Caustics in galactic halos

Aravind Natarajan
Pierre Sikivie
University of Florida

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Outline :

1. Introduction.

2. What are caustics ? Caustics with dark matter.

3. Choosing initial conditions.

4. What do they look like ? The structure of caustics.

5. Conclusion.

- Caustics are regions of high density.
- The existence of caustics requires :

  (a) Collisionless matter.

  (b) Low velocity dispersion.

- Caustics have implications for :

(a) Axion searches : Peaks in the energy spectrum of microwave photons from $a \longrightarrow \gamma$ conversion *(Duffy et al, 2005).*

(b) WIMP direct detection : Plateaux in the recoil energy spectrum *(Vergados, 2001; Green, 2001; Gelmini and Gondolo, 2001; Ling et al, 2004; Bernabei et al, 2005)*

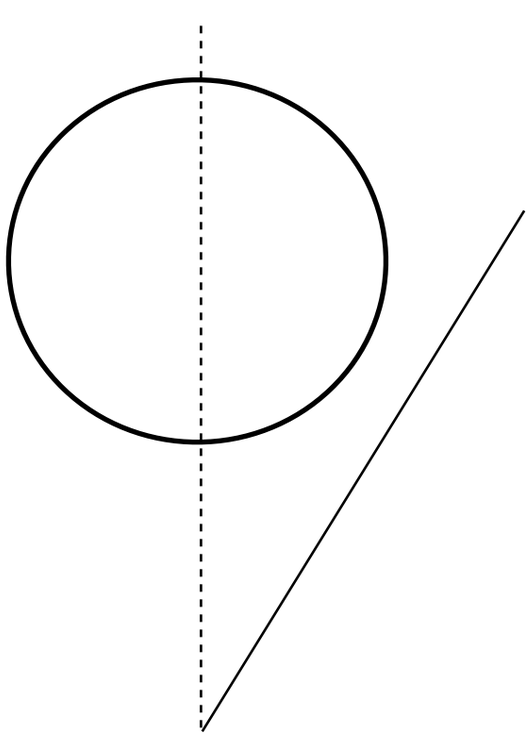
Some previous work on detection of caustics...

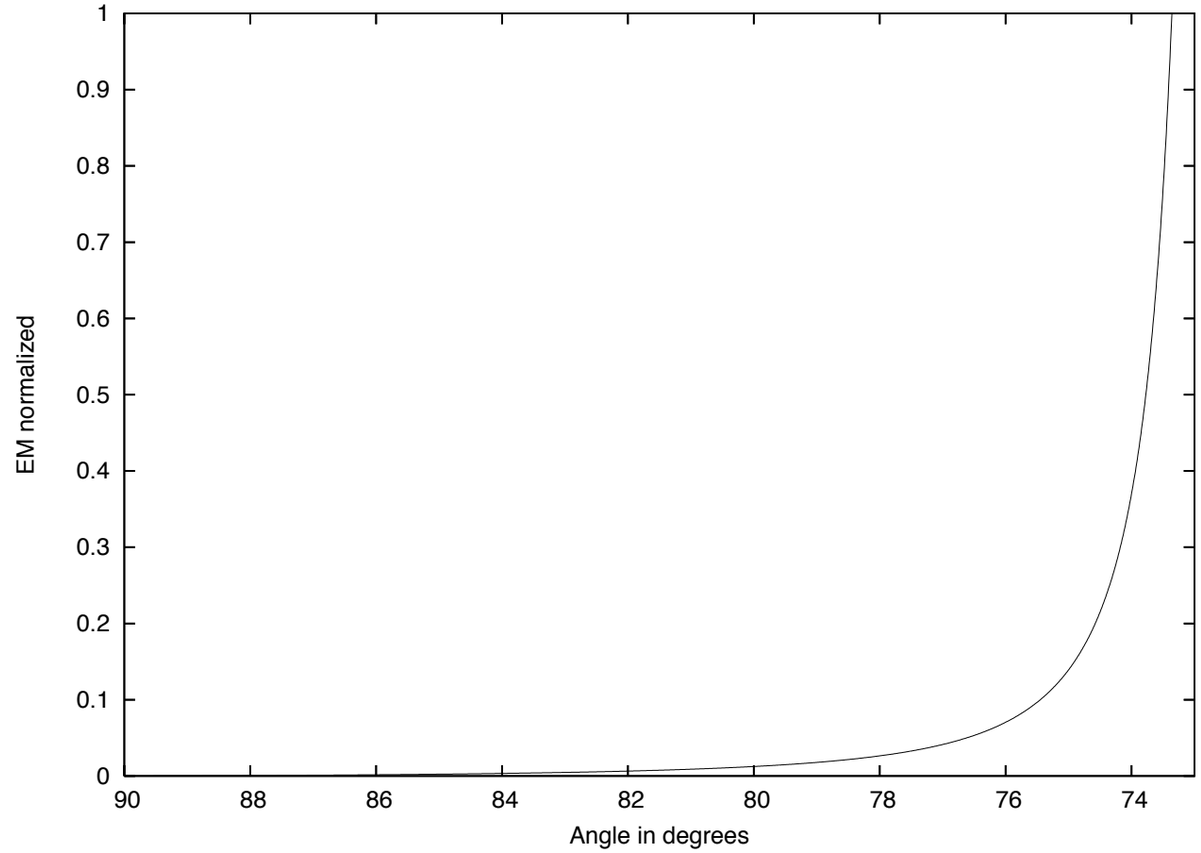
1. By particle annihilation *(Hogan, 2001)* :

   (a) In caustic rings.

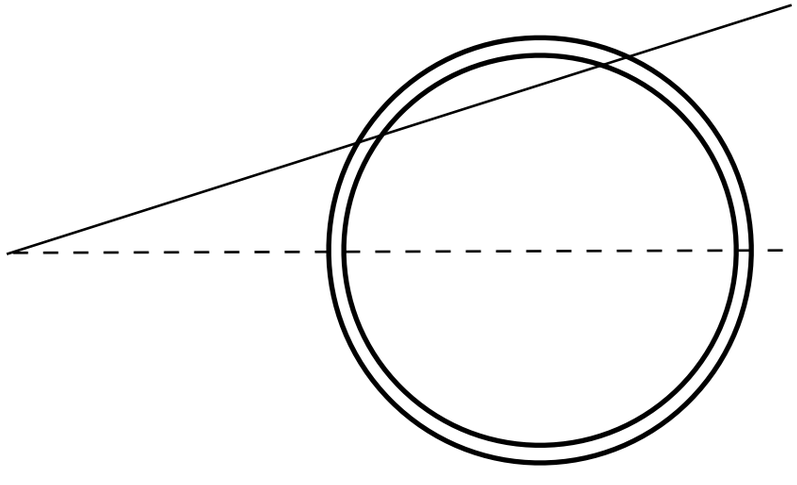
(Bergstrom et al, 2001; Pieri and Branchini, 2005)

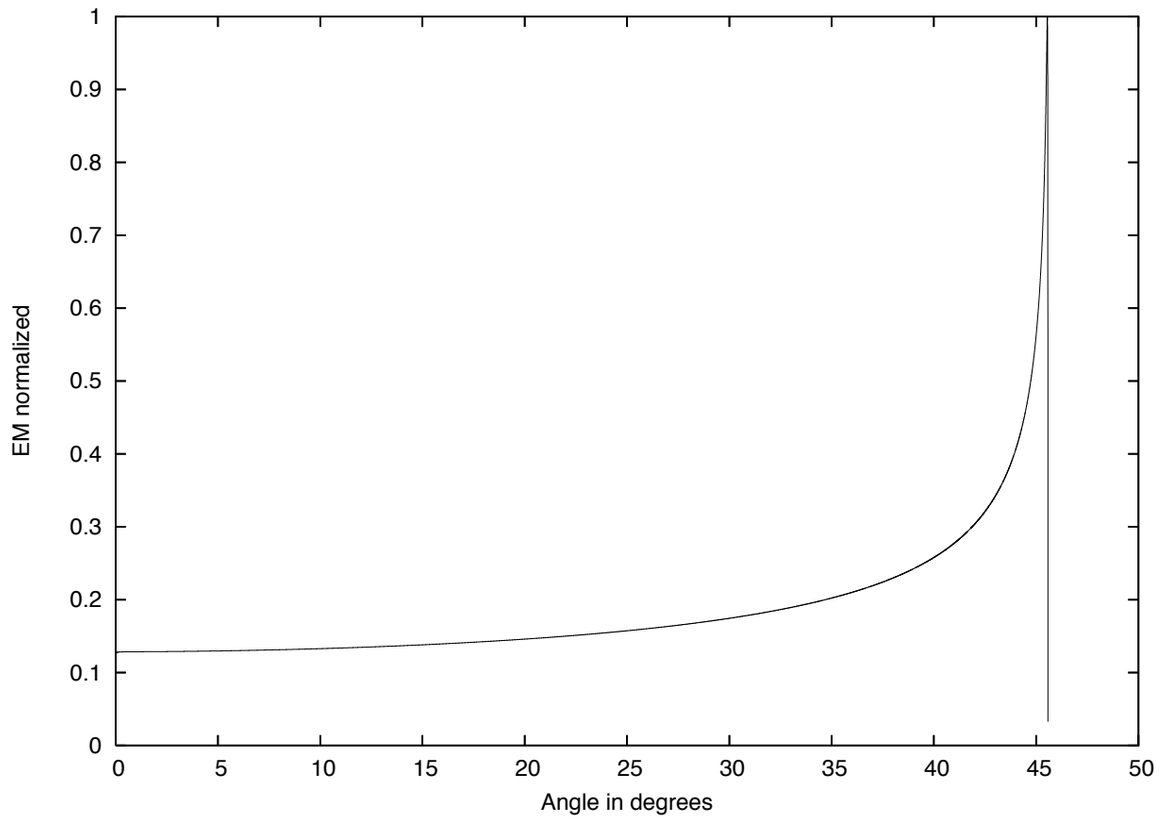
$$EM = \int_{los} \rho^2(\lambda) d\lambda$$



