The Impact of Monetary Policy on Asset Prices[1]

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Abstract

Estimating the response of asset prices to changes in monetary policy is complicated by the endogeneity of policy decisions and the fact that both interest rates and asset prices react to numerous other variables. This paper develops a new estimator that is based on the heteroskedasticity that exists in high frequency data. We show that the response of asset prices to changes in monetary policy can be identified based on the increase in the variance of policy shocks that occurs on days of FOMC meetings and of the Chairman's semi-annual monetary policy testimony to Congress. The identification approach employed requires a much weaker set of assumptions than needed under the "event-study" approach that is typically used in this context. The results indicate that an increase in short-term interest rates results in a decline in stock prices and in an upward shift in the yield curve that becomes smaller at longer maturities.

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1 Introduction

There is a considerable amount of interest in understanding the interactions between asset prices and monetary policy. In previous research (Rigobon and Sack (2001)), we have found that short-term interest rates react significantly to movements in broad equity price indexes, likely reflecting the endogenous response of monetary policy to the impact of stock price movements on aggregate demand. This paper attempts to estimate the other side of the relationship: how asset prices react to changes in monetary policy.

This relationship is an important topic for several reasons. From the perspective of monetary policymakers, having reliable estimates of the reaction of asset prices to the policy instrument is a critical step in formulating effective policy decisions. Much of the transmission of monetary policy comes through the influence of short-term interest rates on other asset prices, as it is the movements in these other asset prices—including longer-term interest rates and stock prices—that determine private borrowing costs and changes in wealth, which in turn importantly influence real economic activity.

Understanding the response of asset prices to changes in monetary policy is also of great importance to financial market participants. Monetary policy has a considerable influence on financial markets, as evidenced by the extensive attention that the Federal Reserve receives in the financial press. Thus, having accurate estimates of the responsiveness of asset prices to monetary policy is an important component of making effective investment decisions and formulating appropriate risk management strategies.

Several difficulties arise in estimating the responsiveness of asset prices to monetary policy, though. First, short-term interest rates are simultaneously influenced by movements in asset prices, resulting in a difficult endogeneity problem. Second, a number of other variables, including news about the economic outlook, likely have an impact on both short-term interest rates and asset prices. These two considerations complicate the identification of the responsiveness of asset prices under previously used methods.

To address these issues, we develop an estimator that identifies the response of asset prices based on the heteroskedasticity of monetary policy shocks. In particular, we assume that the variance of monetary policy shocks is higher on days of FOMC meetings and of the Chairman's semi-annual monetary policy testimony to Congress, when a larger portion of the news hitting markets is about monetary policy. We show that the shift in the variance of the policy shocks on those dates is sufficient to measure the responsiveness of asset prices to monetary policy.

Our approach allows us to identify the parameter of interest under a weaker set of assumptions than required under the approach that other papers have taken in this context. In particular, other papers have typically estimated ordinary-least-squares (OLS) regressions on FOMC dates, which has been called the "event-study" method. We show that the event-study approach is an extreme case of our heteroskedasticity-based estimator in which the shift in the variance of the policy shock is large enough to dominate all other shocks. In contrast, the heteroskedasticity-based estimator that we develop requires only a shift in the relative importance of the policy shock. Thus, our estimator can be used to test whether the stronger assumptions under the event-study approach are valid, and, correspondingly, the extent to which the event-study estimates are biased.

The paper proceeds as follows. Section 2 discusses the problems of simultaneous equations and omitted variables in estimating the responsiveness of asset prices, demonstrating that some bias may remain in the coefficients estimated under the event-study approach unless some strong assumptions are met. Section 3 describes our identification approach based on the heteroskedasticity of monetary policy shocks and compares the assumptions needed to those required under the event-study approach. Section 4 demonstrates that the identification method can be interpreted and implemented as a simple instrumental variables regression. Results on the responsiveness of stock prices and longer-term interest rates to monetary policy using both the event-study and the heteroskedasticity procedures are presented in section 5, and section 6 concludes.

2 Event-Study and the Estimation Problem

The two main problems in estimating the interactions between monetary policy and asset prices are the endogeneity of the variables and the existence of omitted variables. First, while asset prices are influenced by the short-term interest rate, the short-term interest rate is simultaneously affected by asset prices (primarily through their influence on monetary policy expectations). Second, a number of other variables likely influence both asset prices and short-term interest rates, such as variables that provide information about the macroeconomic outlook or changes in risk preferences.

These issues can be captured in the following simplified system of equations:

$$\Delta i_t = \beta \Delta s_t + \gamma z_t + \varepsilon_t \tag{1}$$

$$\Delta s_t = \alpha \Delta i_t + z_t + \eta_t, \tag{2}$$

where Δi_t is the change in the short-term interest rate and Δs_t is the change in an asset price. Equation (1) represents a monetary policy reaction function that captures the expected response of policy to a set of variables z_t and to the asset price.[1] We consider a case in which z_t is a single variable for notational simplicity, but the results can be easily generalized to the case where z_t is a vector of variables. Equation (2) is the asset price equation, which allows the asset price to be affected by the interest rate and also by the other variables z_t. In this paper we are interested in the parameter α, which measures the impact of a change in the short-term interest rate Δi_t on the asset price Δs_t. The variable ε_t is the monetary policy shock, and η_t is a shock to the asset price. Those disturbances are assumed to have no serial correlation and to be uncorrelated with each other and with the common shock z_t.

