

# **Dark Matter** from Hidden Dimensions

**Tim M.P. Tait**



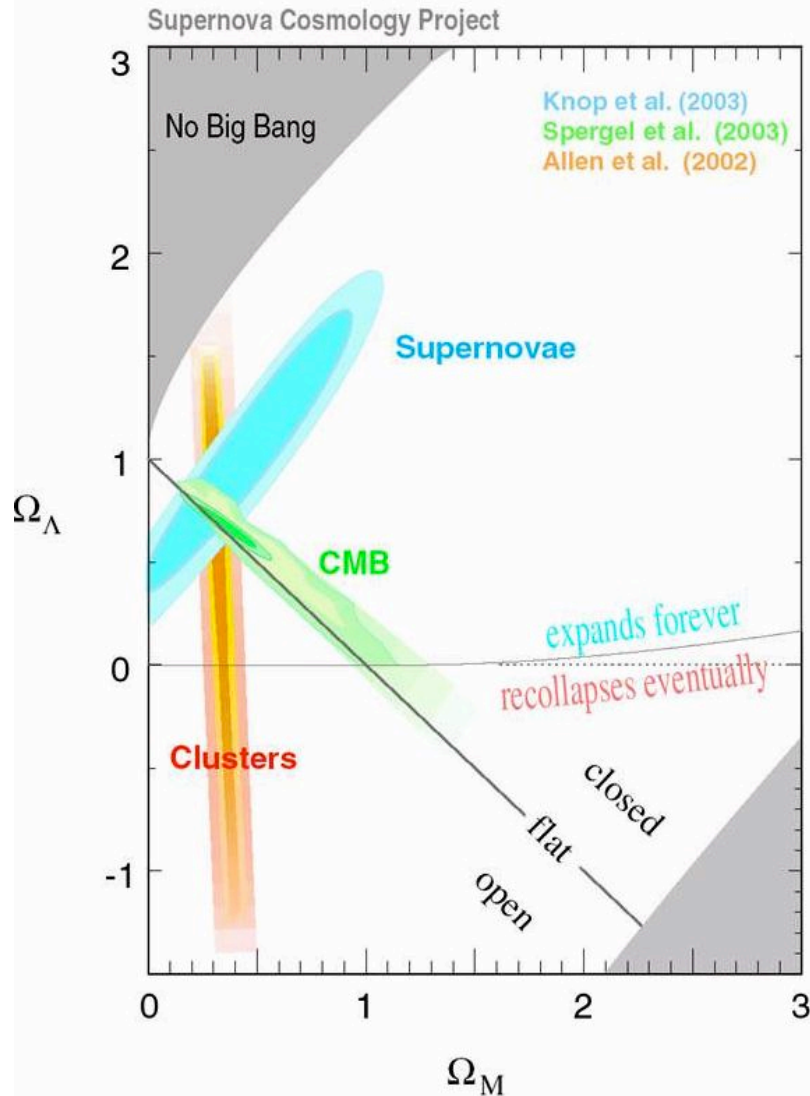
**Argonne National Laboratory**

TeV Particle Astrophysics  
Fermilab  
July 14, 2005

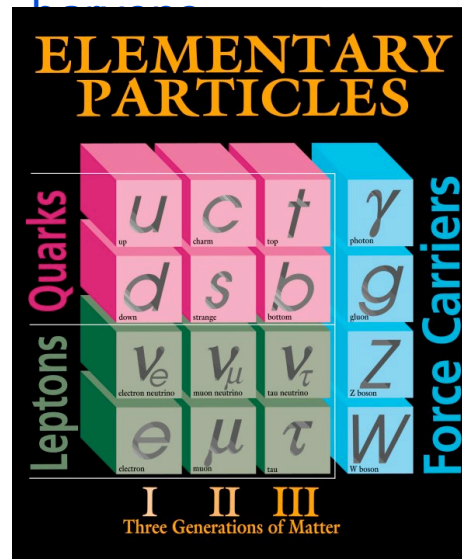
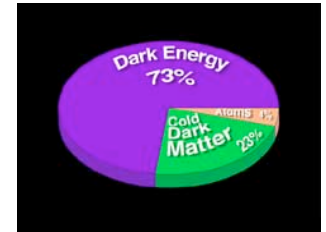
# Outline

- Particle Physics in Extra Dimensions
- Universal Extra Dimensions
  - KK “photon” as Dark Matter.
- Warped Extra Dimensions
  - KK “neutrino” as Dark Matter.
- The Light Radion
  - Extra dimensional “Gravity” as Dark Matter.
- Outlook

# The Dark Side



The supernova data combined with the CMB points to a universe which is roughly **73% dark energy**, **23% cold matter**, and only a few %



“Cold Dark Matter: An Exploded View”  
by Cornelia Parker

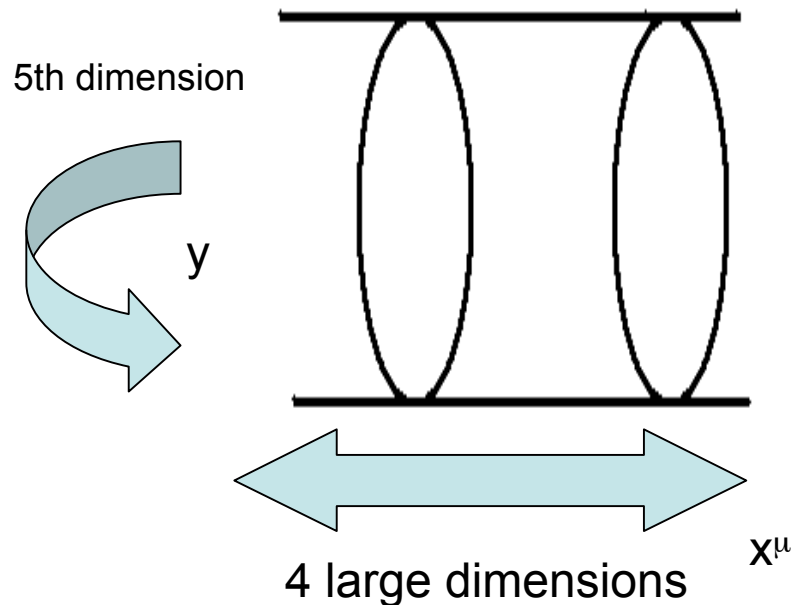
**What is this stuff?**

Particle physics as we know it has no answer.

# Extra Dimensions?

- In recent years, HEP theory has begun to explore theories with weak scale extra dimensions.
  - The picture is that our familiar large **3 + 1** dimensions may be supplemented with more space-like directions.
  - An immediate question is: why do the force laws we observe scale like  $1 / r^2$ ? In more dimensions, they should fall off more quickly.
  - This forces us to consider extra dimensions which are small (of order  $\text{TeV}^{-1} \sim 10^{-17}\text{cm}$ ), or a brane world scenario.
- The main challenge extra dimensions face in making contact with dark matter is explaining why the new states should still exist in the universe today.
  - Naively, we expect such heavy new states should be very short-lived.
  - This expectation can be over-turned either by a symmetry which prevents the new states from decaying, or from a particle which is so weakly coupled that the dark matter does decay, but it has not had time to do so in large quantities today.
  - Extra dimensional theories will exploit both of these possibilities in presenting dark matter candidates.

# Life on a Circle



- How to define a model with extra dimensions..
  - Number of Extra Dimensions
  - Topology
    - Line, circle, torus,...
  - Geometry
    - Flat, warped,...
  - I will confine myself to 5d
    - Most results easily extrapolated to more
    - Simple, only one real choice for the topology.

# Field Theory in 5 Dimensions

- Our extra dimension is a circle ( $S^1$ ).
- This requires wave functions of any states to be periodic as one traverses the extra dimension.
- Mathematically, this is the particle-in-a-box problem familiar from basic Quantum Mechanics.
- The 5<sup>th</sup> component of Momentum ( $p_5$ ) is quantized in units of  $1 / R$ :

$$p_0^2 - \vec{p}^2 - p_5^2 = 0 \quad \longrightarrow \quad p_0^2 - \vec{p}^2 = p_5^2 = m_{eff}^2$$

- States with  $p_5$  different from zero appear massive to an observer who does not realize the extra dimension is there.
- We (and all low energy physics) are composed of the lowest modes.
- Each field has a tower of massive states with the same charge and spin as the zero mode, but with masses given by  $n / R$ .

