Simplified procedure for computing the absorption of sound by the atmosphere

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(Received 2006 August 18; revised 2007 October 06; accepted 2007 October 07)

This paper describes a study that resulted in the development of a simplified method for calculating attenuation by atmospheric-absorption for wide-band sounds analyzed by one-third octave-band filters. The new method [referred to herein as the Volpe Method] utilizes the accurate pure-tone sound absorption algorithms of two published standards, the International Standard, "Acoustics-Attenuation of Sound During Propagation Outdoors-Part 1: Calculation of the Absorption of Sound by the Atmosphere," ISO 9613-1 and, the American National Standard, "Method for Calculation of the Absorption of Sound by the Atmosphere," ANSI S1.26-1995. The purpose of the study was to extend the useful attenuation range of the Approximate Method outlined in the ANSI document, and provide a basis for replacing the current Society of Automotive Engineers Aerospace Recommended Practice 866A, "Standard Values of Atmospheric Absorption as a Function of Temperature and Humidity" (SAE ARP 866A). The Volpe Method was found to be useable to mid-band absorption levels up to 500 dB with errors of less than ±0.5 dB or ±5% (of mid-band attenuation levels) to 100 dB, and ±7% to 500 dB.

Primary subject classification: 24.2; Secondary subject classification: 76.1.1

1 INTRODUCTION

The United States Department of Transportation, John A. Volpe National Transportation Systems Center (Volpe Center), Environmental Measurement and Modeling Division, in support of the Federal Aviation Administration's (FAA) Office of Environment and Energy (AEE), and working under the auspices of the SAE International Aircraft Noise Committee (A-21) Atmospheric Absorption Project Working Team (PWT), has completed a study of a proposed new method to modernize the requirements for calculating the absorption of sound by the atmosphere. The new method, referred to herein as the "Volpe Method", utilizes the pure-tone sound absorption algorithms of two published standards, the International Standard, "Acoustics-Attenuation of Sound During Propagation Outdoors—Part 1: Calculation of the Absorption of Sound by the Atmosphere," ISO 9613-1[1] and, the American National Standard, "Method for Calculation of the Absorption of Sound by the Atmosphere," ANSI S1.26-1995[2]. References 1 and 2 are herein referred to as ISO/ANSI[1,2].

This paper presents the results of the study, along with an introduction to the topic of atmospheric absorption, as it relates to aircraft noise certification. Reference 3 is a technical report examining all aspects of the study from development to sensitivity analysis.

1.1 Background

Aircraft noise certification in the United States is performed under the auspices of the Federal Aviation Regulation, Part 36, "Noise Standards: Aircraft Type and Airworthiness Certification" (FAR 36)[4]. The international counterpart to FAR 36 is the International Civil Aviation Organization (ICAO) Annex 16[5]. FAR 36 requires that aircraft position, performance and noise data be corrected to the following, homogeneous, reference atmospheric conditions for the purposes of noise certification:

- Sea level pressure of 2116 ps (76 cm of mercury, 101.325 kPa);
- Ambient temperature of 77 degrees Fahrenheit (25 degrees Celsius);
- Relative Humidity of 70 percent; and

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- Zero wind.

An integral component of the FAR 36 noise data correction process is the computation of the absorption of sound over the propagation path. FAR 36 requires that atmospheric absorption, as a function of propagation path distance, be computed in one-third octave-bands from 50 Hz to 10 kHz (25 Hz to 10 kHz for helicopters) using the method described in the Society of Automotive Engineers Aerospace Recommended Practice 866A, "Standard Values of Atmospheric Absorption as a Function of Temperature and Humidity," (SAE ARP 866A)[6]. Herein this method is referred to as the "SAE 866A Method." The SAE 866A Method includes an empirical means of adapting its pure-tone equations for use in a fractional octave-band analysis. Specifically, it states that for one-third octave-bands with mid-band frequencies at or below 4 kHz, the sound attenuation rates should be computed at the mid-band frequency of the nominal one-third octave frequency band; and for higher frequency bands, a lower-band edge-frequency should be used.

The two referenced published standards (ISO/ANSI[1,2]) present theoretically-founded and experimentally-validated empirical algorithms for computing atmospheric absorption. A unique characteristic of the ISO/ANSI[1,2] algorithms, as compared to the algorithms of the SAE 866A Method, is that the ISO/ANSI algorithms take into account the effects of atmospheric pressure on sound absorption, in addition to the effects of temperature and relative humidity.

The ISO/ANSI equations for computing sound attenuation as a function of propagation distance are arithmetically identical to one another, and specify computation as a function of temperature, relative humidity and atmospheric pressure for *single, discrete frequencies or pure-tones*. However, FAR 36 requires that noise data be analyzed in one-third octave-bands. Recognizing this fact, the authors of these two standards included methods for adapting the pure-tone algorithms for use in a fractional-octave-band analysis, e.g., one-third octave-band analysis.

Annex D of both the ISO[1] and ANSI[2] standards present a relatively complex, but technically sound, method of adapting the pure-tone algorithms for use in a fractional octave-band analysis. This method is referred to herein as the spectrum integration or "Exact Method". The Exact Method requires knowledge of both the narrow-band characteristics of the sound source and the frequency response characteristics of the one-third octave-band filters used in the analysis. Due to such complex and labor-intensive requirements, its use for aircraft noise certification is not realistic.

Annex E of the ANSI standard presents a more empirical method of adapting the pure-tone algorithms to one-third octave bands, known as the Approximate Method[2]. The Approximate Method does not require knowledge of the narrow-band characteristics of the sound source. It uses a simple 2nd order equation to approximate one-third octave-band level-attenuation based on the frequency response characteristics of a third-order Butterworth filter[7]. As stated in the ANSI standard, it is accurate only for total path-length absorptions of less than 50 dB. Consequently, the Approximate Method is not considered appropriate for the adjustment of aircraft noise certification data or for development of noise-power-distance data to be used in a computer model such as FAA's Integrated Noise Model (INM) where absorption levels far greater than 50 dB are commonplace. The ISO standard does not present a method analogous to the Approximate Method of the ANSI standard.

Currently the computation of atmospheric absorption for aircraft noise certification is performed using a two-step, **reciprocal** process. First, absorption is computed for each one-third octave-band based on the temperature, humidity and propagation distance at the time of the certification test (test-day absorption). Second, absorption is computed for each one-third octave-band based on the reference temperature, humidity and reference propagation distance (reference-day absorption). The as-measured noise data are then corrected to reference-day atmospheric conditions by arithmetically adding the test-day absorption and subtracting the reference-day absorption, taking into account differences in spherical spreading losses, as well as other physical effects. The process is **reciprocal** in the sense that a user can take the reference-day results and work backward to recalculate the original test-day data.

This correction process is performed on a one-third octave-band basis, and the individual bands are later combined into required noise descriptors, typically the sound exposure level (SEL), denoted by the symbol L_{AE}, or the effective perceived noise level (EPNL), denoted by the symbol L_{EPN}. For the purpose of this study, L_{PN} (perceived noise level) was used as a surrogate to L_{EPN}. The net result is a sound level adjusted to a specified reference distance and a reference-day[6] temperature and humidity of 70 degrees Fahrenheit (25 degrees Celsius) and 77 percent relative humidity (%RH), respectively.