This model is clearly an oversimplification of the relationship between movements in interest rates and asset prices. It imposes no structure that might arise from an asset pricing model. However, this is also an advantage, as it allows the interaction between the variables to be fairly unrestricted. Similarly, VARs have often been used to capture the dynamics of asset prices without having to impose many restrictions (see, for example, Campbell and Shiller (1987)). In the current context, we can allow for more complicated dynamics by adding lagged terms to equations (1) and (2), in which case estimating the responsiveness amounts to (partially) identifying the VAR. However, we found that allowing for a richer lag structure had little effect on the results. Moreover, the above system of equations is

[1] Rigobon and Sack (2001) focus on the parameter β measuring the response of monetary policy to the asset price—the stock market in particular. Their results suggest that this parameter is positive and of the magnitude that would be expected if the Federal Reserve were reacting to the stock market to the extent that it affects aggregate demand.

sufficiently rich to demonstrate the problems that arise in identifying the parameter α.

As is well known, equations (1) and (2) cannot be estimated consistently using OLS due to the presence of simultaneous equations and omitted variables. The simultaneity problem is demonstrated in Figure 1, which shows both the policy reaction function (1) and the asset price function (2). Realizations of the interest rate and the asset price will be determined by the intersection of these two schedules and therefore may not provide any information about the slope of either schedule. Moreover, the two schedules are being frequently shifted by realizations of the variable z_t, and thus the observations will be influenced by the coefficients on those variables in the two equations (which determine the relative magnitude of the shifts).

To see the econometric problems formally, consider running an OLS regression on equation (2). The estimated coefficient will be biased because the shock term η_t is correlated with the regressor Δi_t as a result of the response of the interest rate to the stock market, as determined by parameter β in equation (1). Moreover, if some of the variables z_t are not observed, then the exclusion of those variables from the specification would also generate some bias depending on the value of γ. Indeed, if one simply ran OLS on equation (2) above, the estimated parameter would be given by:

$$\widehat{\alpha} = \alpha + (1 - \alpha\beta) \frac{\beta\sigma_\eta + (\beta + \gamma)\,\sigma_z}{\sigma_\varepsilon + \beta^2\sigma_\eta + (\beta + \gamma)^2\,\sigma_z}, \tag{3}$$

where σ_x represents the variance of shock x. Again, according to equation (3), the estimate would be biased away from its true value α due to both simultaneity bias (if $\beta \neq 0$ and $\sigma_\eta > 0$) and omitted variables bias (if $\gamma \neq 0$ and $\sigma_z > 0$).

Researchers have typically addressed these problems by focusing on periods immediately surrounding changes in the policy instrument—what has been often referred to as the event-study approach. This literature largely follows Cook and Hahn (1989), whose approach was to regress daily changes in market interest rates on changes in the federal funds rate for a sample of dates on which the federal funds rate changed. Their work has been followed by a large number of papers applying a similar approach, including Bomfim (2001), Bomfim and Reinhart (2000), Kuttner (2001), Roley and Sellon (1996, 1998), and Thornton(1998).

These more recent papers have modified the work of Cook and Hahn in various directions, including focusing on more recent periods and isolating the surprise component of funds rate changes. Nevertheless, the basis of the approach—estimating OLS regressions on dates of FOMC meetings or policy moves—has remained the same.

The rationale underlying the event-study approach is that the bias in the OLS estimate $\widehat{\alpha}$ will be limited if the sample contains periods in which the innovations to the system of equations (1) and (2) are driven primarily by the policy shock. In fact, as is evident from equation (3), the event-study approach requires the following assumptions to minimize the bias of the estimator:

$$\sigma_\varepsilon \gg \sigma_z \tag{4}$$

$$\sigma_\varepsilon \gg \sigma_\eta, \tag{5}$$

in which case $\widehat{\alpha} \cong \alpha$. In the limit, if the variance of the monetary policy shock becomes infinitely large relative to the variances of the other shocks, or $\sigma_\varepsilon/\sigma_\eta \to \infty$ and $\sigma_\varepsilon/\sigma_z \to \infty$, then the bias goes to zero, and the OLS estimate is consistent. This property of the OLS estimate is what Fisher (1976) referred to as "near identification." However, it should be clear that some bias remains if these ratios are finite. Unfortunately, the event-study approach does not provide any evidence about whether these conditions hold, and thus the magnitude of the bias that remains in those estimates is unclear from the event-study literature.

In the next section, we demonstrate that the parameter α can be estimated under a much weaker set of assumptions by relying on the heteroskedasticity in the data to identify the parameter. This identification approach does not require the variance of one of the shocks to become infinitely large, but instead relies on the change in the covariance of interest rates and asset prices at times when the variance of the policy shocks increases. In effect, this approach can be thought of as estimating α from the *change* in the bias in equation (3) as the variance of policy shocks *changes*, rather than requiring that the *level* of the bias goes to zero. The approach also allows one to measure the bias in the event-study estimates, which can be used to test whether assumptions (4) and (5) are valid.

3 Identifying the Response of Asset Prices

To estimate the response of asset prices to monetary policy, we employ a technique called identification through heteroskedasticity.[2] This approach relies on looking at changes in the co-movements of interest rates and asset prices when the variance of one of the shocks in the system is known to shift. By doing so, the response of asset prices to monetary policy can be identified under a fairly weak set of assumptions.

The intuition for this approach is shown in Figure 2. Suppose one could identify a period of time in which the variance of the policy shocks was higher than at other times, but the variances of the other shocks in the system remained unchanged. As is evident in the figure, the pattern of realized observations would then shift to move more closely along the asset price reaction schedule. That shift in the co-movement of interest rates and asset prices towards the schedule of interest is the basis for the identification.