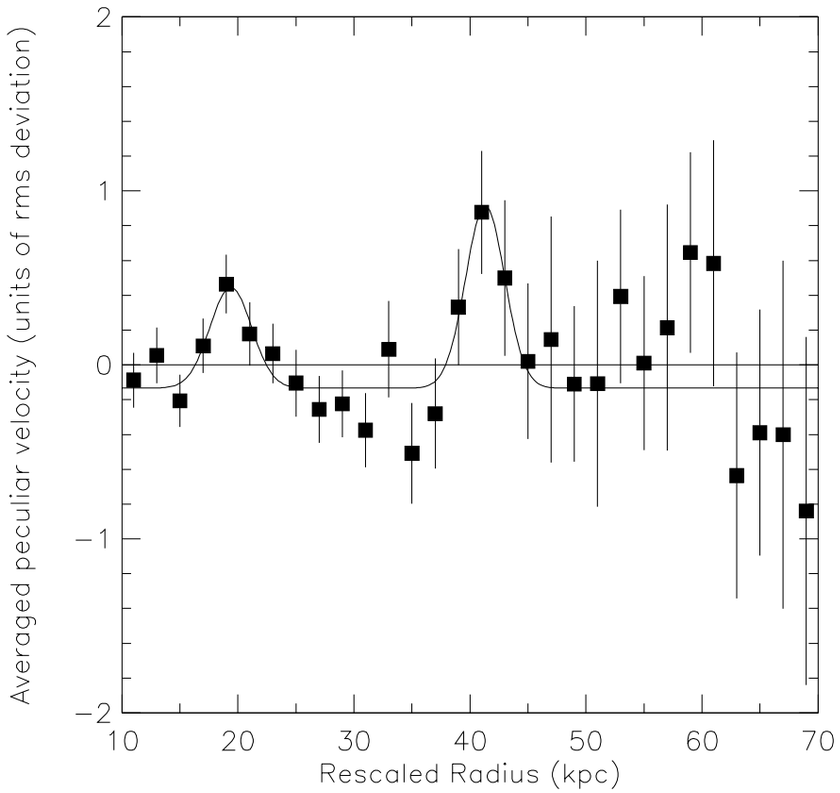


(b) In caustic spheres *(Mohayaee and Shandarin, 2005)*





3. Using rotation curve data *(Kinney and Sikivie, 2003).*



4. By gravitational lensing

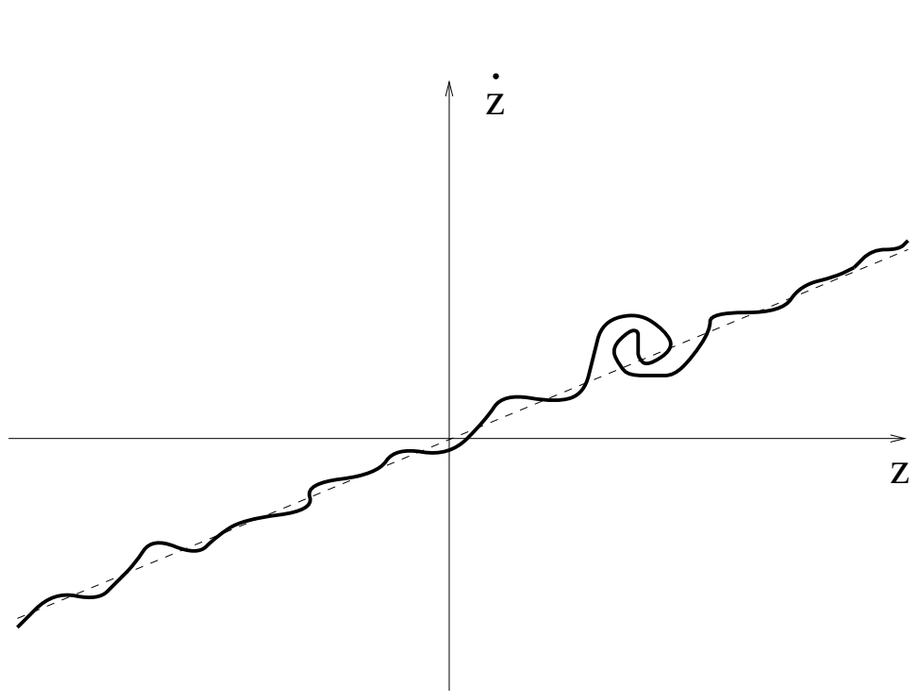
(Hogan, '99; Charmousis et al, 2003; Gavazzi et al, 2005)

Dark matter caustics are of two kinds :

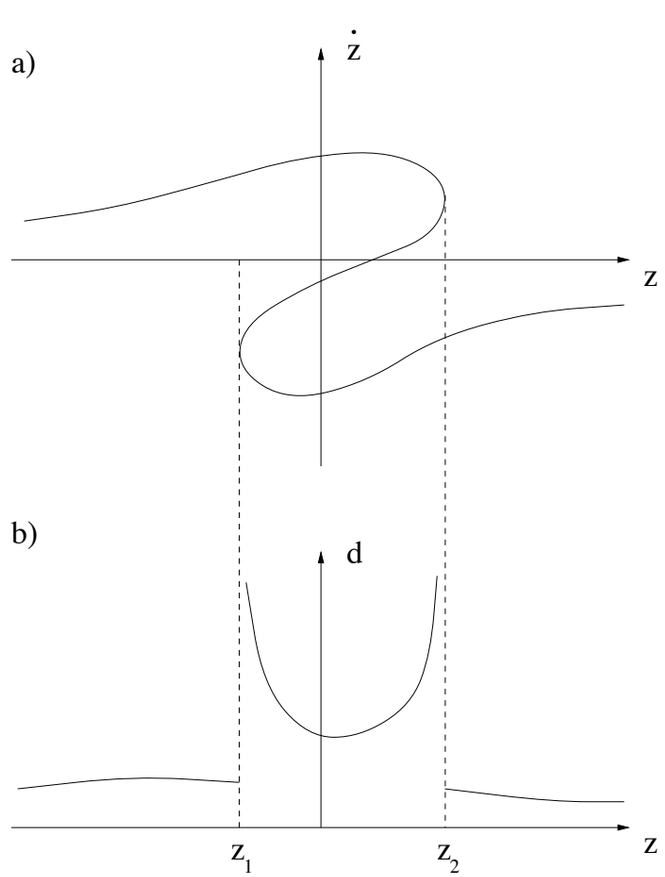
1. Outer Caustics :

(a) First few typically occur at scales of 100's of kpc.

(b) They are topological spheres surrounding galaxies.



(Sikivie '99)

