A Computer Program for Predicting Recharge with a Master Recession Curve

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Abstract

Water-table fluctuations occur in unconfined aquifers owing to ground-water recharge following precipitation and infiltration, and ground-water discharge to streams between storm events. Ground-water recharge can be estimated from well hydrograph data using the water-table fluctuation (WTF) principle, which states that recharge is equal to the product of the water-table rise and the specific yield of the subsurface porous medium. The water-table rise, however, must be expressed relative to the water level that would have occurred in the absence of recharge. This requires a means for estimating the recession pattern of the water-table at the site. For a given site there is often a characteristic relation between the water-table elevation and the water-table decline rate following a recharge event. A computer program was written which extracts the relation between decline rate and water-table elevation from well hydrograph data and uses it to construct a master recession curve (MRC). The MRC is a characteristic water-table recession hydrograph, representing the average behavior for a declining water-table at that site. The program then calculates recharge using the WTF method by comparing the measured well hydrograph with the hydrograph predicted by the MRC and multiplying the difference at each time step by the specific yield. This approach can be used to estimate recharge in a continuous fashion from long-term well records. Presented here is a description of the code including the WTF theory and instructions for running it to estimate recharge with continuous well hydrograph data.

Introduction

In unconfined aquifers a number of factors cause the fluctuation of the water-table, primarily the addition of water to the saturated zone from above (recharge) and the release of water to surface water bodies downstream (discharge). The water-table fluctuations are often recorded in wells, providing a useful set of data with which to estimate ground-water recharge rates. The principle upon which the recharge estimate is based is known as the water-table fluctuation (WTF) principle (Sophocleous, 1991; Healy and Cook, 2002), which states that recharge, R [L], is the product of water-table rise, ΔZ_{WT} [L], and specific yield, Y_s [-]:

$$R = \Delta Z_{WT} * Y_s \qquad [1]$$

For the purposes of this report specific yield is considered synonymous with “fillable porosity”, which is a more accurate name for the quantity that appears in [1].

One difficulty that arises when applying the WTF method of recharge estimation is that the term ΔZ_{WT} can be difficult to estimate because water-tables typically are in a transient state of decline between storm or recharge events. Therefore the true ΔZ_{WT} for a given recharge event is greater than simply the difference between the low and high stages of the event. The issue is analogous to the problem of baseflow / stormflow separation in gauged streams. A solution to this problem is found in the use of a master recession curve (MRC) which is a characteristic water-table decline hydrograph. Once the MRC is determined for a certain site it can be used to predict what the water-table elevation should be in the absence of recharge. The difference between the predicted elevation and the observed (measured) elevation is ΔZ_{WT}.

The MRC often takes the form of a linear relation between water-table elevation and water-table decline rate. The resulting water-table recession hydrograph has the familiar exponential decline shape. Hantush (1967) presented analytical solutions to the unconfined ground-water flow equations for cases of circular and rectangular surface recharge patterns, which show the aforementioned behavior. This typical recession behavior, faster decline rates when the water-table is higher, is caused by the larger hydraulic head gradient between the ground-water mound and the discharge point during high stands of the water-table. The strength of this correlation, and thus the validity of the MRC, depends on many factors including the local relief and topography, seasonality of the climate, hydraulic properties of the subsurface porous medium, and depth to the water-table.

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The purpose of this report is to present a computer program, MRCR (Master Recession Curve Recharge), which allows one to estimate recharge from well hydrograph data using the WTF principle. The code uses a master recession curve (MRC), either specified by the user or extracted from the measured data, to predict what the water-table elevation would be at the succeeding time step in the absence of recharge. The predicted water-table elevation is compared to the measured water-table elevation and the difference between these two values is multiplied by the specific yield to get recharge for that time step.

The assumptions inherent to this approach are: 1) there exists a characteristic functional relation between water-table elevation and water-table decline rate in the absence of recharge (the basis for the MRC); 2) the measured well hydrograph data depict only natural water-table fluctuations caused by ground-water recharge and discharge; and 3) specific yield is known and constant over the interval of water-table fluctuations.

The MRCR code was written using the commercially-available Matlab programming environment and therefore requires the user to have Matlab installed on his/her computer in order to run the program. The program is meant to be easy to use, with minimal data requirements and flexible parameterization options.