To implement this approach, we only need to identify two subsamples, denoted F and $\tilde{F}$ (for reasons that become clear below), for which the parameters of equations (1) and (2) are stable and the following assumptions on the second moments of the shocks hold:

$$\sigma_{\varepsilon}^{F} > \sigma_{\varepsilon}^{\tilde{F}} \tag{6}$$

$$\sigma_{\eta}^{F} = \sigma_{\eta}^{\tilde{F}} \tag{7}$$

$$\sigma_{z}^{F} = \sigma_{z}^{\tilde{F}}. \tag{8}$$

In words, these assumptions imply that the "importance" of policy shocks increases in the subsample F. Note, however, that innovations to the asset price equation and the common shocks continue to take place even in subsample F, but those shocks are assumed to occur with the same intensity as in the other subsample. These conditions are much weaker than the near-identification assumptions (4) and (5) required under the event-study approach. In particular, we do not required the variance of the policy shock to become infinitely large, but only that it increases relative to the variances of the other shocks.[3]

[2]This identification approach is described in detail in Rigobon (1999). There is a relatively new literature using this identification method, including King, Sentana and Wadhwani (1994), Sentana and Fiorentini (2000), and Klein and Vella (2000a,b).

[3]Bomfim (2001) explores patterns of volatility around FOMC meeting dates, finding that the variance of

We use institutional knowledge of the Federal Reserve to identify circumstances in which assumptions (6) to (8) are plausible. In particular, days of FOMC meeting and of the Chairman's semi-annual monetary policy testimony to Congress are likely to contain a greater amount of news about monetary policy than other days.[4] Note that other types of shocks still take place these days, but the relative importance of policy shocks is likely to increase dramatically, as required under our identification. Thus, we take those dates as the set of dates F, which will be referred to as the set of "policy dates" to indicate that the variance of the policy shock is elevated.[5] For the set of non-policy dates $\tilde{F}$, we take the set of days just prior to the days included in F, which keeps the samples the same size and minimizes any effects arising from changes in the variances of the shocks over time.

The identification can be shown analytically by first solving for the reduced form of equations (1) and (2):[6]

$$
\begin{aligned}
\Delta i_t &= \frac{1}{1-\alpha\beta}\left[(\beta+\gamma)\, z_t + \beta\eta_t + \varepsilon_t\right] \\
\Delta s_t &= \frac{1}{1-\alpha\beta}\left[(1+\alpha\gamma)\, z_t + \eta_t + \alpha\varepsilon_t\right].
\end{aligned}
$$

These variables can be divided up into the two subsamples, with the covariance matrix of the variables in each subsample as follows:

$$
\begin{aligned}
\Omega_F &= \frac{1}{(1-\alpha\beta)^2}\begin{bmatrix} \sigma_\varepsilon^F + \beta^2\sigma_\eta^F + (\beta+\gamma)^2\sigma_z^F & \alpha\sigma_\varepsilon^F + \beta\sigma_\eta^F + (\beta+\gamma)(1+\alpha\gamma)\,\sigma_z^F \\ . & \alpha^2\sigma_\varepsilon^F + \sigma_\eta^F + (1+\alpha\gamma)^2\sigma_z^F \end{bmatrix} \\
\Omega_{\tilde{F}} &= \frac{1}{(1-\alpha\beta)^2}\begin{bmatrix} \sigma_\varepsilon^{\tilde{F}} + \beta^2\sigma_\eta^{\tilde{F}} + (\beta+\gamma)^2\sigma_z^{\tilde{F}} & \alpha\sigma_\varepsilon^{\tilde{F}} + \beta\sigma_\eta^{\tilde{F}} + (\beta+\gamma)(1+\alpha\gamma)\,\sigma_z^{\tilde{F}} \\ . & \alpha^2\sigma_\varepsilon^{\tilde{F}} + \sigma_\eta^{\tilde{F}} + (1+\alpha\gamma)^2\sigma_z^{\tilde{F}} \end{bmatrix}.
\end{aligned}
$$

the shock from the stock market equation increases on FOMC meeting dates even controlling for the direct effect of policy news on the stock market. In the view of our model, the reason why some heteroskedasticity remains in the stock market values is because the simultaneity was not fully solved.

[4]This testimony accompanies the release of the Federal Reserve's Monetary Policy Report to the Congress. It used to be referred to as the "Humphrey Hawkins" testimony when it was mandated under the Full Employment and Balanced Growth Act of 1978.

[5]One could imagine a broader set of dates to be included in the set of policy dates, such as dates of policy-related speeches by FOMC members.

[6]This approach can also be implemented by first estimating a VAR that includes interest rates and asset prices, and then focusing on the reduced form residuals in place of Δi_t and Δs_t. The results obtained under this approach are very similar to those reported below.

Note that we have assumed, in addition to (6) to (8), that the parameters α, β, and γ are stable across the two set of dates, which is a necessary condition for the identification.