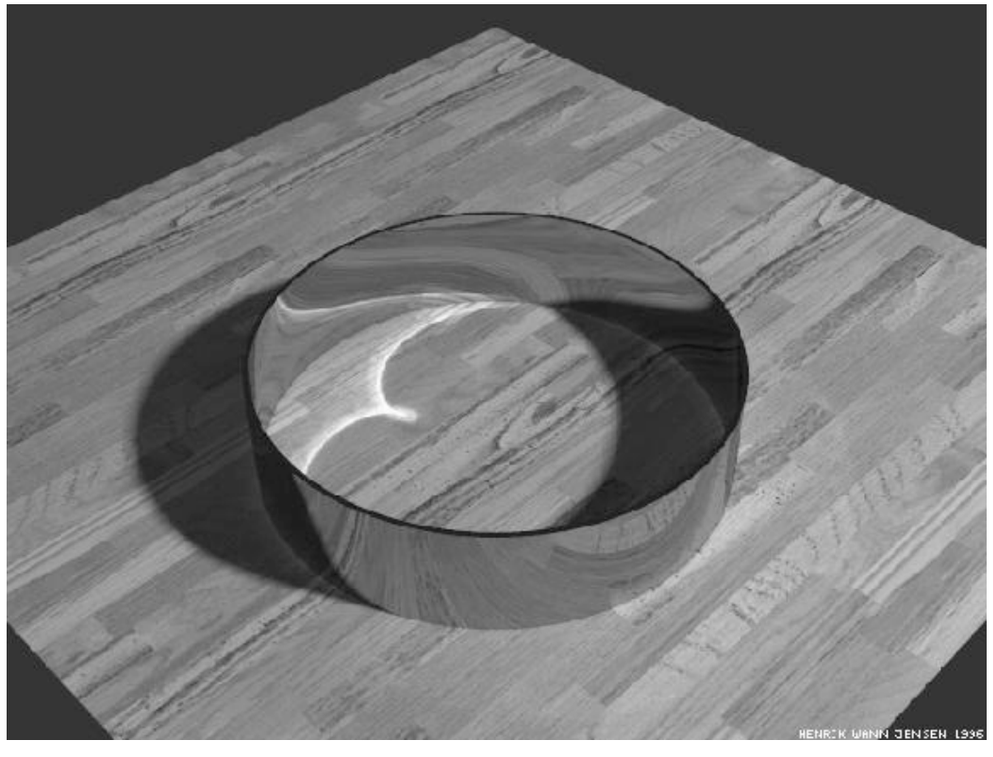


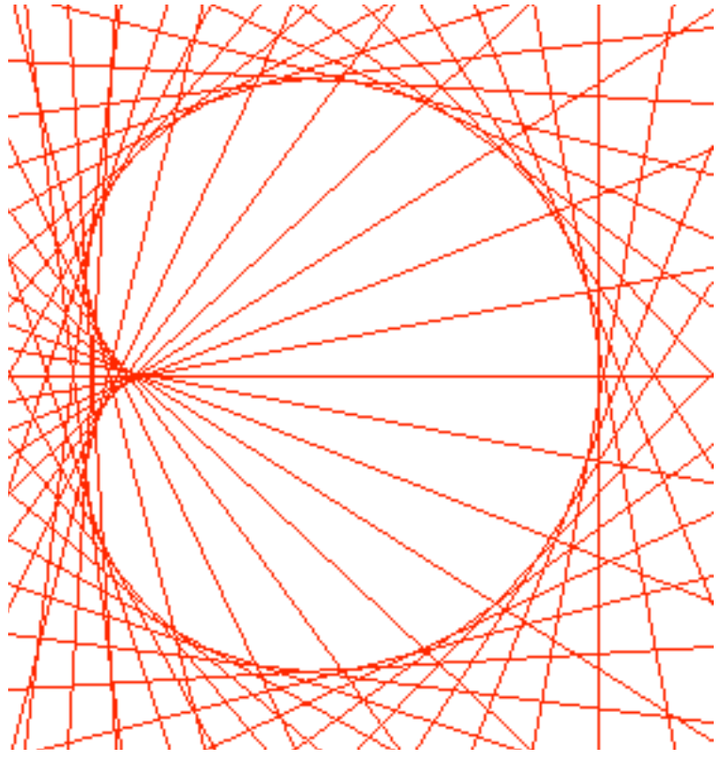
2. Inner Caustics :

(a) First few typically occur at 10's of kpc.

(b) The structure of inner caustics depends on the angular momentum distribution of the dark matter.

Let's first look at a simple example with light ...





(courtesy Henrik Jensen)

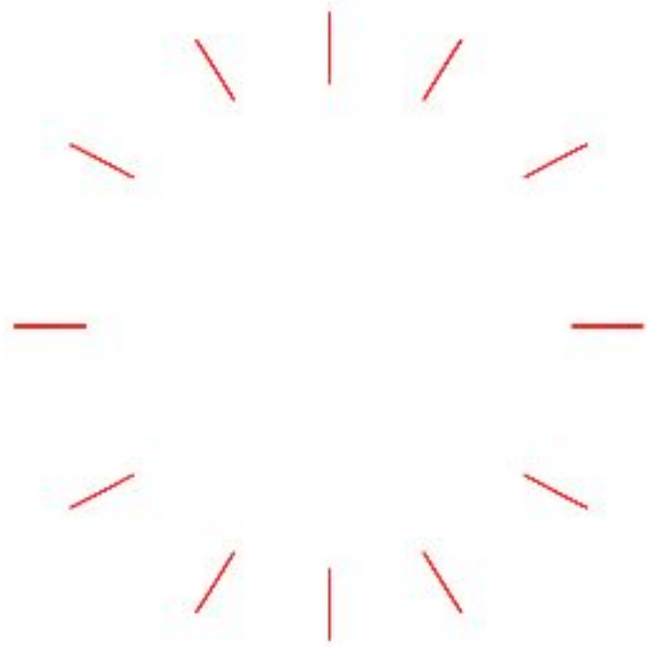
$$
\begin{aligned}
S &= \tan 2\theta - \frac{\sin\theta - y}{\cos\theta - x} = 0 \\
\frac{\partial S}{\partial \theta} &= 0
\end{aligned}
$$

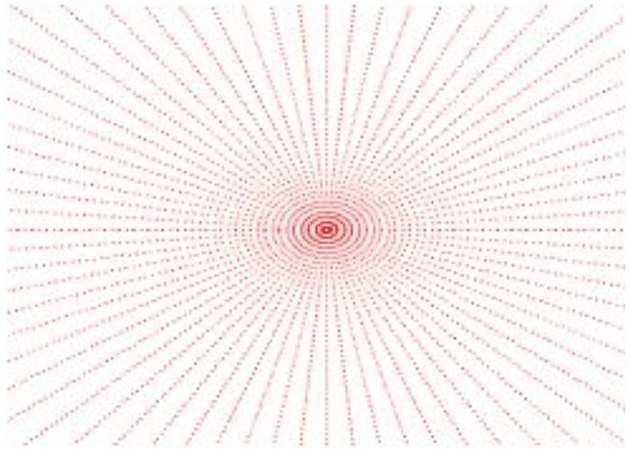
Catastrophes with light -

For a poster showing the geometry of the catastrophes, see

www.phy.bris.ac.uk/research/theory/Berry/pictures/poster1.pdf

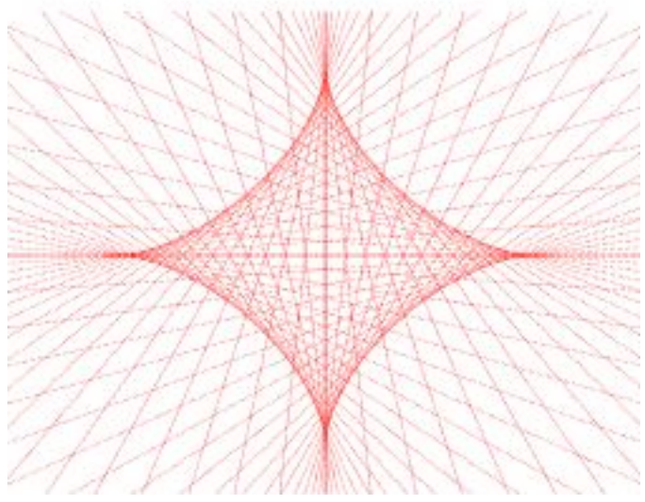
Now let's do the same thing with dark matter -





$$\begin{aligned} x &= x(t,\theta,\phi) \\ y &= y(t,\theta,\phi) \\ z &= z(t,\theta,\phi) \end{aligned}$$

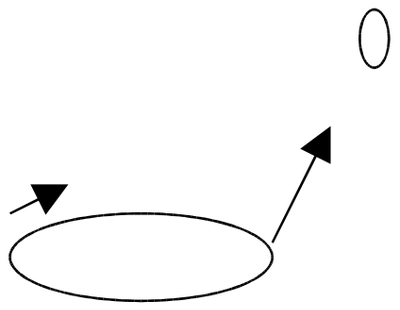
$$\mathsf{J} = \begin{vmatrix} \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial \phi} & \frac{\partial x}{\partial t} \\ \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial \phi} & \frac{\partial y}{\partial t} \\ \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial \phi} & \frac{\partial z}{\partial t} \end{vmatrix} = 0$$



- The caustic is the set of points (x,y,z) where J=0.

- The structure of the caustic depends on the initial conditions, i.e. on the spatial distribution of angular momentum.

The initial conditions : Tidal Torque theory -



$$\mathbf{r}(t,\mathbf{q}) = a(t)\mathbf{q} - D(t)\nabla\varphi(\mathbf{q})$$ *(Zeldovich)*

$$\varphi(\mathbf{q}) = \varphi(\bar{\mathbf{q}}) + (q_i - \bar{q}_i)\frac{\partial\varphi(\bar{\mathbf{q}})}{\partial q_i} + \frac{1}{2}(q_i - \bar{q}_i)\frac{\partial^2\varphi(\bar{\mathbf{q}})}{\partial q_i\,\partial q_j}(q_j - \bar{q}_j) + \dots$$

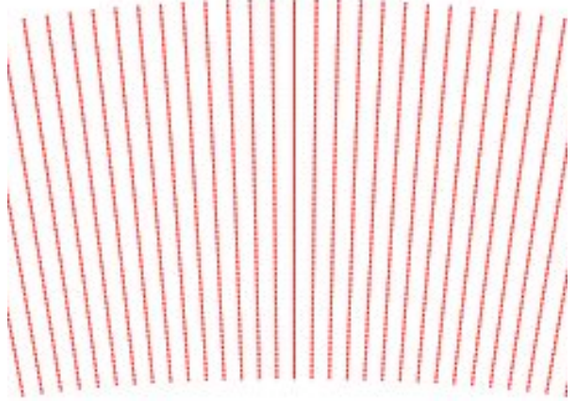
(White '84)

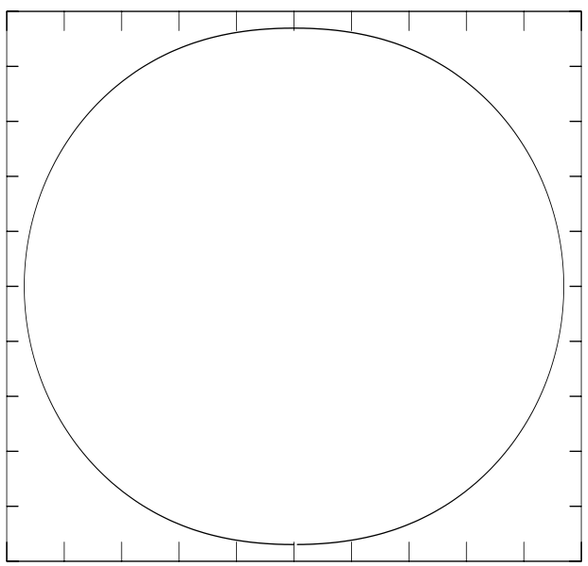
$$\vec{v}(r) = M\,\vec{r}$$

In general, $\mathsf{M} = \mathsf{S} + \mathsf{A} + \mathsf{T}$

For TTT, $M = M^T$

Infall of a shell ($M = M^T$)