Methods

To illustrate how the program MRCR calculates recharge from measured data an example problem based on a hypothetical 30-day well hydrograph is presented below. In this example the MRC is created using several different methods, all of which are options available to the user.

The 30-day hydrograph data are in table 1. The user is required to supply time, *t* [T], and water level, Z_{WT} [L], data (the first two columns in Table 1) as input to the program. The required input is in the form of a tabulated text file (refer to Observed Data File section, below, for more information). From these data the average time, t´ [T], average elevation, $Z_{WT}´$ [L], and the water-table fluctuation rate, dZ_{WT}/dt [L/T], are calculated with each pair of successive points in the time series. The points with negative fluctuation rates (from periods during which the water-table is declining) are extracted and constitute the basis for the MRC. Figure 1 shows the measured hydrograph and the water table fluctuation rate for the hypothetical period.

The next step taken depends on which type of MRC is desired. The four MRC types are linear, power, bin-averaged, and user-defined tabulated. The decision to use a particular MRC type should be based on an examination of the extracted data (see, for example, fig. 2) and knowledge of the site. If the data are sparse but exhibit a clear pattern then one of the functional forms may be best. If there are many data but they have an irregular pattern when plotted then the bin-averaged type may be best. The program also allows for a minimum and maximum decline rate to be specified. Regardless of the MRC type used, all decline rates will fall between these two values.

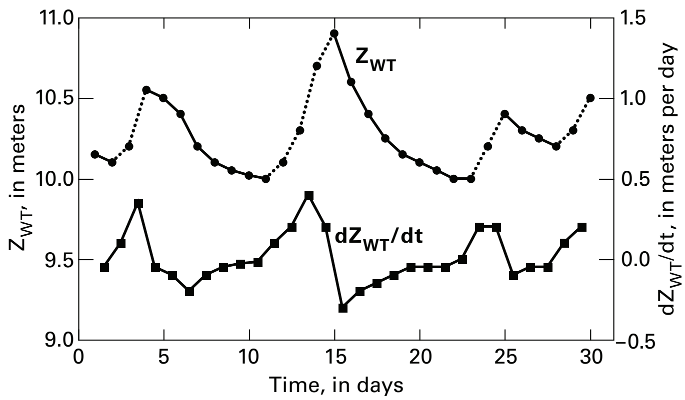

Figure 1. Example problem: water-table elevation and water-table fluctuation rate versus time. Periods of water table decline are shown in solid lines.

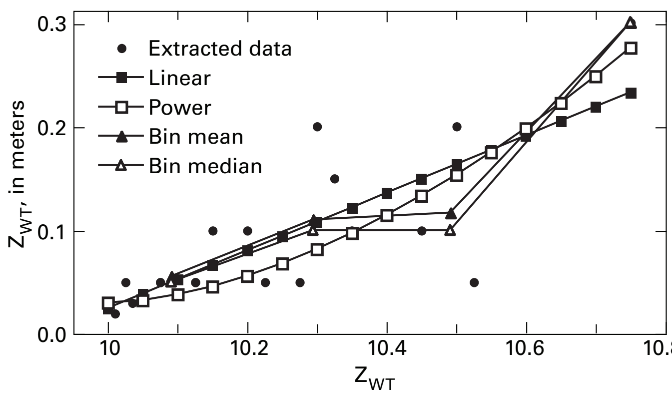

Figure 2. Water-table decline rate versus water-table elevation (extracted data) and four MRCs.

MRC Types:

Linear

The linear function option allows the user to specify a linear relation between water-table elevation and decline rate, according to the following equation:

$$Decline\ rate = dZ_{WT}/dt = aZ_{WT} + b \qquad [2]$$

where *a* is the slope [T $^{-1}$] and *b* is the intercept on the decline rate axis [L/T]. The user is responsible for choosing coefficients, *a* and *b*, that give the best fit to the extracted data.

Power

The power function option allows the user to specify a power-type relation between water-table elevation and decline rate, according to the following equation:

$$Decline\ rate = dZ_{WT}/dt = -(c + d(Z_{WT} - e)^{f}) \qquad [3]$$

where *c* is the intercept on the decline rate axis [L/T], *d* is a multiplier of elevation [–], *e* is an elevation modifier (e.g., a

Table 1. Example problem: user-supplied, model calculated and extracted data.