# KK Decomposition

5d action:

$$\int d^5x \partial_M \Phi \partial^M \Phi \Rightarrow \int d^5x \left\{ \partial_\mu \Phi \partial^\mu \Phi - \partial_5 \Phi \partial_5 \Phi \right\}$$

We perform a Kaluza-Klein decomposition of the 5d field:

$$\Phi(x^\mu, y) = \sum_n f^n(y) \Phi^n(x^\mu)$$

Resulting in:

$$\int d^4x \sum_{n,m} \partial_\mu \Phi^n \partial^\mu \Phi^m \times \underbrace{\left( \int dy f^n f^m \right)}_{\delta_{nm}} - \Phi^n \Phi^m \times \underbrace{\left( \int dy \partial_5 f^n \partial_5 f^m \right)}_{M_n^2 \delta_{nm}}$$

With 4d action:

$$\int d^4x \sum_n \partial_\mu \Phi^n \partial^\mu \Phi^n - M_n^2 \Phi^n \Phi^n$$

# Kaluza-Klein Particles

- Particles:

- $p_5 = 0$

- $M = 0$

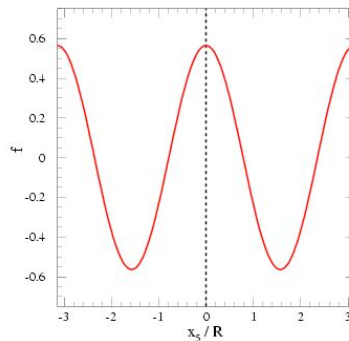
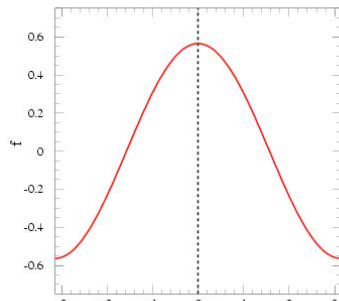
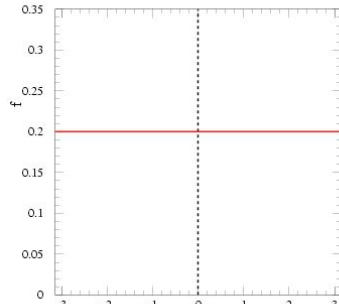
- $p_5 = 1/R$

- $M = 1/R$

- $p_5 = 2/R$

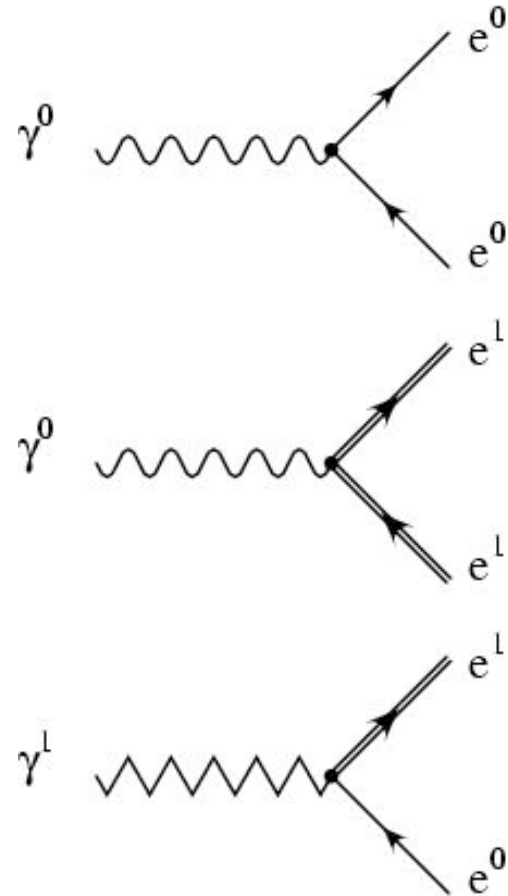
- $M = 2/R$

...and so on!



- Interactions:

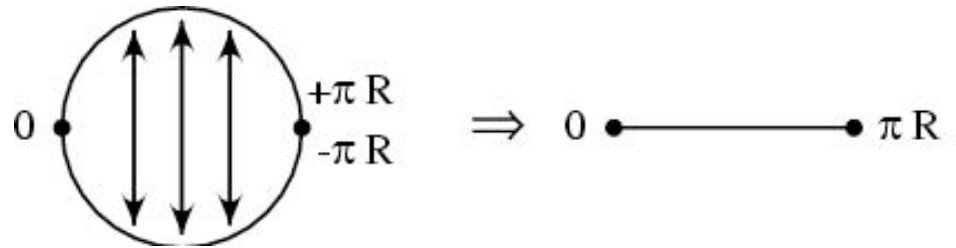
- Conservation of  $p_5$  becomes conservation of **KK number**.





# Orbifold

- 5d vector bosons contain a 4d vector  $V_\mu$  and scalar  $V_5$ .
- Massless 5d spinors have 4 components, leading to mirror fermions at low energies.
- Orbifold boundary conditions project out the unwanted degrees of freedom.
- Instead of a circular extra dimension, we fold the circle, identifying  $y$  with  $-y$ .
- This results in a line segment, with the points  $0$  and  $\pi R$  at the endpoints.
- Chiral fermions result from boundary conditions:



$$V_\mu(-y) = V_\mu(y)$$

$$V_5(-y) = -V_5(y)$$

$$\Psi(-y) = \gamma_5 \Psi(y)$$

# KK Decomposition (II)

- We expand fields in KK modes:

$$\Phi(x^\mu, y) = \sum_n f^n(y) \Phi^n(x^\mu)$$

- Even fields ( $A_\mu, \Psi_L$ ) have zero modes:

$$\Phi(x^\mu, y) = \sqrt{\frac{1}{\pi R}} \Phi^0(x^\mu) + \sum_{n \geq 1} \sqrt{\frac{2}{\pi R}} \cos\left(\frac{ny}{R}\right) \Phi^n(x^\mu)$$

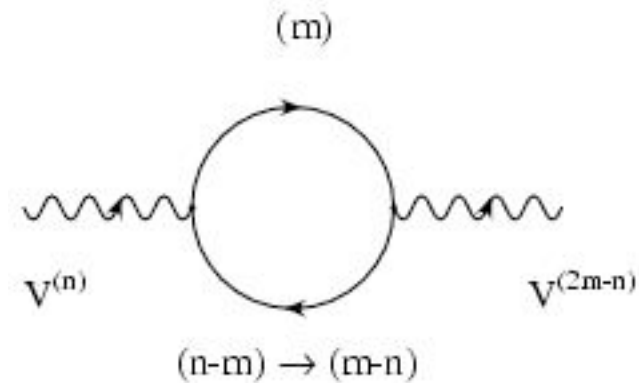
- Odd fields ( $A_5, \Psi_R$ ) don't:

$$\Phi(x^\mu, y) = \sum_{n \geq 1} \sqrt{\frac{2}{\pi R}} \sin\left(\frac{ny}{R}\right) \Phi^n(x^\mu)$$

- Each SM field has a tower of “partner fields” with mass  $n / R$  but the same charge and spin.

# Orbifolds are Opaque

- Even theories without localized fields have terms on boundaries.
- The orbifold, identifying ( $y$  and  $-y$ ), implies the theory can't tell one direction from another.
- Loops of fields generate  $p_5$  non-conserving terms.
- In position space, these are equal terms on both boundaries.
- The loops are **log**-divergent, indicating that they are incalculable parameters of the effective theory.



$$-\frac{r_c}{4} [\delta(y) + \delta(y-L)] F_{\mu\nu} F^{\mu\nu}$$

$$r_c \ddot{y} \frac{\alpha_5}{4\pi} \log \left[ \frac{\Lambda}{\mu} \right]$$

# Boundary Kinetic Terms

- The terms living on the boundaries change the physics.
- They alter the KK decomposition:

$$\frac{1}{g_5^2} \int dy \left\{ 1 + r_c [\delta(y) + \delta(y-L)] \right\} f_n(y) f_m(y) = \delta_{nm}$$

$$\frac{1}{g_5^2} \int dy f_n'(y) f_m'(y) = m_n^2 \delta_{nm}$$

- KK wave functions satisfy:

$$\left[ \partial_5^2 + m_n^2 + r_c m_n^2 \{ \delta(y) + \delta(y-L) \} \right] f_n = 0$$

# Opaque Orbifolds

M. Carena, T. Tait, C. Wagner, APP B33, 2355 (2002)

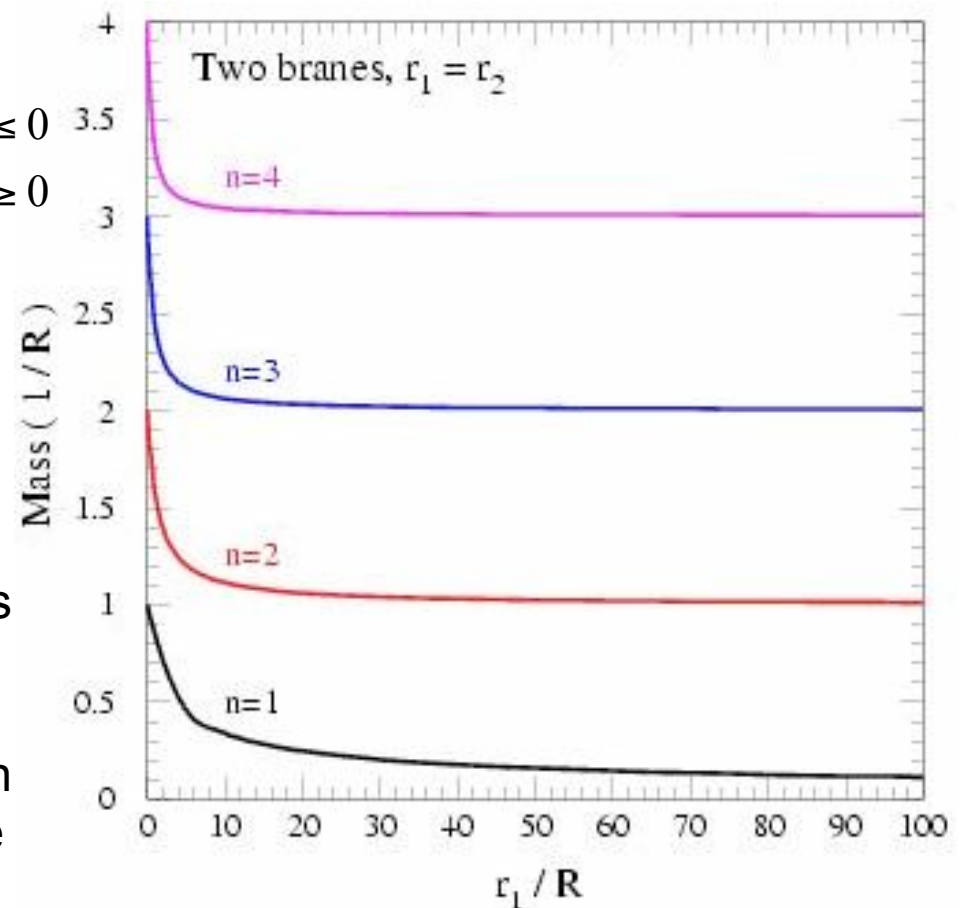
The wave functions are:

$$f_n(y) = \begin{cases} \cos(m_n y) + (m_n r_c / 2) \sin(m_n y) & y \leq 0 \\ \cos(m_n y) - (m_n r_c / 2) \sin(m_n y) & y \geq 0 \end{cases}$$

With quantized masses:

$$0 = (r_c^2 m_n^2 - 4) \tan[m_n \pi R] - 4 r_c m_n$$

The boundary terms change the masses and wave functions. Since each field has a potentially different boundary term, this splits the degeneracy between the entire KK level and allows i.e., some KK modes of the first level to decay.



