UNITED STATES GOVERNMENT

memorandum

DATE: May 1, 2002

TO: Interested Media Bureau Members

FROM: Peter Alexander

SUBJECT: Cable Model

At Judy Herman's request, I am making copies of my working paper (no title) on the cable industry available for your consideration. I do not anticipate that you will gain any insight from reading this - it is distributed as a courtesy.

Caveat and disclaimer: The enclosed draft is extremely crude, superficial, incomplete, and contains numerous errors and mistakes. At best, it should be viewed as my set of working notes. Please do not distribute or cite in any fashion. (Comments are always welcome.)

1 Multiple Buyers and Multiple Sellers: Product Value to Buyers is Non-Stochastic.

Assume that risk neutral content providers (also known as cable networks; hereafter, sellers) have positive fixed costs of producing and zero marginal costs of distributing their product. There are I sellers. The sellers earn revenue by selling their product to cable owners (hereafter, buyers).

We begin by assuming that the seller makes a 'take it or leave it' offer to each prospective buyer. We denote by $T_{1,i}, T_{2,i}, ..., T_{M,i}$ the total payments to seller i from buyers $1, 2, ..., M$ respectively, if the product is sold. There are M buyers, each of whom has $N_1, N_2, ..., N_M$ subscribers, where

$$\sum_{m=1}^{M} N_m = N. \tag{1}$$

We assume that buyer m has positive fixed costs F_m and zero program provision costs (an assumption we relax later in the paper). We note that given I sellers with I products, every buyer has 2^I possible programming choices. We denote a programming choice of buying only seller i's program by E_1^i, where subscript 1 denotes the program package consisting of only one program and the superscript i denotes seller i. The programming package consisting of 2 products, e.g., products from seller k and seller l, is given by:

$$E_2^{k,l} \equiv E_1^k + E_1^l \equiv E_1^k \cup E_1^l \tag{2}$$

The program package that includes all programs from all sellers is denoted by E_I or $E_I^{1,2,...,I}$. The value of programming package $\tilde{E}$ for buyer m is denoted by $V_m(\tilde{E})$. Buyer m's objective is to maximize profits:

$$(V_m(\tilde{E}) - C(\hat{E})) \tag{3}$$

by choice of programming package $\hat{E}$. We assume that the value of any combination of programs is positive, and that the value correspondence satisfies decreasing marginal returns.

ASSUMPTION 1 (Decreasing marginal value): For any buyer m, and any two programming packages $\hat{E}$(non-empty) and $\tilde{E}$; and for any seller i's program $E_1^i \not\subseteq \tilde{E} \cup \hat{E}$, the following inequality holds:

$$V_m(\tilde{E} + E_1^i) - V_m(\tilde{E}) \geq V_m(\hat{E} + \tilde{E} + E_1^i) - V_m(\hat{E} + \tilde{E}) > 0 \tag{4}$$

Claim 1 *With M buyers and I sellers, the unique Nash Equilibrium transfer price for each seller k for buyer m is given by:*

$$T_{m,k} = V_m(E_I) - V_m(E_I - E_1^k) \tag{5}$$

and all buyers buy programs from all sellers.

Proof. We proceed in several steps. First, we show that if there is a Nash Equilibrium, it is an equilibrium where all buyers buy from all sellers. Second, we show that in the equilibrium where all buyers buy from all sellers, the equation:

$$T_{m,k} = V_m(E_I) - V_m(E_I - E_1^k) \tag{6}$$

must hold. Finally, we prove by induction that the transfer price $T_{m,i}$ is in fact a unique Nash Equilibrium transfer price.

Step 1. By contradiction, assume that in some Nash Equilibrium, buyer m did not buy the program from seller i. Then, seller i's payoffs from buyer m are zero. Denote by E^* the set of programs bought by buyer m. Since

$$V(E^* + E_1^i) > V(E^*) \tag{7}$$

seller i is strictly better off (i.e., obtains positive payoffs) by charging any transfer price:

$$T \in (0, V(E^* + E_1^i) - V(E^*)] \tag{8}$$

Thus, buyer m finds it optimal to buy from seller i.