The difference in these covariance matrices is

$$\Delta\Omega = \Omega_F - \Omega_{\tilde{F}} = \frac{\left(\sigma_\varepsilon^F - \sigma_\varepsilon^{\tilde{F}}\right)}{(1-\alpha\beta)^2} \begin{bmatrix} 1 & \alpha \\ \alpha & \alpha^2 \end{bmatrix}. \tag{9}$$

As is evident from equation (9), α is easily identified from the change in the covariance matrix. In fact, α can be estimated in two different ways:

$$\alpha_{het} = \frac{\Delta\Omega_{12}}{\Delta\Omega_{11}} \tag{10}$$

$$\alpha_{het} = \frac{\Delta\Omega_{22}}{\Delta\Omega_{12}}, \tag{11}$$

where $\Delta\Omega_{ij}$ represents the (i,j) element of the change in the Ω matrix. Moreover, as shown in Appendix B, these estimators are consistent even if the shocks have heteroskedasticity over time, as long as the volatility of the policy shock accounts for the shift in the covariance matrix on policy dates (and some additional assumptions are met).

The estimates in equations (10) and (11) have, in spirit, the same interpretation as the event-study estimator. In our case, the event (a policy day) is an increase in the variance of the policy shock, which changes the covariance structure of the observed variables. Under our assumptions, this is enough to estimate some of the underlying coefficients. If the shift in the variance of the policy shocks were infinitely large, then the estimators (10) and (11) would in fact converge to the standard event-study estimates. However, as described above, the heteroskedasticity-based estimators α_{het} do not require such a strong assumption to be consistent. As a result, the heteroskedasticity-based estimates can be used to assess the bias in the event-study estimates, as described in the next section.

If all of the assumptions of the model hold, the two estimates of α should be identical. We can therefore use the two estimates of α to construct a test of the overidentifying restrictions of the model. Differences in the estimates could indicate that the variance of other shocks increased on policy dates or that the parameters of the equations are not stable across the two subsamples. The only assumption that is not testable in our setup is the zero correlation across the structural shocks, which is a maintained assumption in the overidentification tests. This test is described in more detail in the next section.

4 Implementation through Instrumental Variables

A nice feature about this identification method is that it can be implemented using an instrumental variables technique, which makes it simple to apply using any standard econometrics software package.

4.1 Estimators for an individual asset

To arrive at the instrumental variables interpretation of the estimators, define the following variables to include the interest rate and the asset price on all days in our sample, including policy and non-policy dates:

$$\begin{aligned} \Delta i &\equiv \{\Delta i_t, t \in F\} \bigcup \{\Delta i_t, t \in \tilde{\ } F\} \\ \Delta s &\equiv \{\Delta s_t, t \in F\} \bigcup \{\Delta s_t, t \in \tilde{\ } F\}, \end{aligned}$$

which are both $2T \times 1$ vectors (where T is the number of policy dates). Consider the following two instruments:

$$\begin{aligned} w_i &\equiv \{\Delta i_t, t \in F\} \bigcup \{-\Delta i_t, t \in \tilde{\ } F\} \\ w_s &\equiv \{\Delta s_t, t \in F\} \bigcup \{-\Delta s_t, t \in \tilde{\ } F\}. \end{aligned}$$

It turns out that the two estimates for α from the analysis above can be obtained by regressing the change in the asset price Δs_t on the change in the interest rate Δi_t over the combined sample period using the standard instrumental variables approach with the instruments w_i and w_s:

$$\widehat{\alpha}_{het}^{i} = \left(w_i{}' \, \Delta i\right)^{-1} \left(w_i{}' \, \Delta s\right) \quad (12)$$

$$\widehat{\alpha}_{het}^{s} = \left(w_s{}' \, \Delta i\right)^{-1} \left(w_s{}' \, \Delta s\right). \quad (13)$$

To see that, note that the IV coefficients can be written as

$$
\begin{aligned}
\widehat{\alpha}_{het}^{i} &= \tfrac{\{\Delta i_F, -\Delta i_{\tilde{F}}\}'\{\Delta s_F, \Delta s_{\tilde{F}}\}}{\{\Delta i_F, -\Delta i_{\tilde{F}}\}'\{\Delta i_F, \Delta i_{\tilde{F}}\}} &= \frac{Cov(\Delta i_F, \Delta s_F) - Cov(\Delta i_{\tilde{F}}, \Delta s_{\tilde{F}})}{Var(\Delta i_F) - Var(\Delta i_{\tilde{F}})} \\
\widehat{\alpha}_{het}^{s} &= \tfrac{\{\Delta s_F, -\Delta s_{\tilde{F}}\}'\{\Delta s_F, \Delta s_{\tilde{F}}\}}{\{\Delta s_F, -\Delta s_{\tilde{F}}\}'\{\Delta i_F, \Delta i_{\tilde{F}}\}} &= \frac{Var(\Delta s_F) - Var(\Delta s_{\tilde{F}})}{Cov(\Delta i_F, \Delta s_F) - Cov(\Delta i_{\tilde{F}}, \Delta s_{\tilde{F}})},
\end{aligned}
$$

which are the same estimators

$$
\begin{aligned}
\widehat{\alpha}_{het}^{i} &= \frac{\Delta\Omega_{12}}{\Delta\Omega_{11}} \\
\widehat{\alpha}_{het}^{s} &= \frac{\Delta\Omega_{22}}{\Delta\Omega_{12}}
\end{aligned}
$$

from equations (10) and (11) above.[7] See Appendix A for a more complete derivation and analysis of the properties of these estimators.

An advantage of implementing the identification technique through instrumental variables is that all of the properties of IV estimators, including the asymptotic distribution of the coefficient, apply. Moreover, it can be shown that w_i and w_s are valid instruments for estimating α under the assumptions underlying the heteroskedasticity approach—that the parameters are stable, that the asset price shocks are homoskedastic, and that the monetary policy shocks are heteroskedastic (see the appendix).

