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Octet Enhancement*

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ABSTRACT

A simple universal current-current theory of weak interactions now accounts fairly well for all weak phenomena except the nonleptonic $|\Delta \underset{\sim}{I}| = 1/2$ rule, which presumably should be generalized to a unitary octet rule, covering not only the familiar $|\Delta Y| = 1$ nonleptonic interactions but also the $|\Delta Y| = 0$ nonleptonic interactions (for which experimental evidence in heavy nuclei has been presented by Boehm and Kankeleit).

One may account for the octet nonleptonic rule either (a) by a theory that adds extra current-current products for strongly interacting particles alone, or else (b) by a dynamical mechanism that enhances octets by means of strong interactions. It is interesting that we can distinguish any reasonable theory of type (a) from any theory of type (b) by the amount of $|\Delta \underset{\sim}{I}| = 1$ in the $\Delta Y = 0$ nonleptonic interaction; the $|\Delta \underset{\sim}{I}| = 1$ component is large in the former case and small in the latter case. Difficult experiments involving light nuclei may be able to resolve the two possibilities.

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If octet enhancement is indeed the explanation, then the same mechanism can operate (as remarked by Coleman and Glashow) to enhance the octet portion of electromagnetic mass splittings in various supermultiplets, and can also build up the octet violation of SU(3) symmetry of strong interactions.

In the bootstrap theory of strong interactions, a particularly simple description of enhancement is available, which we illustrate for the electromagnetic mass differences. The electromagnetic interaction is introduced as a driving term, which affects the masses of external particles and exchanged particles in the diagrams that account for the structure of particles. Crudely, one has a matrix relation like

$$\Delta m = D + A\,(\Delta m) \quad ,$$

where D is the direct effect of the driving force and $A\,(\Delta m)$ is the effect on external and exchanged particles. Strong octet enhancement corresponds to having octet eigenvalues ${}_8a$ of A close to unity, while 27-plet eigenvalues ${}_{27}a$ are far from unity.

In the case of the breaking of SU(3) symmetry of the strong interactions (for which a related approach was originally suggested by Cutkosky) one might consider spontaneous violation, which corresponds in the linear approximation to $D = 0$ and ${}_8a = 1$. However, the direction in SU(3) space of the octet violation seems very difficult to explain in terms of the directions defined by the electromagnetic and weak perturbations. Therefore, even in an otherwise pure bootstrap theory, a driving term something like Ne'eman's "fifth interaction" may be needed. Weak nonleptonic effects, electromagnetic mass differences, and strong violations of SU(3) symmetry would then really be analogous to one another.

I. THE WEAK INTERACTION

Let us start from a simple picture of the weak interaction, which explains nearly all of the experimental evidence. The weak coupling is represented, in the local limit, as the product of a single current and its hermitian conjugate:

$$\frac{G}{\sqrt{2}}\, J_\alpha^{+}\, J_\alpha \quad . \tag{1}$$

The interaction may, of course, be nonlocal -- that is, mediated by an intermediate boson of finite mass rather than infinite mass -- for our purposes we need not emphasize this distinction.

The current J_α may be written in the form

$$J_\alpha = J_\alpha^{\ell} + J_\alpha^{h} \quad , \tag{2}$$

where J_α^{ℓ} is the leptonic part and has, as far as we know, the simple structure

$$J_\alpha^{\ell} = \bar{\nu}_e\, \gamma_\alpha\, (1+\gamma_5)\, e + \bar{\nu}_\mu\, \gamma_\alpha\, (1+\gamma_5)\, \mu \quad , \tag{3}$$

where J_α^{h} is the part that concerns hadrons (strongly interacting particles) and is assumed to be given by the formula

$$J_\alpha^{h} = (\mathcal{F}_{1\alpha} + i\, \mathcal{F}_{2\alpha} + \mathcal{F}^{5}_{1\alpha} + i\, \mathcal{F}^{5}_{2\alpha}) \cos\theta + (\mathcal{F}_{4\alpha} + i\, \mathcal{F}_{5\alpha} + \mathcal{F}^{5}_{4\alpha} + i\, \mathcal{F}^{5}_{5\alpha}) \sin\theta , \tag{4}$$

where $\mathcal{F}_{i\alpha}$ $(i = 1 \ldots 8)$ is the current of the F-spin and $\mathcal{F}^{5}_{i\alpha}$ $(i = 1 \ldots 8)$ is an octet of axial vector currents.[1-4] As Cabibbo[2] has shown, Eq. (4) gives a good description of leptonic weak interactions with θ of the order of 15°.

