Calculation of Age-conditional Probability of Dying of A Specific Cancer

To calculate the age-conditional probability of dying of a specific cancer we use standard competing risks methodology (see e.g., Kalbfleisch and Prentice, 1980). First, we repeat (almost indentically) the description of the problem from the beginning of Section 2 of Fay, Pfeiffer, Cronin, Le, and Feuer (2003):

> We observe the time until one of several events, T, and an indicator of the type of event that occurred, J. [The value] T is a random variable denoting the age at death and J has one of 2 values, $J = d$ means death from the event of interest (e.g., breast cancer), and $J = o$ means death from other causes. For ease of exposition, we use the term "cancer" to denote the event of interest. The cause specific hazard function for $J = j$ is
>
> $$\lambda_j(a) \quad = \quad \lim_{\epsilon \to 0^+} \frac{Pr\left[a \leq T < a + \epsilon, J = j | T \geq a\right]}{\epsilon}.$$
>
> Thus $\lambda_d(a)$ is the rate of cancer deaths per person-years alive at age a, and $\lambda_o(a)$ is the rate of other (i.e., non-cancer) deaths per person-years alive at age a. The overall failure rate at age a is $\lambda(a) = \lambda_d(a) + \lambda_o(a)$, and the overall survival function is $S(a) = Pr[T > a] = \exp\left(-\int_0^a \lambda(u)du\right)$. The probability of dying from cause j in the age interval $[x, y)$ given survival until just prior to x is
>
> $$Pr\left[x \leq T < y, J = j | T \geq x\right] \quad = \quad \frac{\int_x^y \lambda_j(u) S(u-) du}{S(x-)}.$$
>
> where $S(a-) = \lim_{\epsilon \to 0} S(a - \epsilon)$.

Thus, the result depends on the method for estimating the $\lambda_d(a)$ and $\lambda_o(a)$ functions. In version 5.0 of DevCan, we use the piecewise mid-age joinpoint model for the rates described in Fay (2003), where the pieces are $\frac{1}{2}$ year long. Previous versions of the DevCan software used the simple 5-year piecewise constant rate model described in Fay, et al. (2003). The method for confidence interval estimation is described in Fay, et al. (2003).

Note: The previous description from the DevCan website for the calculation of the age-conditional probability of dying used notation similar to that used in multiple decrement life tables (see e.g., Elandt-Johnson and Johnson, 1980). Although the notations look very different, the calculations are the same. This is not true for the calculation of the age-conditional probability of **developing** cancer, where the new method described in Fay, et al. (2003) is different from the method described in Wun, Merrrill, and Feuer (1998), even though both methods use the simple 5-year piecewise constant rate model in their calculations (see Fay, et al., 2002).

References

Elandt-Johnson, R.C., and Johnson, N.L. (1980). *Survival Models and Data Analysis.* Wiley, New York, Chapter 10.

Fay, M.P., Pfeiffer, R., Cronin, K.A., Le, C. and Feuer, E.J. (2002). Comparison of Two Methods for Calculating Age-Conditional Probabilities of Developing Cancer. Technical Report #2002-01, Statistical Research and Applications Branch, National Cancer Institute (Accessed at http://srab.cancer.gov/reports on September 3, 2002).

Fay, M.P., Pfeiffer, R., Cronin, K.A., Le, C. and Feuer, E.J. (2003). Age-Conditional Probabilities of Developing Cancer. *Statistics in Medicine* **22**(11):1837-1848.

Fay, M.P. (2003). Estimating Age Conditional Probability of Developing Cancer using a Piecewise Mid-Age Group Joinpoint Model for the Rates. Statistical Research and Applications Branch, NCI, Technical Report # 2003-03. (Available at http://srab.cancer.gov/reports).

Kalbfleisch, J.D. and Prentice, R.L. (1980). *The Statistical Analysis of Failure Time Data.* Wiley, New York 1980, pp. 163-178.

Wun, L-M, Merrill, R.M., and Feuer, E.J. (1998). Estimating lifetime and age-conditional probabilities of developing cancer. *Lifetime Data Analysis* **4,** 169-186.