1.2 Objective

The objectives of this study are: (1) to develop an empirical algorithm utilizing the pure-tone sound absorption algorithms of the ISO/ANSI[1,2] standards; (2) to simplify the computational process of the Exact Method; and (3) to extend the useful absorption range

Table 1—One-third octave-band error data, predicted minus exact levels (4 data slopes)

Mid-Band Attenuation (dB)	Error (Predicted Level) 10 T/H points (dB)	(32 °C/95% RH) (dB)
0–10	≤0.5	≤0.5
10–30	≤1.0	≤1.0
30–60	≤3.5	≤3.5
60–100	≤10	≤15
100–150	≤15	≤25
150–300	≤20	≤35
300–500	≤30	≤35
500—700	≤60	no data

of the Approximate Method. The resultant approach would replace the current SAE 866A Method for correcting sound level data for specific atmospheric conditions.

1.3 Overview

An empirical algorithm (the Volpe Method) utilizing the pure-tone sound absorption algorithms of the ISO/ANSI[1,2] standards was developed, to simplify the computational process of the Exact Method, and is recommended for mid-band attenuation up to 500 dB.

Section 5 summarizes the one-third octave-band (50 Hz to 10 kHz) level difference data for the Volpe Method and the Exact Method, for a variety of environmental conditions, using a representative aircraft noise spectrum, spanning an altitude range of 75 to 7620 meters. Tests outlined in Ref. 3 indicate the method is usable 25 Hz to 20 kHz.

The Volpe Method is seen to be more accurate than the SAE 866A Method, and unlike the SAE 866A Method, it takes into account the effects of changes in atmospheric pressure on sound absorption, as well as the effects of temperature and relative humidity.

2 IMPLEMENTATION

Implementation of the one-third octave-band adaptations of the ISO/ANSI[1,2] pure-tone equations is described. The general ISO/ANSI equations for computing sound attenuation rates are arithmetically identical. These equations provide a means of computing attenuation rates at single, discrete frequencies, i.e., for pure-tones. The common equations used for computing pure-tone sound attenuation by atmospheric absorption are shown in Eqns. (1)–(6). The pure-tone equations are the foundation of the Exact Method, the Approximate Method and the Volpe Method described herein.

The sound attenuation rate, α in decibels per meter, is computed as follows:

$$\alpha(f) = 8.686f^2[1.84 \times 10^{-11}(p_a/p_r)^{-1}(T/T_r)^{1/2}] + (T/T_r)^{-5/2}\{0.01275[\exp(-2239.1/T)][f_{rO}/(f_{rO}^2 + f^2)] + 0.1068[\exp(-3352.0/T)][f_{rN}/(f_{rN}^2 + f^2)]\} \quad (1)$$

where: f = pure-tone frequency for which the sound attenuation rate is to be computed, in Hz; p_a = ambient atmospheric pressure in kPa (either test- or reference-day pressure, as appropriate); p_r = 101.325 kPa, reference pressure of one standard atmosphere; T = ambient atmospheric temperature in °K (either test—or reference-day, as appropriate); T_r = 293.15 °K, reference ambient temperature;

$$f_{rO} = p_a/p_r\{24 + [(4.04 \times 10^4h)(0.02 + h)/(0.391 + h)]\}; \quad (2)$$

and

$$f_{rN} = (p_a/p_r)(T/T_r)^{-1/2} \times (9 + 280(h) \times\exp\{-4.170[(T/T_r)^{-1/3} - 1]\}. \quad (3)$$

In Eqns. (2) and (3) for f_{rO} and f_{rN}, h is equivalent to the molar concentration of water vapor, as a percentage, and is computed as follows:

$$h = h_{rel}(p_{sat}/p_r)(p_a/p_r)^{-1} \quad (4)$$

where: h_{rel} = relative humidity in percent (either test- or reference-day relative humidity, as appropriate);

$$p_{sat} = (p_r)10^V; \quad (5)$$

and

$$V = 10.79586[1 - (T_{01}/T)] - 5.02808 \times \log_{10}(T/T_{01}) + 1.50474 \times 10^{-4}\{1 - 10^{-8.29692[(T/T01)-1]}\} + 0.42873 \times 10^{-3}\{-1 + 10^{4.76955[1-(T01/T)]}\} - 2.2195983 \quad (6)$$

where: T_{01} = 273.16 °K, triple-point isotherm temperature.

Although Eqns. (1)–(6) are common to all the methods described herein, the procedure used for adapting the pure-tone attenuation rate for use in a one-third octave-band analysis of wideband sounds is quite different. The procedures adapted for use in this study are described in Secs. 2.1–2.4 and in Ref. 3.

2.1 Exact Method

The Exact Method is a relatively complex, technically sound approach adapting pure-tone sound absorption algorithms (Eqns. (1)–(6)) for one-third octave-band analysis. Annex D of both the ISO[1] and ANSI[2] standards provide general guidance for implementing this method, but leave several parameters and assumptions to the discretion of the user. In addition to its non-definitive nature, it also requires knowledge of both the narrow-band characteristics of the sound source and the frequency response characteristics of the one-third octave-band filters used in the analysis. Due to such complex and labor-intensive requirements, it places an undo burden on applicants for aircraft certification, and its use for aircraft noise certification is not realistic. Therefore, the Exact Method is not considered a viable option for regulatory adoption. It is used herein as a point of reference because it is considered the most technically sound approach. Specific definitive assumptions made by the Volpe Center in implementing the Exact Method are discussed.

In the absence of narrow-band data, the first step in the process used herein was to use the as-measured one-third octave-band sound pressure level data to derive equivalent pressure spectrum level data for each band. The pressure spectrum level data at the exact mid-band frequency, $L_S(f_{m,i})$, were computed as follows:

$$L_S(f_{m,i}) = L_{BS}(f_{m,i}) - 10\log_{10}(B_i/B_0) \tag{7}$$

where: L_{BS} $(f_{m,i})$=the as-measured sound pressure level for the one-third octave-band, i; i=integers 1-24 representing one-third octave-band from 50 Hz to 10 kHz; and

$$f_{m,i} = (10^{x/10})(1000). \tag{8}$$

where: $f_{m,i}$ is the exact mid-band frequency for one-third octave-band i, in Hz, for base-10 design one-third octave-band filters; x=0 for the 1 kHz one-third octave-band; and is incremented by 1 for each successive one-third octave-band; and, is decremented by one for each one-third octave-band below the 1 kHz band, index x ranges from −13 to 10 for one-third octave-bands 50 Hz to 10 kHz; B_i=0.23077 $f_{m,i}$, the exact bandwidth for base-10 designed one-third octave-band filter, i; and B_0=1 Hz, normalizing bandwidth.