It is useful to introduce[5)] a unitary transformation in the space of SU(3), which transforms the familiar isotopic spin and hypercharge operators as follows:

$$I_1 = F_1 \to I_1' = F_1' = F_1 \cos\theta + F_4 \sin\theta \quad ,$$

$$I_2 = F_2 \to I_2' = F_2' = F_2 \cos\theta + F_5 \sin\theta \quad ,$$

$$I_3 = F_3 \to I_3' = F_3' = F_3 \cos^2\theta + (\frac{\sqrt{3}}{2} F_8 + \frac{1}{2} F_3) \sin^2\theta - F_6 \sin\theta \cos\theta \quad ,$$

$$\frac{\sqrt{3}}{2} Y = F_8 \to \frac{\sqrt{3}}{2} Y' = F_8' = F_8 (1 - \frac{3}{2} \sin^2\theta) + F_3 (\frac{\sqrt{3}}{2} \sin^2\theta) + \sqrt{3} F_6 \sin\theta \cos\theta \quad . \tag{5}$$

The nonleptonic part of the weak coupling (1) can then be written

$$\frac{G}{\sqrt{2}} \left\{ (\mathcal{J}_{1\alpha}' + \mathcal{J}_{1\alpha}^{5'})(\mathcal{J}_{1\alpha}' + \mathcal{J}_{1\alpha}^{5'}) + (\mathcal{J}_{2\alpha}' + \mathcal{J}_{2\alpha}^{5'})(\mathcal{J}_{2\alpha}' + \mathcal{J}_{2\alpha}^{5'}) \right\}$$

$$\equiv Z_{1\alpha}' \; Z_{1\alpha}' + Z_{2\alpha}' \; Z_{2\alpha}' \quad . \tag{6}$$

The main experimental fact that is not accounted for by the simple theory we have sketched is the $|\Delta I| = 1/2$ rule for the $|\Delta Y| = 1$ part of the nonleptonic interaction. The coupling (6) contains, of course, both $|\Delta I| = 1/2$ and $|\Delta I| = 3/2$. From the point of view of the eightfold way, the expression (6) consists of three parts, transforming like **1**, **8**, and **27** respectively. The $|\Delta I| = 3/2$ contribution comes entirely from the **27** part, and it is natural to suppose that whatever is responsible for the $|\Delta I| = 1/2$ nonleptonic rule actually makes the entire octet contribution much more important than the entire **27** contribution.[5-8)] We shall henceforth assume that the $|\Delta I| = 1/2$ rule is actually a rule of predominance of **8** (and perhaps **1**) over **27**.

If we display the interaction (6) explicitly as a sum of $\underset{\sim}{1}$, $\underset{\sim}{8}$, and $\underset{\sim}{27}$ terms, it becomes:

$$Z'_{1\alpha} Z'_{1\alpha} + Z'_{2\alpha} Z'_{2\alpha} = (\tfrac{1}{4})\ S + (\tfrac{2}{5})\ O_8 + (\tfrac{1}{20})\ T \tag{7}$$

where

$$S = Z'_{1\alpha} Z'_{1\alpha} + Z'_{2\alpha} Z'_{2\alpha} + Z'_{3\alpha} Z'_{3\alpha} + Z'_{4\alpha} Z'_{4\alpha} + Z'_{5\alpha} Z'_{5\alpha} + Z'_{6\alpha} Z'_{6\alpha} + Z'_{7\alpha} Z'_{7\alpha} + Z'_{8\alpha} Z'_{8\alpha} \quad , \tag{8}$$

$$O_8 = Z'_{1\alpha} Z'_{1\alpha} + Z'_{2\alpha} Z'_{2\alpha} + Z'_{3\alpha} Z'_{3\alpha} - \tfrac{1}{2}\ Z'_{4\alpha} Z'_{4\alpha} - \tfrac{1}{2}\ Z'_{5\alpha} Z'_{5\alpha} - \tfrac{1}{2}\ Z'_{6\alpha} Z'_{6\alpha} - \tfrac{1}{2}\ Z'_{7\alpha} Z'_{7\alpha} - Z'_{8\alpha} Z'_{8\alpha} \quad , \tag{9}$$

$$T = 7\ Z'_{1\alpha} Z'_{1\alpha} + 7\ Z'_{2\alpha} Z'_{2\alpha} - 13\ Z'_{3\alpha} Z'_{3\alpha} - Z'_{4\alpha} Z'_{4\alpha} - Z'_{5\alpha} Z'_{5\alpha} - Z'_{6\alpha} Z'_{6\alpha} - Z'_{7\alpha} Z'_{7\alpha} + 3\ Z'_{8\alpha} Z'_{8\alpha} \quad . \tag{10}$$