User-supplied Data		Model Calculated			Extracted for MRC
Elapsed time	Water-table elevation	Average elapsed time	Average water-table elevation	Water-table fluctuation rate	Points during water-table decline
t	Z_{WT}	t'	$Z_{WT'}$	dZ_{WT}/dt	dZ_{WT}/dt
(days)	(m)	(days)	(m)	(m/day)	(m/day)
1	10.15	—	—	—	—
2	10.1	1.5	10.125	–0.05	–0.05
3	10.2	2.5	10.15	0.1	—
4	10.55	3.5	10.375	0.35	—
5	10.5	4.5	10.525	–0.05	–0.05
6	10.4	5.5	10.45	–0.1	–0.1
7	10.2	6.5	10.3	–0.2	–0.2
8	10.1	7.5	10.15	–0.1	–0.1
9	10.05	8.5	10.075	–0.05	–0.05
10	10.02	9.5	10.035	–0.03	–0.03
11	10	10.5	10.01	–0.02	–0.02
12	10.1	11.5	10.05	0.1	—
13	10.3	12.5	10.2	0.2	—
14	10.7	13.5	10.5	0.4	—
15	10.9	14.5	10.8	0.2	—
16	10.6	15.5	10.75	–0.3	–0.3
17	10.4	16.5	10.5	–0.2	–0.2
18	10.25	17.5	10.325	–0.15	–0.15
19	10.15	18.5	10.2	–0.1	–0.1
20	10.1	19.5	10.125	–0.05	–0.05
21	10.05	20.5	10.075	–0.05	–0.05
22	10	21.5	10.025	–0.05	–0.05
23	10	22.5	10	0	—
24	10.2	23.5	10.1	0.2	—
25	10.4	24.5	10.3	0.2	—
26	10.3	25.5	10.35	–0.1	–0.1
27	10.25	26.5	10.275	–0.05	–0.05
28	10.2	27.5	10.225	–0.05	–0.05
29	10.3	28.5	10.25	0.1	—
30	10.5	29.5	10.4	0.2	—

datum) with units of elevation [L], and *f* is the exponent. The user chooses values of the coefficients, *c, d, e,* and *f* that give the best fit to the extracted data.

Bin-Averaged

If the bin-averaged MRC type is chosen, the user specifies the number of elevation bins into which the data should be split. A bin is simply a range of elevation. For this example problem we have specified five bins. The program then generates equally-spaced elevation bins ranging from the lowest to highest observed water-table elevations. The extracted data from points during water level decline are then tallied in their respective elevation bins. The mean (arithmetic average) and median decline rates for points in each bin (table 2) and the mean elevation for points in each bin (table 3) are calculated. The user has the option to use the mean or median decline

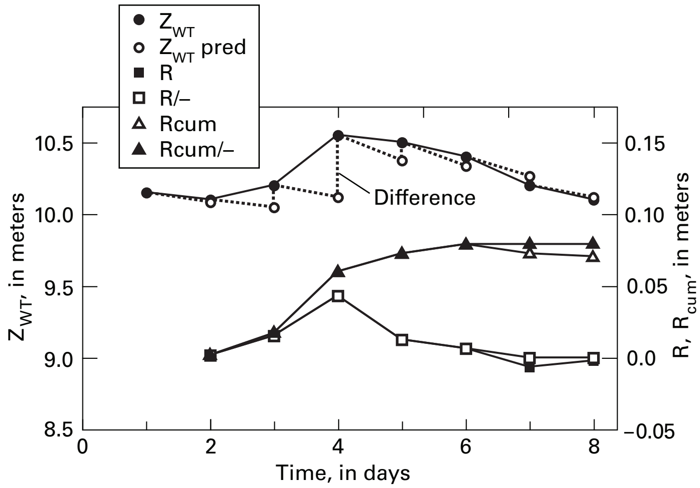

Figure 3. Example problem: recharge calculation for the first 8 days.

rate when using the bin-averaged MRC type. The MRC is then constructed using the bin-averaged elevation and decline rate. During the recharge calculation the program interpolates between these average points to calculate the decline rate.