Note: B_0 defines the reference bandwidth for the derived, pressure spectrum level data. A bandwidth of 1 Hz was selected for this study.

The pressure spectrum level was computed at discrete frequencies encompassed by each one-third octave-band (every 1/24 of a bandwidth, B_i/24). The intermediate pressure spectrum levels, $L_S(f_{k,i})$ were obtained by linear interpolation using the mid-band pressure spectrum levels $L_S(f_{m,i})$ and the corresponding slope of the pressure spectrum which was computed using the $L_S(f_{m,i})$ values between adjacent one-third octave-bands. The slope for the upper portion of the highest one-third octave-band was obtained using the extrapolated slope derived from the two highest bands.

The number of levels calculated at discrete frequencies was determined by recalculating the overall sound pressure level for each one-third octave-band using a summation process. In the summation process, each pressure spectrum level, $L_S(f_{k,i})$ including the mid-band level $L_S(f_{m,i})$, was: (1) adjusted for filter response, $A(f_{k,i})$, (2) converted to acoustic energy, (3) multiplied by 1/24 the bandwidth of the corresponding one-third octave-band filter (B_i/24), and (4) summed on an energy basis to produce the calculated one-third octave-band level, L_{CS} (f_i). The process of multiplying the energy-equivalent of each level by B_i/24 is analogous to integrating acoustic energy between subsequent values using a simple trapezoidal approximation[8].

The process begins with the level at the exact mid-band frequency, $f_{m,i}$ and continues alternately adding level data at the discrete frequency $kB_i/24$ lower than the mid-band frequency, $f_{m,i}$ (negative k values) and then at $kB_i/24$ frequencies higher than the mid-band frequency, $f_{m,i}$ (positive k values). The alternating process continues toward the lower and upper band-edge frequency of the one-third octave-band i, decrementing and incrementing k by 1, until the calculated (i.e., reconstructed) one-third octave-band level, $L_{CS}(f_i)$ equals the value of the original as-measured sound pressure level, (L_{BS} (f_i)):

$$L_{CS}(f_i) = 10\log_{10}((B_i/24)\times 10^{L'S(fm,i)} + (B_i/24) \times 10^{(\Sigma L'_{LS}(fk,i) + \Sigma L'_{US}(fk,i))}) \tag{9}$$

where:

$$L'_S(f_{k,i}) = L_S(f_{k,i}) - A(f_{k,I});$$

and

$$A(f_{k,i}) = 10\log_{10}\{1 + (4.5229)^6(f_{k,i}/f_{m,i} - f_{m,i}/f_{k,i})^6\}; \tag{10}$$

$L'_{LS}(f_{k,i})$=level adjusted for filter at frequencies lower than $f_{m,i}$, k=−1 to −24; $L'_{US}(f_{k,i})$=level adjusted for filter at frequencies higher than $f_{m,i}$, k=+1 to +24; $f_{k,i}=f_{m,i}+kB_i/24$, discrete frequency within the one-third octave-band i, in Hz, where, k=0 for mid-band, and decremented by one for frequencies lower than mid-band and incremented by one for frequencies higher than mid-band; $f_{1,i}=(10^{-1/20})f_{m,i}$, lower edge for base-10 design one-third octave-band filters; $f_{2,i}=(10^{1/20})f_{m,i}$, upper edge for base-10 design one-third octave-band filters; and f_i=nominal mid-band frequencies for one-third octave-band filters in accordance with IEC 1260[9], 50 Hz to 10 kHz, i=1,24.

A second summation is simultaneously performed as above, steps (1) through (4). However, included in step (1) of this second summation is the adjustment for the attenuation effects of atmospheric absorption using the above pure-tone algorithms, Eqns. (1)–(6), $L''_S(f_{k,i})$. Note the absorption is additive for receiver-to-source distances (Eqn. (12)) and subtractive for source-to-receiver distances (Eqn. (13)). The resultant summation on an energy basis produces the calculated one-third octave-band level with both filter and atmospheric absorption effects, $L_{AS}(f_i)$:

$$L_{AS}(f_i) = 10 \log((B_i/24) \times 10^{L''S(fm,i)} + (B_i/24) \times 10^{(\Sigma L''LS(fk,i)+\Sigma L'US(fk,i))}) \quad (11)$$

where:

$$L''_S(f_{k,i}) = L_S(f_{k,i}) - A(f_{k,I}(f) \times S) + \alpha \quad (12)$$

S = receiver-to-source distance in meters; and

$$L''_S(f_{k,i}) = L_S(f_{k,i}) - A(f_{k,I}(f) \times S) - \alpha \quad (13)$$

S = source-to-receiver distance in meters.

A base-10 designed third-order Butterworth[7] filter shape was assumed. The adjustment for the attenuating effects, A $(f_{k,i})$, of a one-third octave-band filter at discrete frequencies, is based upon the well-known response equation for filters meeting the requirements of Type 1-X one-third octave-band Butterworth filter (Eqn. (10)). The one-third octave-band Butterworth filter is a common filter design in use in many of today's analyzers. It was also the filter response equation used by the authors of ANSI S1.26-1995 in the development of the Approximate Method[2].

Typically, summation of the pressure spectrum levels for each one-third octave-band ends before the lower and upper band-edge frequencies, ($f_{L,i}=0.2f_{1,i}$ and $f_{U,i}=2f_{2,i}$). Specifically, in this paper computation stopped when the calculated one-third octave-band level, $L_{CS}(f_i)$ was equal to the value of the original as-measured sound pressure level, ($L_{BS}(f_i)$).

Finally, the difference between the calculated one-third octave-band level with filter effects, $L_{CS}(f_i)$ and the calculated one-third octave-band level with both filter and atmospheric absorption effects, $L_{AS}(f_i)$ yielded the effective one-third octave-band attenuation by absorption, $\delta_{eff}(f_i)$ for the given temperature, humidity, pressure, and distance.

$$\delta_{eff}(f_i) = L_{CS}(f_i) - L_{AS}(f_i) \quad (14)$$

It was found that the process used in the Exact Method deviated from a true reciprocal process. In the receiver-to-source case the calculated absorption values diverge to unrealistically high absorption values versus those calculated for the source-to-receiver case under the same conditions over the same propagation distance (see Figs. 1 and 2). Since absorption is additive in the receiver-to-source case, large corrected levels are computed. These are used in the energy summation process to extract the effective absorption. This is especially true at frequencies higher than mid-band in each one-third octave-band. The result is the high frequency portion of the one-third octave-band controls the absorption value calculated by dominating and overshadowing the contributions to the calculated

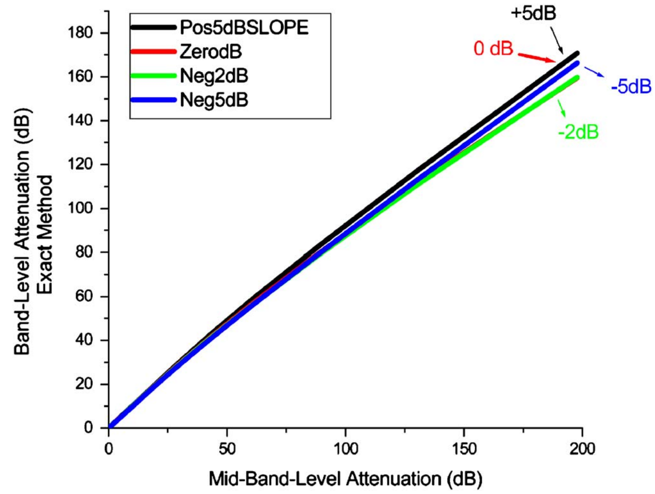

Fig. 1—One-third octave-band-level attenuation vs mid-band attenuation source-to-receiver propagation data slopes, +5, 0, −2, −5 dB, 25 °C, 70% RH, static pressure 101.325 kPa.

