Code 4351 4016
SW Branch

April 1, 1974

SURFACE WATER BRANCH TECHNICAL MEMORANDUM NO. 74.14

Subject: COMPUTATIONS – Bridge contraction ratio

A graphical method of determining the contraction ratio of bridge openings m is suggested in TWRI 3A4, "Measurement of peak discharge at width contractions by indirect methods," by Howard Matthai. Recent study of this method of determining m indicates erroneous results may be obtained; the study shows m is a function of the graph scales chosen to plot the cumulative area and cumulative conveyance curves such as shown in figure 30 of TWRI 3A4.

The suggested method for computing the contraction ratio is as follows:

Using the definitions of conveyance K_a, K_q, and K_b in the approach section given in TWRI 3A4, let

$$m = 1 - \frac{K_q}{K_a + K_q + K_b} = \frac{K_a + K_b}{K_a + K_q + K_b}$$

and compute these conveyances exactly, realizing depths at verticals representing ends of the subsection defining K_q may have to be interpolated.

Note that the sum $K_a+K_q+K_b$ may not be equal to K_1, the total conveyance of the approach section, when K_1 is computed in the usual manner. The difference between $K_a+K_q+K_b$ and K_1 will not be unreasonable.

The suggested method of determining m is illustrated in the following example:

Let $K_{i\text{-}j}$ be the conveyance in the approach section for a subsection between stations i and j feet from left bank.

Using data listed in table 1, the conveyance fro the subsection this n = 0.080 is

$$K_{0-63} = \frac{1.486}{0.080}(926.6)\left(\frac{926.6}{69.0}\right)^{2/3}$$
$$= 97{,}246$$

The conveyance for the subsection with n = 0.045 is

$$K_{63-178} = \frac{1.486}{0.045}(2464.1)\left(\frac{2464.1}{126.7}\right)^{2/3}$$
$$= 589{,}119$$

The total conveyance of the approach section K_1, computed in the usual manner, is

$$K_1 = K_{0-63} + K_{63-178} = 97{,}246 + 589{,}119$$
$$= 686{,}365$$

The conveyances K_a, K_q, and K_b are computed as follows:

$$K_a = K_{165-178} = \frac{1.486}{0.045}(65.8)\left(\frac{65.8}{16.1}\right)^{2/3}$$
$$= 5{,}563$$
$$K_q = K_{26-16}5 = \frac{1.486}{0.080}(665.6)\left(\frac{665.6}{37.2}\right)^{2/3} + \frac{1.486}{0.045}(2398.3)\left(\frac{2398.3}{110.6}\right)^{2/3}$$
$$= 64{,}690 + 616{,}560 = 681{,}250$$
$$K_b = K_{0-26} = \frac{1.486}{0.080}(261.0)\left(\frac{261.0}{31.8}\right)^{2/3}$$
$$= 19{,}732$$

Hence,

$$K_a + K_q + K_b = 706{,}545$$

The contraction ratio is

$$m = \frac{K_a + K_b}{K_a + K_q + K_b} = \frac{5{,}563 + 19{,}732}{706{,}545} = 0.0357$$

Graphical Determination of Contraction Ratio

The inadequacy of the graphical method in determining m may be understood by considering how area, conveyance, and stationing are actually plotted in a graph such as figure 30 in TWRI 3A4.

Suppose K is the conveyance for the first subsection from the left bank, X is the distance from the left bank at which K is computed, and A is the area of this subsection.

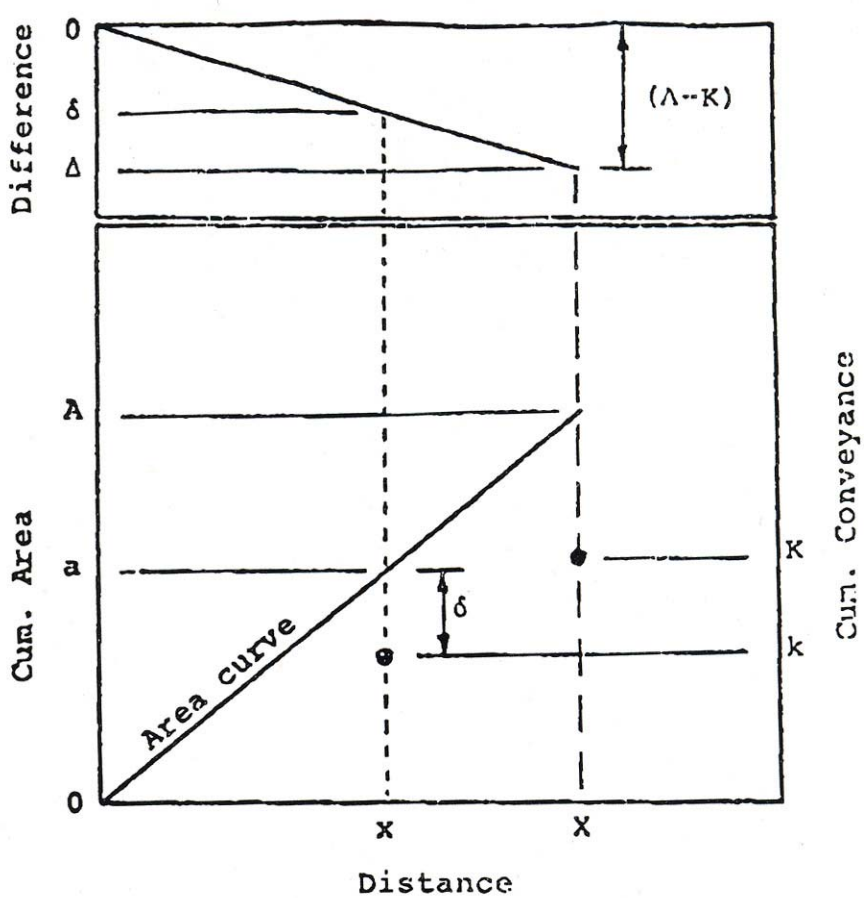
And, the conveyance k is to be estimated for a subsection which ends at a distance x from the left bank, $x<X$, and has an area a.