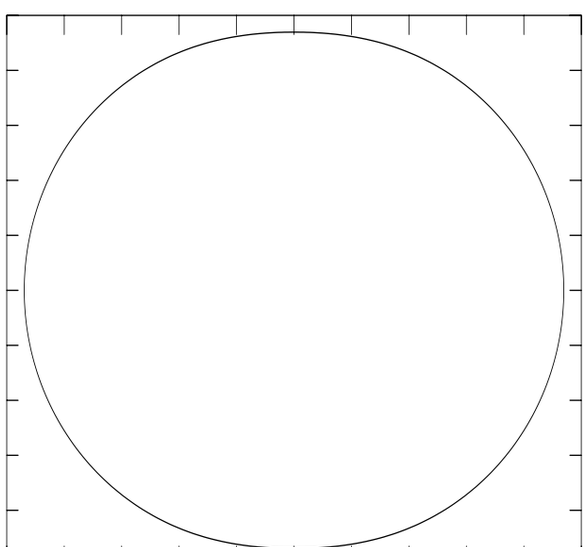


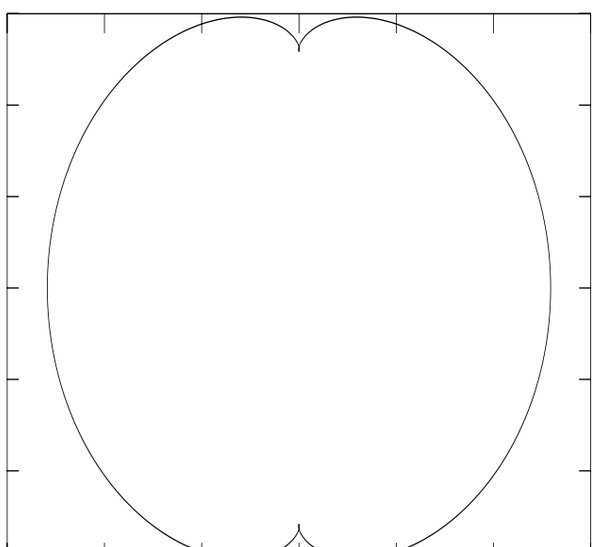


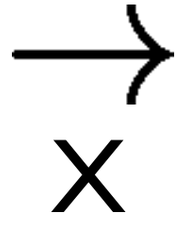
z

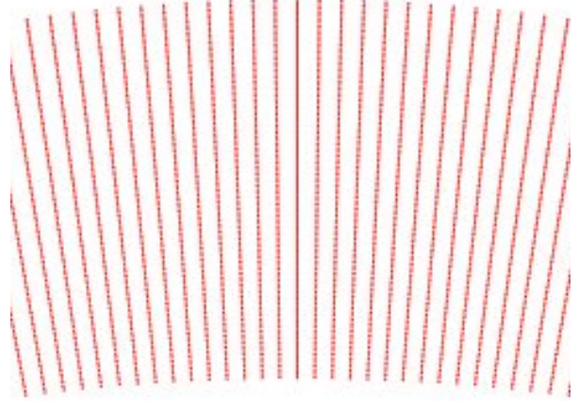
x

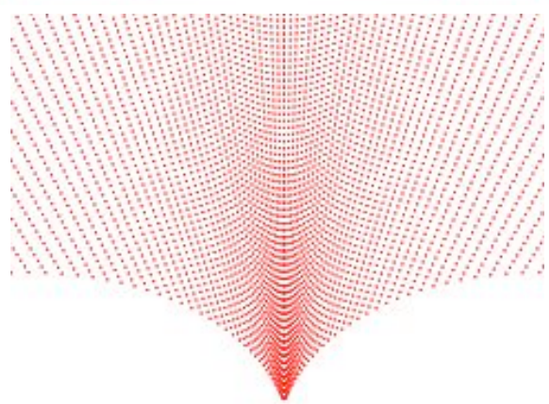
Infall of a shell ($M = M^T$)