(Note that O_8 transforms like the eighth component of a primed octet.) There are, of course, two different ways to assure predominance of $\underset{\sim}{8}$ over $\underset{\sim}{27}$. One is to add to our basic interaction (1) some new products of currents (presumably involving hadrons alone) so as to cancel the $\underset{\sim}{27}$ portion. The other is to find a mechanism that enhances the octet (and perhaps singlet) contributions to nonleptonic processes.

Let us begin with the first mechanism, which involves adding extra current products to the expression $Z'_{1\alpha} Z'_{1\alpha} + Z'_{2\alpha} Z'_{2\alpha}$. We assume that these further products also have positive coefficients, so as to correspond to positive probabilities for exchanged intermediate bosons. Then, if we are restricted to linear combinations of the unitary singlet S and the eighth component O_8 of a primed unitary octet, the only expressions we can obtain are sums of either seven or eight current products. No theory of this type seems to have any distinctive features to recommend it. We may, however, also make use of the third component of a primed octet, proportional to

$$O_3 = \frac{\sqrt{3}}{2}\left(Z'_{4\alpha} Z'_{4\alpha} + Z'_{5\alpha} Z'_{5\alpha} - Z'_{6\alpha} Z'_{6\alpha} - Z'_{7\alpha} Z'_{7\alpha}\right) + 2\, Z'_{3\alpha} Z'_{8\alpha} \quad . \tag{11}$$

In that case we can obtain the more elegant interaction of d'Espagnat:[9)]

$$\frac{2}{3}\, S + \frac{1}{3}\, O_8 - \frac{1}{\sqrt{3}}\, O_3 = Z'_{1\alpha} Z'_{1\alpha} + Z'_{2\alpha} Z'_{2\alpha}$$

$$+ \left(Z'_{3\alpha} - \frac{1}{\sqrt{3}} Z'_{8\alpha}\right)\left(Z'_{3\alpha} - \frac{1}{\sqrt{3}} Z'_{8\alpha}\right) + Z'_{6\alpha} Z'_{6\alpha} + Z'_{7\alpha} Z'_{7\alpha} \quad . \tag{12}$$

Here, there are only five current products; moreover, if the interaction is expressed in terms of intermediate bosons, they belong to a triplet and anti-triplet and the coupling transforms under SU(3) like $\underset{\sim}{3}$ and $\underset{\sim}{\bar{3}}$.

We have seen, then, that an attractive theory involving extra current products involves not just S and O_8 (Eqs. (8) and (9)) but also an admixture, with coefficient of order unity, of O_3 (Eq. (11)). Now O_3 has the property $|\Delta \underset{\sim}{I}'| = 1$, while S and O_8 have $|\Delta \underset{\sim}{I}'| = 0$. Translating into the language of ordinary isotopic spin and hypercharge, we may say that the $\cos^2\theta$ term in O_3 has $\Delta Y = 0$, $|\Delta \underset{\sim}{I}| = 1$, while the $\cos^2\theta$ term in O_8 (like all of S) has $\Delta Y = 0$, $|\Delta \underset{\sim}{I}| = 0$. Thus in an elegant theory of added current products, the $\cos^2\theta$ term in the $\Delta Y = 0$ nonleptonic interaction should have an admixture of $|\Delta \underset{\sim}{I}| = 1$ of order unity.

Now let us contrast such a situation with the case in which the $\underset{\sim}{27}$ part of the nonleptonic interaction is unimportant because of octet (and perhaps singlet) enhancement. Such enhancement by strong interactions cannot introduce any $|\Delta \underset{\sim}{I}| = 1$ into the $\cos^2\theta$ term in the original interaction (6), which contains[10] only $|\Delta \underset{\sim}{I}| = 0$ and $|\Delta \underset{\sim}{I}| = 2$. The $|\Delta \underset{\sim}{I}| = 0$ part of the $\cos^2\theta$ term will be enhanced and the $|\Delta \underset{\sim}{I}| = 2$ part will not.