# Universal Extra Dimensions

- The framework is that all fields live in all dimensions:
  - Quarks & Gluons
  - Leptons
  - Photons and Gauge Bosons
  - Higgs
  - Gravity
- This is unlike the “[brane world](#)” scenario where everything except gravity is stuck to some point.
- This universality implies a translational invariance along the [5<sup>th</sup>](#) dimension, and thus conservation of momentum in that direction.
- The result is a stable particle, necessary to have a dark matter candidate.

# Why Universal Extra Dimensions?

- **String Theory:**
  - String theories require **supersymmetry** and **10 dimensions** to be consistent.
  - So, extra dimensions are (from a low energy point of view), the “**other half**” of stringy phenomenology.
- TeV extra dimensions provide a natural setting for **top seesaw** models:
  - A theory without a Higgs can still exhibit spontaneous symmetry-breaking driven by KK modes of gluons.
  - At high energies, the bound state Higgs breaks into quarks and gluons with no quadratic divergences.
- **Number of generations:**
  - Cancellation of anomalies in six dimensions requires the number of families to be a multiple of three!

# Effective Theory for UED

- To define a model of UED, one **must** specify:

- Bulk Interactions, i.e.:

$$\frac{1}{g_5^2} F_{\mu\nu}(x^\mu, y) F^{\mu\nu}(x^\mu, y)$$

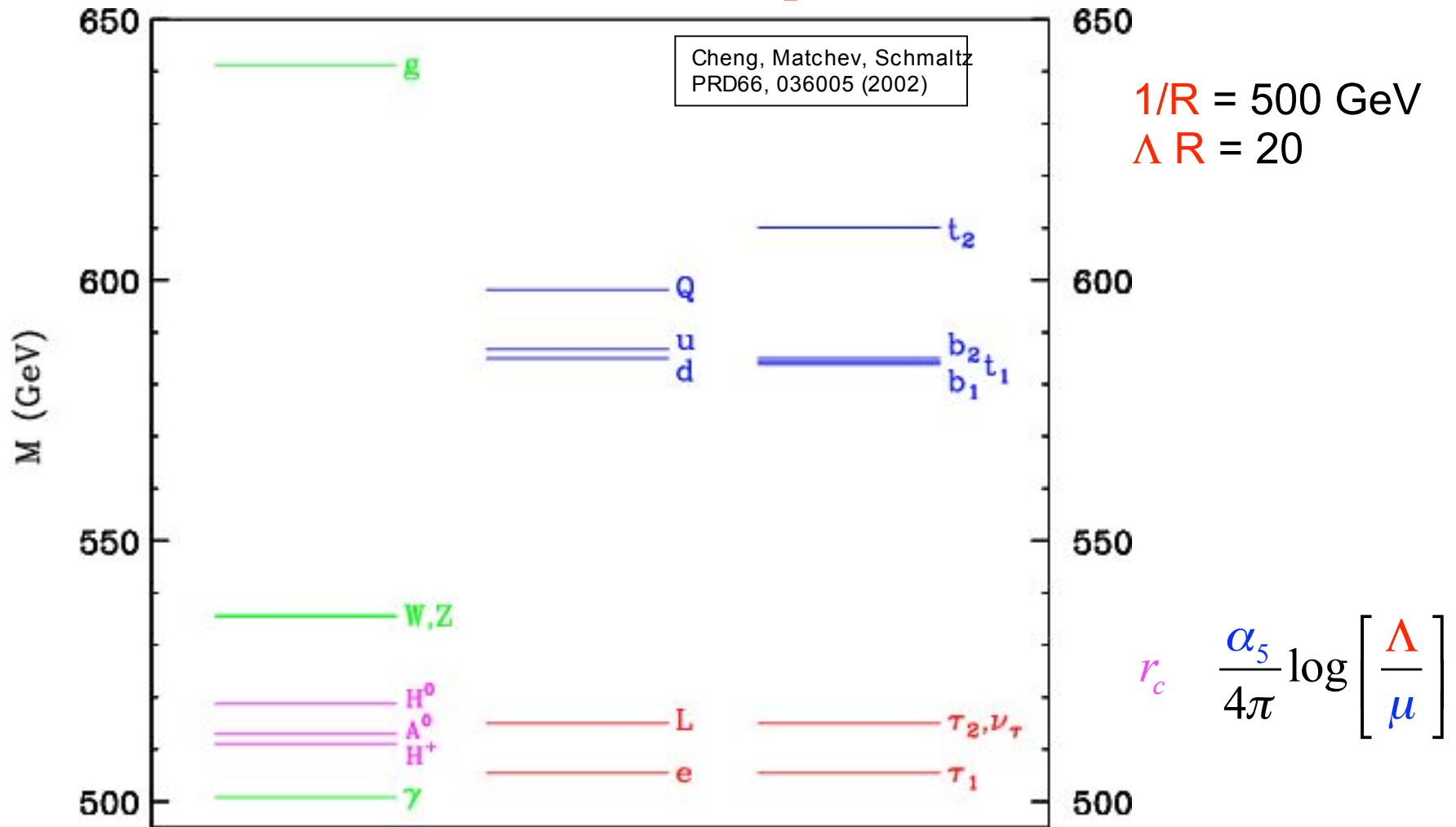
- Boundary terms, i.e.:

$$\frac{r_c}{g_5^2} F_{\mu\nu}(x^\mu) F^{\mu\nu}(x^\mu) [\delta(y) + \delta(y-L)]$$

- Each field has a (potentially different) kinetic term living on the boundaries of the extra dimension. These free parameters play an important role in the resulting phenomenology of the universal extra dimension.



# KK Mode Spectrum



# KK Parity

- Conservation of KK number is broken to conservation of KK parity:  $(-1)^n$ .
- KK parity requires odd KK modes to couple in pairs:
  - The lightest first level KK mode is stable.
  - First level KK modes must be pair-produced.
- The Lightest Kaluza-Klein Particle plays a crucial role in phenomenology, similar to the LSP of SUSY:
  - All relic KK particles decay to LKPs.
  - Any first level KK mode produced in a collider decays to zero modes and an LKP.

# Identity of the LKP

- Boundary terms play a role similar to **soft masses**, determining **masses** and **couplings** for the entire **KK tower**.
- If we imagine the terms are zero at the cut-off, they will be induced at loop size.
- Since  $\alpha_1 \ll \alpha_2 \ll \alpha_3$ , we imagine the smallest corrections will be to the U(1) gauge boson.
- Since  $\delta M \sim 1/R \gg v$ , the **LKP** is (almost) purely a KK mode of the U(1) gauge boson,  $B_\mu^{(1)}$ .
- Following this line of reasoning, the **NLKP** is the right-handed electron,  $e^{(1)}_R$ .

$$\delta M^2 = \frac{1}{R^2} \frac{\alpha}{4\pi} \log(\Lambda R)$$

$$\begin{pmatrix} \frac{1}{R^2} + \frac{1}{4} g_1^2 v^2 + \delta M_1^2 & \frac{1}{4} g_1 g_2 v^2 \\ \frac{1}{4} g_1 g_2 v^2 & \frac{1}{R^2} + \frac{1}{4} g_2^2 v^2 + \delta M_2^2 \end{pmatrix}$$

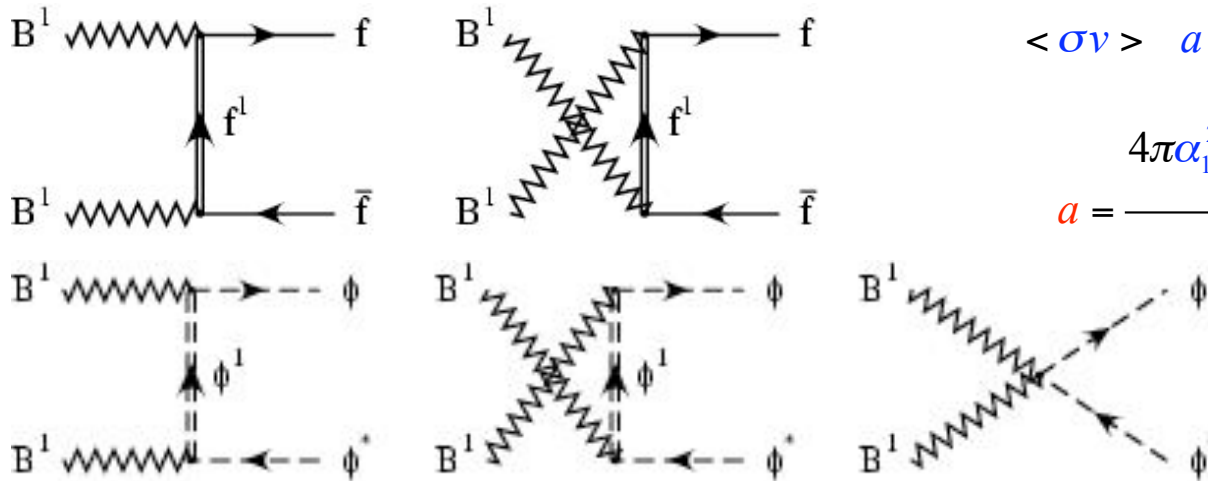
$B^1 - W_3^1$  Mass<sup>2</sup> matrix

# LKP as Dark Matter

- We know how much dark matter the universe seems to require.
- The question, then, is for which regions of parameter space the LKP is a good dark matter candidate.
- The couplings of the LKP to matter are fixed by the structure of the theory.
- The LKP couples to one zero-mode matter particle, and one KK mode matter particle, with coupling given by the (measured) U(1) coupling of the Standard Model,  $g_1$ , times the corresponding hypercharge,  $Y$ .
- Thus, the only parameter we don't already know is the size of the extra dimension, or in other words, the masses of the KK particles.
- I will explore this single parameter to find the cosmologically interesting values.

