SABES Math Bulletin

Building Research Into Practice

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In This Issue

Some New Ideas for the New Year

Adult education students are brave enough to enter our classrooms in order to continue learning. We need the same courage to keep ourselves alive as learners.

In the hopes of generating some new brain cells for our readers, we present in particular a summary of an article penned by Arnold Packer, the former executive director of the Secretary's Commission on Achieving Necessary Skills (SCANS) and current Chair of the SCANS 2000 Center, in which he argues for an entirely new approach to teaching mathematics.

Packer makes a powerful argument for discarding old textbook notions of what and how to teach mathematics. Read on to see if you agree....

—Tricia Donovan, Editor

Thinking Outside the Box: Arnold Packer on the Value of Teaching "Empirical Mathematics"

All excerpts are from: Packer, Arnold, "What Mathematics Should 'Everyone' Know and Be Able to Do?" retrieved from www.maaa.org/ql/pgs33_42.pdf January 12, 2009

"Two hundred years ago, only merchants, engineers, surveyors, and a few scientists were mathematically literate," writes Arnold Packer (p.33), now, he asserts, we need everyone to develop mathematical or quantitative literacy skills because of the "rigors of international competition." (p. 34) and the demands of modern life.

Packer argues for a radical restructuring of mathematics teaching in order to achieve quantitative literacy among the general population. He concludes that it is better to teach mathematics "inductively" through projects and real-life problems that create the need to know mathematics than to teach a long series of mathematical topics deemed essential for mathematical preparation for later application.

"*Although they may not know the reasons why, generations of American students have been*

Continued on page 2

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Thinking Outside the Box...
Continued from page 1

convinced that something was amiss with mathematics classes. Many a parent has heard their teenage children complain, "I hate math; it's boring and hard. Why do I have to learn math, it's so useless...." Many parents are sympathetic. They themselves finished their last required mathematics course in high school or college with expressions of relief, not commitments to take another mathematics course as an elective.

These parents often are mathematically inadequate at their own jobs and in other aspects of their lives. They do not understand statistical quality processes, cannot follow political candidates who speak of 'weighted averages,' (See page 3) and cannot make sense of alternative strategies for financing their own retirement.

...Our society pays a high cost for the general lack of mathematical competence. (p. 34)

Cost of Math Incompetence

In schools, according to Packer, the problem surfaces in low test scores and the need to take developmental math courses.

National Assessment of Educational Progress (NAEP) scores in mathematics (for 17 year-olds) in 1996 were only 3 percent higher than in 1982. The average NAEP score was 307, meaning that the average 17 year-old can compute with decimals, use fractions and percentages, recognize geometric figures, solve simple equations, and use moderately complex reasoning. The averages among blacks (286) and Hispanics (292) were below 300, meaning the ability to do no more than perform the four arithmetic operations with whole numbers and solve one-step problems.

"Our society pays a high cost for the general lack of mathematical competence."

...Over one in four college freshmen feels a need for tutoring or remedial work in mathematics. This compares to one in 10 for English, science, and foreign language.

The Need for Context

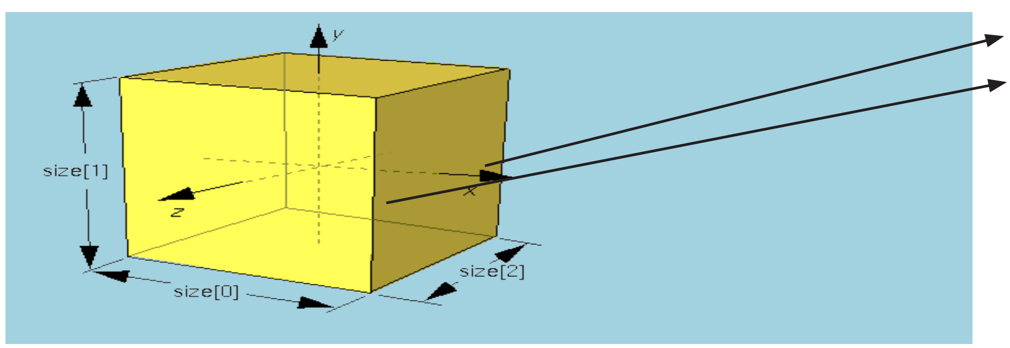
What is wrong? The way middle school teachers teach fractions provides a clue. They teach their students to add fractions by:

- *First finding the lowest common denominator.*
- *Then converting all fractions to that denominator.*
- *Then adding the numerators.*
- *Finally, reducing the answer, if possible.*

Nobody does that outside the schoolroom. Imagine a school cafeteria in which the selected items totaled three quarters and three dollars and four dimes. The school room method would be to change all these in for nickels. Or go to the shop. Maybe the problem is adding one foot and 8 and 1/16 inches to 6 and $\frac{1}{4}$ inches. Would any carpenter change it all into sixteenths? . . .