Since $\cot^2\theta \approx 15$, the amount of $|\Delta \underset{\sim}{I}| = 1$, $\Delta Y = 0$ in the extra current model is around 15 times larger than in the octet enhancement model. If the isotopic spin dependence can be tested experimentally, it would help to distinguish the two possibilities. At present, our evidence comes entirely from the $|\Delta Y| = 1$ nonleptonic decays, and consists mostly in the observation that the $|\Delta \underset{\sim}{I}| = 1/2$ octet amplitudes seem to be rather larger than one might expect from crude estimates based on the current-current product without enhancement, while the $|\Delta \underset{\sim}{I}| = 3/2$, 27-plet amplitudes are roughly of the expected order of magnitude.[11] Such evidence, of course, supports the octet enhancement model, but is far from conclusive.[12]

II. NUCLEAR PHYSICS TESTS OF THE STRUCTURE OF NON-LEPTONIC WEAK INTERACTIONS

The effects of nonleptonic $\Delta Y = 0$ weak interactions are hard to study experimentally because they are generally masked by the stronger interactions. Nevertheless, many attempts have been made, and recently Boehm and Kankeleit[13] have claimed detection of circular polarization in 482-keV γ-rays emitted by unpolarized Ta^{181}, thus providing evidence for parity violation in nuclear forces. Although information about the isotopic spin of the weak interaction cannot be gained from such a heavy nucleus as Ta^{181}, the work of Boehm and Kankeleit encourages the hope that similar experiments can eventually be done on light nuclei, where weak $|\Delta \underset{\sim}{I}| = 0$, 1, and 2 effects can be distinguished from one another.

In evaluating possible experiments on light nuclei, we need to estimate the magnitude of parity-mixing to be expected in nuclear states on the basis of the octet enhancement and extra-current models respectively. The usual steps in such an estimate are to give a weak two-nucleon potential, deduce an effective weak potential for one nucleon in nuclear matter, and then study the parity-mixing that results in nuclear wave functions. The last step requires detailed analysis of specific nuclear states, but the first two steps can be treated in a slightly more general fashion.

One finds the two-nucleon potential as a function of internucleon distance by summing the various possible exchanges such as π, η, ρ, ω, ϕ, W, W+π, and so forth. (Here, W is the hypothetical intermediate boson.) This procedure closely parallels the usual treatment of strong interactions except that now one vertex is weak (or, as in W exchange, each vertex is half-weak). Exchanges of more massive systems can be neglected with better assurance than usual because they contribute to the weak potential mainly inside the repulsive core of the strong interactions, where the nucleons rarely penetrate.[14]

Classifying the various long-range, parity-violating terms, one finds that CP forbids π^0 and η exchange and forces a purely $|\Delta \underset{\sim}{I}| = 1$ character upon $\pi^\pm$ exchange,[10] whereas ρ, ω, and ϕ exchange contribute only to $|\Delta \underset{\sim}{I}| = 0$ or 2. The long-range effects of $W^\pm$ exchange are already included in $\rho^\pm$ exchange, since the NNW vertex acquires a form factor, extending the original range of W exchange, from $W \to \rho \to N\bar{N}$. These contributions from $\rho^\pm$ exchange have been estimated by Michel.[15] The space-spin dependence turns out to have the important property that the interactions of a single unpaired nucleon with the other nucleons, including those in closed shells, add constructively.

For the $|\Delta I| = 1$ potential, we consider $\pi^{\pm}$ exchange and again find that the interactions of a single unpaired nucleon with the other nucleons add. The magnitude of the weak $NN\pi$ vertex does not follow immediately from the current-current formalism, but experimental numbers are available for its $\Delta Y = 1$ counterparts $\Xi \to \Lambda + \pi$, $\Sigma \to N + \pi$, and $\Lambda \to N + \pi$, and each of the models we are comparing relates these values for $\Delta Y = 1$ to the $\Delta Y = 0$, $|\Delta I| = 1$ amplitude. Working out the implications of the d'Espagnat model, one finds that in nuclear matter $\pi^{\pm}$ exchange contributes a $|\Delta I| = 1$, parity-violating, single-nucleon effective potential with just about the same strength as the $|\Delta I| = 0$ parity-violating potential estimated by Michel.[15)] By contrast, in the octet enhancement model with no extra currents, the $|\Delta I| = 1$ potential is weaker by the factor $\cot^2\theta \approx 15$, as described previously. The two models also lead to different expectations for the absolute magnitude of the $|\Delta I| = 0$ amplitude, which would be raised above Michel's estimate by octet enhancement,[16)] and the $|\Delta I| = 2$ amplitude, which may be wiped out completely by extra currents but not by octet enhancement.