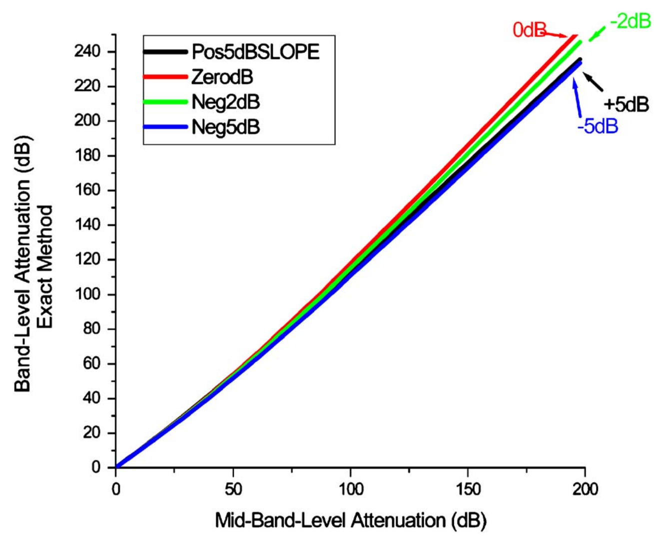

Fig. 2—One-third octave-band-level attenuation vs mid-band attenuation receiver-to-source propagation data slopes, +5, 0, −2, −5 dB, 25 °C, 70% RH, static pressure 101.325 kPa.

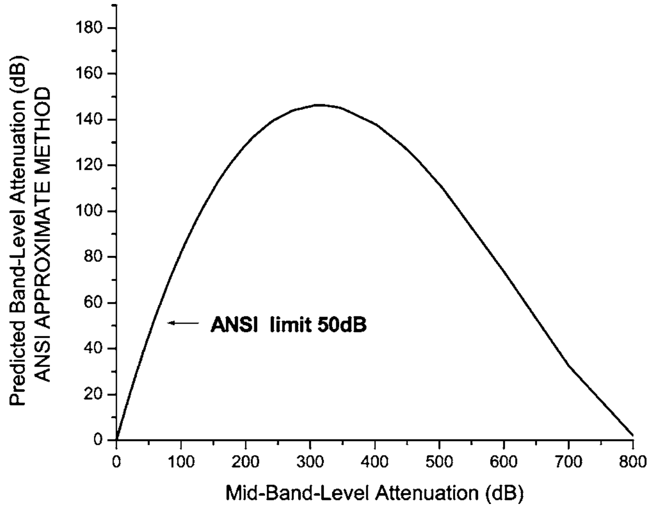

Fig. 3—ANSI function: approx. method, predicted vs. mid-band-level attenuation.

value in the low frequency portion of the band. A less drastic process is evident in the source-to-receiver case where almost equal weighting is given to contributions of both the low and high frequency ends of the one-third octave-bands. For this reason, all comparisons made in this paper versus the Exact Method are source-to-receiver comparisons.

2.2 Approximate Method

Annex E of the ANSI[2] standard presents a simplified method for adapting the pure-tone algorithms to one-third octave bands, known as the Approximate Method". As shown in the standard, the sound absorption, δ_B (f_i), for any one-third octave-band is computed as follows:

$$\delta_B(f_i) = [\alpha(f_{m,i})][s]\{1 + (B_r^2/10)[1 - (0.2303)[\alpha(f_{m,i})] \times(s)]\}^{1.6} \tag{15}$$

where: α ($f_{m,i}$)=the sound attenuation rate computed at the exact mid-band frequency f_m, for one-third octave-band i, using Eqns. (1)–(6); $f_{m,i}$=as defined in Eqn. (8); f_i=as defined in Eqn. (8); s=propagation distance in meters; $B_r^2/10=0.0053254$, for base-10 designed one-third octave-band filters; and $B_r=(10^{1/20}-10^{-1/20})$ for base-10 designed filters.

The ANSI standard[2] specifies that the above adaptation of pure-tone sound absorption provides an excellent measure of one-third octave-band absorption for the test spectra chosen in the development of the Approximate Method, assuming that the pure-tone attenuation over the total propagation path distance is less than 50 dB at the associated exact mid-band frequency.

The calculated attenuation (Eqn. (15)) plotted in Fig. 3, is seen to increase from a minimum of 0 dB, with increasing mid-band attenuation up to a mid-band attenuation of approximately 250 dB, beyond which the calculated attenuation decreases with increasing mid-band attenuation. The attenuation calculated with Eqn. (15) was found in this study to be in good agreement with the Exact Method up to the prescribed 50 dB ANSI limit[2].

Specifically, the difference between the Approximate Method and the Exact Method was found to be less than ±0.5 dB up to a mid-band level of 10 dB; increasing to ±5 dB up to a mid-band level of 50 dB. These results were measured using the four spectrum shapes of Sec. 3 and the eleven Temperature/Humidity (T/H) points of Sec. 3.1. Above 50 dB (the ANSI limit[2]), the calculated attenuation diverges to unrealistically low values due to the limitations of the 2nd order equation.

2.3 Volpe Method

The Volpe Method is a simplified procedure for calculating attenuation by atmospheric absorption on wideband sounds analyzed by one-third octave-band filters. It was developed (Sec. 3) using an approach similar to that used in the development of the Approximate Method. Specifically, the Exact Method presented in the ISO/ANSI[1,2] standards was considered the reference by which the Volpe Method was judged. A base-10 designed third-order Butterworth[7] filter shape was assumed. The same assumption was made by Joppa et al.[10], and appears to be reasonable, since the traditional third-order Butterworth algorithms are used in common filter designs by many analyzer manufacturers.