# Thermal Production & Freeze Out

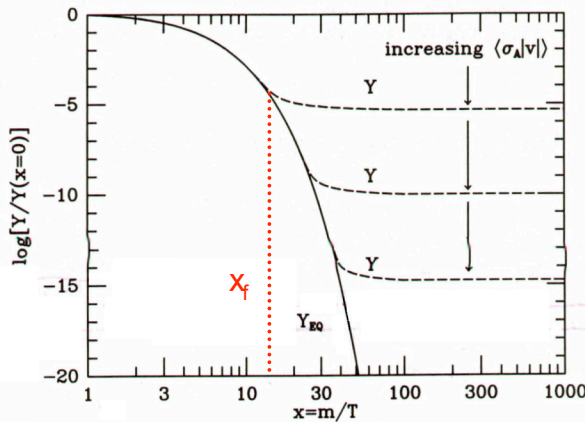
- To estimate the relic density of the **LKP**, we assume it was originally in thermal equilibrium in the early universe.
- As the universe expands, eventually the density is small enough that they can no longer interact with one another, and fall out of equilibrium.
- Below this *freeze-out temperature*, the density of **WIMPs** per co-moving volume is fixed.



$$\langle \sigma v \rangle = a + b \langle v^2 \rangle + \dots = a + 6b/x + \dots$$

$$a = \frac{4\pi\alpha_1^2 \left( 2 \sum_F Y_F^4 + 3Y_H^4 \right)}{9m_{KK}^2}$$

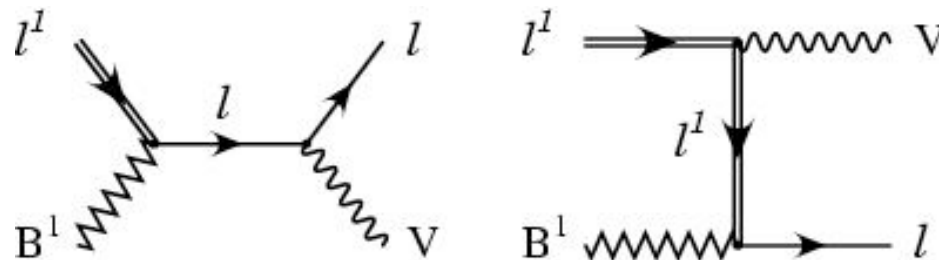
59% charged leptons
35% hadrons
4% neutrinos
2% Higgs



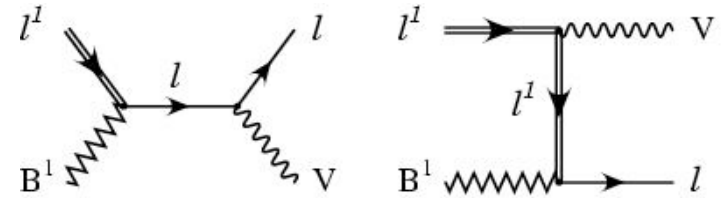
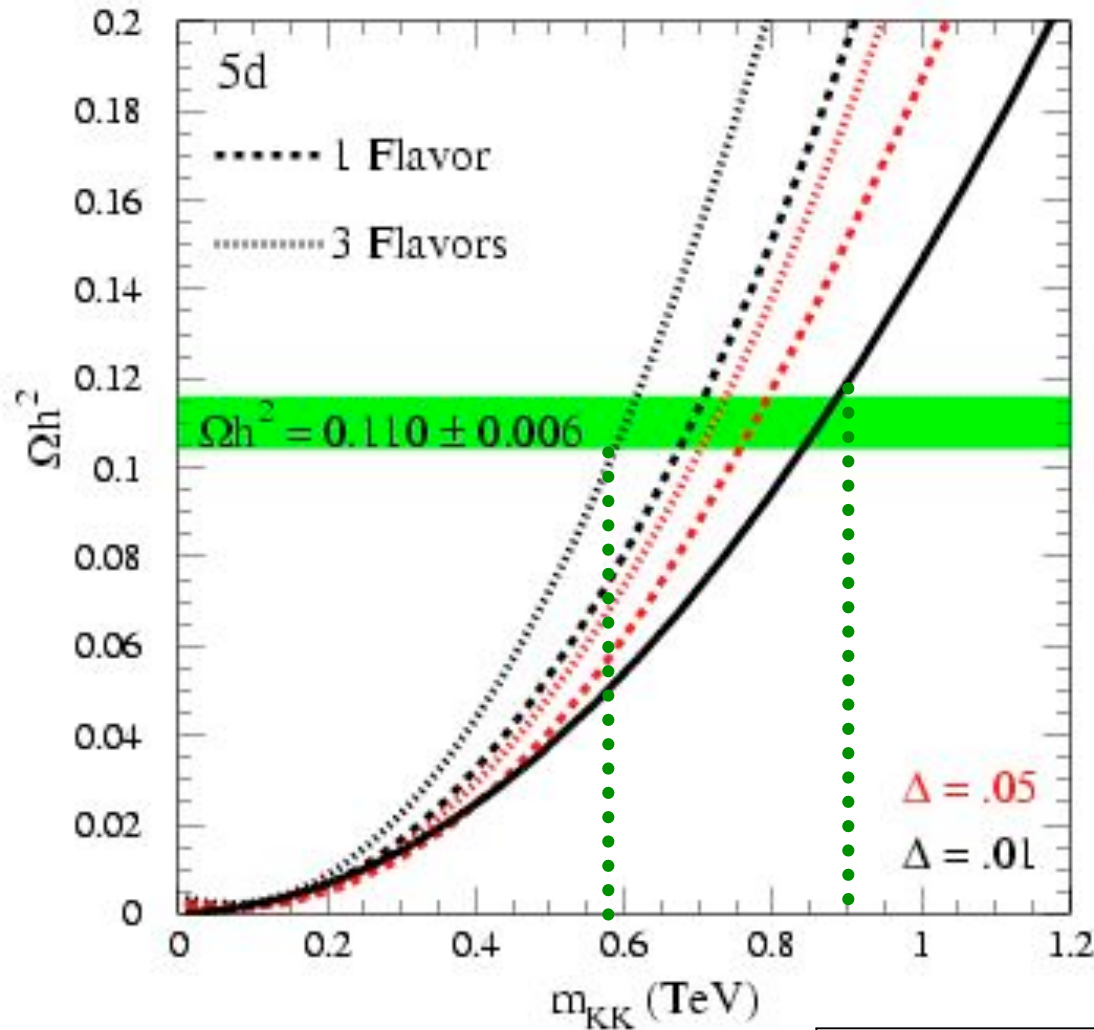
$x_f \sim 25$   
 For  $m_{KK} \sim 1 \text{ TeV}$  :  $T_f \sim 40 \text{ GeV}$

# Coannihilation

- If the mass of  $e^{(1)}_R$  is close to  $B^{(1)}$ , it may substantially affect the relic density.
- They interact roughly with the same efficiency.
- The freeze-out temperature is basically unchanged,
- Some  $e^{(1)}_R$  left over after freeze-out, and eventually decay into  $B^{(1)}$  and  $e^{(0)}$ .
- The net relic density of  $B^{(1)}$  is increased.
- Quite different from SUSY.



# Relic Density



$\Delta$  is the splitting between the  $B^{(1)}$  and  $e_R^{(1)}$  masses.

$$\Delta \equiv \frac{m_{e_R^{(1)}} - m_{B^{(1)}}}{m_{B^{(1)}}}$$

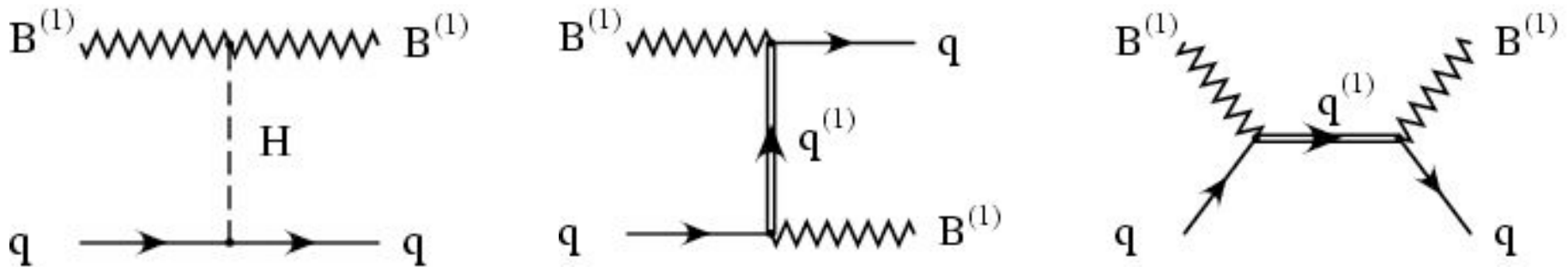
Coannihilation favors the 5d range **600-900 GeV**.