Mathematics teachers might say they want a 'general solution' so that students could add twentieths and sixteenths. A thoughtful student might respond, "Yeah ... like when?" ...The practical result is not a universe of students who can solve a universe of fraction problems. Instead it is a great many students who learn (about the sixth grade) that they 'can't do math' and demonstrate that truth by being unable to solve either the cafeteria or the shop problems. (Idem.)

Continued on page 4



Weighted Average or Means

(Adapted from Wikipedia)

The weighted mean is similar to an arithmetic mean (the most common type of average), where instead of each of the data points contributing equally to the final average, some data points contribute more than others. The notion of weighted mean plays a role in descriptive statistics and also occurs in a more general form in several other areas of mathematics.

The term weighted average usually refers to a weighted arithmetic mean. If all the weights are equal, then the weighted mean is the same as the arithmetic mean.

Example

Given two school classes, one with 20 students, and one with 30 students, the grades in each class on a test were:

Morning class = 62, 67, 71, 74, 76, 77, 78, 79, 79, 80, 80, 81, 81, 82, 83, 84, 86, 89, 93, 98

Afternoon class = 81, 82, 83, 84, 85, 86, 87, 87, 88, 88, 89, 89, 89, 90, 90, 90, 90, 91, 91, 91, 92, 92, 93, 93, 94, 95, 96, 97, 98, 99

The straight average for the morning class is 80 and the straight average of the afternoon class is 90. The straight average of 80 and 90 is 85, the mean of the two class means. However, this does not account for the difference in number of students in each class, and the value of 85 does not reflect the average student grade (independent of class). The average student grade can be obtained by either averaging all the numbers without regard to classes, or weighting the class means by the number of students in each class:

$$X = \frac{430}{50} = 86$$

Or, using a weighted mean of the class means:

$$X = \frac{(20)\,80 + (30)\,90}{20+30} = 86$$

The weighted mean makes it possible to find the average student grade also in the case where only the class means and the number of students in each class are available.

Thinking Outside the Box...

Continued from page 2

Packer begs for change. "Let students first learn the power of mathematics in specific examples. Later they can appreciate mathematics' power to generalize. The inductive approach is more likely to succeed than the current deductive process…." he argues (p.33)

An Empirical Approach

The curriculum needs to be less abstract, he claims, because "…the current approach to achieving widespread mathematical competence is failing." (p. 35) He summarizes his point by saying, "The real issue is whether mathematics should be taught inductively – from the concrete to the abstract—or the other way around…." He believes there would be more students taking and continuing in mathematics if it "were less abstract in the earlier years of school." (Idem.)

In his introduction, Packer also shares some data about the effectiveness of this approach, which focuses on project-based collaborative learning, with context-rich problems and the use of computers:

Over the last few years, inner-city Baltimore students were taught quantitative literacy in their algebra courses. They out-performed traditionally taught students by a wide margin. They took and passed Algebra II at a greater rate, received higher grades, were absent less, and were more likely to graduate and to go on to college." (p. 33)

A specific plan of action that would maximize the benefits under the constraints of time, talent, and materials would "Think first of the student's professional life." (p.36)

…The optimizing school would seek to maximize the benefit that is a function of:

1. *The frequency of having any particular job or career;*
2. *The probability that any particular problem will arise in that job;*
3. *The criticality of having the appropriate mathematics skills to solve the problem; and*
4. *The economic importance of solving the problem properly.*

Useful Problems

The challenge is to identify important, frequently encountered problems that cannot be efficiently solved without using mathematics. (Idem.)

Developing a file of useful, mathematically dependent problems for students at all levels would require an enormous outpouring of effort. According to Packer, he and others are starting to develop those problems for teachers in order to save them time and to eliminate the excuse for keeping to the status quo.