The Volpe Method predicts band level attenuation ($\delta_B(f_i)$ in decibels) by atmospheric absorption using two equations as follows: For mid-band attenuation levels, $\delta_t(f_{m,i}) < 150$ dB

$$\delta_B(f_i) = (A \times [\delta_t(f_{m,i})] \times (1 + B \times (C - D \times [\delta_t(f_{m,i})]))^E , \tag{16}$$

For mid-band attenuation levels, δ_t ($f_{m,i}$) $\geqslant 150$ dB

$$\delta_B(f_i) = F + G \times [\delta_t(f_{m,i})], \tag{17}$$

where: $\delta_t(f_{m,i})=[\alpha(f_{m,i})][s]$, mid-band attenuation (decibels); $\alpha(f_{m,i})$=mid-band attenuation coefficient, in decibels per meter, see Eqn. (1); s=path length distance, in meters;

A: 0.867942;
B: 0.111761;
C: 0.95824;
D: 0.008191;
E: 1.6;
F: 9.2; and

G: 0.765;
f_i=nominal mid-band frequencies for one-third octave-band filters in accordance with IEC 1260[9], 50 Hz to 10 kHz, i=1,24; i=integers 1-24 representing one-third octave-band from 50 Hz to 10 kHz; and $f_{m,i}$=as defined in Eqn. (8).

The Volpe Method is a reciprocal process and can be used for both source-to-receiver and receiver-to-source propagation distances.

2.4 SAE 866A Method

The SAE 866A Method[6] has served, to date, as the basis for many analyses of noise propagation and for correcting sound propagation levels measured under given atmospheric conditions to specified reference conditions. It is a pure-tone method based on the theory of Kneser[11].

The SAE 866A Method is a reciprocal process and is used for both source-to-receiver and receiver-to-source propagation distances. Readers are referred to Ref. 6 for further discussion of the SAE 866A methodology.

3 DEVELOPMENT

Spectral analysis of wideband sounds using one-third octave-band filters yields sound pressure levels in one-third octave frequency bands. This analysis is a requirement of FAR 36 for aircraft noise certification. The sound pressure levels include the effects of filter attenuation response characteristics as well as the attenuation introduced by atmospheric absorption. The magnitude of these effects was found to vary with the slope of the test spectrum.

The Volpe Method was developed using a set of shaped broadband (non-tonal) spectra chosen to represent a wide range of spectrum slopes that may be encountered in typical aircraft data. Pink noise was artificially shaped to produce a set of four non-tonal test spectra with data slopes of +5, 0, −2, and −5 dB per one-third-octave, respectively. These shaped source spectra were adjusted to fixed receiver distances using the Exact Method. The result was a set of data points relating one-third octave-band attenuation by absorption, with mid-band-level attenuation for each one-third octave-band. Least-squares regression techniques were used on these data points with the goal of developing an empirical equation relating representative one-third-octave-band atmospheric attenuation with mid-band attenuation for a particular one-third-octave-band. The general form of the equation for the ANSI S1.26-1995 Approximate Method[2] was used as a starting point because it was found to be in good agreement with the Exact Method for small values of absorption. The goal of the analysis, using the data derived from the Exact Method, was to increase the accuracy and extend the applicability of the Approximate Method to well beyond the limited 50 dB path-length absorption.

3.1 Temperature/Humidity/Static Pressure

From a grid of 22 T/H points covering the FAR 36 T/H window, a more manageable eleven fixed T/H points were selected. The points were selected as representative of the attenuation characteristics within the window after comparing the band level versus mid-band level attenuation for each of the 22 T/H points. The selected points were at the extremes and at the center of the FAR 36 accepted T/H window and include the FAR 36 reference T/H (25°C/70%RH) and the (6°C/35%RH) T/H point used by Joppa et al.[10] in the development of the Approximate Method.

The eleven selected T/H points are:
(25°C/70%RH), (6°C/35%RH),
(6°C/49%RH), (6°C/95%RH),
(10°C/42%RH), (15°C/60%RH),
(15°C/95%RH), (21°C/27%RH),
(21°C/95%RH), (32°C/20%RH),
(32°C/95%RH)

For the purpose of the development of this procedure, the atmosphere was assumed to be homogeneous and uniform with constant atmospheric pressure at the ISA, sea level value of 101.325 kPa.

3.2 Data Processing

Data were processed and attenuation by atmospheric absorption was calculated with the Exact Method as described in Sec. 2.1 above. Each of the sloped test spectra was processed at each of the eleven T/H data points.

The receiver-to-source distances were chosen to produce attenuation values in the 200 to 500 dB ranges in the upper frequency bands. From plots of the data, it was determined that data processed at 25°C/70%RH produced data curves that reasonably approximated data from the other ten T/H points. The 25°C/70%RH one-third octave-band attenuation versus mid-band attenuation is shown plotted in Figs. 1 and 2 for the four shaped test spectra for both the source to receiver and receiver-to source case.

Note in Fig. 2, the receiver-to-source case, the calculated absorption values diverge to unrealistically high absorption values versus those calculated for the source-to-receiver case, Fig. 1, under the same conditions over the same propagation distance. For this reason, all comparisons made in this report versus the Exact Method are source-to-receiver comparisons. (See Sec. 2.1)

Table 2—2σ statistics 1/3 octave-band data level-difference versus mid-band attenuation Volpe method minus exact method 75 to 7620 meters

Mid-Band Attenuation dB	Static Pressure Section 4.1 dB	Lapsed Pressure Section 4.2 dB	30 m Layers w/Lapsed P. Section 4.3 dB	LD_{long} Filter Static Pres. Section 4.4 dB	SAE Method[a] Static Pres. Section 4.5 dB	20 kHz[b] Static Pres. Section 4.6 dB	Appox. Method[c] Static Pres. Section 4.1 dB
0–10	±0.5	±0.5	±0.2	±0.5	±2.0	±0.5	±0.5
10–20	±1.0	±1.0	±0.5	±1.0	±4.0	±1.0	±1.0
20–50	±2.0	±2.0	±1.5	±2.0	±6.0	±3.0	±5.0
50–100	±5.0	±5.0	±2.5	±7.0	±10	±5.0	
100–200	±8.0	±8.0	±5.0	±10	±15	±12	
200–500	±25	±25	±10	±16	±38	±30	
500–700	±40	±40	±17	±22	±60	±45	

[a]—SAE ARP 866A Method minus VOLPE Method
[b]—20 kHz data limited to a distance of 2400 meters
[c]—Approximate Method minus Exact Method

3.3 Algorithm Generation

Least-squares regression techniques were applied to atmospheric absorption data versus mid-band attenuation data, obtained from processing the four shaped spectra at 25°C/70%RH (see Fig. 1) at distances up to 2500 ft with a goal of developing an appropriate algorithm. The equations were evaluated by comparing their relative goodness-of fit, using the R^2 coefficient of determination.

Because the equation for the ANSI S1.26-1995 Approximate Method was found to be in good agreement with the Exact Method for levels below 10 dB, its general form was used as a starting point in the regression analysis with a variable initially introduced for data slope. Preliminary results and Fig. 2 show that the complexity of an equation with slope as an added variable would not provide a worthwhile improvement in accuracy over an equation with a fixed slope. A final regression equation was thus normalized with a fixed slope of 3 dB per one-third octave-band for mid-band levels less than 150 dB and is shown in Sec. 2.3, Eqn. (16). The test spectra were further processed at large propagation distances (large value of atmospheric attenuation) to extend the usefulness of the Volpe Method for mid-band attenuation levels ⩾150 dB. This yielded a second regression equation (Eqn. (17)) tangent to the first (Eqn. (16)) at the 150 dB point. The discontinuity at 150 dB is less than 0.05 dB.