Thus the isospin properties of the effective parity-violating potential in nuclear matter closely parallel those of the basic weak interaction discussed in Section I, and experimental information on the magnitudes of $|\Delta I| = 0$, 1, and 2 effects would be of great value for deciding whether extra currents exist. Although we have no immediately feasible experiments to propose, we will mention briefly a couple of examples[17)] of the kind of experiment which may prove successful after further work. The general idea is to find cases where the parity-conserving amplitude is strongly inhibited and dominates the parity-violating amplitude by less than the usual factor of 10^6 or 10^7.

One case which has already been studied intensively[18)] is the parity forbidden α decay from the 8.8 MeV 2^-, $I = 0$ level of O^{16} to $C^{12}(0^+, I=0)+\alpha$. Upper limits of order 10^{-12} of normal parity-conserving α decay rates have been placed on its occurrence. If any decays were seen they would provide sure evidence that the initial or final state has an admixture of the opposite parity and same I spin. Thus if the search can be pushed somewhat farther and parity-violating α decays found, it should be possible to deduce the $|\Delta \underset{\sim}{I}| = 0$ weak interaction strength which will be particularly large if the octet is enhanced.

Needless to say, developments making it possible to observe a weak amplitude instead of a rate in α decay, or to observe $|\Delta \underset{\sim}{I}| = 1$ terms, would also be very useful.

For an example of another line of attack, which can be directed towards either $|\Delta \underset{\sim}{I}| = 0$ or $|\Delta \underset{\sim}{I}| = 1$ terms, consider the nucleus B^{10}. Gamma rays emitted in transitions from the 5.11-MeV, $I = 0$, 2^- level to lower 1^+, 2^+, or 3^+ levels with $I = 0$ are likely to exhibit an especially large circular polarization because of the following factors: (i) the parity-conserving E1 matrix element is inhibited. If the states were pure $I = 0$, the E1 matrix element would vanish for long wavelength; in practice, it is rescued from this fate by the small Coulomb admixture of $I = 1$ in the states.

(ii) The parity-violating M1 amplitude, which interferes with E1 to produce circular polarization, is enhanced by parity mixing between our 5.11-MeV, $I = 0$, 2^- level and the very nearby 5.16-MeV, $I = 1$, 2^+ level <u>provided</u> the weak interaction has a large $|\Delta \underset{\sim}{I}| = 1$ component to connect the two states. As a bonus, the M1 electromagnetic transition is then

$|\Delta \underset{\sim}{I}| = 1$ and makes use of the large nucleon isovector magnetic moment.

Thus, a survey of the gamma transition rates from the 5.11-MeV level of B^{10}, which are not very well understood at present, might suggest a favorable case for detection of a large $|\Delta \underset{\sim}{I}| = 1$ component in the weak interaction.

III. OCTET ENHANCEMENT IN WEAK AND ELECTROMAGNETIC INTERACTIONS

Coleman and Glashow[19)] have pointed out the analogy between the nonleptonic weak decay amplitudes and the electromagnetic mass differences of strongly interacting particles. In each case, the interaction is a product of an octet current with itself; the product contains $\underset{\sim}{27}$ as well as $\underset{\sim}{8}$. In the case of weak nonleptonic decays, the $\underset{\sim}{8}$ part has been found to be much larger than the $\underset{\sim}{27}$ part in all known processes. In the electromagnetic case, there are two supermultiplets for which good information is available. For the $J = 1/2^+$ baryon octet, the $\underset{\sim}{8}$ part of the electromagnetic mass difference strikingly predominates over the $\underset{\sim}{27}$, as evidenced by the approximately equal spacing of the Σ masses; for the $J = 0^-$ meson octet, the $\underset{\sim}{8}$ part of the difference in mass squared is only slightly bigger than the $\underset{\sim}{27}$ part. Despite the counterexample of the pseudoscalar octet, we may guess that there is a systematic tendency toward octet enhancement for the electromagnetic perturbation as well as the weak nonleptonic one.