4.2 Estimators for multiple assets

Of course, we are interested in the response of a number of asset prices to monetary policy. The method described above can be generalized to allow for more than one asset price (as will be the case in the empirical implementation below). Under the IV interpretation, if we consider K different assets, we will have available $K+1$ different instruments—one for the interest rate (w_i), and one (w_s) for each asset price $s \in S$, where S is the set of all asset

[7] More specifically, this is the case if the sets F and $\tilde{F}$ have the same number of observations. If the number of observations in these sets differs, the instruments and the variables have to be divided by the square root of the number of dates in that set.

prices included. Denote the set of all possible instruments as

$$W_t = \omega_i \bigcup_{s \in S} \omega_s,$$

which is a $2T \times (K+1)$ matrix. We can then consider an estimator that uses all possible instruments to estimate the coefficients:

$$\widehat{\alpha}_{het}^{all} = \left(\widehat{\Delta i}\ '\ \Delta i\right)^{-1} \left(\widehat{\Delta i}\ '\ \Delta s\right), \tag{14}$$

for each $s \in S$, where

$$\widehat{\Delta i} = W_t \left(W_t\ '\ W_t\right)^{-1} W_t\ '\ \Delta i. \tag{15}$$

As discussed in Appendix A, this instrument set is again valid under the maintained assumptions.[8]

4.3 Hypothesis tests

If the assumptions of the model are correct, then all the IV estimators $\widehat{\alpha}_{het}^{i}$, $\widehat{\alpha}_{het}^{s}$, and $\widehat{\alpha}_{het}^{all}$ will asymptotically yield the true parameter value α. This implies that the system is overidentified and allows us to perform a test of the underlying assumptions of the model by comparing any two estimates. To limit the scope of the analysis, we focus on the estimators $\widehat{\alpha}_{het}^{i}$ and $\widehat{\alpha}_{het}^{all}$. We first stack the estimates for each asset price $s \in S$ into vectors, so that $\widehat{\alpha}_{het}^{i}$ and $\widehat{\alpha}_{het}^{all}$ are now both $K \times 1$. The test of overidentifying restrictions then is as follows:

$$\widehat{\delta}_{all,i} = \tfrac{1}{K} \left|\widehat{\alpha}_{het}^{all} - \widehat{\alpha}_{het}^{i}\right| \ M_{all,i}^{-1} \ \left|\widehat{\alpha}_{het}^{all} - \widehat{\alpha}_{het}^{i}\right|$$

where $M_{all,i}$ is the variance of the difference of the estimators. A rejection of the hypothesis that the two estimates are equal would indicate that at least one of the maintained assumptions—that the parameters are stable, or that the stock market or the common shock are homoskedastic—is not valid.

[8] We implement optimal IV when all instruments are used.

We are also interested in testing whether the stronger assumptions required under the event-study approach are valid. To do so, we compare the estimates under the heteroskedasticity-based approach to the event-study estimates obtained by running an OLS regression. Formally, the event-study estimator is

$$\widehat{\alpha}_{es} = \left(\Delta i_F{}' \ \Delta i_F\right)^{-1} \left(\Delta i_F{}' \ \Delta s_F\right), \tag{16}$$

where only those observations corresponding to policy dates (that is, $t \in F$) are included. The event-study estimator is consistent and efficient under the assumption that endogeneity is not a problem, which would be the case if equations (4) and (5) hold. Otherwise, the event-study estimator is inconsistent, but the heteroskedasticity-based estimators are still consistent. Thus, the validity of the event-study assumptions can be tested with a Hausman (1978) specification test:

$$\begin{aligned} \widehat{\delta}_{es,all} &= \tfrac{1}{K} \left| \widehat{\alpha}_{het}^{all} - \widehat{\alpha}_{es} \right| \ M_{es,all}^{-1} \left| \widehat{\alpha}_{het}^{all} - \widehat{\alpha}_{es} \right| \\ M_{se,all} &= Var\left(\widehat{\alpha}_{het}^{all}\right) - Var\left(\widehat{\alpha}_{es}\right) \end{aligned}$$

where the event-study estimates have been stacked into a $K \times 1$ vector $\widehat{\alpha}_{es}$.[9] This test statistic has an F distribution with $K, K(T-1)$ degrees of freedom. Note that for this test statistic the variance of the difference in the estimators is the difference in the variances, given the efficiency of the OLS estimator under the null hypothesis that the event-study assumptions hold. A significant test statistic would indicate a rejection of the assumption that the variance of the policy shock on policy dates is sufficiently large for near-identification to hold.

[9] A similar test statistic can be computed for the other heteroskedasticity-based estimator $\widehat{\alpha}_{het}^{i}$. To narrow the discussion, we will focus only on the test statistic $\widehat{\delta}_{es,all}$ in the results below.

5 Results

In the following results we focus on the effect of monetary policy on stock market indexes and longer-term interest rates. The data on stock indexes include the Dow Jones Industrial Average (DJIA), the S&P 500, the Nasdaq, and the Wilshire 5000. The longer-term interest rates considered include Treasury yields with maturities of six months, one, two, five, ten, and thirty years.[10] Because the six Treasury rates considered are not sufficient to derive a complete set of forward rates, we also look at the response of eurodollar futures rates expiring every three months from six months to five years ahead.