Step 2. Assume that there is a Nash Equilibrium where all buyers buy from all sellers. Then, it should be the case that buyer m prefers buying from all sellers to buying from any set of $(I-1)$ sellers; i.e., the following condition must hold for all m and k:

$$V_m(E_I) - \sum_{i=1}^{I} T_{m,k} \geq V_m(E_I - E_1^k) - \sum_{i=1}^{I} T_{m,i} - T_{m,k} \tag{9}$$

Assume (9) holds with a strict inequality for any seller l. Then, seller l can increase it's payoffs by increasing the transfer price by an ϵ small amount, while condition (9) still holds for all $k = 1, ..., I$. This is a contradiction. Therefore (9) must hold with equality:

$$V_m(E_I) - \sum_{i=1}^{I} T_{m,i} = V_m(E_I - E_1^k) - \sum_{i=1}^{I} T_{m,i} - T_{m,k} \tag{10}$$

which simplifies to:

$$T_{m,k} = V_m(E_I) - V_m(E_I - E_1^k) \tag{11}$$

Step 3. We have shown that for all sellers it is optimal to charge $T_{m,k}$. In order to ensure that this is in fact a Nash Equilibrium, we must check that for any buyer m the value of buying from all sellers is greater than or equal to the value of any programming package from the remaining 2^{I-1} possibilities. To begin, denote by $T_{m,k}^n$ the transfer price defined in (11) when there are a total of $I = n$ sellers. Clearly, when $I = 1$,

$$T_{m,k}^1 = V_m(E_1^1) \tag{12}$$

is a Nash Equilibrium of the game, and all buyers buy from the seller.

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Now, assume that $T_{m,k}^n$ is a Nash Equilibrium outcome for some $I = n \geq 1$. Then, it suffices to show that $T_{m,k}^{n+1}$ is also a Nash Equilibrium, which we do by showing that buyer m's benefit from buying all available $n+1$ programs is positive. We note that:

$$V_m(E_{n+1}) - \sum_{i=1}^{n+1} T_{m,i}^{n+1} \tag{13}$$

equals:

$$\sum_{i=1}^{n+1} V_m(E_{n+1} - E_1^i) - n * V_m(E_{n+1}) \tag{14}$$

or:

$$\sum_{i=1}^{n+1} V_m(E_{n+1} - E_1^i) - \sum_{i=1}^{n} (V_m(E_{n+1} - E_1^i) + T_{m,i}^{n+1}) \tag{15}$$

which equals:

$$V_m(E_{n+1} - E_1^{n+1}) - \sum_{i=1}^{n} T_{m,i}^{n+1} \tag{16}$$

We then note that:

$$V_m(E_{n+1} - E_1^{n+1}) - \sum_{i=1}^{n} T_{m,i}^{n+1} \geq V_m(E_{n+1} - E_1^{n+1}) - \sum_{i=1}^{n} T_{m,i}^n \geq V_m(E_n) - \sum_{i=1}^{n} T_{m,i}^n \geq 0 \tag{17}$$

where the last inequality holds due to our assumption that:

$$T_{m,i}^{n+1} = T_{m,i}^n \tag{18}$$

Any buyer m's payoffs are positive when there are $n+1$ sellers charging $T_{m,i}^{n+1}$, and this buyer is better off buying $n+1$ programs than any program packages consisting of n programs. But we know from our induction assumption for $I = n$ that when there are n sellers, buying from all sellers is preferred to all other choices. Therefore, with $n+1$ sellers, buying from all $n+1$ sellers is preferred to any other programming package. Then for $I = n+1$, a Nash Equilibrium consists of sellers charging $T_{m,i}^{n+1}$ and all buyers buying from all sellers. By construction this Nash Equilibrium is unique. ∎

The interpretation of the claim above is straightforward. When there are no capacity restraints, cable operators will buy all network programs, and, as a result, maximum program diversity will be achieved. However, in actual practice, we observe that cable operators do not buy from all sellers. We offer several explanations. First, we argue that there may exist capacity constraints on cable operators. While in reality physical constraints on program carriage may not be approached, finite audience size, for any given level of cost, may constrain the profit-maximizing level of program carriage due to substitution effects. Second, we explore the possible effects on program carriage in the presence of so-called 'most favored customer' clauses. In these cases, larger buyers are able to obtain prices charged the smaller buyers, i.e., smaller buyers do not obtain asymmetric price discounts.