The usual mechanism of enhancement, often referred to in terms of "tadpoles", has been discussed for many years in a variety of ways.[19-21)] We may describe it briefly as follows. The nonleptonic weak interaction acts like a scalar and a pseudoscalar "spurion" carrying zero energy and momentum. We may, however, write an unsubtracted dispersion relation in the four-momentum squared t carried by the nonleptonic weak interaction and evaluate the dispersion integral at $t = 0$. We may furthermore suppose that the integral is dominated by low-lying meson states with suitable quantum numbers. Now if scalar and pseudoscalar meson octets of low mass exist, while 27-plets do not, then we have an explanation of octet predominance over the $\underset{\sim}{27}$. Moreover, this explanation can be generalized, more or less, to the electromagnetic as well as strong mass differences.[19)]

An important feature[8)] of the "tadpole" mechanism is that the charge-conjugation properties of the parity-violating spurion octet and of the known pseudoscalar meson octet are opposite. For the $|\Delta Y| = 1$ part of the nonleptonic interaction, this means that the "tadpole" mechanism of octet enhancement is forbidden by SU(3) symmetry; of course, the enhancement effect could still be considerable despite the SU(3)-forbiddenness, like the ratio of $K_1^0 \to 2\pi$ and $K^+ \to 2\pi$ rates.[12,22)] If we turn to the $\Delta Y = 0$ nonleptonic interaction, however, we see that the disagreement in charge conjugation properties between the parity-violating spurion and the pseudoscalar mesons completely forbids the tadpole mechanism of octet enhancement. For $\Delta Y = 0$, then, tadpoles are powerless to make $|\Delta \underset{\sim}{I}| = 0$ more important than $|\Delta \underset{\sim}{I}| = 2$ in parity-violating weak forces.

A somewhat different mechanism for octet enhancement has been proposed by Dashen and Frautschi,[23)] extending earlier suggestions by Cutkosky and Tarjanne.[24)] For purposes of describing the mechanism, we consider the specific case of electromagnetic mass corrections.

We assume the bootstrap theory and describe each strongly interacting particle as a bound state of strongly interacting particles. For simplicity, we consider two-body channels and single-body exchange forces only, treating unstable particles and stable ones on the same footing. Also, again for simplicity, we ignore strong violations of SU(3) symmetry.

Each supermultiplet now appears as a set of degenerate poles in the various two-body scattering amplitudes with appropriate quantum numbers. The electromagnetic interaction causes shifts in the positions of the poles, in each supermultiplet, from their original common value, by amounts that can be expressed by a mass shift matrix[25] δM_{ij}. Here, $i = 1, \ldots \nu$, where ν is the supermultiplicity. The shift in mass of the bound state occurs in response to electromagnetic mass shifts of the external particles and of the exchanged particles, as well as more direct electromagnetic effects involving photon exchange. To order e^2 in the electromagnetic perturbations, we have, for the various supermultiplets such as B ($J = 1/2^+$ baryon octet) and Π ($J = 0^-$ meson octet), a relation of the form

$$\delta M_{ij}^{B} = A_{ij,k\ell}^{BB}\ \delta M_{k\ell}^{B} + A_{ij,k\ell}^{B\Pi}\ \delta M_{k\ell}^{\Pi} + \ldots\ldots + \ {}_{ij}^{B} \quad ,$$

$$\delta M_{ij}^{\Pi} = A_{ij,k\ell}^{\Pi B}\ \delta M_{k\ell}^{B} + A_{ij,k\ell}^{\Pi\Pi}\ \delta M_{k\ell}^{\Pi} + \ldots\ldots + \ {}_{ij}^{\Pi} \quad , \tag{13}$$

etc.,

where the photon exchange effects have been lumped together in the driving terms . In these equations, the A coefficients, because of SU(3) symmetry, have the important property that the various representations $\underset{\sim}{1}$, $\underset{\sim}{8}$, $\underset{\sim}{27}$, etc., do not mix. Let us consider one of these representations, say $\underset{\sim}{8}$. We enumerate all the relevant sets of matrices transforming like 8, namely D_{ij}^n and F_{ij}^n for the baryons B, D_{ij}^n for the mesons Π, and so forth, with $n = 1, \ldots 8$. We may now write

$$\text{Octet part of } \delta M_{ij}^B = {}_8\epsilon_n^{(1)} D_{ij}^n + {}_8\epsilon_n^{(2)} F_{ij}^n \quad ,$$

$$\text{Octet part of } \delta M_{ij}^\Pi = {}_8\epsilon_n^{(3)} D_{ij}^n \quad , \quad \text{etc.} \tag{14}$$

where the index α in $\epsilon_n^{(\alpha)}$ runs over all possible independent octets of mass matrices. Likewise, we break up the driving force in the same way:

$$\text{Octet part of } \Delta_{ij}^B = {}_8d_n^{(1)} D_{ij}^n + {}_8d_n^{(2)} F_{ij}^n \quad ,$$

$$\text{Octet part of } \Delta_{ij}^\Pi = {}_8d_n^{(3)} D_{ij}^n \quad , \quad \text{etc.} \tag{15}$$

