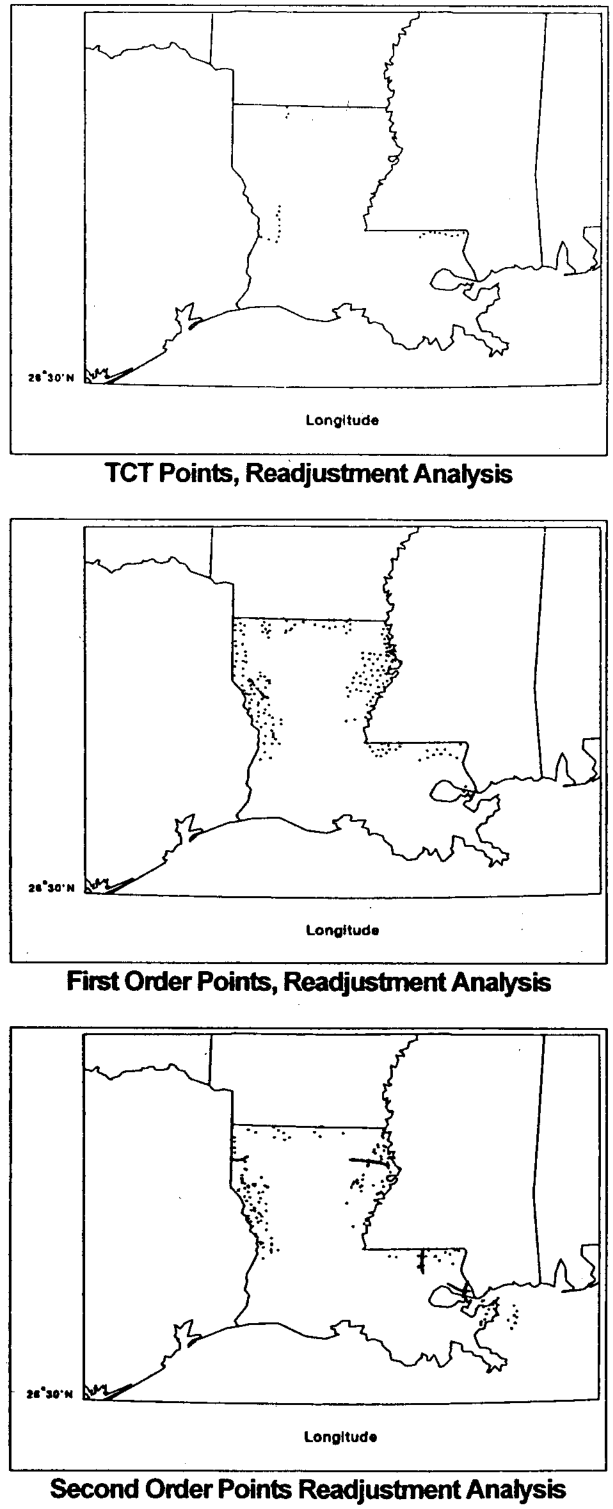
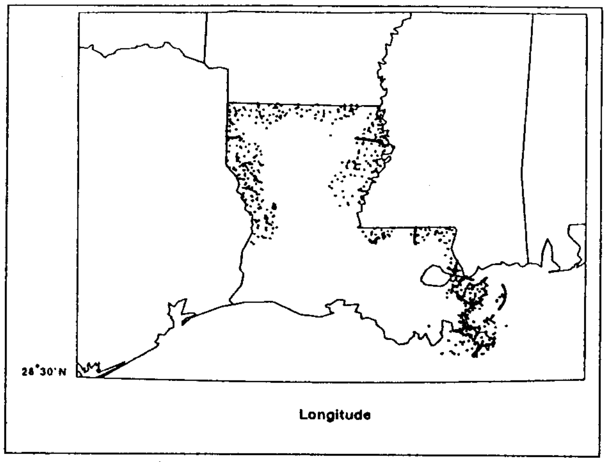
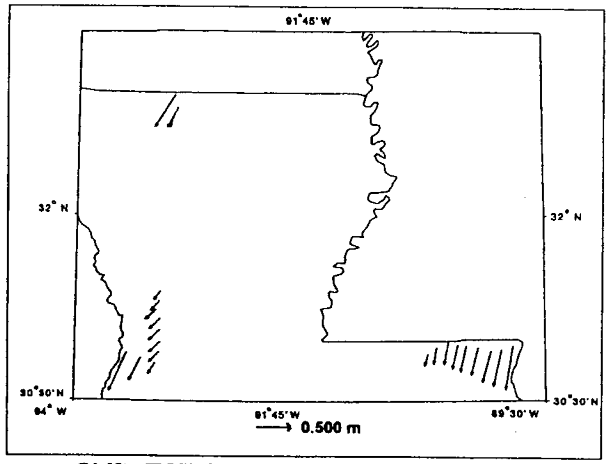