3.4 Error Analysis

The algorithms (Eqns. (16) and (17)) developed for the Volpe Method were used to predict atmospheric absorption for each of the four-shaped spectra at each of the eleven T/H data points of Sec. 3.1. The predicted attenuation values were compared against data obtained using the Exact Method assuming a homogeneous atmosphere at a static pressure of 101.325 kPa for each of the eleven T/H data points.

Error curves (Predicted minus Exact Method) are shown in Ref. 3 for each of the eleven T/H points for mid-band levels up to 1000 dB for each of the four data slopes. The error in the predicted data from these curves is summarized in Table 1.

As seen in Table 1, the error in the predicted data using the Volpe Method for ten of the data points is less than 10% of the mid-band attenuation values. The 32°C/95%RH data point at the high temperature/high humidity extreme of the FAR 36 testing window is the exception with errors up to 20% of the mid-band attenuation

4 SENSITIVITY TESTING

A measure of the sensitivity of the Volpe Method to aircraft data was obtained using an approximation to a true spectrum shape. Fig. 4 depicts an approximation, at the source, of a true one-third octave-band spectrum shape for typical commercial jet aircraft at high engine power settings. This test spectrum, used by Joppa et al.[10] in the development of the Approximate Method, was also used herein. Six conditions were tested as follows:

1) (Sec. 4.1), Static atmospheric pressure of 101.325 kPa;
2) (Sec. 4.2), Lapsed pressure changes with altitude;
3) (Sec. 4.3), Lapsed pressure changes using 30 meter altitude layers;
4) (Sec. 4.4), Changing filter shapes;
5) (Sec. 4.5), Comparison versus the SAE 866 A Method); and
6) (Sec. 4.6), Extending the applicable frequency range to 20 kHz.

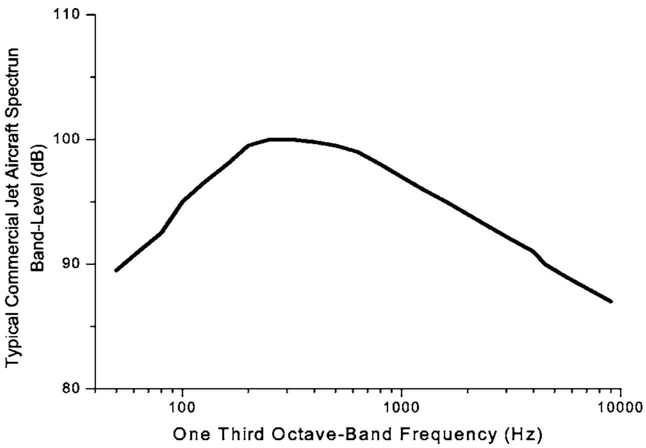

Fig. 4—Commercial jet aircraft, typical 1/3 *octave-band noise levels.*

Atmospheric absorption calculations were performed at eight altitudes from 75 to 7620 meters taking into account atmospheric pressure as appropriate at the following four temperature / relative humidity points at the extremes and center of the FAR 36 T/H window:

32°C/20%RH,
32°C/95%RH,
6°C/49%RH,
25°C/70%RH

Adjustments were made, to the one-third octave-band data for the absorption effects of temperature, humidity, and pressure using the Volpe Method, the Exact Method and the SAE 866A Method. The adjusted one-third octave-band data were compared and level difference data curves prepared versus altitude.

The adjusted one-third octave-band data (50 Hz to 10 kHz) were further used to compute the Perceived Noise Level denoted by the symbol L_{PN}, and the A-weighted Noise Level denoted by the symbol L_A. Tabulations of the level-difference data ΔL_A and ΔL_{PN} (Volpe Method minus Exact Method) for these four-temperature/humidity points were prepared. Statistical summaries of these data are included in Figs. 5–8 for five of the conditions tested.

4.1 Homogeneous Atmosphere with Constant Pressure (101.325 kPa)

For these tests, a homogenous atmosphere was assumed, i.e., temperature, humidity and pressure did not vary with altitude. Atmospheric pressure was set to a static, ISA, sea level value of 101.325 kPa and used in the appropriate atmospheric absorption calculation for the four T/H conditions tested at eight altitudes from 75 to 7620 meters. The L_A and L_{PN} level-difference data tabulations (Volpe Method minus the Exact Method) obtained at the four T/H points (see Ref. 3) were combined and are presented in Figs. 5 and 6. Included are the mean, maximum, minimum, and $\pm 2\sigma$ (± 2 standard deviations (SD)) range of values for eight altitudes from 75 to 7620 meters at the four selected temperature / humidity points.

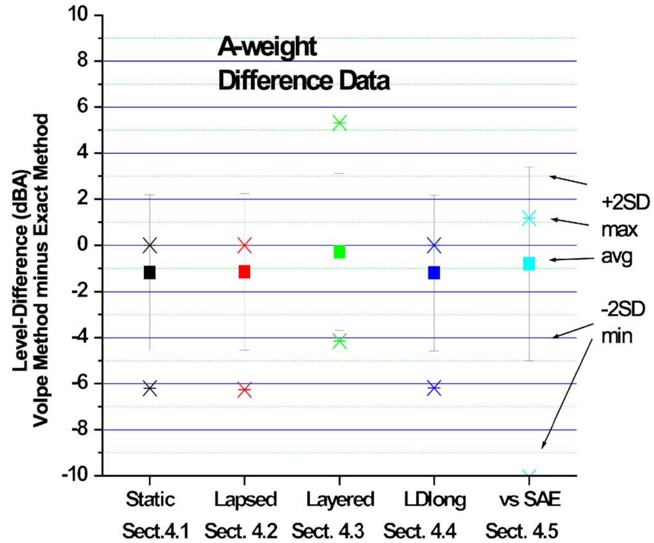

Fig. 5—ΔL_A-level-difference data, combined data—4 T/H points, eight altitude 75 to 7620 meters Volpe method minus exact method see Secs. 4.1–4.5.

Note in Figs. 5 and 6 that over the altitude range of 7620 meters the mean difference for both ΔL_A and ΔL_{PN} is -1.2 dB with a 2σ range of less than ± 4.5 dB. Over the altitude range of 4800 meters, the mean difference for both ΔL_A and ΔL_{PN} is -0.7 dB with a 2σ range of less than ± 2.6 dB.

