Cosmic Calibration - Statistical Modeling for Dark Energy

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Abstract

The fact that the Universe is expanding has been known since the 1920's. If the Universe was filled with ordinary matter, the expansion should be decelerating. Beginning in 1998, however, observational evidence has been accumulating in favor of an accelerating expansion of the Universe. The unknown driver of the acceleration has been termed dark energy. The nature of dark energy can be investigated by studying its equation of state, that is the relationship of its pressure to its density. The equation of state can be measured via a study of the luminosity distance-redshift relation for supernovae. In this study, we employ supernovae data, including measurement errors, to determine whether the equation of state is constant or not. Our method is based on Bayesian analysis of a differential equation and modeling w(z) directly, where w(z) is the equation of state parameter. This work stems from collaboration between UCSC and Los Alamos National Laboratory (LANL) in the context of the Institute for Scalable Scientific Data Management (ISSDM) project.

Equations and Parameters of Interest

The main parameter of interest is w(z) there are also two other unknown parameters: $H_0 = 71.0 \pm 2.6$ and $\Omega_{m0} = 0.265 \pm 0.03$. Where the uncertainty shown is one standard deviation. The main equation of interest is a transformation:

$$T(z, H_0, \Omega_m) = 25 + 5\log_{10}\left(\frac{c(1+z)}{H_0} \int_0^z \left(\Omega_m (1+s)^3 + (1-\Omega_m)(1+s)^3 e^{3\int_0^s \frac{w(u)}{1+u} du} \right)^{-0.5} ds \right)$$

To be able to use this equation we will need to specify a form for w(z). This also leads to a likelihood as follows:

$$L(\sigma, w_0, H_0, \Omega_m) \propto \left(\frac{1}{\tau_i \sigma} \right)^n e^{\frac{-1}{2} \sum_{i=1}^{n} \left(\frac{\mu_i - T(z_i, H_0, \Omega_m)}{\tau_i \sigma} \right)^2}$$

To be able to use this likelihood we will need priors for σ, Ω_m, H_0, and whatever parameters we used to specify w(z). As a note the τ's are the standard deviations for μ.

Bayesian Modeling

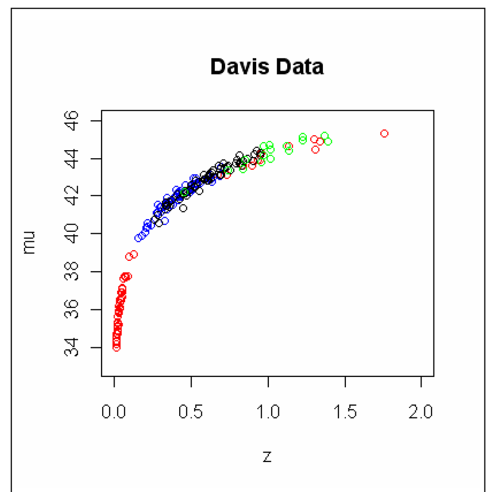
For Bayesian modeling we will need priors for each of the parameters. σ will receive a rather straightforward Inverse-Gamma prior with mean about one. However, we want to examine six priors for w_0 and their sensitivity. At first, we will hold Ω_m and H_0 constant. We found that the N(-1,1), Unif(-2,0), and Unif(-25,0) had nearly identical posteriors; so we will conclude that the prior choice does not effect the posteriors significantly in this case (see Table 1).

Table 1 - Posteriors for Three Priors of Interest on w_0

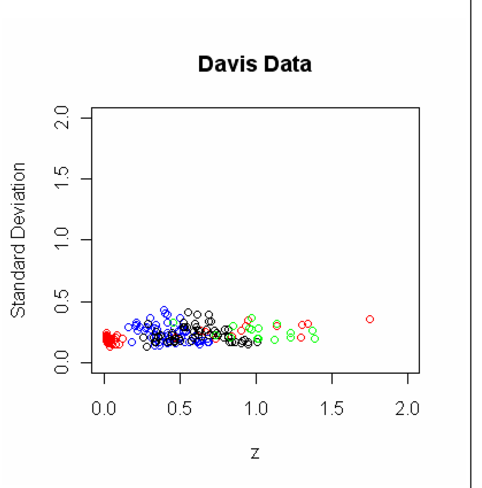
Prior	95% Probability interval for w0	95% Probability Interval for σ
N(-1,1)	(-1.11, -0.98)	(0.95, 1.08)
Unif((-2,0)	(-1.11, -0.98)	(0.95, 1.08)
Unif(-25,1)	(-1.11, -0.98)	(0.95, 1.08)

We also examined three other priors that included point masses. The first one was a point mass at -1 and a Unif(-5,0); the second was a point mass at -2 and a Unif(-5,0); and the third prior had three point masses at -1/3, -2/3, and -1. These were helpful in doing a type of Bayesian hypothesis testing. In simulated data sets these priors gave in simulated data, these priors produced posteriors with high posterior probabilities for the true values of w_0.