We may now rewrite the octet part of the system of equations (13), replacing the coefficients $A_{ij,k\ell}^{BB}$ and so on by new coefficients ${}_8a_{\alpha\beta}$:

$${}_8\epsilon_n^{(\alpha)} = {}_8a_{\alpha\beta} \; {}_8\epsilon_n^{(\beta)} + {}_8d_n^{(\alpha)} \quad , \quad n = 1, \ldots 8 \quad , \tag{16}$$

where the ${}_8\epsilon$'s correspond to octet mass shifts and the ${}_8d$'s to octet driving terms.

In the same fashion, we can let an index λ run over all the independent 27-plets of mass matrices, and obtain a set of equations

$${}_{(27)}\epsilon_m^{(\lambda)} = {}_{(27)}a_{\lambda\mu} \; {}_{(27)}\epsilon_m^{(\mu)} + {}_{(27)}d_m^{(\lambda)} \quad , \quad m = 1, \ldots 27 \quad , \tag{17}$$

and so forth. For the electromagnetic perturbation in order e^2, there are driving terms only for $\underset{\sim}{27}$ and $\underset{\sim}{8}$, apart from the trivial singlet portion which does not split supermultiplets.

It is easy to see what is a natural enhancement mechanism in this formalism. We invert one of the equations for ϵ, say (16), obtaining

$$ {}_8\epsilon_n^{(\alpha)} = \left(\frac{1}{1 - {}_8\underline{a}}\right)_{\alpha\beta} {}_8d_n^{(\beta)} \quad , \tag{18} $$

with an octet mass difference operator pointing in the electric charge direction in SU(3) space, just like the octet part of the electromagnetic perturbation. If one (or more) of the eigenvalues of ${}_8\underline{a}$ is near unity, then ${}_8\epsilon_n^{(\alpha)}$ will contain a large term multiplying the associated eigenvector (s). If the matrix ${}_{27}\underline{a}$ lacks eigenvalues near unity, then the octet is preferentially enhanced.

Dashen and Frautschi have studied this mechanism in the particular case of the B octet. Using their N/D perturbation theory,[26)] they can make rough estimates of the driving terms and the eigenvalues of $\underline{a}$, and they indeed find a strong octet enhancement. In their estimate, the B octet is treated as a bound state of ΠB only, with static model kinematics, strong forces largely from B, decimet, and ρ exchange, and the F/D ratio in BBΠ coupling suggested both by bootstrap calculations and by inference from observations. They investigate one eigenvalue for ${}_{27}\underline{a}$, which comes out negative, and two eigenvalues for ${}_8\underline{a}$, one of which is very close to unity; its associated eigenvector has a large component in the direction of B mass difference, with a well-defined F/D ratio. Thus they predict a strongly enhanced octet effect in baryon electromagnetic mass differences, with an F to D ratio which turns out to agree with experiment.

The formalism we have discussed for mass differences can evidently be generalized to include changes in coupling constants together with mass differences, so that our equations would be replaced by linear equations involving δg's and δM's. This extended formalism can presumably be applied to the weak nonleptonic perturbation and made to yield weak parity-conserving and parity-violating coupling constants. Again, octet enhancement can result from finding octet eigenvalues near unity, with 27-plet eigenvalues far away.

If this mechanism does in fact predict octet enhancement for the nonleptonic weak interaction, then the enhancement, even in the parity violating case, should apply to both $\Delta Y = 0$ and $|\Delta Y| = 1$ amplitudes, and without necessarily violating SU(3) symmetry. This is in contrast to the "tadpole" mechanism utilizing the Π octet, which fails to enhance the $|\Delta \underset{\sim}{I}| = 0$, $\Delta Y = 0$ amplitude above the $|\Delta \underset{\sim}{I}| = 2$, and fails to give the sum rule derived in Ref. 8 for $|\Delta Y| = 1$ parity-violating nonleptonic baryon decays.