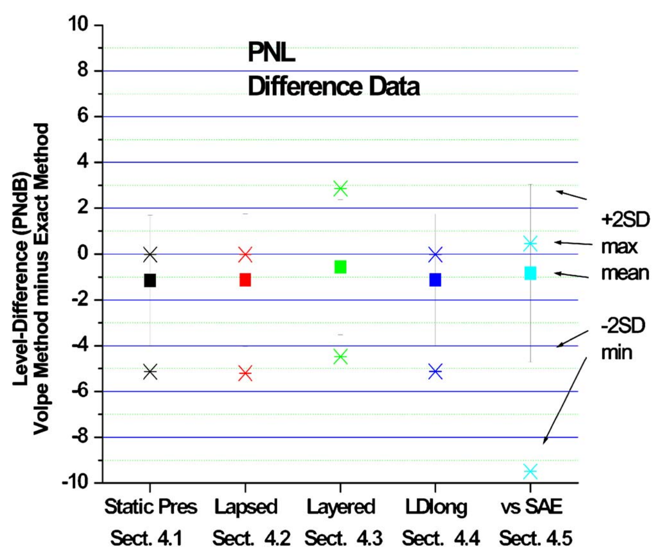

Fig. 6—ΔL_{PN}-level-difference data, combined data—4 T/H points, eight altitude 75 to 7620 meters, Volpe method minus exact method see Secs. 4.1–4.5.

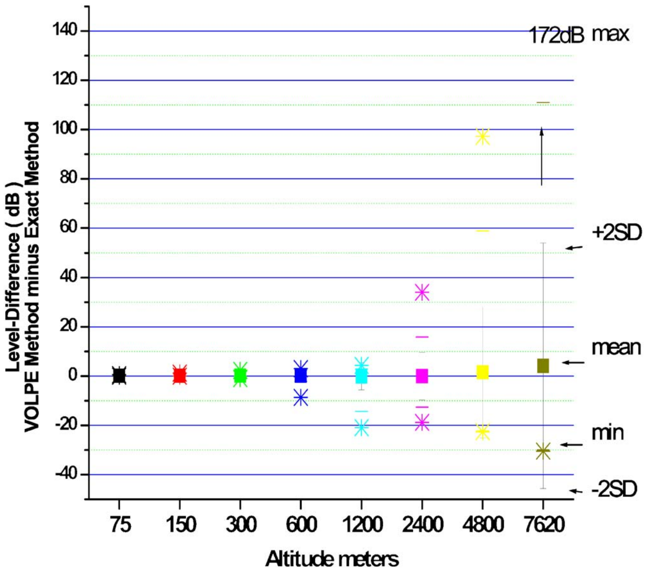

Fig. 7—1/3rd octave—level-difference data, combined data—4 T/H points, Volpe method, minus exact method static pressure 101.325 kPa.

The one-third octave-band level-difference data (Volpe Method versus the Exact Method), for the four T/H conditions tested, were also combined and are presented in Fig. 7. Shown in Fig. 7 is the level-difference over the altitude range of 75 to 7620 meters. Mid-band level data is presented in Fig. 8. Included are the mean, maximum, minimum, and $\pm 2\sigma$ range of values.

The $\pm 2\sigma$ statistics of Figs. 7 and 8 are summarized in Table 2 for the static pressure condition of 101.325 kPa. Note in Table 2 that through 50 dB of mid-band attenuation an error of ±2 dB was observed, which is less than one half the error noted in Sec. 2.2 for the Approximate Method (±5 dB).

In addition the useable range is seen to extend to over 500 dB of mid-band attenuation with errors less than ±0.5 dB or ±5% (of mid-band attenuation) to 100 dB, and ±7% to 500 dB.

Similar results were obtained extending the frequency range of the Volpe Method down to the 25 Hz one-third octave-band. This frequency range would be applicable for helicopter noise data processing in accordance with FAR 36.

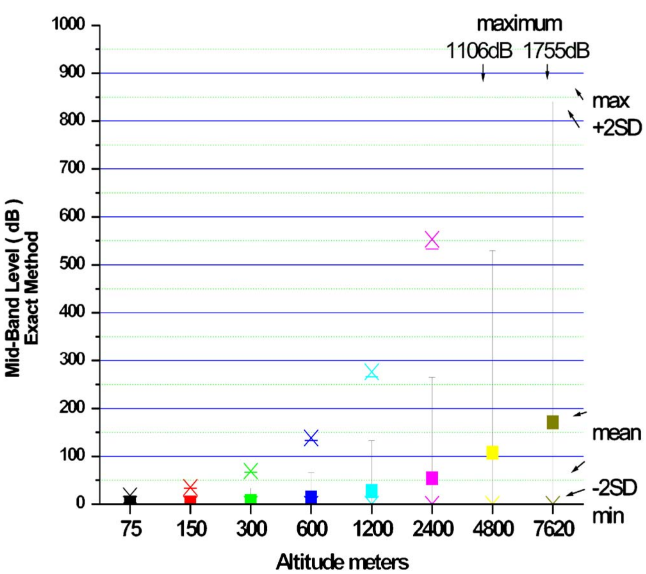

Fig. 8—Mid-band level data, combined data—4 T/H points exact method, static pressure 101.325 kPa.

4.2 Homogeneous Atmosphere with Lapsed Pressure

For these tests, a homogenous atmosphere was assumed, i.e., temperature, humidity did not vary with altitude. Atmospheric pressure was taken into account to an altitude of 7620 meters. Lapsed pressure, calculated at the appropriate altitude, was used to represent the pressure in the calculations for both the Volpe and Exact Methods for the four T/H conditions tested at eight altitudes from 75 to 7620 meters.

The standard ISO pressure lapse in kPa was calculated as follows:

$$\text{Pressure} = 101.325 \times 10^{(-5.256\text{E}-05 \times \text{alt})} \qquad (20)$$

where alt=altitude above mean sea level [meters].

As in Sec. 4.1, the level-difference data (Volpe Method minus the Exact Method) for the L_A and L_{PN} metrics are included in Figs. 5 and 6 and $\pm 2\sigma$ statistics for the one-third octave-band level-difference data (Figs. 7 and 8) are included in Table 1.

Note introducing lapsed pressure resulted in no statistical difference in computed levels when compared with the static pressure case.

4.3 30 meter Layers with Lapsed Pressure

For these tests, the atmosphere was divided into equal altitude layers of 30 meters each. A homogenous atmosphere was assumed, i.e., temperature and humidity did not vary within each layer or with altitude.

Atmospheric pressure changes were taken into account to an altitude of 7620 meters. Lapsed pressure was calculated. Using Eqn. (20) at the center of each layer at the appropriate altitude. That pressure was used to represent the pressure for that layer in the calculations for both the Exact and Volpe methods for the four T/H conditions tested over the altitude range of 75 to 7620 meters.

As in Sec. 4.1, the level-difference data (Volpe Method minus the Exact Method) for the L_A and L_{PN}

metrics are included in Figs. 5 and 6 and $\pm 2\sigma$ statistics for the one-third octave-band level-difference data (Figs. 7 and 8) are included in Table 2.

Note the average level-difference is improved when compared to the results of the static pressure case (Sec. 4.1) and the lapsed pressure case (Sec. 4.2). This is to be expected since the Volpe Method is repeatedly used for the calculations in each 30-meter altitude layer in the region of its greatest accuracy, i.e. in the mid-band attenuation range of 0 to 50 dB.

4.4 Filter Shape

To obtain a measure of the sensitivity of the Volpe Method to one-third-octave filter shapes, the base-10 designed third-order Butterworth[7] filter shape (used in Exact Method in the above sections) was modified to represent an "ideal" filter, i.e., a filter with infinite attenuation characteristics outside of the pass-band.