Packer's problems are designed with the five SCANS competencies and four adult roles (worker, consumer, citizen, and personal) in mind. (See SCANS at a Glance on page 6.) He acknowledges that such "teaching through modeling" is difficult, with students often devoting more time to understanding the model than doing mathematics. However, he is quick to point out that "Brain research has shown again and again that retention of information requires context." (p. 38)

Call for Change

He foretells the need for many significant changes in pedagogy, curriculum, and assessment, which will be needed to transform from "you got it wrong, let's move on" to multiple drafts of written and oral presentations. Each of these changes "will take resources, from political leadership to money. Imbuing all students with quantitative literacy cannot take place without additional instructional materials, substantial teacher training, and new assessment instruments." (Idem.) However, if such changes are not undertaken, Packer predicts that students' "careers and our society will suffer from it (because) …current approaches are failing too many students." (Idem.)

"Teachers in technical college programs complain that students cannot do mathematics. We have enough history to know that students are not going to change. We must, therefore change what and how we teach mathematics." (Idem.)

For more on the idea of "context" and its importance in adult numeracy, see SABES Math Bulletin Volume 1, Issue 3, May 2007, for a review of "The Components of Numeracy," a NCSALL occasional paper autjhored by Lynda Ginsburg, Myrna Manly, and Mary Jane Schmitt.

www.sabes.org/resources/publications/mathbulletin/index.htm

SCANS' Competencies Requiring Math

The 5 SCANS domains and the sub-domains that require quantitative (mathematical) literacy include:

Planning Problems
Allocating money (budgeting), time (scheduling), space, and staff

Systems and Processes Problems
Understanding, monitoring, and designing social, physical, or business systems

Interpersonal Problems
Working in teams, negotiating, teaching, and learning

Information Problems
Gathering and organizing data, evaluating data, and communicating, both in written and oral form

Technology Problems
Using, choosing, and maintaining equipment of any type

Empirical Problems: Planning

Note: In his appendix, Packer outlines the types of 'empirical' math problems that might be presented to students to address each of the SCANS' competencies listed above.

We share part of one of those elaborated lists—**for Planning**—below.

Planning (For a budget)

Worker: Using a spreadsheet with algebraic formulas, develop a budget for a retail store, construction project, manufacturing operation, or personal services (e.g., dental) office. The budget should include wages, benefits, material (or inventory), rent, and interest costs on borrowed funds.

Consumer: Using pencil and paper (with a calculator) and given a set of criteria and prices, develop a monthly budget for a family of four. Develop a budget for a party.

Citizen: Given an agency or organization budget for the past five years, write a two-page letter explaining and criticizing it. Include information on the growth or decline of the budget components themselves and as shares of the total. Relate to other variables, such as inflation and population growth.

Personal: Be able to understand the effects of budgets on historical events. Was the Athenian budget for its navy an excessive burden?

Planning (For a Schedule)

Worker: Using a spreadsheet (or other software) with algebraic formulas, develop a schedule for a construction project, advertising campaign, conference, medical regime or software project. Require conversions from hours to workweeks. Understand the difference between activities done in sequence and simultaneously. Understand PERT and Gantt charts.

Customer: Using pencil and paper without a calculator, plan a party or a meal. Convert hours to minutes.

Citizen: Understand why it takes so long to build a road or school.

Personal: Appreciate why Napoleon was beaten by the weather nad Russia.

Planning (For Space)

Worker: Using a computer graphics package, lay out a store room or office space in three dimensions. Develop a graphic for a brochure. Lay out material for a garment or a steel product. Lay out a restaurant or hotel space. Place paintings in a gallery.

Consumer: Look at a builder's plans and modify them. Understand your own living space.

Citizen: Understand plans for a public building.

Personal: Appreciate good design in products and building. Hang paintings in your house.