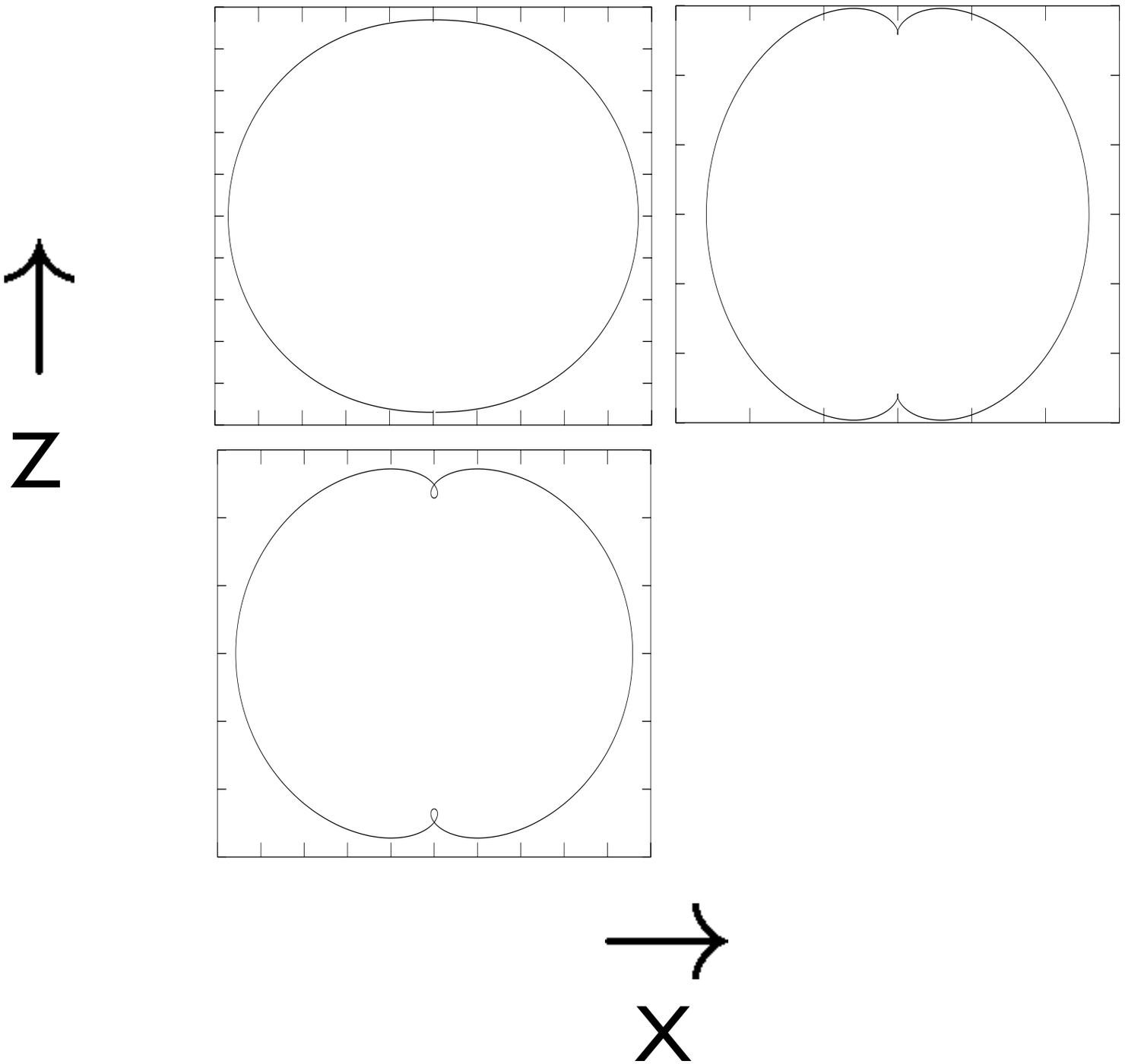


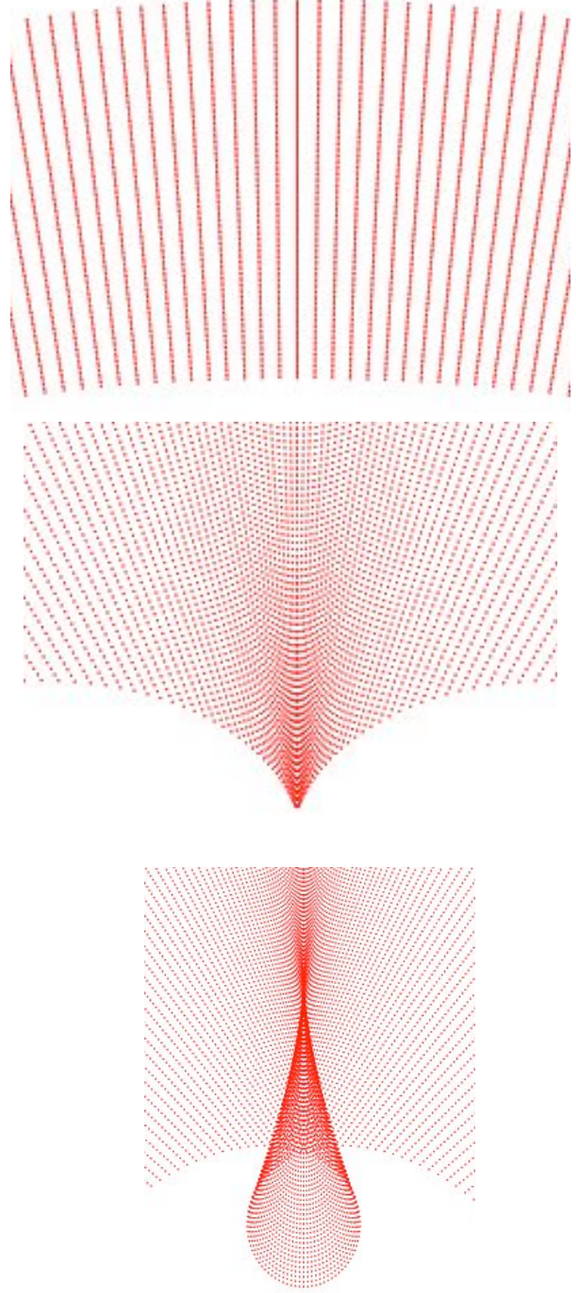




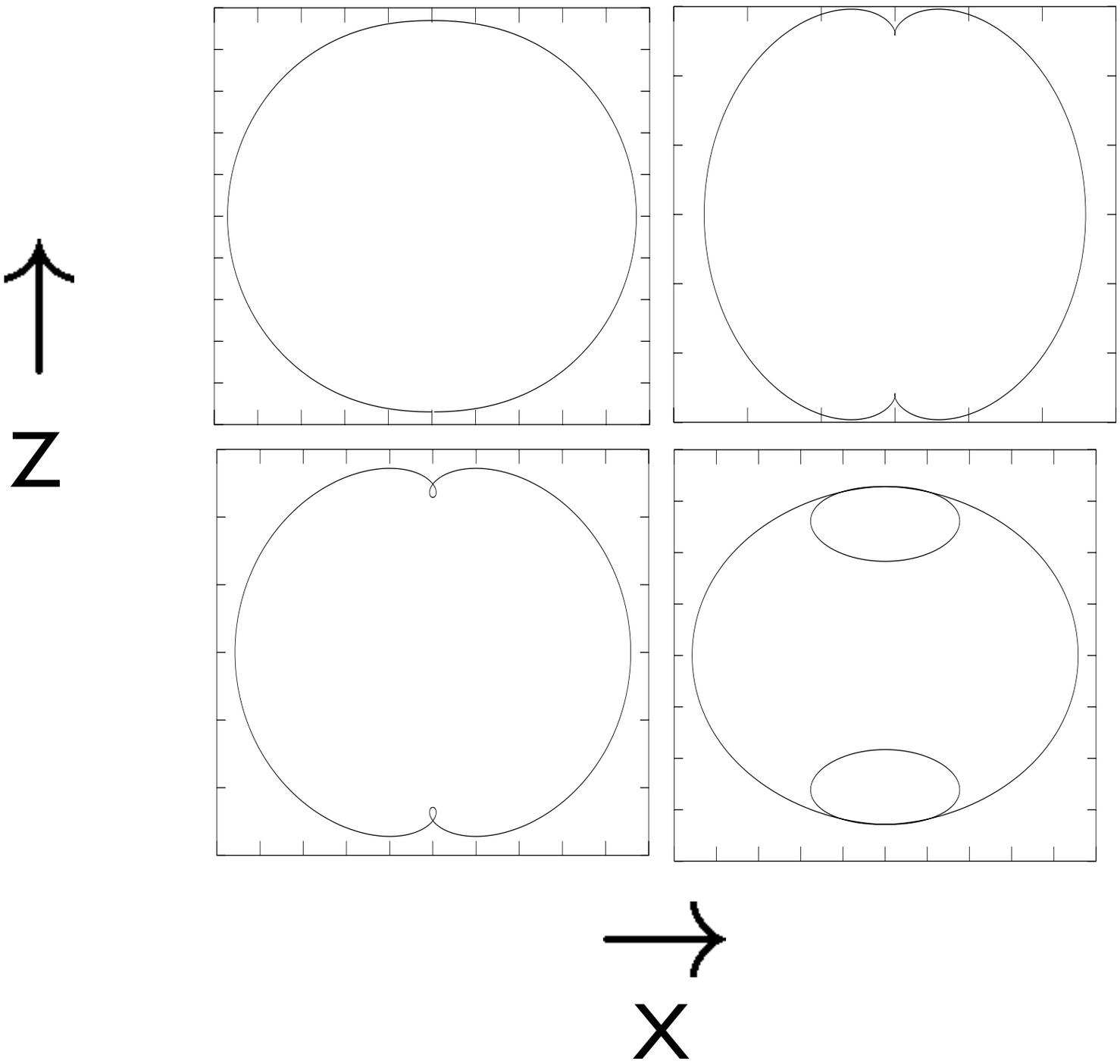


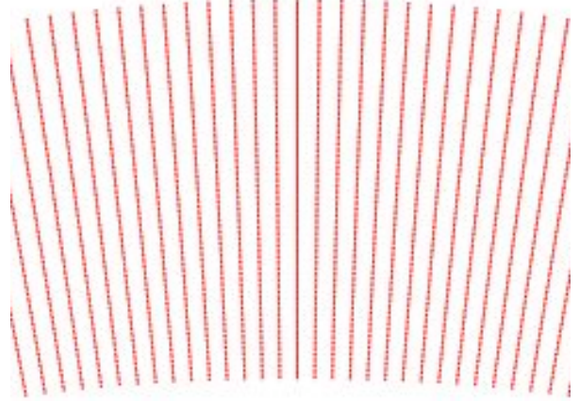
Infall of a shell ($M = M^T$)

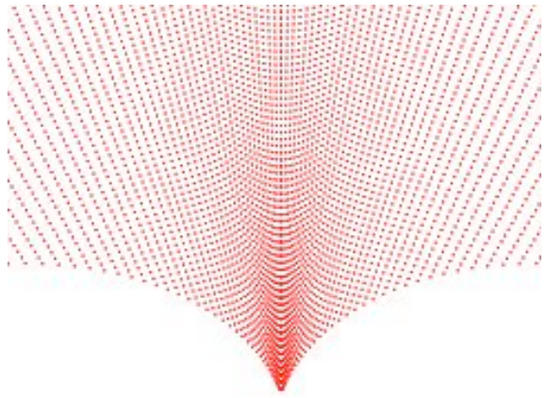


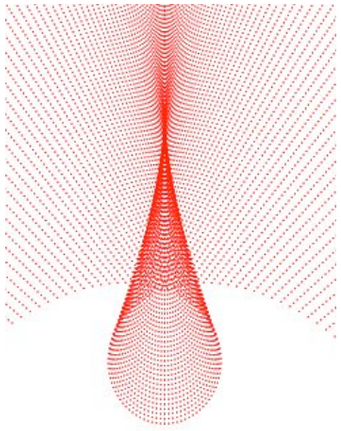


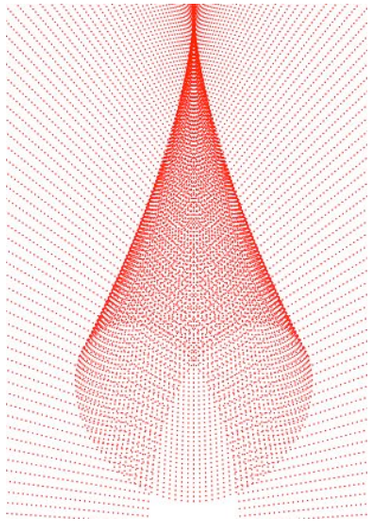
Infall of a shell ($M = M^T$)