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and the buyer buys from all sellers.

ii) if:

$$C_m(1) \geq V_m(E_1^1) \tag{27}$$

then buyer m does not buy from any seller regardless of the transfer price.

iii) if:

$$C_m(I) > V_m(E_I) - V_m(E_I - E_1^I) \tag{28}$$

and

$$C_m(1) < V_m(E_1^1) \tag{29}$$

then: *A) There exists a $k \in \{1, 2, ..., I-1\}$ such that:*

$$V_m(E_k^{1,2,...,k}) - V_m(E_k^{1,2,...,k} - E_1^k) \geq C_m(k) \tag{30}$$

and

$$C_m(k+1) > V_m(E_{k+1}^{1,2,...,k,k+1}) - V_m(E_{k+1}^{1,2,...,k,k+1} - E_1^{k+1}) \tag{31}$$

B) The transfer price is given by:

$$T_{m,i} = V_m(E_k^{1,2,...,k}) - V_m(E_k^{1,2,...,k} - E_1^i) - max\{C_m(k), V_m(E_k^{1,2,...,k} - E_1^i + E_1^{k+1}) - V_m(E_k^{1,2,...,k} - E_1^i) \tag{32}$$

for all $\leq i \leq k$, and $T_{m,i} \geq 0$ for $k+1 \leq i \leq I$

C)Buyer m buys from the first k sellers.

Proof.

i) This is a direct extension of previous claim. The condition on the cost function implies that there is a positive value to be obtained by including an additional program regardless of the current combination of programs. Therefore, all programs will be bought in the unique Nash Equilibrium. The transfer price charged by a seller will be such that the buyer is indifferent between buying and not buying this additional program, i.e,

$$T_{m,k} = V_m(E_I) - V_m(E_I - E_1^k) - C_m(I) \tag{33}$$

ii) The condition placed on the cost structure implies that the net benefit from buying any program is negative. Clearly, no programs will be bought in equilibrium.

iii) This condition states that the net value of buying just one program is positive, and the net value of buying the last program after buying all other $I-1$ programs in negative. Clearly, there exists a k between 1 and $I-1$ such that the net value of buying from first k sellers (ignoring transfer prices) is positive and the net value of buying from $k+1$'s seller (ignoring transfer prices) is negative. In mathematical terms:

There exists a $k \in \{1, 2, ..., I-1\}$ such that:

$$V_m(E_k^{1,2,...,k}) - V_m(E_k^{1,2,...,k} - E_1^k) \geq C_m(k) \tag{34}$$

and

$$C_m(k+1) > V_m(E_{k+1}^{1,2,...,k,k+1}) - V_m(E_{k+1}^{1,2,...,k,k+1} - E_1^{k+1}) \tag{35}$$

Clearly, the buyer will buy, at most, k programs. Since the value of seller i's program is never less that the value of seller $(i+1)$'s program, it is straightforward to see that if seller i is served then seller $i+1$ should also be served in any Nash Equilibrium. This implies that sellers $k+1, ..., I$ are not served in any Nash Equilibrium. Seller k must be served in any Nash Equilibrium, since it can always charge $T_{m,k} = 0$ and the buyer buys k's seller, either by replacing some of its programs by program k or by keeping all other programs. Therefore, if there is a Nash Equilibrium, then all k programs should be bought. If there is a Nash Equilibrium with k sellers served, then it should be the case that the buyer is indifferent between buying from any seller i as compared to not buying from that seller, and to replacing it with other program from any of remaining $I - k$ sellers' programs. i.e. for $1 \leq i \leq k$

$$T_{m,i} = V_m(E_k^{1,2,...,k}) - V_m(E_k^{1,2,...,k} - E_1^i)$$
$$- \max\{C_m(k), V_m(E_k^{1,2,...,k} - E_1^i + E_1^{k+1}) - V_m(E_k^{1,2,...,k} - E_1^i)\}$$

Just like in claim 1, $T_{m,i} \geq 0$ and $V_m(E_k^{1,2,...,k}) - \sum_{i=1}^{k} C_m(i) - \sum_{i=1}^{k} T_{m,i} \geq 0$ and both buyer and the sellers prefer these transfer prices. ∎

The optimality implies that all programs that has marginal value above marginal cost should be broadcasted. The claim above shows that under our assumptions, the optimal program diversity is achived. The market achieves the first best outcome.