IV. OCTET VIOLATION OF SU(3) SYMMETRY

We now turn to the violation of SU(3) symmetry in strong interactions. As is well known, the mass splitting among members of a supermultiplet occurs in an octet pattern, and we wish to emphasize how this property may be related to the octet violations of SU(3) symmetry in electromagnetic and weak interactions.

One possible explanation of SU(3) symmetry breaking, advanced by Cutkosky and Tarjanne[24)] and by Glashow,[25)] is that of spontaneous violation. To first order in such a violation, their theory can be described with the aid of our Eqs. (13) to (18); there are no driving terms "d" and one or more eigenvalues of ${}_{8}\underline{a}$ must be unity, while eigenvalues corresponding to other representations are not close to one. (When we take into account nonlinear effects, the octet eigenvalue in the linear perturbation theory need not necessarily be exactly one.)

If only one eigenvalue of ${}_{8}\underline{a}$ lies near one and provides the dominant term, the relative amounts of mass splitting in the π octet, Δ decimet, B octet with D and F terms, and so forth can be read off from the components of the associated eigenvector. Dashen and Frautschi[23)] have extended their treatment of B to a reciprocal bootstrap on B and Δ, and find that in this simple model there are actually two eigenvalues of ${}_{8}\underline{a}$ lying near one. The correspondence between the Δ and B components of the associated eigenvectors and the observed Δ and B mass splittings is promising, although a detailed comparison with experiment must await more precise determination of the eigenvalues. Note that this result, while encouraging for the bootstrap mechanism, does <u>not</u> prove that spontaneous violation has occurred; a non-zero driving term would also be enhanced preferentially along the same eigenvectors.

In an effort to compare spontaneous violation with SU(3) mass splittings induced by a special force, let us turn from the orientation of the octet perturbation eigenvector in the space of Δ, B, π, etc., to the direction of the perturbation in the space of SU(3). We have seen that in the linear theory with a driving term, the resultant mass splitting has the same direction in SU(3) space as the perturbing force. If there is a special force responsible for the strong violations of SU(3), it must transform like the eighth component of an octet. In spontaneous breakdown of symmetry without any electromagnetic or weak perturbation, the linear approximation provides a homogeneous equation for the mass splitting;

with an eigenvalue of ${}_{8}\underset{\sim}{a}$ equal to unity, the octet splitting can point in any direction in SU(3) space and can have any size.

We must, however, consider the nonlinearity of the problem and also the presence of electromagnetic and weak perturbations, which define directions in SU(3) space that have no simple relation to the direction finally chosen by the large mass splitting. The nonlinearity alone presumably introduces a scale for the splitting, but does not affect its arbitrariness of direction. The perturbations alone would be more and more enhanced the closer the eigenvalue of ${}_{8}\underset{\sim}{a}$ is to unity, and the net result would be a splitting oriented in accordance with these perturbations. What happens when both are included is not clear, but it is not easy to see how the splitting finally emerges as the eighth component of an octet.

In an attempt to avoid this problem, Ne'eman[27)] has suggested that the bootstrap mechanism does not give rise to SU(3) violation, and must be supplemented by a special force of the current-current type that induces the large mass splittings. He refers to this force as the "fifth interaction", although we might just as well think of it as the "fourth interaction", since in the bootstrap scheme there is no other explicit strong coupling. The current presumably would transform partly like a singlet and partly like the eighth component of an octet, so that the interaction would contain $\underset{\sim}{1}$, $\underset{\sim}{8}$, and $\underset{\sim}{27}$. If we forget[28)] the suggestion that the force be long-range, there is no reason why the singlet part of the current should not be bigger than the octet part, so that the $\underset{\sim}{8}$ part of the interaction is more important than the $\underset{\sim}{27}$. In any case, the $\underset{\sim}{8}$ part would be preferentially enhanced.

We do not know whether the mass splitting can be described as spontaneous in the bootstrap theory, with some explanation of the direction in SU(3) space, or whether a force like Ne'eman's will turn out to cause it, but in the latter case we can say that octet enhancement is a common feature of weak nonleptonic effects, electromagnetic mass differences, and strong violations of SU(3) symmetry.

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17. We wish to thank Professor C. A. Barnes for discussions of these problems in nuclear physics.

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