User-Defined Tabulated

The user-defined tabulated MRC option allows the user to specify the name of a file containing two columns: the left column is the elevation, and the right column is the decline rate. These tabulated data are read in and used as the MRC. This option is useful if the user has already created a MRC through other means, or if one wants to use an MRC from a previous bin-averaged calculation (i.e., the format of the input file is the same as the format of the output file from the bin-averaged MRC type option).

Figure 2 shows the extracted decline rate versus elevation data points along with MRCs created with each method (linear, power function, bin-averaged mean, and bin-averaged median). It is clear that when using the bin-averaged method it is desirable to have as many data points as possible (i.e., a long observed record) and to split them into the maximum number of bins possible that will still allow a meaningful average to be computed within each bin.

Table 2. Example problem: decline rates (m/day) for points in each elevation bin

Bin 10–10.2 m	10.21–10.4 m	10.41–10.6 m	10.61–10.8 m	10.81–11 m
0.05	0	0	0	0
0	0	0.05	0	0
0	0	0.1	0	0
0	0.2	0	0	0
0.1	0	0	0	0
0.05	0	0	0	0
0.03	0	0	0	0
0.02	0	0	0	0
0	0	0	0.3	0
0	0	0.2	0	0
0	0.15	0	0	0
0.1	0	0	0	0
0.05	0	0	0	0
0.05	0	0	0	0
0.05	0	0	0	0
0	0.1	0	0	0
0	0.05	0	0	0
0	0.05	0	0	0
Average decline rate (m/day)				
0.056	0.11	0.117	0.3	—
Median decline rate (m/day)				
0.05	0.1	0.1	0.3	—

Table 3. Example problem: elevations (m) for points in each elevation bin.

Bin 10–10.2m	10.21–10.4m	10.41–10.6m	10.61–10.8m	10.81–11m
10.125	0	0	0	0
0	0	10.525	0	0
0	0	10.45	0	0
0	10.3	0	0	0
10.15	0	0	0	0
10.075	0	0	0	0
10.035	0	0	0	0
10.01	0	0	0	0
0	0	0	10.75	0
0	0	10.5	0	0
0	10.325	0	0	0
10.2	0	0	0	0
10.125	0	0	0	0
10.075	0	0	0	0
10.025	0	0	0	0
0	10.35	0	0	0
0	10.275	0	0	0
0	10.225	0	0	0
Average elevation				
10.091	10.295	10.492	10.75	–

Table 4. Example problem: recharge computations.

Time	Water-table elevation	Predicted water-table elevation	Difference	Recharge	Recharge with no negatives	Cumulative Recharge	Cumulative Recharge with no negatives
t	Z_{WT}	Z_{WT} pred	Z_{WT}-Z_{WT} pred	R	R/-	R_{cum}	R_{cum}/-
(days)	(m)	(m)	(m)	(m)	(m)	(m)	(m)
1	10.15						
2	10.1	10.0836	0.0164	0.0016	0.0016	0.0016	0.0016
3	10.2	10.0474	0.1526	0.0153	0.0153	0.0169	0.0169
4	10.55	10.1198	0.4302	0.0430	0.0430	0.0599	0.0599
5	10.5	10.3729	0.1271	0.0127	0.0127	0.0726	0.0726
6	10.4	10.3368	0.0633	0.0063	0.0063	0.0790	0.0790
7	10.2	10.2644	–0.0644	–0.0064	0.0000	0.0725	0.0790
8	10.1	10.1198	–0.0198	–0.0020	0.0000	0.0705	0.0790

Recharge Calculation

The calculation of recharge using the WTF method is demonstrated here for the first 8 days of the 30-day hypothetical well hydrograph. For this example the specific yield is 0.1 and the *linear* MRC function has coefficients *a* = 0.2767 day^{-1} and *b* = -2.7421 m day^{-1}. Table 4 shows how recharge is calculated and figure 3 depicts the recharge computation graphically. The predicted water-table elevation, Z_{WT} *pred*, is subtracted from Z_{WT} and the difference is multiplied by Y_s to compute recharge, *R*. The column titled 'Recharge with no negatives' simply excludes any negatively-valued recharge estimates. The option to include or exclude negative recharge estimates is available. Including negative recharge estimates can be helpful when working with very erratic hydrograph data as it effectively filters out high frequency fluctuations.