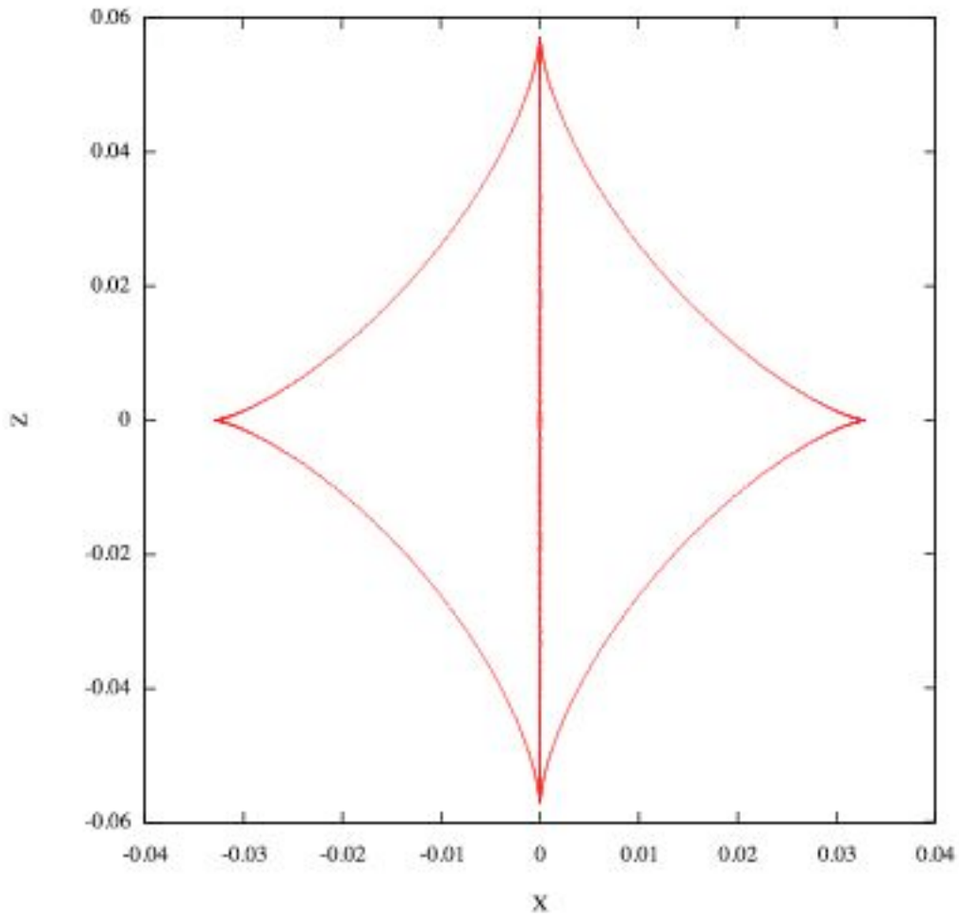




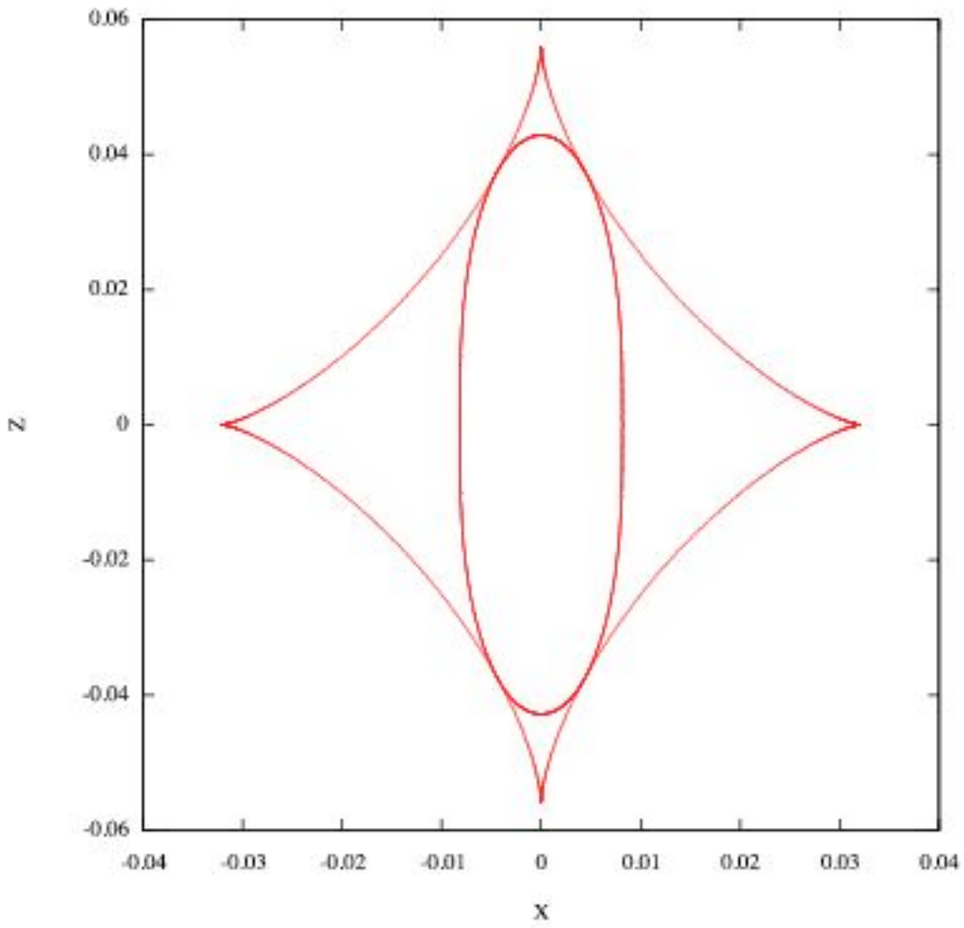




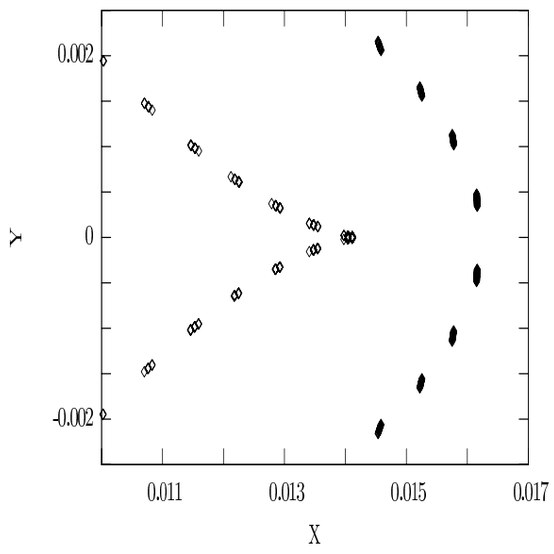


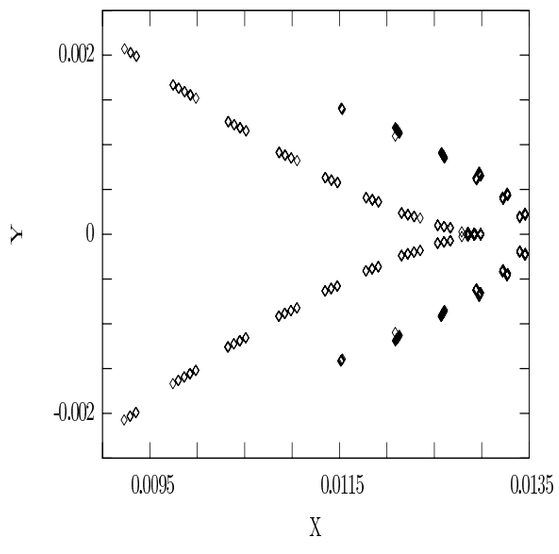


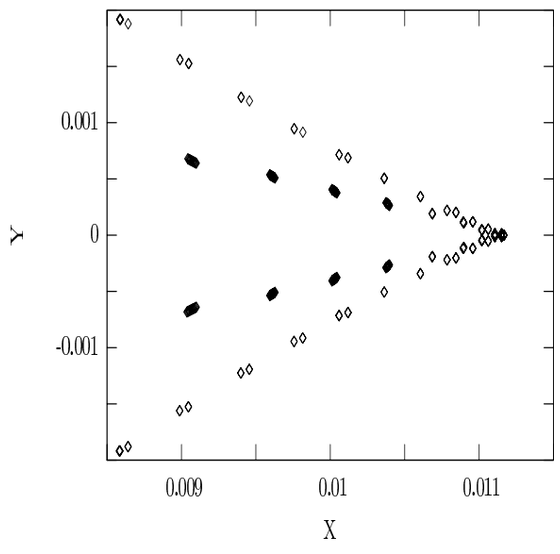
Perturbation

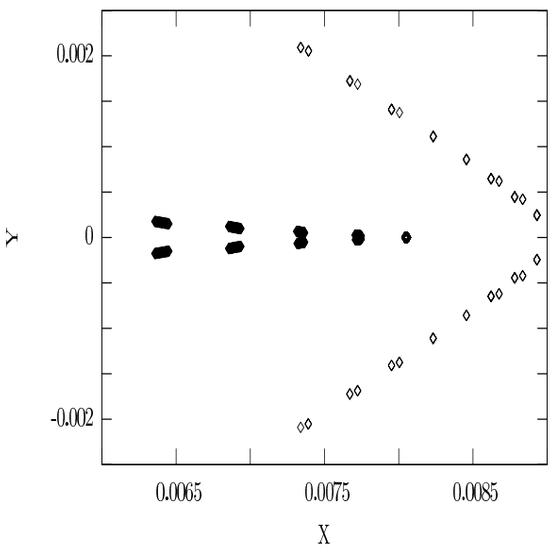


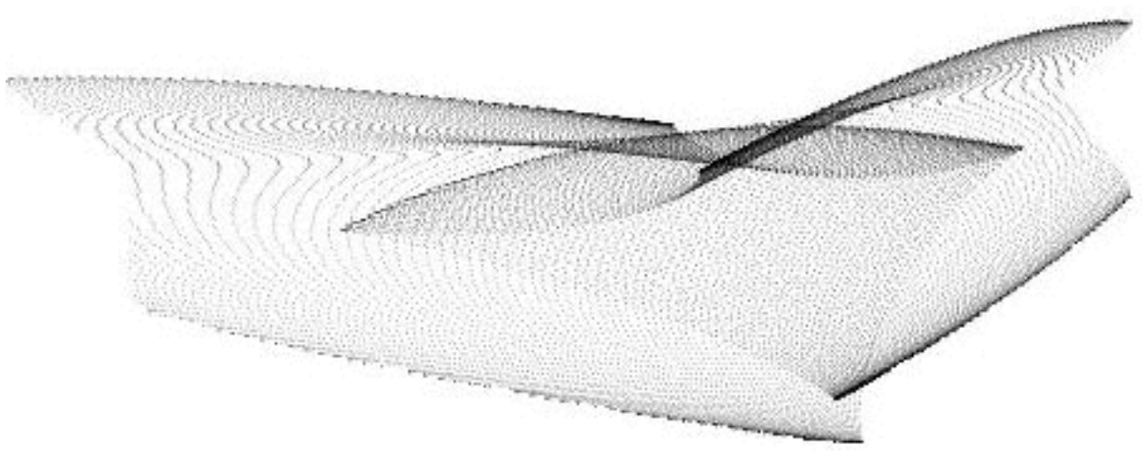
Different x-y cross sections of the caustic -