The 6d range is **425-625 GeV**.

G. Servant, T. Tait, NPB650, 351 (2003)

# Direct Detection

- Direct detection of dark matter attempts to see WIMP-nuclei scattering.
- At the fundamental level, the LKP scatters with quarks.



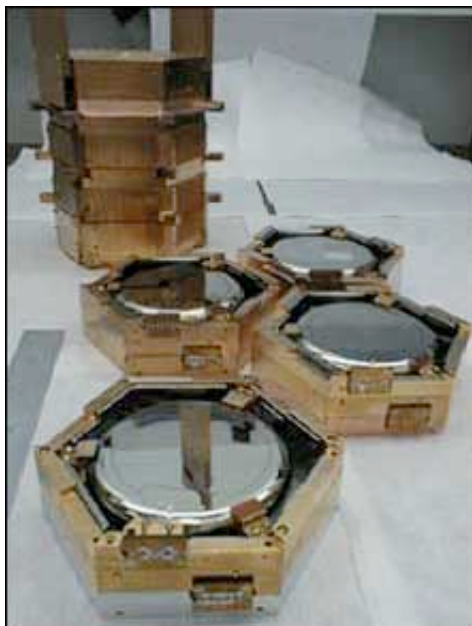
- Form factors relate quark interactions to nucleon scattering.
- Nuclear physics relates nucleon scattering to nuclear interactions.
- Energy is deposited in the nucleus target:

$$\frac{dR}{dE_r} = \frac{\rho}{m_{KK}} \frac{\sigma_0}{2\mu^2 v^2} F^2(E_r, v) f(v) v dv$$

$\rho$	: WIMP halo density
$v$	: WIMP velocity
$f$	: WIMP $v$ distribution
$\sigma_0$	: nucleon $\sigma$
$F$	: nuclear form factor

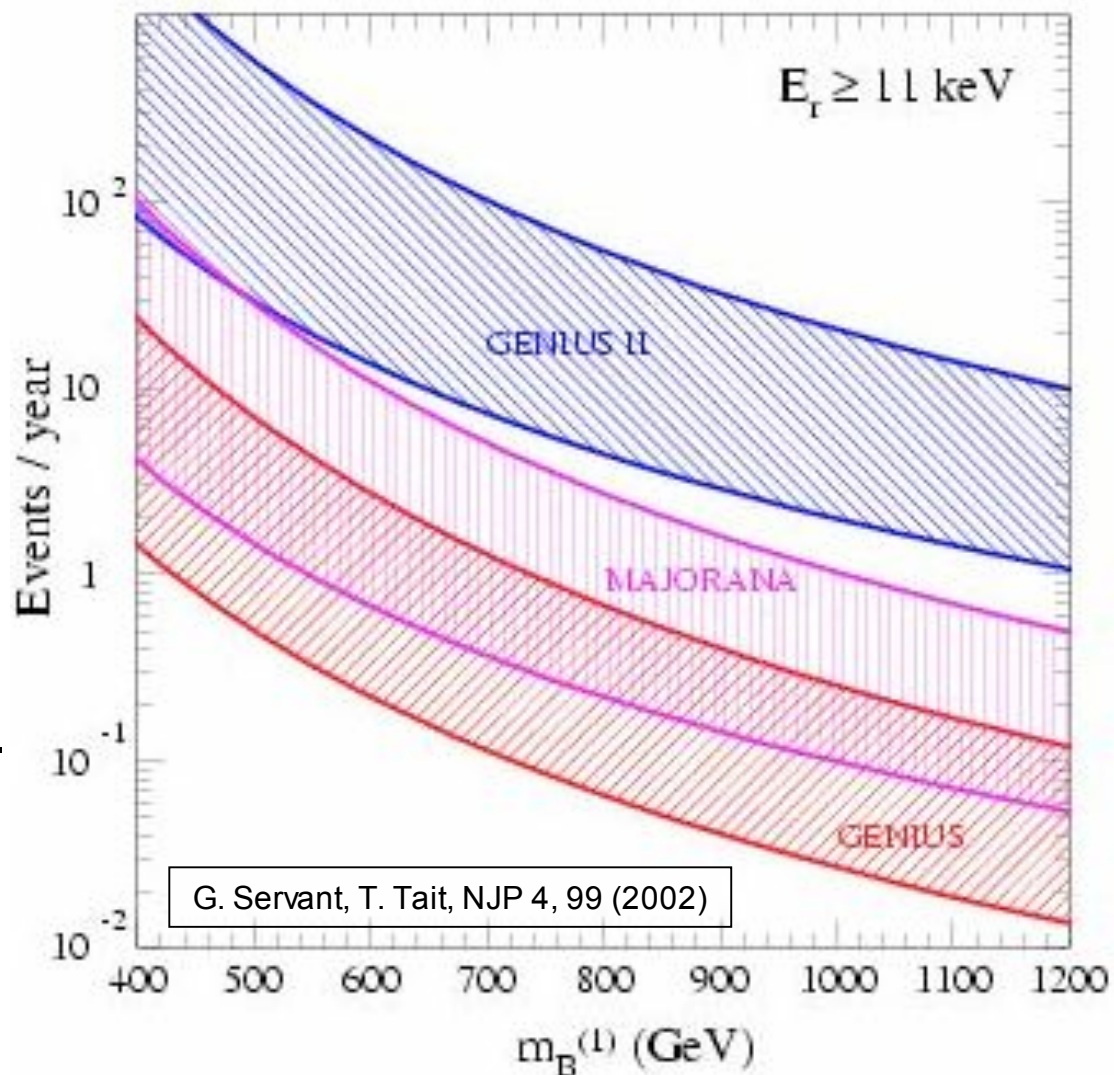


# Germanium Detectors



Very precise calorimeter, in order to see a clear excess.

**GENIUS** : 100 kg  $^{73}\text{Ge}$   
**GENIUS-2** :  $10^4$  kg  $^{76}\text{Ge}$   
**MAJORANA** : 500 kg  $^{76}\text{Ge}$



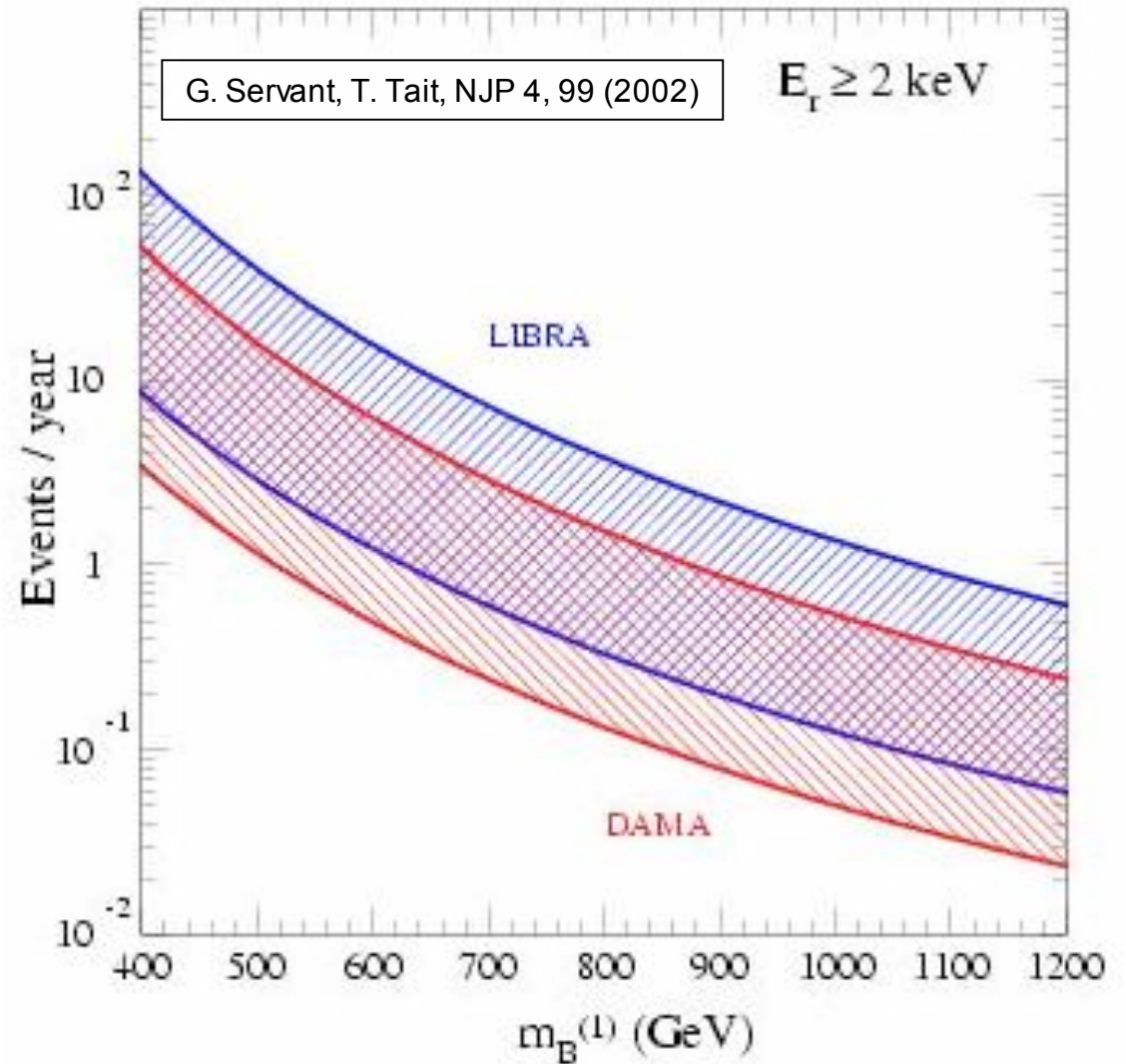
# Nal Detectors



Search strategy is to see an annual modulation of events as the earth revolves around the sun.