Instructions for Running MRCR

The program MRCR is an "m.file" which is a file that is executable in the Matlab programming environment. Matlab is commercially-available scientific computing software that runs on PC, Mac and Unix platforms. The user must have Matlab installed and running on his/her computer in order to run MRCR. The program is executed by typing MRCR at the Matlab command prompt. The file MRCR.m must be in the directory in which Matlab runs or in Matlab's "path".

The program may be run with 1) an input file, or 2) with interactive prompts. The program functions the same way in both cases. The difference lies in how the input parameters (described in the *Input Parameters* section, below) are supplied to the code. Both options require the user to specify 1) whether to use the input file or prompt method, and 2) the name of the file containing the well hydrograph data.

The first method, running with an input file, requires the creation of a text file containing the input parameters. The format of the file is described in the *Input File Format* section below. All parameters must be entered into the input file, regardless of whether they will be used in any particular run. The user is also required to enter the name of the observed data file that contains the well hydrograph data. Both the input file and the observed data file can be in the same directory as the code itself, or in another directory, which requires the use of folder navigation symbols before the file name (e.g., ../../data.txt for a data file located two directory levels above the current directory).

The second method of running the code requires the user to interactively enter values for input parameters, including the name of the observed data file. The advantage to this method is that only parameters for the relevant options are required.

Observed Data file

The observed data file contains the well hydrograph data. This file should be in two columns, with time (in days or seconds) in the first column, and water-table elevation (in feet or meters) in the second. There should be a header line (used to label the columns) before the first line of data. As an example, refer to the first two columns of table 1.

Input Parameters

The following is a list of all the input parameters used in the code.

t_unit_input = the units of time used in the observed data file. The two options are 1 = days, and 2 = seconds.

t_unit_desired = the units of time desired by the user for all output calculations and figures. The two options are 1 = days, and 2 = seconds.

l_unit_input = the units of length used in the observed data file. The two options are 1 = feet, and 2 = meters.

l_unit_desired = the units of length desired by the user for all output calculations and figures. The two options are 1 = feet, and 2 = meters.

t_step_type = the type of time step in the observed data file. The two options are 1 = constant, and 2 = variable. A constant time step means that all hydrograph observations (i.e., water-table elevation data) are equally spaced through time. A variable time step means that observations are unequally spaced through time.

t_step_observed = the time step of the observed data, in the case that *t_step_type* = 1 (constant). The units of this number are the same as the *t_unit_desired*.

t_step_desired = the time step desired by the user for calculations in the code, including generation of the MRC by the average bin method, in the case that *t_step_observed* = 1 (constant). This number must be an integer multiple of *t_step_observed*. The units of this number are the same as the *t_unit_desired*. The purpose of this parameter is to allow the user to remove unwanted high-frequency noise from the observed data by reducing the temporal resolution of the data.

MRC_type = the type of MRC the user wishes to use in the recharge computation. There are four options: 1 = linear function, 2 = power function, 3 = average bins, and 4 = tabulated input file.

a = the slope of the linear relation between elevation and decline rate [2], in the case that *MRC_type* = 1. The units of this number are decline rate per length [L/T/L = T^{-1}].

b = the intercept on the decline rate axis of the linear relation between elevation and decline rate [2], in the case that *MRC_type* = 1. The units of this number are [L/T].

c = the intercept on the decline rate axis in the power law relation between elevation and decline rate [3], in the case that *MRC_type* = 2. The units of this number are [L/T].

d = the multiplier of elevation in the power law relation between elevation and decline rate [3], in the case that *MRC_type* = 2. This number is dimensionless.

e = the elevation modifier in the power law relation between elevation and decline rate [3], in the case that *MRC_type* = 2. The units of this number are elevation [L].

f = the exponent in the power law relation between elevation and decline rate [3], in the case that *MRC_type* = 2. This number is dimensionless.

min_rate = the minimum decline rate, specifiable by the user. The units of this number are decline rate [L/T]. Note: decline rates are all negative so the minimum decline rate will be a small negative number.

max rate = the maximum decline rate, specifiable by the user. The units of this number are decline rate [L/T]. Note: decline rates are all negative so the maximum decline rate will be a large negative number.

num_bins = the number of elevation bins into which the user wishes the observed elevation-decline rate data to be split, in the case that *MRC_type* = 3.

mean_or_med = a parameter specifying which type of average to use in the calculation of average decline rate for bins, in the case that *MRC_type* = 3. The two options are 1 = arithmetic mean, and 2 = median.