3 MFC Clause

3.0.1 Marginal Seller, MFC

Suppose without MFC provisions some seller i was charging $t_1^*, t_2^*, t_3^*, ..., t_M^*$ per customer transfer prices to buyers $1, 2, 3, ..., M$ respectively.

Buyer 1 has the most customers, i.e. $N_1 > N_m$ for all $m \geq 2$.

Suppose buyer 1 forces this seller to MFC terms requiring the seller to charge per customer price no more than the minimum of prices charged to other buyers, i.e. $t_1 \leq \min\{t_2, t_3, ..., t_M\}$

If $t_m^* \geq t_1^*$ for all $m \geq 2$, then MFC provision will have no affect on seller's decision.

For simplicity assume that t^*'s take 4 possible values (as long as the number of buyers is finite, this analysis applies).

$$0 = t_4^* < t_3^* < t_1^* < t_2^*$$

There are some buyer's with no MFC transfer prices above t_1^*, there are some buyer's with no MFC transfer prices below t_1^*, and there are some buyers who do not buy from seller i, denoted by $t_4^* = 0$.

Denote customers served by different transfer price t_k^* by

$$n_1 = N_1$$
$$n_2 = \sum_{t_m^* = t_2^*} N_m$$

$n_3 = \sum_{t^*_m = t^*_3} N_m$

$n_4 = \sum_{t^*_m = t^*_4} N_m, \qquad \sum_{k=1}^{4} n_k = N$

Clearly, MFC arrangements will not affect the buyers who are paying above buyer 1's price. Given MFC restraints, seller 1 has 2 options:

1) Charge $t_1 = t_3 = t^*_1$, $t_2 = t^*_2$.

The seller serves only first and second type of buyers and seller's revenue is

$r_1 = n_1 \cdot t^*_1 + n_2 \cdot t^*_2$

2) Charge $t_1 = t_3 = t^*_3$, $t_2 = t^*_2$.

The seller serves all the buyers that it would serve without MFC and seller's revenue is

$r_2 = (n_1 + n_3) \cdot t^*_3 + n_2 \cdot t^*_2$

The program diversity is below optimal (i.e. only 1st and 2nd buyer types are served) if

$r_1 > r_2 \Leftrightarrow \frac{n_1}{n_1 + n_3} > \frac{t^*_3}{t^*_1}$.

Notice, the higher n_1 (and thus the market share of buyer 1), the more likely that the market will undersupply programs. Also buyer 1 always buys the product and pays at most the price under no MFC provisions.

3.0.2 2 Sellers, MFC

Assume there are 2 sellers and 2 types of buyers. Buyer 1 is large enough and imposes MFC conditions to both sellers. Denote:

$v_1(1)$-buyer 1's per customer valuation of seller 1's product.

$v_1(1+2)$- buyer 1's valuation of having both buyers' products.

$v_2(2)$- buyer 2's valuation of seller 2's product.

Also assume that assumption 1 holds. i.e.

$v_1(1) + v_1(2) > v_1(1+2)$ \qquad and

$v_2(1) + v_2(2) > v_2(1+2)$

We know that NE prices under no MFC provisions are

$t^*_{11} = v_1(1+2) - v_1(2)$

$t^*_{12} = v_1(1+2) - v_1(1)$

$t^*_{21} = v_2(1+2) - v_2(2)$

$t^*_{22} = v_2(1+2) - v_2(1)$

We consider following 4 cases:

i) $t^*_{11} \leq t^*_{21}$ and $t^*_{12} \leq t^*_{22}$.

In this case, MFC and no MFC cases give the same prices and outcomes since MFC provisions do not restrict seller's behavior.

ii) $t^*_{11} > t^*_{21}$ and $t^*_{12} \leq t^*_{22}$. MFC clause affects the first seller.