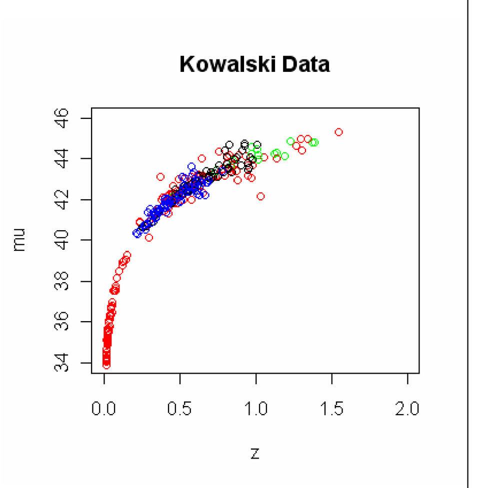
Original Data



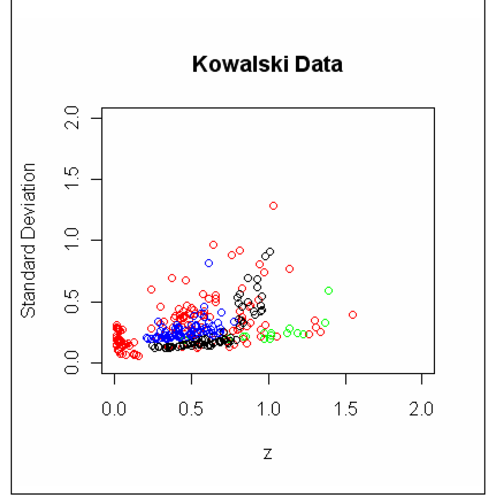
The Davis data are a collection of 192 supernovae observations (SNe Ia.) The data has a redshift (z) value for each supernova and a value for μ (observed distance modulus.) The first plot is of z vs μ. The four colors mark different observers of the supernovae; some of the telescopes are looking for supernovae at particular values of redshift.



The second plot is of z vs the standard deviation of μ. μ has a standard deviation associated with it; μ is derived by astronomers by fitting light curves to raw data. We will use the inverse of the standard deviations as weights in our likelihood equations.

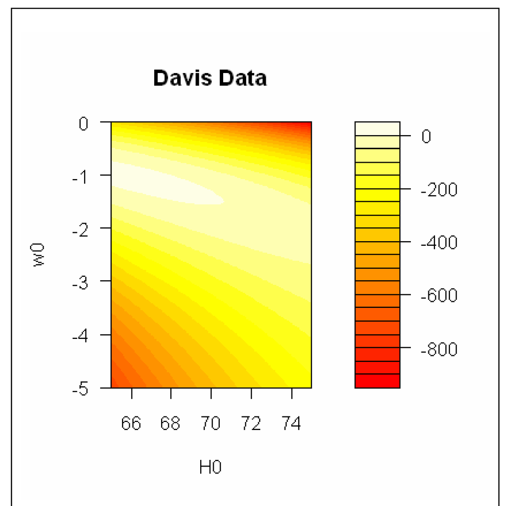


The Kowalski data are 307 SNe Ia observations. More supernova observations are included in the Kowalski data, in part this is due to the fact that Kowalski's dataset is newer and has had more years to collect more supernova data. There are 177 supernovae observations in common in these two data sets. Different methods of light curve fitting result in different values of μ for the same supernova, this can be seen in the differences between the Davis and Kowalski datasets.