SCANS At a Glance

Mathematics Required to Solve Frequently Occurring Problems in Four Roles and Five SCANS Competencies (Appendix, p. 41)

Problem Domains	**Planning** • Budget • Schedule • Space • Staff	**Systems and Processes** • Understand • Monitor • Design	**Interpersonal** • Negotiate • Teach and learn	**Information** • Gather and organize • Evaluate • Communicate	**Technology** • Use • Choose • Maintain
Worker Role	Four arithmetic operations, estimation, geometry, algebra, exponential functions, spreadsheets, conversions. Concept of trade-offs. Awareness of tools such as linear programs and calculus for making trade-off decisions.	Model-building. Concept of first and second derivative and of integral, average, and standard deviation.	Mental arithmetic, fractions, percentages.	Create and read graphs, tables, and explanatory text.	Read graphs, tables, and explanatory text. Concept of trade-offs. Geometry.
Consumer Role	Four arithmetic operations, geometry, exponential functions, spreadsheets,. Concept of trade-offs.	Concept of first and second derivative and of integral, average, and standard deviation.	Mental arithmetic, fractions, percentages.	Read graphs, tables, and explanatory text	Read graphs, tables, and explanatory text. Geometry.
Citizen Role	Four arithmetic operations, geometry, concept of trade-offs.	Concept of first and second derivative and of integral, average, and standard deviation.	Mental arithmetic, fractions, percentages.	Read graphs, tables, and explanatory text.	Read graphs, tables, and explanatory text. Geometry.
Personal Role	Geometry, concept of trade-offs.	Concepts of calculus and statistics. History of mathematical discovery.		Read graphs, tables, and explanatory text.	Geometry.

Spatial Sense Impacts Problem Solving

Years ago I asked the Numeracy Listserv:
If you can form no picture of (a math) problem, can you solve it?

I received this reply from Ron Kindig (former Math Committee Chairman for the LA Unified School District):

"A recent research article in our California Math Council Journal summarized a comparison by Booth and Thomas (2000) of students with low and average spatial skills. Students with higher spatial skills performed significantly better on "arithmetic word problems."

They found two spatial domains needed for success with these kinds of problems:

"An ability to translate word problems into diagrammatic or pictorial representations, and

an ability to interpret the relevance of a given diagram or picture to a world problem."

The findings suggest that mathematically disadvantaged students may encounter difficulties with visual presentations of information, and that "it may be necessary to supplement these presentations with explicit verbal explanations to ensure the presentations are useful for those whose spatial skills are less well developed."

"Students with higher spatial skills performed significantly better on "arithmetic word problems."

This exchange came to mind as I read Packer's article in Quantitative Literacy: Why Numeracy Matters for Schools and Colleges. In this article, "What Mathematics should 'Everyone' Know and Be Able to Do," Packer quotes physicist Albert Einstein as saying "No scientist thinks in equations." He proceeds to write "Einstein employed visual images and muscular feelings. The mathematician S. M. Ulam said that he used 'mental images and tactile sensations to perform calculations,' replacing numerical values with the weights and sizes of imagined objects." It appears that spatial sense does impact mathematical problem solving.

As adult education teachers, then, we should work to develop our students' visual and spatial sense, not only as it relates to literature, but also as it relates to mathematics.

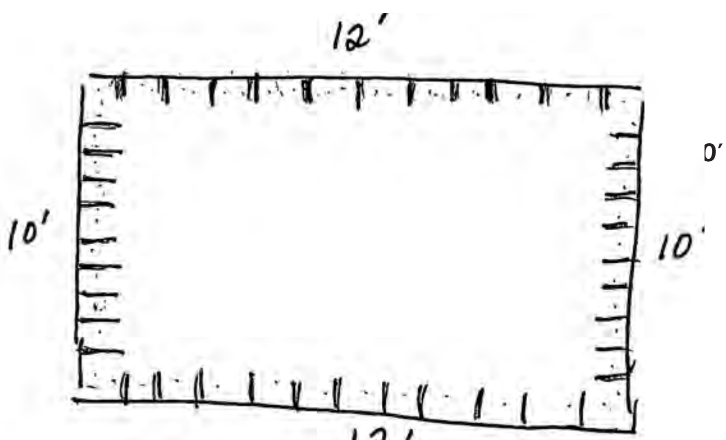
References:(from King's email)

CMC ComMuniCator, *CMC Research Notes: An Integrated Mathematics Curriculum: Problem Solving*, by Jack Carter, CSU Hayward.

Booth and Thomas, "Visualization in Mathematics Learning: Arithmetic Problem Solving and Student Difficulties." Journal of Mathematical Behavior 18 (Winter 2000): 169-190)

Translating a Word Problem Into a Picture

How many 1' square tiles do you need to cover the floor of a 10' X 12' kitchen?



12' in one row

10 rows

so 12x 10