Seller 1 has 2 options:

A) Charge $t_{11} = t_{21} = t_{21}^*$ and both buyers buy from seller 1.

Seller 1's revenue is:

$N \cdot t_{21}^* = (\sum_{m=1}^{M} N_m) \cdot t_{21}^*$

Seller 2's best response is to charge $t_{12} = t_{12}^*$, $t_{22} = t_{22}^*$

B) Charge $t_{11} = t_{21} = t_{11}^*$ and only the buyer 1 is served.

Seller 1's revenue is:

$N_1 \cdot t_{11}^*$

Seller 2's best response is to charge

1) $t_{12} = t_{12}^*$, $t_{22} = v_2(2)$ if $v_2(1) - t_{11}^* < 0$

2) $t_{12} = t_{12}^*$, $t_{22} = v_2(2) - v_2(1) + t_{11}^*$ if $v_2(1) - t_{11}^* \geq 0$

Seller 1 prefers B to A if $N \cdot t_{21}^* < N_1 \cdot t_{11}^*$

$\Leftrightarrow \quad \frac{N_1}{N} \cdot (v_1(1+2) - v_1(2)) > v_2(1+2) - v_2(2)$

$\Leftrightarrow \quad$ (buyer 1's market share) $\cdot v_1(1+2) - v_1(2)) > v_2(1+2) - v_2(2)$

iii) $t_{11}^* \leq t_{21}^*$ and $t_{12}^* > t_{22}^*$. This case is symmetric to case (ii) and therefore the results will be symmetric.

iv) $t_{11}^* > t_{21}^*$ and $t_{12}^* > t_{22}^*$. **MFC arrangements restrict both sellers. In this case, each seller has 3 choices: provide the product only to buyer 1, only to buyer 2, and to both buyers. seller 1**

	Seller 2		
	buyer 1	buyer 2	bo
buyer 1	outcome a	outcome b	o
buyer 2	d	e	
both buyers	g	h	

a)

b)

c)

d)

e) Cannot be a NE. If both sellers serve only buyer 2, then $t_{21} = t_{21}^*$ and $t_{22} = t_{22}^*$. Then also $t_{11} = t_{21}^*$ and $t_{12} = t_{22}^*$. But at such tranfer prices, buyer 1 finds it optimal to buy from both sellers.

f) Cannot be a NE by the same reason as outcome (e).

g)

h) Cannot be a NE by the same reason as outcome (e).

i)

4 Single Seller and Multiple Buyers

A risk neutral content provider (hereafter, seller) has a positive fixed cost of producing and zero marginal cost of distributing its product. The seller gets revenue from the cable owners (hereafter, buyers), as well as some additional advertising revenue (if the product is sold). The seller makes a 'take it or leave it' offer to each prospective buyer.

We denote by T_1, T_2,...,T_M the total payment from buyers 1, 2,...,M respectively if the product is sold. Also, we denote by A_1, A_2,...,A_M the total amount of the seller's advertising revenue from selling the product to buyers 1, 2,...,M respectively.

The seller does not observe the exact value of it's programming to the buyers. However, the seller knows that the value of it's programming for any given buyer m is distributed by a probability density function $F_m(.)$. In other words, the seller only knows the *approximate* value of its programming for any buyer, but its estimate is not exact. We assume the individual buyer always has a precise estimate of the value of the seller's product to that buyer.

There are M buyers, each of whom has N_1, N_2,...,N_M subscribers, where

$$\sum_{m=1}^{M} N_m = N. \tag{36}$$

Buyer m observes the value of the seller's program, V_m, (where V_m belongs to the p.d.f. $F_m(.)$), and the transfer price T_m given by the seller. Thus, we assume that V_m is the expected value of the product for buyer m. Notice that the buyer does not have the incentive to truthfully reveal its estimate of the true value of the product, since the seller would extract all of the gains from trade by charging $T_m = V_m$.

The buyer's objective is to maximize its profits $(V_m - T_m)$. Thus, the decision rule for firm m is straightforward: buy the program if $T_m \leq V_m$, and do not buy the program if $T_m > V_m$.