These conveyances and areas cannot be directly plotted against distance on a graph. First, each variable must be converted to some convenient plotting scale by a scaling factor.

Let α = scaling factor in units of inches/foot be used to determine plotting position for distance measured in feet,

β = scaling factor in units of inches/square foot be used to determine plotting position for area measured in square feet, and

γ = scaling factor in units of inches/cubic foot per second be used to determine plotting position for conveyance measured in cubic feet per second.



Hence, X is plotted at a distance αX inches from the origin, A at βA inches from origin, and K at γK inches from origin.

The auxiliary graph at the top of the figure represents the variation in the lineal difference between the area and conveyance curves. At the distance X the difference is Δ inches. Assuming the variation in the difference between the area and conveyance curves is linear, the difference at distance x is

$$\delta = \frac{\alpha x}{\alpha X}\Delta = \frac{x}{X}\Delta \text{ inches}$$

where $\Delta = \beta A - \gamma K$

The difference is laid off from the area curve to plot k. Hence,

$$\gamma k = \beta a - \delta \text{ inches}$$

Substituting into this expression, we obtain

$$\begin{aligned}\gamma k &= \beta a - \frac{x}{X}\Delta \\ &= \beta a - \frac{x}{X}(\beta A - \gamma k)\end{aligned}$$

Then, the expression is divided through by γ,

$$k = \frac{\beta}{\gamma}a - \frac{x}{X}\left(\frac{\beta}{\gamma A - K}\right), \text{ and}$$

the terms are rearranged,

$$k = \frac{x}{X}K + \frac{\beta}{\gamma}\left(a - \frac{x}{X}A\right)$$

Because x, S, a, A, and K are all constants, k is a function of the ratio of scaling factors β and γ.

Example:

The convenient set of scaling factors for the data listed in table 1 is

$$\alpha = \frac{1 \quad \text{in}}{40 \quad \text{ft}} = 0.025 \text{ in/ft}$$

$$\beta = \frac{1 \quad \text{in}}{400 \quad \text{ft}}2 = 0.0025 \text{ in/ft}^2\text{, and}$$

$$\gamma = \frac{1 \quad \text{in}}{100{,}000 \quad \text{cfs}} = 0.\varnothing.00001 \text{ in/cfs.}$$

To determine K_b, we find from the table x = 26 ft, X = 63 ft, a = 261 ft^2, A = 926.6 ft^2, and K = 97,246 cfs. Hence, by the preceding equation

$$K_b = k = \frac{26}{63}(97{,}246) + \frac{0.0025}{0.\varnothing.00001}\left(261 - \frac{26}{63}(926.6)\right) = 9{,}782 \quad \text{cfs}$$

If the scaling factor for conveyance is changed to

$$\gamma\beta = \frac{1 \quad \text{in}}{200{,}000 \quad \text{cfs}} = 0.000005 \quad \text{in/cfs}$$

then,

$$K_b = k = \frac{26}{63}(97{,}246) + \frac{0.0025}{0.000005}(261 - \frac{26}{63}(926.6)) = -20{,}570 \quad \text{cfs.}$$

/signed/
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WRD Distribution: A,B,S,FO,PO

Table 1.—Data for approach section

Sta.	Dist.	Mean Depth	Area	Cum. Area	WP	Cum. WP
LB						
0	0	0	0	0	0	0
2	2	0.95	1.9	1.9	2.8	2.8
7	5	3.9	19.5	21.4	6.4	9.2
15	8	8.9	71.2	92.6	10.0	19.2
19	4	13.05	52.2	144.8	4.6	23.8
26	7	16.6	116.2	261.0	8.0	31.8
35	9	17.6	158.4	419.4	9.0	40.8
46	11	18.05	198.6	618	11.1	51.9
63	17	18.15	308.6	926.6	17.1	69.0
68	5	17.9	89.5	1016.1	5.1	74.1
74	6	20.5	123.0	1139.1	7.3	81.4
75	1	23.1	23.1	1162.2	1.4	82.8
82	7	24.55	171.8	1334.0	7.3	90.1
89	7	25.9	181.3	1515.3	7.0	97.1
118	29	26.35	764.2	2279.5	29.0	126.1
126	8	25.7	205.6	2485.1	8.1	134.2
141	15	24.3	364.5	2849.6	15.1	149.3
148	7	23.1	161.7	3011.3	7.1	156.4
153	5	22.15	110.8	3122.1	5.1	161.5
157	4	19.9	79.6	3201.7	5.4	166.9
160	3	16.9	50.7	3252.4	3.8	170.7
165	5	14.5	72.5	3324.9	5.5	176.2
165	0	11.6	0.0	3324.9	3.4	179.6
171	6	7.9	47.4	3372.3	7.2	186.8
173	2	4.9	9.8	3382.1	2.8	189.6
175	2	2.9	5.8	3387.9	2.8	192.4
178	3	0.95	2.8	3390.7	3.3	195.7
RB						

n=0.080 b n=0.045