**DAMA: 100 kg NaI**

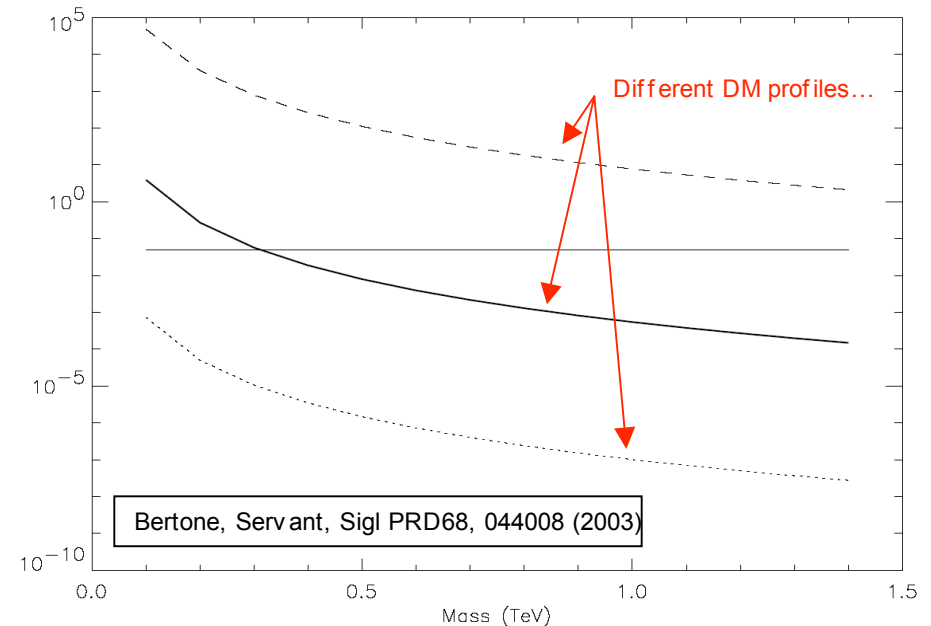
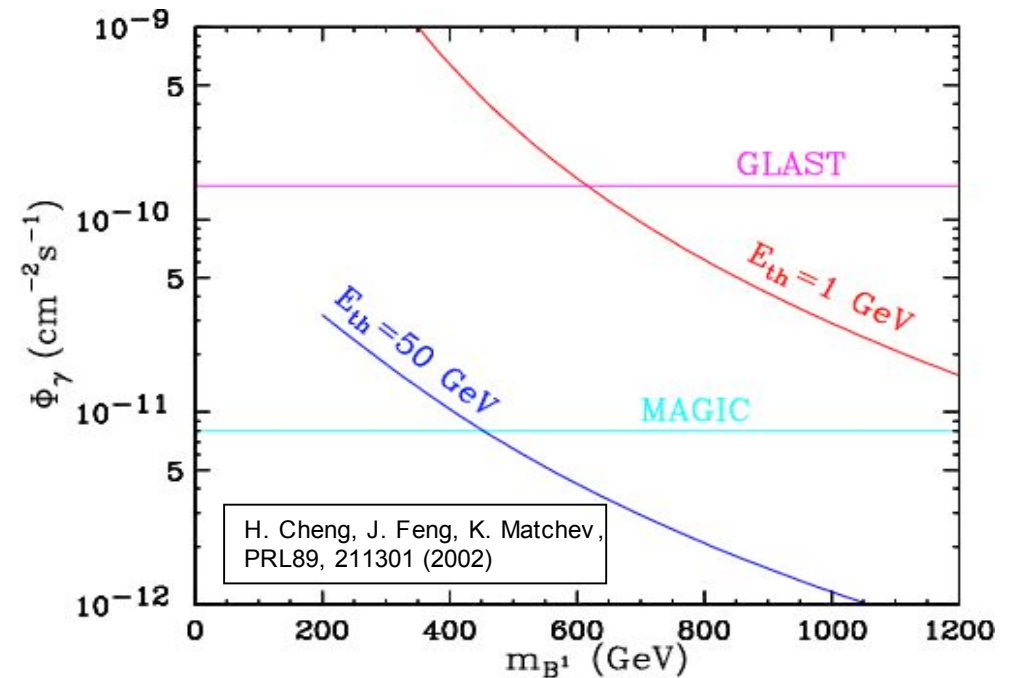
**LIBRA: 250 kg NaI**



# Indirect Detection: $\gamma$

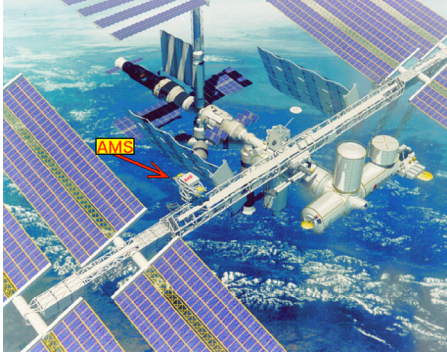
- WIMPs in the galactic halo can annihilate into high energy gamma rays.
- Rates are rather sensitive to the profile of dark matter in our galaxy, which is not very well understood.
- Energetic photons from synchrotron radiation of charged particles is also possible, though this further depends on galactic magnetic fields.
- This may be observable at **GLAST** or **MAGIC** for the lower range of LKP masses.