The seller's objective is to maximizes its total expected profits. The expression for this is given by:

$$\sum_{m=1}^{M} (\text{probability offer accepted by firm } m) \cdot (T_m + A_m) \tag{37}$$

which we re-write as:

$$\sum_{m=1}^{M} (1 - F_m(T_m)) \cdot (T_m + A_m) \tag{38}$$

where the choice variable for the seller is the price sequence given by T_1, ... T_M. The first-order conditions for T_m are given by:

$$(1 - F_m(T_m^*)) - T_m^* \cdot f_m(T_m^*) - A_m \cdot f_m(T_m^*) = 0 \tag{39}$$

where $f_m(.)$ is the probability density function. We note that (4) is maximized when:

$$T_m^* = \frac{(1 - F_m(T_m^*))}{f_m(T_m^*)} - A_m \tag{40}$$

The second-order conditions for a maximum are given by:

$$-f_m(T_m) - f_m(T_m) - T_m \cdot f_m^1(T_m) - A_m \cdot f_m^1(T_m) < 0 \tag{41}$$

which holds if:

$$f_m^1(.) \leq 0 \qquad (42)$$

where f_m^1 is the first derivative of the p.d.f.. If (6) holds, then T_m^* is unique for each firm m.

The assumption of a uniform probability distribution function automatically satisfies (6). Thus, for example, let us assume that the quality of the program is distributed uniformly in $[V_m, \bar{V}_m]$ for each firm m. Then:

$$F_m(T_m) = \frac{(T_m - \underline{V}_m)}{(\bar{V}_m - \underline{V}_m)} \qquad (43)$$

and

$$f_m(T_m) = \frac{1}{(\bar{V}_m - \underline{V}_m)}. \qquad (44)$$

Recall that the seller does not know the exact V_m, but the buyer does. In this case, the optimal price charged by the seller is:

$$T_m^* = \frac{\bar{V}_m - A_m}{2} \qquad (45)$$

if $T_m^* \in [\underline{V}_m, \bar{V}_m]$, and $T_m^* = \underline{V}_m$ otherwise (a corner solution).

The seller's expected profits from selling to firm m are:

$$\pi = \frac{(\bar{V}_m + A_m)^2}{4(\bar{V}_m - \underline{V}_m)} \qquad (46)$$

if $T_m^* \in [\underline{V}_m, \bar{V}_m]$, and $V_m \cdot (T_m + A_m)$ otherwise. Thus, the seller supplies the programming if, and only if, the expected profits are higher than F.

The expected number of subscribers served is (if all solutions are interior):

$$S = \sum_{m=1}^{M} N_m \cdot \frac{(\bar{V}_m + A_m)}{2 \cdot (\bar{V}_m - \underline{V}_m)} \qquad (47)$$

Next, we assume that some of the large buyers have 'Most Favored Customers' (hereafter, MFC) arrangements. The arrangements we explore are directly analogous to Most Favored Nation (MFN) trade arrangements whereby

a larger trading partner is guaranteed a price no higher than another smaller trading partner. For the current case, we denote the price per subscriber as:

$$p_m = T_m/N_m \tag{48}$$

In our model the most favorite customer clause means that the firm m under the MFC will pay at most $min\{p_1, ..., p_{m-1}, p_{m+1}, ..., p_M\}$ per subscriber, or $N_m \cdot min\{p_1, ..., p_{m-1}, p_{m+1}, ..., p_M\}$ overall. In the single seller model, the seller cannot do better with the MFC clause than without, because the MFC restricts the seller's possible price choices. In what follows, we explore two possible cases. In the first, the MFC covers all buyers. In the second, the MFC covers some, but not all buyers.