Figure 9: Subsets of control points showing excessive relative shifts:
(top) Transcontinental Traverse (TCT) control points
(middle) First order control points
(bottom) Second order control points

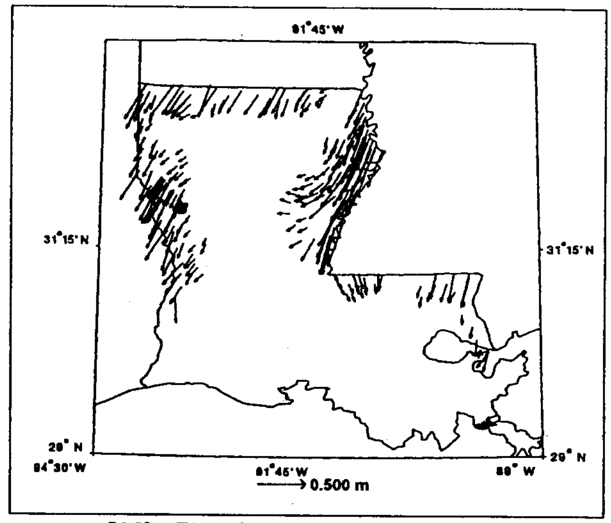


Large Shifts, Constrained

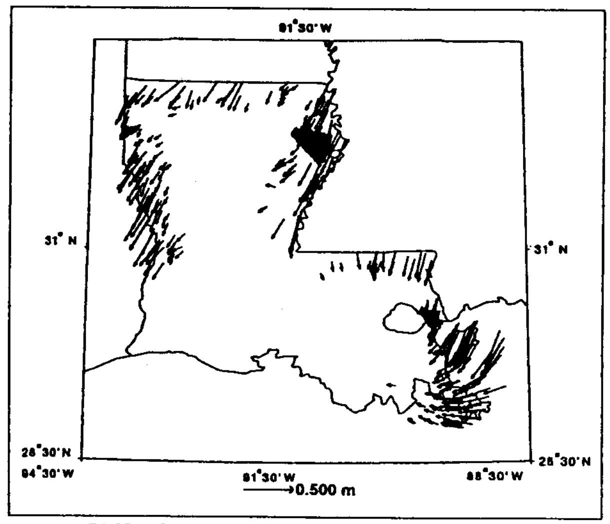
Figure 6: Locations of coordinate shifts, dS[A][L], in excess of 0.1 m, between test adjustment [A] and the Louisiana state readjustment [L].



Shifts TCT Greater than 1 Decimeter LA



Shifts First Order > 1 Decimeter LA



Shifts Second Order > 1 Decimeter LA

Figure 7: Subsets of coordinate shifts, dS[A][L], in excess of 0.1 m:
(top) Transcontinental Traverse (TCT) control points
(middle) First order control points
(bottom) Second order control points

fixed control was similar to that shown in Figure 3. This initial adjustment is denoted [0]. Note that the FBN/CBN was fixed in the state to prevent the GPS results from being distorted.

Then, position shifts dS[0][N] (between the initial adjustment and NAD 83 (86) coordinates) were examined at the points in the adjacent states. If a station is shifted by less than a tolerance (0.1 m), it was constrained. If a station shifted by more than the tolerance, then those fixed stations directly connected to the shifting station were freed ("spreading out" the discontinuity). The new adjustment is denoted [1]. This process was iterated, yielding adjustments [2], [3],....,[m]. The shifts dS[m][N] were computed between the current adjustment [m] and the NAD 83 (86) coordinates [N]. A rule was adopted which prohibits freeing a fixed neighbor that was previously free with a small shift. This insures convergence of the iterative process. The general effect was to cause more perimeter points to be fixed as the outer zone of free stations gradually contracts.