Line emission: Bergstrom, Bringmann, Eriksson, Gustafsson, hep-ph/0412001

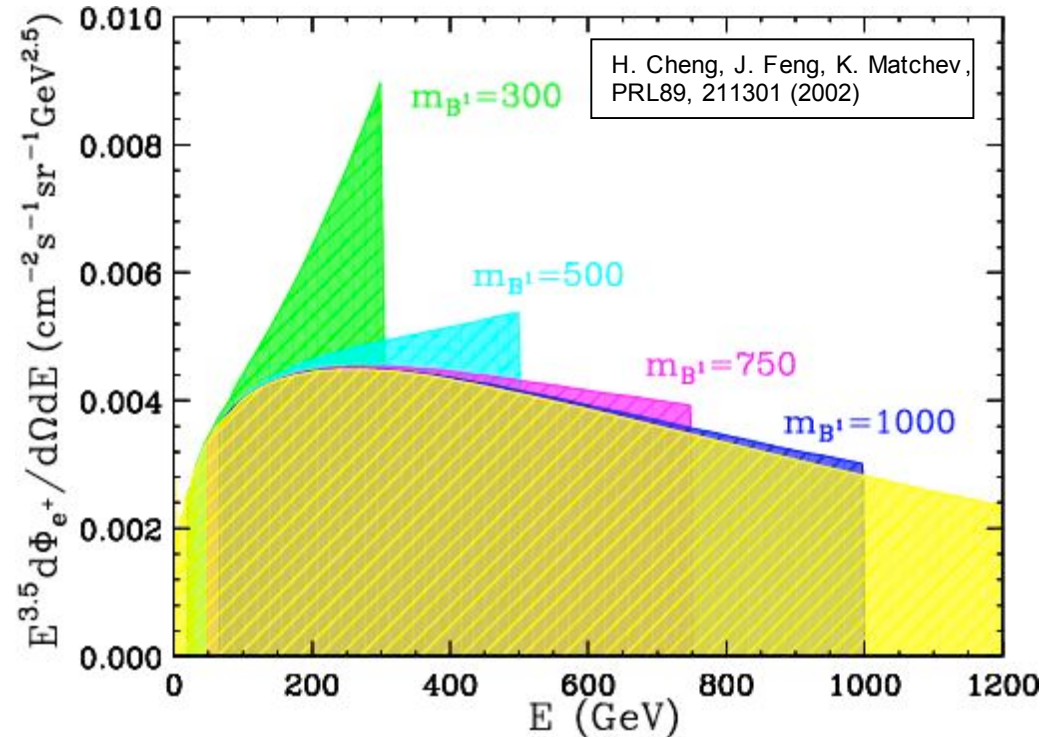




# Indirect Detection: $e^+$



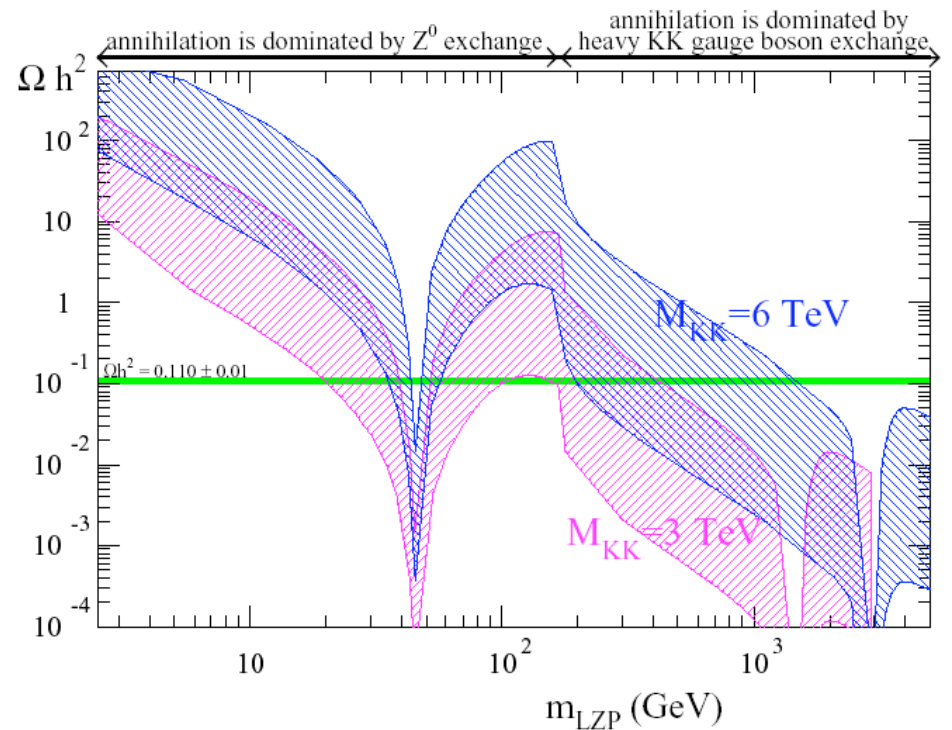
- WIMPs in the galactic halo can annihilate into  $e^+ e^-$ .
- The  $e^+$  can be detected by space-based experiments, such as AMS.
- **LKPs** *prefer* to annihilate into  $e^+ e^-$ , and produce mono-energetic positrons.
- This is a striking signal, clearly visible against backgrounds for  $m_{KK}$  less than about **500 GeV**.



# Warped Extra Dimension?

- Another possibility is that the extra dimension is warped (Randall-Sundrum), with non-zero bulk curvature.
  - Such theories naturally solve the hierarchy problem and lead to GUTs.
  - However, they generically lead to much too rapid proton decay, requiring the imposition of more symmetries.
  - A particular SO(10) model splits the families among three 16s of SO(10), with different  $Z_3$  charges. The lightest Z-odd particle is stable!
  - This turns out to be the right-handed neutrino (KK mode) which lives in the same multiplet as the right-handed top quark (zero mode).
  - It can have the correct thermal relic density for a wide range of masses. The processes maintaining equilibrium can proceed either through the ordinary Z or a Z' KK mode from the broken part of SO(10).
  - Preferred coupling is to the LZP's GUT partner, the ordinary RH top quark.

$$\begin{pmatrix} \mathbf{u}_L, \mathbf{d}_L \\ u_R^c \\ d_R^c \\ \nu_L, e_L \\ e_R^c \\ \nu_R^c \end{pmatrix}_{B=1/3}, \quad \begin{pmatrix} u_L', d_L' \\ \mathbf{u}_R^c \\ \mathbf{d}_R^c \\ \nu_L', e_L' \\ e_R^c \\ \nu_R^c \end{pmatrix}_{-1/3}, \quad \begin{pmatrix} u_L', d_L' \\ u_R^c \\ d_R^c \\ \nu_L, e_L \\ \mathbf{e}_R^c \\ \nu_R^c \end{pmatrix}_0$$



# Light Radion?

- Any theory with an extra dimension contains a scalar field, the radion, which is the modulus describing the size of the extra dimension.

$$ds^2 = \left( e^{-1/3\tilde{\phi}} g_{\mu\nu} + e^{2/3\tilde{\phi}} \tilde{A}_\mu \tilde{A}_\nu \right) dx^\mu dx^\nu + 2e^{2/3\tilde{\phi}} \tilde{A}_\mu dx^\mu dy + e^{2/3\tilde{\phi}} dy^2$$

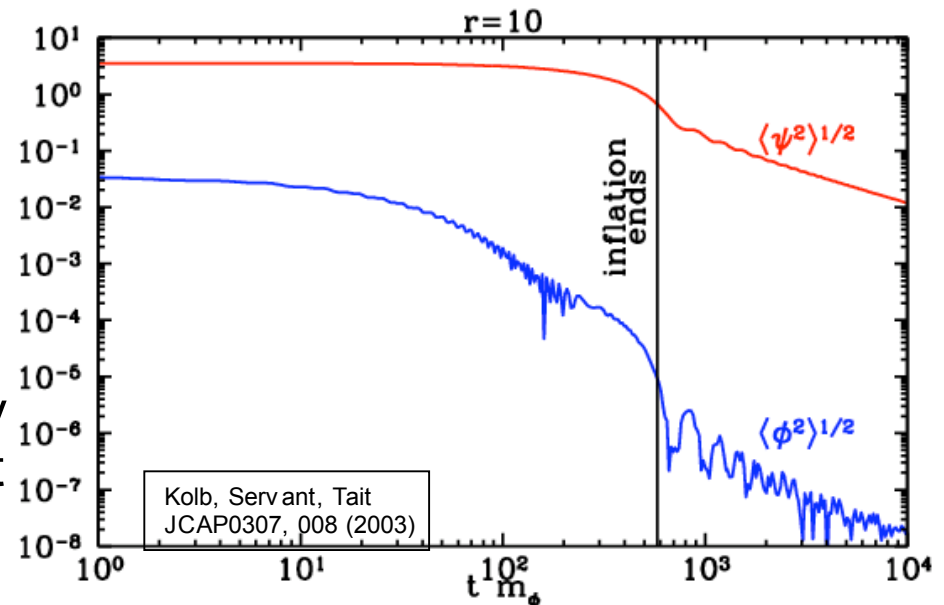
- In 5d, the KK modes of  $g_{55}$  are eaten by the massive spin-2 modes.
- The massless mode is a physical degree of freedom, but a mass to stabilize the size of the extra dimension.

$$m_r \sim \sqrt{3} \frac{M_c^2}{m_{Pl}}$$

- Its couplings are gravitational, and thus highly suppressed, allowing for the possibility that

$$\text{if } \Gamma = \tau^{-1} \simeq \frac{m_r^3}{192\pi m_{Pl}^2} \sim \frac{\sqrt{3} M_c^6}{64\pi m_{Pl}^5},$$

- It can be produced non-thermally through a coherent misalignment during inflation (like the axion).



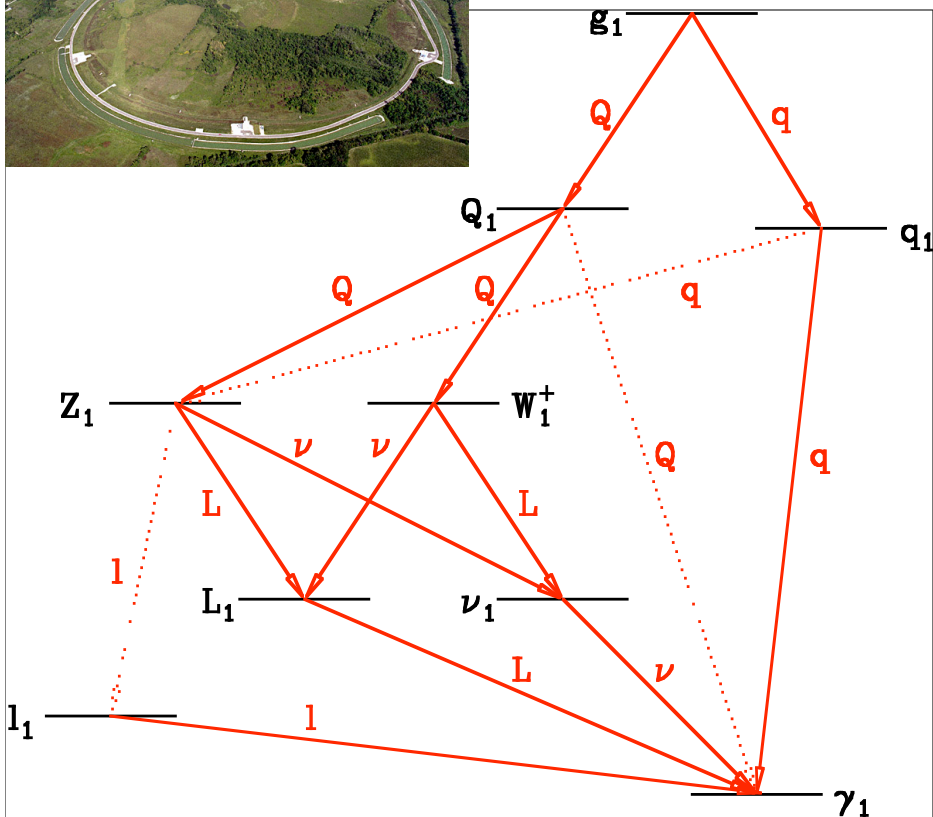
# Outlook

- Extra Dimensions allow us to address many puzzles of particle physics: now including **dark matter!**
- In UED, the dark matter candidate is a massive vector particle, stable because of a remnant of the extra-dimensional space-time symmetries.
- RS has a right-handed neutrino which is stable because of a symmetry which is imposed to avoid too rapid proton decay.
- Both theories produce WIMPs thermally in the correct abundance if the WIMP mass is at the TeV scale.
- Any theory can have a light radion which may survive to the present day because it interacts very weakly. It may be produced non-thermally through vacuum misalignment during inflation.
- All of these theories have interesting **experimental signatures** allowing us to explore the nature of dark matter.
- **Let the exploration of the 5<sup>th</sup> dimension begin!**