Case 1. In the first case, we assume the MFC clause covers all buyers. Clearly, the seller will charge the same price for all buyers. In what follows we transform prices into per subscriber prices. This transformation does not change the analysis in any fashion, but it does allow us to explore in greater detail some of the effects of MFC arrangements. Thus, we denote:

$$\underline{v}_m \equiv \frac{\underline{V}_m}{N_m} \tag{49}$$

$$\bar{v}_m \equiv \frac{\bar{V}_m}{N_m} \tag{50}$$

$$a_m \equiv \frac{A_m}{N_m} \tag{51}$$

$$F_m(T_m) \equiv H_m(p_m) \tag{52}$$

$$f_m(T_m) \equiv h_m(p_m) \tag{53}$$

The seller's objective is to maximize:

$$\sum_{m=1}^{M} (1 - H_m(p_m)) \cdot (p_m + a_m) \cdot N_m \tag{54}$$

which equals:

$$\sum_{m=1}^{M} (1 - F_m(T_m)) \cdot (T_m + A_m)] \tag{55}$$

where $p_1 = p_2 = ... = p$ at the optimum. The first-order conditions for an interior solution are:

$$\sum_{m=1}^{M} \{[(1 - H_m(p^*)) - h_m(p^*) \cdot (p^* + a_m)] \cdot N_m = 0 \tag{56}$$

which yields:

$$p^* = \frac{\sum_{m=1}^{M}[(1 - H_m(p^*)) - h_m(p^*) \cdot a_m] \cdot N_m}{\sum_{m=1}^{M} N_m \cdot h_m(p^*)} \tag{57}$$

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The solution to p^* is unique, and the second-order conditions are satisfied if $h_m^1(.) \le 0$ for all m. As in the previous case, we assume a uniform probability distribution. For simplicity, we write:

$$\bar{v}_m - \underline{v}_m = k \tag{58}$$

Then, the expected number of buyers served is:

$$S_{MFC} = \sum_{m=1}^{M} N_m \cdot \frac{2\bar{V}_m - \frac{N_m^2}{2N} \cdot \sum_{m=1}^{M}(\bar{V}_m - A_m)}{2 \cdot (\bar{V}_m - \underline{V}_m)} \tag{59}$$

which we re-write (using k) as:

$$S_{MFC} = \sum_{m=1}^{M} \frac{2\bar{V}_m - \frac{N_m}{2N} \cdot \sum_{m=1}^{M}(\bar{V}_m - A_m)}{2k} \tag{60}$$

or, finally, as:

$$S_{MFC} = \sum_{m=1}^{M} \frac{(\frac{3}{2}\bar{V}_m + \frac{1}{2}A_m)}{2k} \tag{61}$$

Now we can compare this result to the case with no MFC. Recall that:

$$S_{No\ MFC} = \sum_{m=1}^{M} N_m \cdot \frac{(\bar{V}_m + A_m)}{2 \cdot (\bar{V}_m - \underline{V}_m)} \tag{62}$$

Re-writing (using k) we see that:

$$S_{No\ MFC} = \sum_{m=1}^{M} \frac{(\bar{V}_m + A_m)}{2k} \tag{63}$$

or, finally:

$$S_{No\ MFC} = \sum_{m=1}^{M} \frac{(\bar{V}_m + A_m)}{2k} \tag{64}$$

It is immediately clear that:

$$S_{MFC} > S_{No\ MFC} \tag{65}$$

if $A_m < \bar{V}_m$. Intuitively, we would expect this condition to hold since the advertising revenue for the seller cannot be larger than the largest possible value of the program for the cable operator (the cable operator also profits from

advertising; however, advertising is not the only source of revenue for the cable operator). Also, for an interior solution, the condition $A_m < \overline{V}_m$ has to hold.

Case 2. In this example, we assume that the MFC clause covers some, but not all buyers. First, we consider an example with two buyers and uniform distributions where the seller has an MFC arrangement with buyer one but not buyer two. If there were no MFC at all, then:

$$p_1^* = \frac{\overline{v}_1 - a_1}{2} \tag{66}$$

if $p_1^* \in [v_1, \overline{v}_1]$ and $p_1^* = v_1$ otherwise, and:

$$p_2^* = \frac{\overline{v}_2 - a_2}{2} \tag{67}$$

if $p_2^* \in [v_2, \overline{v}_2]$ and $p_2^* = v_2$ otherwise. If $p_1^* \leq p_2^*$, then the introduction of a MFC clause will not change the prices.