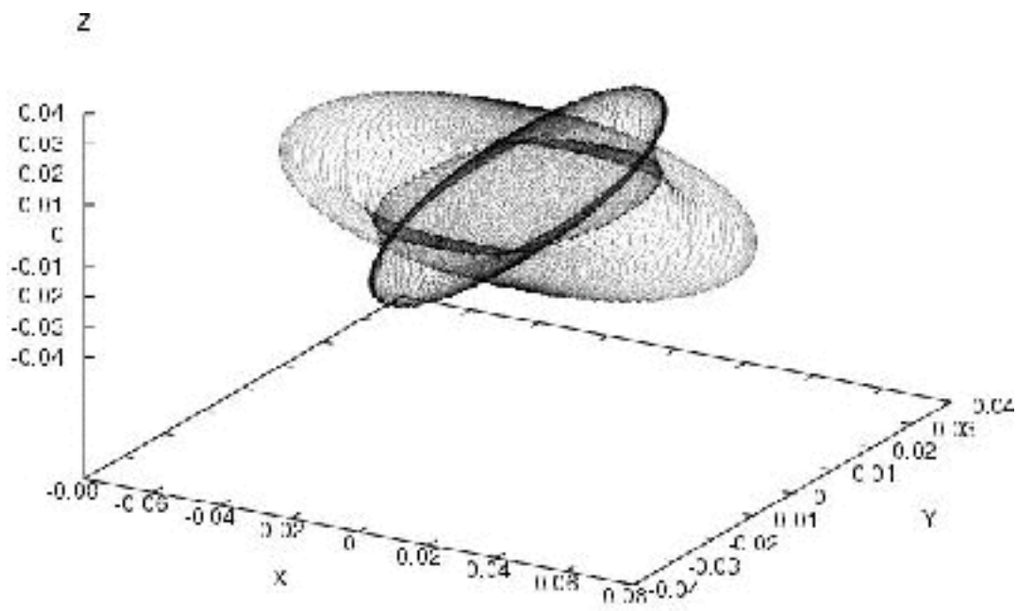






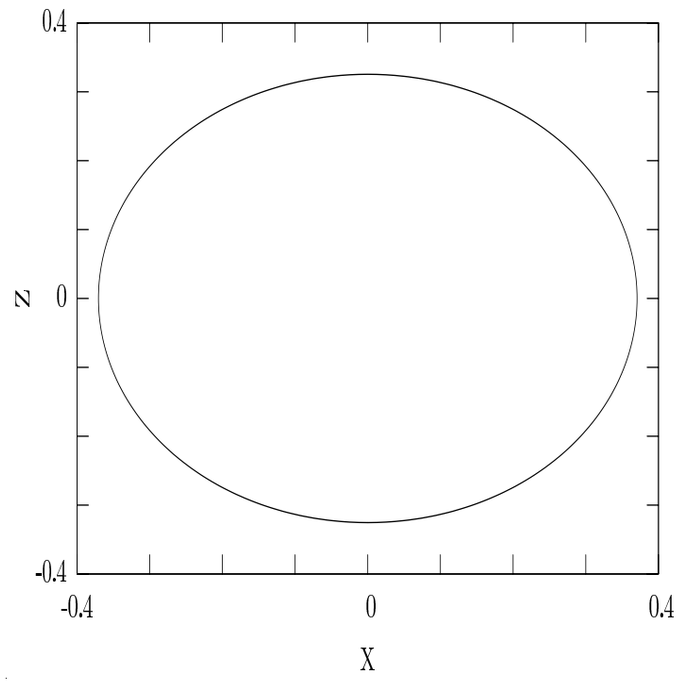


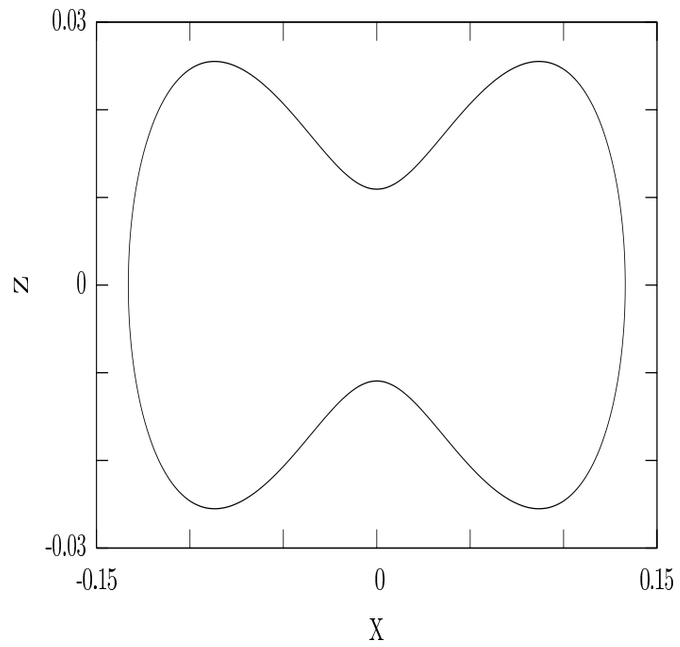


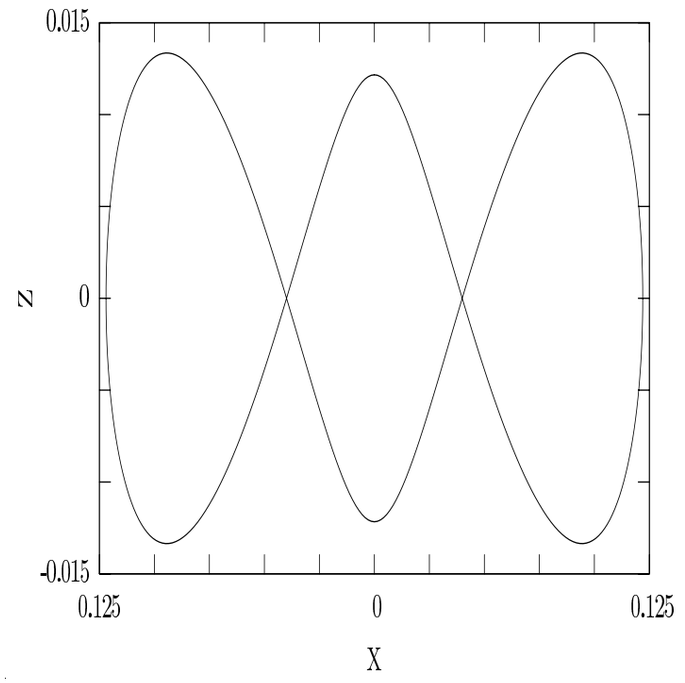


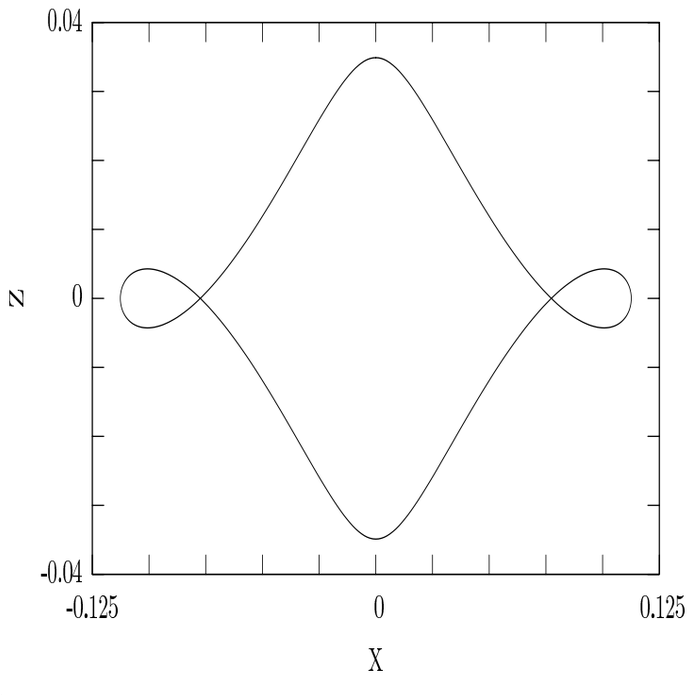
(hyperbolic umbilic catastrophe)