Ys = specific yield to be used in recharge calculations. This number will be multiplied by the difference between observed and predicted water-table elevation to compute recharge. This number is dimensionless and will always be less than one.

neg_recharge = a parameter specifying whether to include negative values for computed recharge in the cumulative recharge total. The two options are 1 = include negative recharge, and 2 = do not include negative recharge.

Input File Format

When the code is run using the input file option a file is read in that has the following format.

t_unit_input
t_unit_desired
l_unit_input
l_unit_desired
t_step_type
t_step_observed
t_step_desired
MRC_type
a
b
c
d
e
f
min_rate
max_rate
mum_bins
mean_or_med
Ys
neg_recharge

An example input file is shown below:
1
1
1
2
1
0.25
1.0
3
0
0
0
0
0
0
-0.0001
-10.0
50
1
0.2
1

Program Output

The program outputs one number, one log file, and several figures, depending on the time-step type and MRC type.

The number that is output is the calculated cumulative recharge [L] for the entire hydrograph period. The user may want to use the Matlab Workspace Browser (a built-in data viewer) to view the values for other variables.

The outputted log file is called "log_file.txt" by default and may be renamed after it is created. It contains information on all the input data files and parameters used in that particular model run. It also includes the calculated cumulative recharge. The figures that are output are as follow:

1) Plot of observed well hydrograph. There are two lines plotted: the original data, and the "reduced" data, where all data points whose elevation is the same as the previous point have been eliminated. The reduced line results in the removal of step-like patterns in the original data.

2) Plot of observed water-table fluctuation rate. There are two lines plotted: the original data and the "reduced" data. This plot demonstrates how reducing the data tends to remove the large spikes in fluctuation rate seen in the original data.

3) Plot of decline rate versus elevation data points, constructed from the "reduced" data set.

4) Plot of decline rate versus elevation data points, constructed from the "reduced" data set, with the MRC superimposed.

5) Plot of the number of data points per bin versus the average elevation of the bin. (Only shown when *MRC_type* = 3.)

6) Plot of the predicted master recession curve hydrograph (water-table elevation versus time), starting at the highest observed elevation and ending at the lowest observed elevation.

7) Plot of predicted recharge rate versus time.

8) Plot of predicted cumulative recharge versus time.

9) Summary plot: Top window is the observed hydrograph. Middle window is the predicted recharge rate versus time. Bottom window is the predicted cumulative recharge versus time.

Summary

A computer program, MRCR, was developed to compute ground-water recharge from well hydrograph data based on the water-table fluctuation principle and a master recession curve. Recharge for each time step in a continuous record is computed as the product of specific yield and rise in water table. The rise in water table at each time step is determined by comparing the measured water-table elevation with a predicted water table elevation. The predicted water-table elevation is computed using a master recession curve, which is a characteristic relation between water-table elevation and decline rate. The master recession curve can be created using a linear function, a power function, a bin-average method, or by reading tabular data.

The program runs in the Matlab programming environment. The input data requirements are 1) well hydrograph data (i.e., a water table elevation time series), and 2) a specific yield estimate. The program output includes a cumulative recharge estimate, a log file, and up to 9 figures depicting the original data and calculated variables.

The program MRCR can be used to estimate recharge continuously from long-term well hydrograph data. The advantages of this approach are its simplicity, its few data requirements, and its efficiency. There are limitations to this approach, however, including: difficulty in assigning a specific yield value, transience in the specific yield (fillable porosity) parameter due to unsaturated zone hydraulics, poor relation between water-table elevation and decline rate, and externally-driven water table fluctuations (e.g., pumping, barometric effects). Mindful of these limitations the user should exercise good judgment when applying this method to recharge estimation at any specific site.

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