The sample runs from January 3, 1994 to November 26, 2001—a period over which the majority of monetary policy actions took place at FOMC meetings. In contrast, over the five years preceding our sample, only about one quarter of policy moves took place on FOMC dates, with other policy actions often taking place on the days of various macroeconomic data releases. Thus, there was greater uncertainty about the timing of policy moves over the earlier period, which makes it more difficult to split it according to the heteroskedasticity of policy shocks. Our sample includes 78 policy dates, of which five are discarded due to holidays in fianncial markets.[11]

The short-term interest rate used in the analysis is the rate on the nearest eurodollar futures contract to expire, which is is based on the three-month eurodollar deposit rate at the time the contract expires.[12] An advantage of using this policy rate is that it moves only to the extent that there is a policy surprise. The importance of focusing on the surprise component of policy moves has been emphasized in recent research, including many of the papers listed in section 2. Those papers typically use the current month's federal funds futures rate to derive a measure of the unexpected component of policy moves.[13] However,

[10] The Treasury series are the constant maturity Treasury yields reported on the Federal Reserve's H.15 data release.

[11] These holidays fall either one or two days before the policy dates. Those observations are needed because of our use of differences in the specification and our speciication of the F sample, as described below.

[12] We use the eurodollar futures rather than using the eurodollar deposit rate because the futures contract is more liquid and trades in U.S. markets, thereby avoiding issues with the timing of its quote relative to those on other asset prices. One drawback of using futures is that the horizon of the contract can vary. Because the contracts expire quarterly, the nearest contract will have between zero and three months to expiration, depending on the timing of the FOMC meeting.

[13] Kuttner (2001) offers a useful discussion of calculating monetary policy surprises from federal funds rate futures.

this measure will be strongly influenced by surprises in the timing of policy moves. For example, if the FOMC is expected to cut rates at next month's FOMC meeting but instead does so at this month's meeting, this shift would be reflected in a possibly sizable federal funds futures shock. But such a shift in the timing of policy actions, if it does not affect the total magnitude of policy action expected in the near term, might not be expected to have as large an impact on asset prices. Using the three-month eurodollar rate as the monetary policy variable reduces the influence of these timing shocks, instead picking up surprises to the level of the interest rate expected over the coming three months.

Table 1: Variances and Covariances on Policy and Non-Policy Dates

	—Std. Dev. of Asset Prices—		—Covar. with Policy Rate—	
	$\tilde{F}$ Dates	F Dates	$\tilde{F}$ Dates	F Dates
Policy Rate	2.62	5.26	-	-
S&P 500	0.88	0.99	-0.20	-1.60
Nasdaq	1.63	1.71	0.08	-2.02
DJIA	0.89	0.92	-0.51	-1.35
i_{6mo}	4.79	5.80	6.13	25.89
i_{1yr}	3.64	6.54	7.47	29.57
i_{2yr}	3.83	7.25	7.58	31.43
i_{5yr}	3.90	7.75	7.29	31.73
i_{10yr}	3.95	7.10	7.22	26.38
i_{30yr}	3.89	6.00	6.36	18.01

The table uses daily percent changes for stock prices (in percentage points) and daily changes in Treasury yields (in basis points).

Table 1 reports some descriptive statistics on daily changes in the policy rate and in other asset prices on policy and non-policy dates. In all of the results that follow, the non-policy dates are taken to be the day before each policy date.[14] The variance of changes in the short-term interest rate rises substantially on the days with higher variance of policy shocks, as expected. More importantly, for the non-policy dates, there is no discernible relationship between stock prices and the policy rate, as evidenced by the relatively small

[14]There is likely to be little news about monetary policy on those dates, as FOMC members appear to refrain from making public comments and the FOMC from taking intermeeting policy actions so close to an FOMC meeting. Similar results are obtained if we define the set of non-policy dates to include the week before each FOMC meeting.

covariances between them. In contrast, a negative relationship between these variables becomes evident on the policy dates, as the higher variance of the policy shocks on those days tends to move the observations along the asset price response function (as suggested in Figure 2). Treasury rates instead have a positive covariance with the policy rule on non-policy dates. But again the relationship between these variables shifts importantly on policy dates, with the positive covariance jumping much higher in that subsample.

As described in the previous two sections, the shift in the covariance between the policy rate and the asset prices that takes place on policy dates can be used to estimate the parameter α from equation (2). We will consider two of the heteroskedasticity-based estimators corresponding to equations (12) and (14).

5.1 Stock market indexes

The results across the four stock market indexes considered are shown in Table 2, which reports the estimates obtained under the heteroskedasticity-based approach using both sets of instruments ($\widehat{\alpha}^{i}_{het}$ and $\widehat{\alpha}^{all}_{het}$) as well as the estimate obtained under the event-study approach ($\widehat{\alpha}_{es}$).

The stock indexes considered have a significant negative reaction to monetary policy. The estimate $\widehat{\alpha}^{all}_{het}$ for the S&P 500 is -7.702, implying that a 25-basis point increase in short-term interest rates results in a 1.9% decline in the S&P index. A similar response is found for the broader market index, the Wilshire 5000. The Nasdaq index shows a considerably larger reaction, perhaps because of the greater duration of those shares (their cash flows are farther in the future, making the share price more sensitive to the discount factor), while the DJIA has the smallest reaction, maybe because it includes companies that have current rather than back-loaded cash streams. In all four cases, the two heteroskedasticity-based estimates $\widehat{\alpha}^{i}_{het}$ and $\widehat{\alpha}^{all}_{het}$ are similar. Indeed, the test statistic $\widehat{\delta}_{all,i}$ indicates that the over-identifying restrictions of the heteroskedasticity-based estimators are easily accepted.