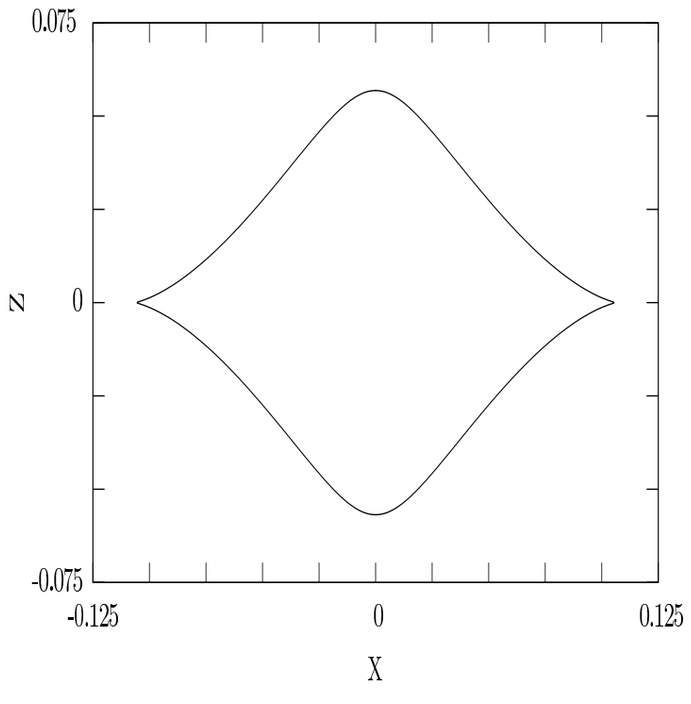
Infall of a shell : $\vec{v}(r) = M\,\vec{r}$ with $\mathsf{M} = -\mathsf{M}^{T}$

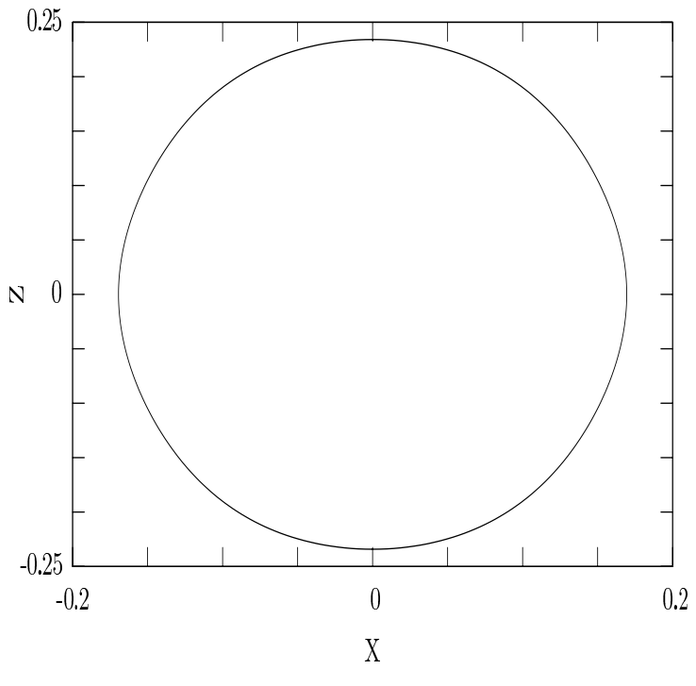


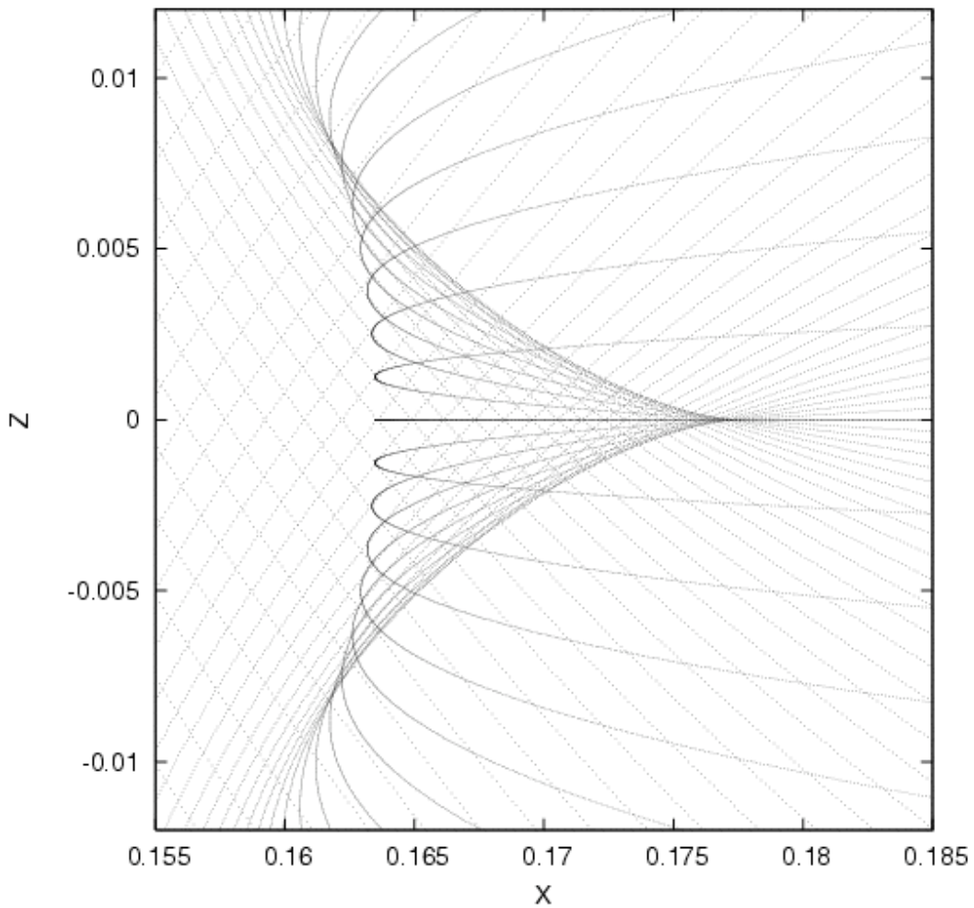




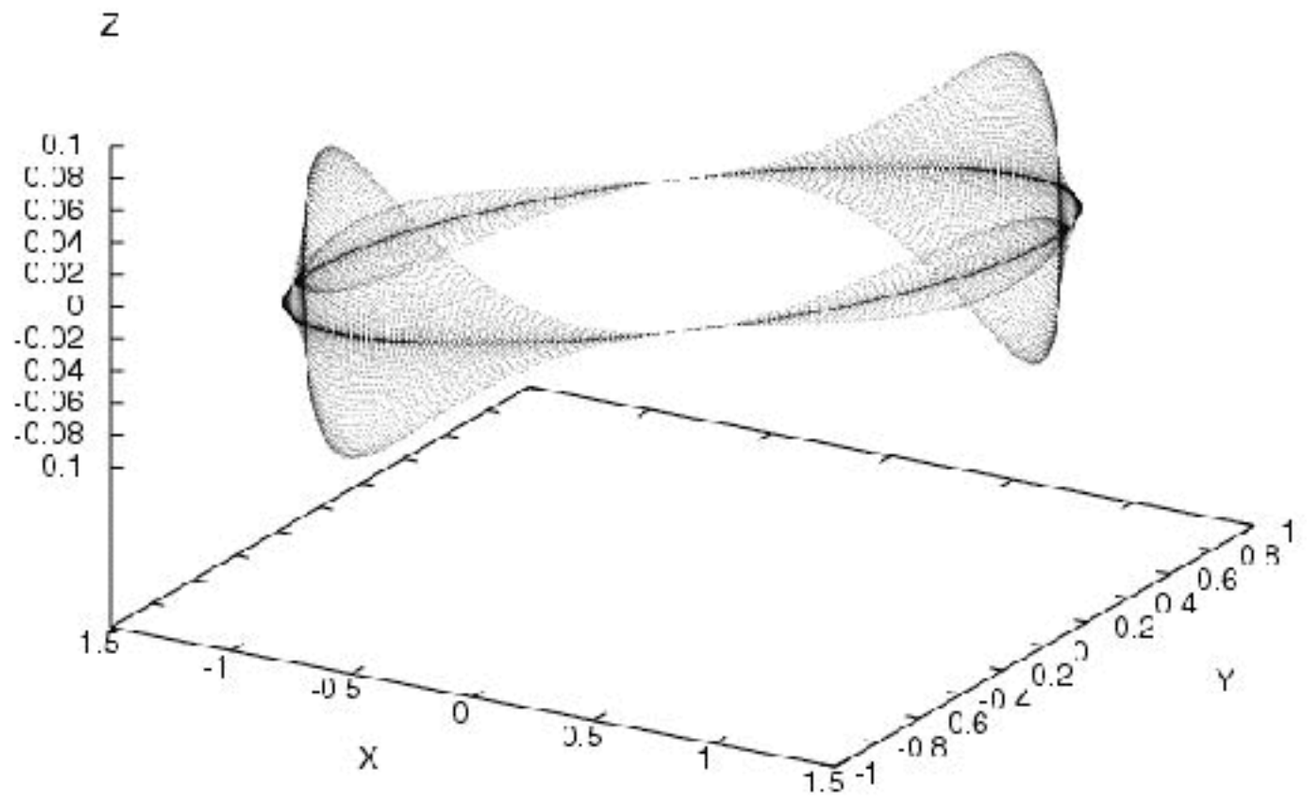








(elliptic umbilic catastrophe)



Conclusions:

- The continuous infall of dark matter forms caustics under two assumptions :

  1. The dark matter is collisionless.

  2. It has low velocity dispersion.

- Outer caustics are spherical shells surrounding galaxies. Inner caustics are made up of sections of the higher order catastrophes.

- The structure of inner caustics depends on the dark matter angular momentum.