The station shift tolerance analysis is performed first, since it involves a simplified decision process, and provides a basic set of fixed control for subsequent analysis. After the global properties of the coordinate discontinuity distribution are established, then the analysis proceeds to local properties (as expressed in relative position shifts) in the second phase.

In the second phase, relative shifts between fixed and readjusted stations, dR[m][N] were tested against a tolerance based on the accuracy classification of the readjusted station. If the relative shift exceeded the tolerance (e.g., 1:90,000 exceeds 1:100,000), then the fixed station was freed. No action was taken for relative shifts within the tolerance. This second phase was also iterated, producing

Relative shifts, *dR*, between a pair of stations, *i* and *j*, and between two adjustments [A][B] are computed as,

$$dR_{ij}[A][B] = \frac{dS_i - dS_j}{D_{ij}}$$

where

D_{ij} = distance between points i and j.

We express relative shifts as a proportion (e.g., 1:100,000) to facilitate comparison with the horizontal control classification standard (FGCC 1984).

A key to the various adjustments is found in Table 1 on page 220.

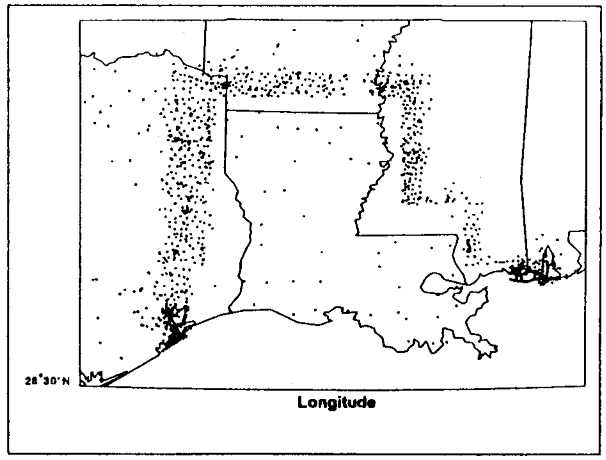
In this paper, the terms "fixed" and "constrained" are used interchangeably. In practice, the fixing of a coordinate is achieved by a 0.1 mm. constraint. Due to the minuscule size of these constraints, the points may be considered fixed. No other constraints are used in this study.

Constraining Louisiana to FBN/CBN

A hierarchical adjustment approach is employed in large readjustments of survey data with dramatically different accuracies, as indicated in Bodnar (1990). A order coordinates are computed with very long GPS connections (nominal 300-km) and constrained to previously existing A order control. Then, B order GPS data are adjusted while constraining the newly derived A order coordinates. This combined set of coordinates (FBN/CBN) is then held fixed in the subsequent readjustment of lower order control. Further information on the Louisiana survey may be found in Love et al. (1993).

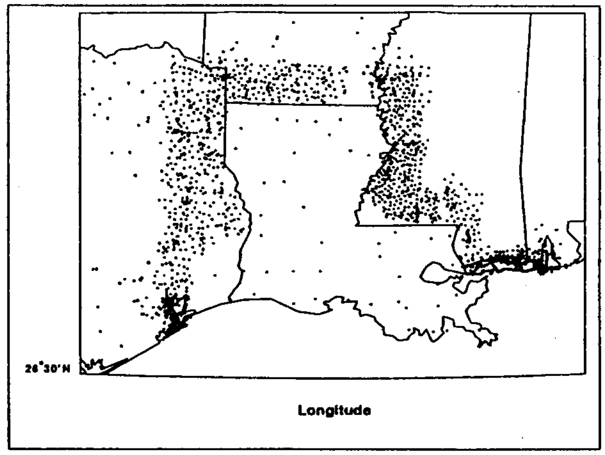
The goal of the Louisiana readjustment was to get the best possible positions in Louisiana (as expressed by the FBN/CBN), while minimizing the effects of the readjustment in the adjacent states of Arkansas, Texas, and Mississippi. At the time of this investigation, FBN/CBN surveys had not been performed in those adjoining states. A two-phase approach, which tested position shifts and relative shifts, was used. A key concern was to readjust no more stations in adjacent states than were absolutely necessary, because FBN/CBN surveys will be performed in the adjacent states in the future. Frequent readjustment of positions is neither understood nor appreciated by many users of our data.