The estimated responses of the stock indexes under the heteroskedasticity-based method are almost always larger (in absolute value) than the corresponding estimates under the event-study approach, and by a considerable amount in some cases. This difference likely reflects the bias in the event-study estimates. Shocks to the stock market generally cause short-term interest rates to respond in the same direction (Rigobon and Sack (2001)), while many other variables, such as news about future economic activity, also tend to induce a positive correlation between the two variables. These shocks therefore generate an upward bias (towards zero) in the estimated coefficient $\widehat{\alpha}_{es}$ under the event-study approach. The hypothesis that the heteroskedasticity-based and event-study estimates are equal across the four stock price indexes, which is tested using the statistic $\widehat{\delta}_{es,all}$, can be rejected at the 0.10 significance level, although not at the 0.05 level. Thus, the results suggest that the assumptions underlying the event-study approach are violated enough to generate a bias in the event-study estimates that is marginally significant.

Table 2: The Response of Stock Prices to Monetary Policy

	Estimator: $\widehat{\alpha}^i_{het}$		Estimator: $\widehat{\alpha}^{all}_{het}$		Estimator: $\widehat{\alpha}_{es}$	
	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev
S&P 500	-7.101	2.883	-7.702	2.748	-6.171	2.087
Wilshire	-7.004	2.834	-7.271	2.698	-5.961	2.039
Nasdaq	-11.045	5.108	-10.023	4.841	-7.356	3.689
DJIA	-4.729	2.823	-5.883	2.681	-5.409	1.985
			F-test	P-value		
Test of O.I. Restrictions: $\widehat{\delta}_{all,i}$			0.750	0.559		
Test of E.S. Assumptions: $\widehat{\delta}_{es,all}$			2.283	0.063		

Both test statistics are distributed F(4,145). The value of the 95th percentile is 2.43.

The finding of a significant response of stock prices to monetary policy actions stands out against the fairly inconclusive findings of the previous literature. Bomfim (2001) also found a significant response for the S&P 500, although smaller in magnitude than the response that we identify, while other papers, including Bomfim and Reinhart (2000) and Roley and Sellon (1998), have found no statistically significant response. Of course, these papers relied exclusively on the event-study approach.

5.2 Treasury yields

Treasury yields also respond strongly to monetary policy, as shown in Table 3. The heteroskedasticity-based coefficients $\widehat{\alpha}_{het}^{all}$ are significant across all maturities except the thirty-year bond. The pattern of the coefficients, which is shown in Figure 3, indicates that monetary policy has the strongest impact on short-term and intermediate-term Treasury yields. The impact falls off fairly sharply for maturities beyond five years. The other set of heteroskedasticity-based estimates, $\widehat{\alpha}_{het}^{i}$ are largely similar, and the test of overidentifying restrictions $\hat{\delta}_{all,i}$ indicates that the model's assumptions are not rejected.

Table 3: The Response of the Term Structure to Monetary Policy

	Estimator: $\widehat{\alpha}_{het}^{i}$		Estimator: $\widehat{\alpha}_{het}^{all}$		Estimator: $\widehat{\alpha}_{es}$	
	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev
i_{6mo}	0.883	0.114	0.843	0.107	0.903	0.066
i_{1yr}	0.802	0.089	0.716	0.087	0.913	0.072
i_{2yr}	0.826	0.107	0.732	0.103	0.930	0.092
i_{5yr}	0.961	0.122	0.872	0.116	1.035	0.110
i_{10yr}	0.639	0.135	0.474	0.132	0.770	0.119
i_{30yr}	0.377	0.135	0.225	0.132	0.527	0.113
			F-test	P-value		
Test of O.I. Restrictions: $\widehat{\delta}_{all,i}$			1.558	0.164		
Test of E.S. Assumptions: $\widehat{\delta}_{es,all}$			2.171	0.049		

Both test statistics are distributed F(7,145). The value of the 95th percentile is 2.16.

Both sets of heteroskedasticity-based estimates fall below the corresponding event-study estimates, likely reflecting an upward bias in the event-study coefficients. Many types of shocks push short-term and long-term interest rates in the same direction, including macroeconomic developments that shift inflation expectations or changes in the value that investors place on the safety and liquidity of their portfolios. These shocks are likely to still be present on policy dates, inducing an upward bias to the event-study estimates. The test statistic $\widehat{\delta}_{es,all}$ indicates that the equality of the event-study and the heteroskedasticity-based estimates can be rejected at the 0.05 significance level.

Based on the point estimates in Table 3, the bias in the event-study coefficients is largest at long maturities. One possible explanation is that the policy shock is less influential on the Treasury yield as the maturity lengthens, thus leaving a larger role for the biases induced by other shocks. Note that one puzzling aspect of the event-study results is the magnitude of the response of long-term interest rates to policy changes, which is surprisingly large if movements in the short-term rate are expected to be transitory. According to the results, this puzzle partly reflects the bias in the event-study estimates, as both sets of heteroskedasticity-based estimates decline more rapidly than the event-study estimates as the maturity lengthens.

The response of the term structure to policy surprises has also been studied by Kuttner (2001), among others. Our results are qualitatively similar to his, in that he finds that Treasury yields respond significantly across most maturities, and that the response diminishes at longer maturities. His estimates differ in magnitude from ours in part because of differences in the sample period and in the definition of policy shocks.[15] Moreover, because he employs the event-study methodology, his results would be subject to the biases discussed above.

5.3 Eurodollar futures rates

Our final set of results is based on eurodollar futures rates expiring every three months out to horizons of five years. Focusing on the responsiveness of futures rates rather than yields provides a more complete reading of the term structure response for short- and intermediate-term maturities. Table 4 reports the estimated coefficients and their standard deviations, and the pattern of coefficients across all maturities is shown in Figure 4.