Assume that $p_1^* > p_2^*$, then the optimal choice of prices involve choosing the same price both for buyer 1 and buyer 2. Let us denote such a price by p^*. Then:

$$p^* = \frac{\overline{v}_1 N_1 + \overline{v}_2\, N_2 - A_1 - A_2}{2N} \tag{68}$$

Case 3. In the next example, we explore the case where there are three buyers, where one buyer has the MFC clause and the other two do not.

5   Multiple Sellers and Multiple Buyers

Now assume that there are I sellers and M buyers. The ith sellers program has a quality of V_{mi} for buyer m. This quality is distributed with p.d.f. $F_{mi}(\cdot)$. Buyer m knows V_{mi} for each seller i, and the seller knows the distribution $F_{mi}(\cdot)$, both for its product, and for its competitors products.

Buyer m now has several options when it observes prices $T_{m1},...,T_{mI}$, since it can buy any combination of programs, and the different combinations of programs gives different value levels that are known only to buyer m.

Specifically, we consider the case where there are two sellers, multiple buyers, and no MFC clauses. Since there is no MFC, the buyers do not compete against each other for each supplier, and the sellers can isolate and bargain individually with each buyer. We begin by considering the case where there is no uncertainty about the value of the programs for the sellers.

First, consider buyer m. Let V_{m1} and V_{m2} be known to all sellers and to buyer m. The buyer can buy programming from seller one or from seller two, or both. In the event it buys from both, the buyer gets value V_m.

Case I. $V_{m1} + V_{m2} = V_m$. This case implies that the buyers decision of buying from seller one is not influenced by the decision of buying from seller

two, and vice versa. This might be the case with programming content that targets customers with distinctly heterogeneous preferences. In this case, the Nash equilibrium solution for the sellers is:

$$T^*_{mi} = V_{mi}$$

Case II. $V_{m1} + V_{m2} < V_m$. In this case, the buyer values programming diversity and is better-off buying two kinds of programming rather than one. This implies that the programming is complimentary. The Nash equilibrium solutions for this game are the set:

$$T^*_{m1} \in [V_m - V_{m2}, V_{m1}]$$

and

$$T^*_{m2} = V_m - T^*_{m1}$$

and the buyer buys from both sellers. Any seller does at least as good as if it was a single seller.

Case III. $V_{m1} + V_{m2} > V_m > max\{V_{m1}, V_{m2}\}$. The programming exhibits significant substitutability in that the broadcast of one type of programming diminishes the value of the other type of programming, e.g., sports programming that broadcasts similar events.

The buyer compares:

$$max\{V_{m1} - T_{m1}, V_{m2} - T_{m2}, V_m - T_{m1} - T_{m2}\}$$

and decides to buy either from seller one, seller two, or from both sellers.

In this case, the price competition does not drive prices to zero because when prices become low enough the buyer buys from both sellers. Notice that buying from only one seller cannot be a Nash Equilibrium since the other seller can always over-price. Thus, in equilibrium both sellers serve the market.

Then:

$$V_m - T_{m1} - T_{m2} \geq V_{m1} - T_{m1}$$

and

$$V_m - T_{m1} - T_{m2} \geq V_{m2} - T_{m2}$$

For example, assume that $V_m - T_{m1} - T_{m2} > V_{m1} - T_{m1}$ in equilibrium. Then seller two has the incentive to raise T_{m2} such that $V_m - T_{m1} - T_{m2} = V_{m1} - T_{m1}$, and the condition $V_m - T_{m1} - T_{m2} \geq V_{m2} - T_{m2}$ still holds. This is a contradiction. Therefore we must have:

$$V_m - T_{m1} - T_{m2} = V_{m1} - T_{m1}$$

From symmetry:

$$V_m - T_{m1} - T_{m2} = V_{m2} - T_{m2}$$

Thus:

$$T_{m1}^* = V_m - V_{m2}$$

and

$$T_{m2}^* = V_m - V_{m1}$$

and the buyer buys from both.

Next, we check the condition that the buyer benefits from buying both products for the transfer prices. We have:

$$V_m - T_{m1}^* - T_{m2}^* =$$
$$V_m - 2V_m + V_{m1} + V_{m2} =$$
$$V_{m1} + V_{m2} - V_m > 0$$

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