In the first phase of the process, an adjustment was computed by constraining all A and B order coordinates (FBN/CBN) and the NAD 83 (86) coordinates of stations in an outer (about 15-minute) boundary around the state. The distribution of the



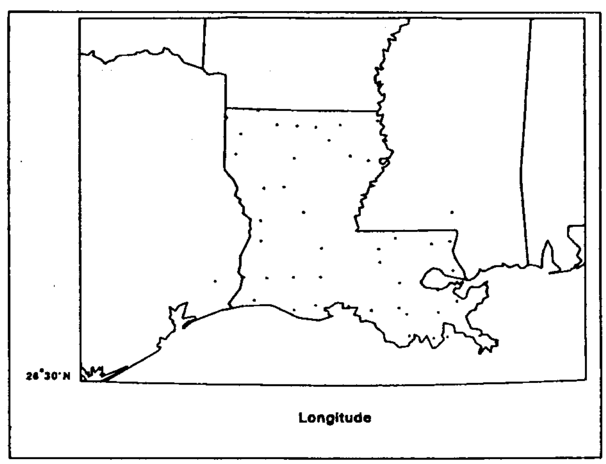
Fixed Control Beyond 15' Zone

Figure 3: Constrained points used in test adjustment [B]. This control distribution is very similar to that used in test adjustment [0] at the start of the feathering process.



Constrained Adjustment, Fixed Control

Figure 4: Constrained points used in the published Louisiana state readjustment [L].



Order B Control

Figure 5: Louisiana FBN/CBN points which also have published NAD 83 (86) coordinates. These points are constrained in test adjustment [A].

For example: 0.5-m shift / 1:100,000 = 50 km. Thus, the retrieval area can be no less than 50 km from the first order control in the upgraded state. Preferably, one should retrieve an extra 25% to 50% further, since geodetic control is irregularly spaced. With a large enough area retrieved, the perimeter constraints should not unduly affect the upgraded state.

A Procedure to Analyze States Adjacent to States Not Adjusted to FBN/CBN Network

The objective is to minimize shifts within the upgraded state. One will readjust as much of the adjacent states as necessary to meet the objective.

We recommend running several test adjustments. The first should be an adjustment that holds only A and B order positions fixed. The second should hold A and B order positions and perimeter points fixed. The third should hold all positions from the edges of the retrieval area to approximately the halfway point to the state boundary, along with A and B order positions. Coordinates within the upgraded state from the first adjustment should be compared to the second and third adjustments, by means of both positional and relative shifts. With this information, decisions can be made concerning how much of an adjacent state should be readjusted.

A Procedure to Analyze States Adjacent to States Adjusted to the FBN/CBN Network

If one is performing a state readjustment, and if one or more adjoining states have been upgraded, then it is a perfect time to check those adjoining states and see if any of their coordinates have been unduly degraded by earlier feathering procedures.

To determine how much readjustment is necessary, we recommend performing a test adjustment that holds only A and B order coordinates fixed. The results of this adjustment should be compared to the published values of stations in the adjacent state that have been previously adjusted to the FBN/CBN network. If shifts are significant, it will be necessary to readjust as many of the previously published positions as necessary.

Closing Remarks

Difficulties arise in the readjustment of states in a piecemeal fashion. The root of the problem is the difference between high accuracy GPS surveys and less accurate triangulation surveys. These difficulties can be managed by taking the proper actions at the time state readjustments are performed.

Fortunately, this research was performed before many state readjustments were performed. Now that we have been alerted to the potential pitfalls, most residual NAD 83 (86) influences will be removed from the states already completed at the time of the state readjustments of their adjacent states to the FBN/CBN. The few remaining problems may be studied separately, and readjustments performed as required.

Of course, an ideal alternate solution would be a simultaneous readjustment of the entire country after the A and B order network is in place. This would certainly correct any remaining problems at the state boundaries.

Finally, once the B order networks are in place in Mississippi, Texas, and Arkansas, we advocate that further studies be performed to verify the recommendations in this paper. With the additional FBN/CBN data, the border points in Louisiana which should be readjusted may be identified unambiguously.

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