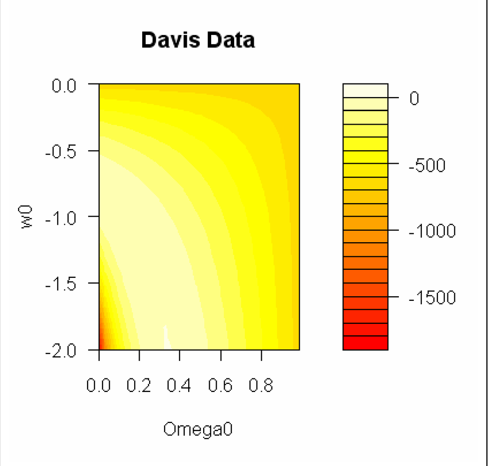


This data plot shows some trend in the standard deviations coming from different observers. This phenomena will be looked into as it is advantageous to the analysis to be able to reduce as much of the uncertainty as possible. The hope would be that the observers with larger uncertainty associated with their measurements could possibly adjust for this.

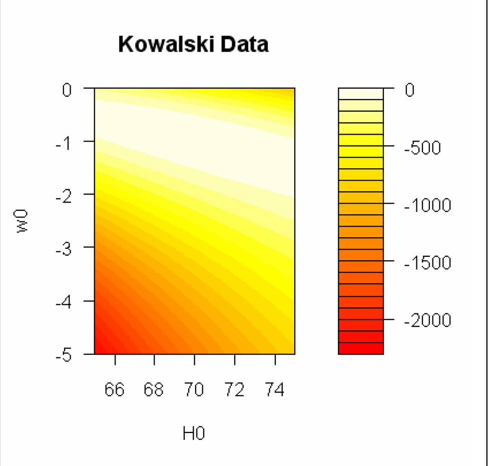
Likelihood Plots



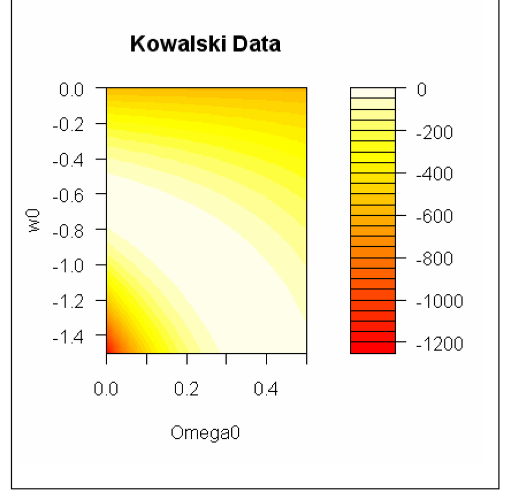
In the full model we will allow all four parameters to be variables but for the graphical output we would just like to examine some of the two dimensional likelihood plots. These likelihood plots are of the model where we assume that the form of w(z) is constant: w(z) = w_0. This first plot is a likelihood plot for the two parameters w_0 and H_0. For this plot we will hold $\Omega_m = 0.265$ and σ = 1. The maximum likelihood estimates are at w_0 = -1.002 and H_0 = 65.63.



This second plot is for the likelihood of the two parameters Ω_m and w_0. For this plot we will hold H_0 = 71.0 and σ = 1. The maximum likelihood estimates are at w_0 = -2.91 and Ω_{m0} = 0.41.



The third and fourth plots are for the Kowalski data. This one is a likelihood plot for the two parameters w_0 and H_0. For this plot we will hold Ω_{m0}= 0.265 and σ = 1 . The maximum likelihood estimates are at w_0 = -0.96 and H_0 = 70.09.



This final plot is for the likelihood of the two parameters Ω_m and w_0. For this plot we will hold H_0 = 71.0 and σ = 1. The maximum likelihood estimates are at w0 = -1.47 and Ω_{m0} = 0.39.

Conclusions

- Fitting models directly to w(z) seems to work well; but requires all Metropolis-Hasting steps in MCMC because of the double integral
- Ω_{m0} and H_0 must be incorporated into the model as unknown parameters with priors
- The raw data is not being used directly and the fitted values of μ for the two datasets have systematic differences.
- Thus far the choice of prior for model w(z) = w_0 does not greatly influence the posterior.

Future Work

- Use a Gaussian process to model w(z).
- Fit a full Bayesian model with 4 unknown parameters for both datasets using Metropolis Hasting steps
- Set up an experimental design to find where more data is need (on the z axis). In the experimental design also test how shrinking uncertainty for μ, Ω_{m0} , and H_0 would help in drawing more conclusive statements about w(z).
- Look into which type of measurement error could be reduced to help make conclusive statements about the parameters of interest; especially the standard deviations associated with μ

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