Using the data of Ref. 12, an equation was developed to simulate the attenuation ($A(f_{k,i})$ characteristics of the "long" filter shape found in the Larson Davis Laboratories, (LD) Model 2900 analyzer. The so-called "long" filter shape (known herein as LD_{Long}) represents the manufacturers attempt to simulate an "ideal" filter. The equation developed is as follows:

$$A(f_{k,i}) = 90 \log_{10}\{1 + (2.8071)^6 (f_{k,i}/f_{m,i} - f_{m,i}/f_{k,i})^6\}; \tag{21}$$

where: $f_{k,i} = f_{m,i} + kB_i/24$, as defined in Eqn. (8).

In this section, Eqn. (21) was substituted for Eqn. (10) for the Exact (LD_{Long}) Method processing. Also, a homogenous atmosphere was assumed, i.e., temperature, humidity and pressure did not vary with altitude. Atmospheric pressure was set to a static, ISA, sea level value of 101.325 kPa and used in the appropriate absorption calculation for the four T/H conditions tested at eight altitudes from 75 to 7620 meters.

The level-difference data (Volpe Method minus the Exact (LD_{Long}) Method) for ΔL_A and ΔL_{PN} obtained for the four T/H points were combined and are presented in Figs. 5 and 6. The $\pm 2\sigma$ statistics for the one-third octave-band level-difference data (Figs. 7 and 8) are included in Table 2.

Note introducing a drastic change in the filter shape did not substantially affect the level-differences when compared with the results obtained in Sec. 4.1 using the third-order Butterworth filter shape.

4.5 SAE 866A Method

The Volpe Method was compared with the SAE 866A Method as described in the Society of Automotive Engineers Aerospace Recommended Practice 866A, "Standard Values of Atmospheric Absorption as a Function of Temperature and Humidity" (SAE ARP 866A)[6].

For these comparisons, a homogenous atmosphere was assumed, i.e., temperature, humidity and pressure did not vary with altitude. Atmospheric pressure was set to a static, ISA, sea level value of 101.325 kPa and used in the appropriate absorption calculation for the four T/H conditions tested at eight altitudes from 75 to 7620 meters.

The level-difference data (SAE 866A Method minus the Volpe Method) for ΔL_A and ΔL_{PN} obtained at the four T/H points were combined and are presented in Figs. 5 and 6. The $\pm 2\sigma$ statistics for the one-third octave-band level-difference data (Figs. 7 and 8) are included in Table 2.

Although not shown herein, the differences comparing the SAE 866A Method directly to the Exact Method were found to be in the same range as the differences shown in Table 2 for the SAE 866A minus Volpe Method case. Thus it is concluded that the Volpe Method is in better agreement with the Exact Method than is the SAE 866A Method.

4.6 Applicability to 20 kHz

The test spectrum was extended one octave to 20 kHz and the Volpe Method was again compared with the Exact Method.

For these comparisons, a homogenous atmosphere was assumed, i.e. temperature, humidity and pressure did not vary with altitude. Atmospheric pressure was set to a static, ISA, sea level value of 101.325 kPa and used in the appropriate absorption calculation for the four T/H conditions tested over the altitude range of 75 to 7620 meters.

As in Sec. 4.1, the one-third octave-band level-difference data for the four T/H points were combined and the $\pm 2\sigma$ statistics for the one-third octave-band level-difference data are included in Table 2 for the limited altitude range of 75 to 2400 meter. Note through 2400 meters the level-differences for the extended frequency range compare favorably to the results in Sec. 4.1 (50 Hz to 10 kHz, 75 to 7620 meters).

5 CONCLUSION

A new simplified procedure (the Volpe Method) is introduced for the calculation of atmospheric-absorption for broadband sounds analyzed by one-third octave-band filters. The proposed method utilizes the accurate pure-tone sound absorption algorithms of the ISO/ANSI[1,2] standards for predicting band-level attenuations and is recommended for mid-band attenuation up to 500 dB. The Volpe Method was evaluated under a variety of temperature/humidity conditions, atmospheric pressure conditions, filter shapes, and over the frequency range of 25 Hz to 20 kHz. The Volpe Method was applied to representative commercial jet aircraft spectra over a range of altitudes from 75 to 7620 meters at four T/H points. Two noise descriptors, the L_{PN}, and L_A were computed. These data were compared with similar data computed with the Exact Method. Level-difference results (Figs. 5 and 6) over the altitude range of 75 to 7620 meters show the mean difference for both the ΔL_{PN} and ΔL_A descriptors to be less than −1.2 dB with a 2σ range of less than ±5 dB for the five conditions used in the evaluation (Secs. 4.1-4.5).

Reherman et al.[13], also performed a sensitivity analysis, comparing the Volpe Method and the SAE 866A procedure. Difference data (ΔL_A) for measured aircraft data processed for distances between 120 and 1200 meters showed a mean difference of less than 0.2 dB with a 1σ range of less than ±0.5 dB. ΔL_A from Ref. 3 is in good agreement with the Reherman data showing a mean difference of −0.1 dB with a 1σ range of less than ±0.3 dB (2σ range less than ±0.6 dB) over the range 75 to 1200 meters.

Table 2 summarizes the one-third octave-band level-difference data observed in Secs. 4.1-4.6. The Volpe Method is compared directly against the Exact Method under a variety of conditions using a representative aircraft noise spectrum over an altitude range of 75 to 7620 meters. Table 2 includes data for the frequency range of 50 Hz to 20 kHz and for the Approximate Method.

The data in Table 2 shows the Volpe Method to be more accurate than the SAE 866A Method and the Approximate Method. Unlike the SAE 866A Method, the Volpe Method can take into account the effects of changes in atmospheric pressure and unlike the Approximate Method it is useable to mid-band attenuation levels to 500 dB. Lapsed pressure had little if any effect on the accuracy of the Volpe method, as did the change in filter characteristics. Extending the frequency range to 20 kHz also had minimal effect through a distance of 2400 meters.

The Volpe Method is seen to be useable to mid-band levels up to 500 dB with errors of less than ±0.5 dB or ±5% (of the mid-band level) to 100 dB, ±7% to 500 dB. The Volpe Method is as easy to apply as the SAE 866A Method, can take into account the effects of changes in atmospheric pressure and it is accurate from 25 Hz through 20 kHz.

It is recommended that the Volpe Method presented herein, combined with the ISO/ANSI[1,2] standards, replace the SAE 866A Method for the computation of attenuation by atmospheric absorption of broadband sounds when analyzed by one-third octave-band filters and that it be adopted as the method of choice in FAR 36 and its international counterpart ICAO Annex 16[12].

6 ACKNOWLEDGMENTS

The authors gratefully acknowledge the support of the members of the International SAE Project Working Team (PWT) that include Airbus Industries, Boeing Commercial Airplane Company, Dytec Engineering, Mestre Grieve and Associates, and Louis Sutherland.

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