# Collider Signatures



Fermilab

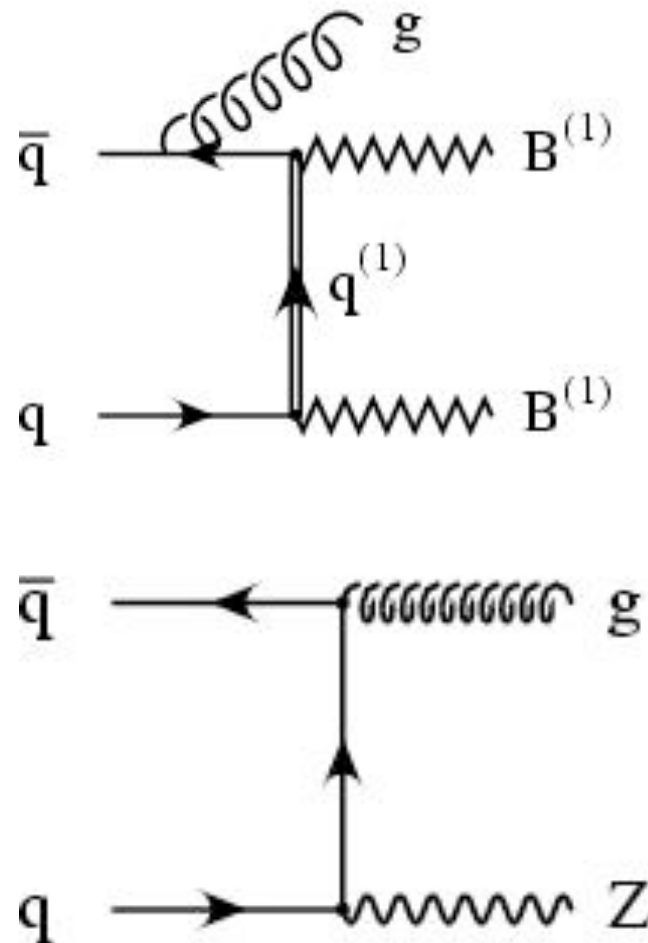


- Another interesting avenue would be to discover LKPs (and other KK modes) at high energy colliders.
- Current precision bounds are  **$1 / R > 200 \text{ GeV}$** .
- One would expect Tevatron limits are on the same order, though no detailed analysis exists.
- Complicated cascade decays are possible, especially for coloured KK modes of  $q$  and  $g$ .
- In order to study the dark matter scenario more directly, I will focus on  $B^{(1)}$  and  $e^{(1)}_R$ , though cross sections may be smaller.



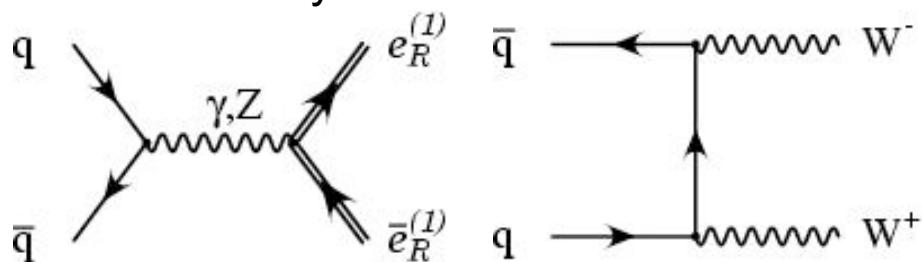
# Monojet Signature

- One could look for pairs of **LKPs** accompanied by initial state radiation.
- The **LKPs** themselves escape from the detector.
- Unfortunately, this is sensitive to the KK quark masses.
- Physics backgrounds come from **Z + jet** production, with **Z** decaying into neutrinos.
- Fake backgrounds are from missed jets, energy mismeasurement, etc.
- At an  **$e^+ e^-$**  collider:  **$\gamma$**  + missing energy.



# Leptons + Missing Energy

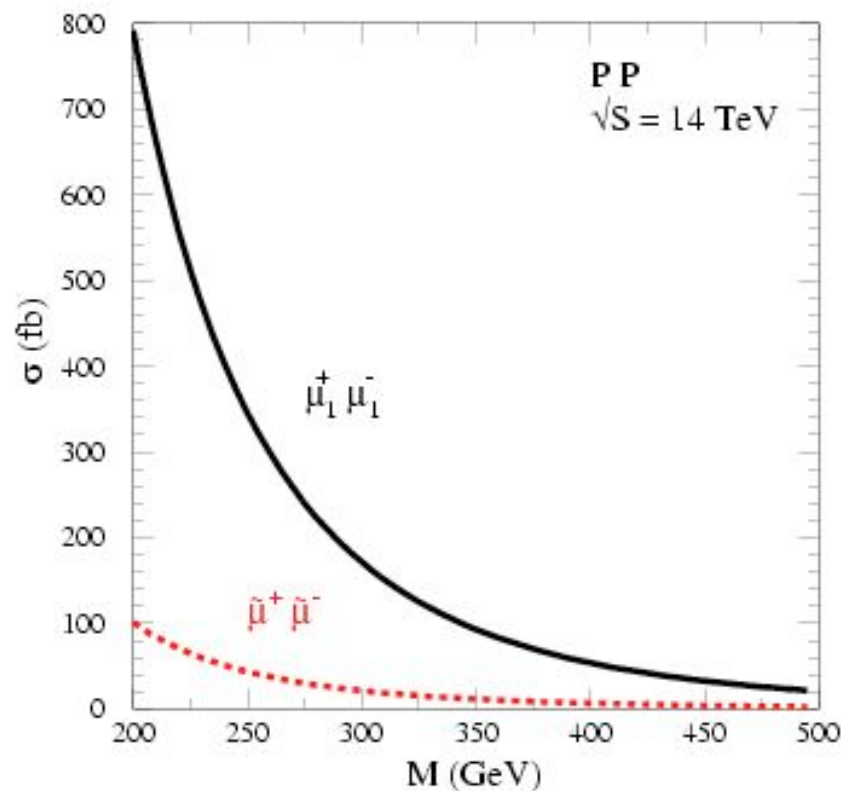
- Another mode is pair production of  $e_R^{(1)}$ , followed by its decay into  $e^{(0)} + B^{(1)}$ .
- This process is not sensitive to KK quark masses.
- Physics backgrounds are diboson production, dominantly **W pairs**.
- Fake backgrounds are from Drell-Yan + missed jets, etc.
- A linear collider is limited by kinematics, but can also search effectively.



Tim Tait

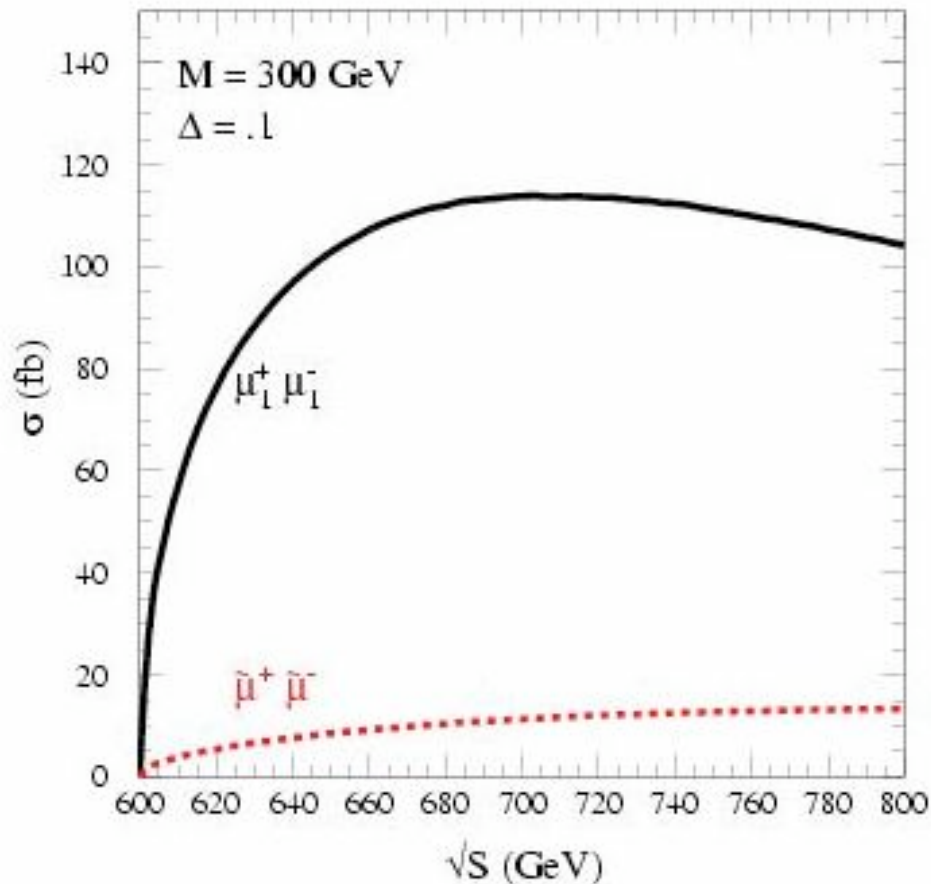
6/14/2005 - TeV Particle Astrophysics

J. Hewett, T. Rizzo, T. Tait, in progress



# $e^+e^-$ Linear Collider

J. Hewett, T. Rizzo, T. Tait, in progress



- One pressing question for particle physics is how we will be able to tell one theory from another.
- In particular, the theory with universal extra dimensions can be very similar to supersymmetry.
- Each theory has partner fields for each SM particle. The difference is the spin of the partner.
- A high energy  $e^+ e^-$  collider can distinguish the spins.