The responses of the futures rates under the heteroskedasticity-based estimator $\widehat{\alpha}_{het}^{all}$ are sizable and strongly significant across all the horizons considered. The responses build over the first several quarters, suggesting that the policy surprise leads to some expectations of a continuation of the short-term interest rate in the same direction, and then gradually

[15] Kuttner identifies policy shocks by looking at changes in the federal funds futures rate rather than the eurodollar futures rate. As discussed above, this is an alternative measure of policy shocks that will, to some extent, include surprises in the timing of monetary policy moves. Perhaps reflecting the presence of such timing surprises, he finds that the three-month interest rate moves only by 0.63 of the policy shock (see his Table 5).

decline at longer horizons. A similar pattern is found under the $\widehat{\alpha}_{het}^{i}$ estimator, and the test statistic $\widehat{\delta}_{all,i}$ indicates that the over-identifying restrictions of the heteroskedasticity-based estimators are easily accepted.

Table 4: The Response of Eurodollar Futures Rates to Monetary Policy

	Estimator: $\widehat{\alpha}_{het}^{i}$		Estimator: $\widehat{\alpha}_{het}^{all}$		Estimator: $\widehat{\alpha}_{es}$	
	Coef	Std Dev	Coef	Std Dev	Coef	Std Dev
ΔED_2	1.223	0.065	1.186	0.058	1.234	0.057
ΔED_3	1.308	0.104	1.259	0.092	1.344	0.096
ΔED_4	1.325	0.126	1.239	0.113	1.377	0.118
ΔED_5	1.230	0.139	1.123	0.125	1.298	0.130
ΔED_6	1.154	0.142	1.047	0.127	1.225	0.132
ΔED_7	1.056	0.144	0.944	0.129	1.142	0.133
ΔED_8	0.985	0.146	0.879	0.131	1.088	0.134
ΔED_9	0.911	0.148	0.808	0.133	1.033	0.134
ΔED_{10}	0.856	0.149	0.755	0.134	0.985	0.134
ΔED_{11}	0.823	0.149	0.728	0.134	0.956	0.133
ΔED_{12}	0.797	0.149	0.698	0.134	0.931	0.133
ΔED_{13}	0.741	0.151	0.649	0.135	0.888	0.132
ΔED_{14}	0.727	0.151	0.633	0.136	0.875	0.132
ΔED_{15}	0.717	0.151	0.618	0.136	0.861	0.131
ΔED_{16}	0.687	0.152	0.592	0.136	0.836	0.132
ΔED_{17}	0.666	0.154	0.569	0.138	0.814	0.132
ΔED_{18}	0.641	0.156	0.552	0.140	0.799	0.134
ΔED_{19}	0.641	0.156	0.553	0.140	0.797	0.133
			F-test	P-value		
Test of O.I. Restrictions: $\widehat{\delta}_{all,i}$			0.626	0.875		
Test of E.S. Assumptions: $\widehat{\delta}_{es,all}$			0.615	0.884		

Both test statistics are distributed F(19,145). The value of the 95th percentile is 1.68. The contracts expire quarterly, with ΔED_2 representing the change in the three-month interest rate one quarter ahead, ΔED_5 the change one year ahead, and so on.

As found above for Treasury yields, the heteroskedasticity-based estimates are below the event-study estimates by a considerable amount, likely reflecting the bias in the event-study estimates that arises for the reasons discussed above. The largest differences occur at longer maturities, suggesting that the event-study assumptions are increasingly violated as the horizon lengthens. In contrast, the event-study estimates are fairly close to the heteroskedasticity-based estimates for eurodollar contracts with very short maturities. The policy news may in fact be the dominant influence over the eurodollar rate at those maturities, in which case the event-study assumtions nearly hold. Looking across all maturities, the test of the equality of the heteroskedasticity-based and event-study estimates $\widehat{\delta}_{es,all}$ is not rejected, although the test does reject the hypothesis if it is restricted to contracts expiring further out.[16]

In summary, the results suggest that some bias exists in the event-study estimates for all of the assets considered, with the heteroskedasticity-based estimators indicating a larger impact of monetary policy on stock prices and a smaller impact on longer-term interest rates. These results are robust to changes in the specification. For example, we obtain qualitatively similar results if we allow for lags in equations (1) and (2) and perform the same analysis on the reduced-form residuals. In that case the point estimates change very little and the rejections of overidentification tests occurred in the same circumstances as found above. In addition, we have repeated the analysis defining the $\tilde{F}$ subsample as the two days or five days prior to the policy dates. Again, the results are very similar.

6 Conclusions

This paper has demonstrated that the response of asset prices and market interest rates to changes in monetary policy can be estimated from the heteroskedasticity of policy shocks that takes place on particular dates, including days of FOMC meetings and of the Chairman's semi-annual monetary policy testimony to Congress. We show that the correlation between the policy rate and these other asset prices shifts importantly on those dates, as one would expect given the greater importance of policy shocks. Using this time series property, we define a heteroskedasticity-based estimator of the response of asset prices to monetary policy. Moreover, we implement this method with a simple instrumental variable approach.

The heteroskedasticty-based estimators are of the same spirit as the event-study esti-

[16]In particular, the equality of the estimates for contracts expiring beyond four years can be rejected at the 0.02 significance level.

mators that have been used in previous studies, only where the "event" is a change in the volatility of policy news. Indeed, the event-study method can be seen as an extreme case of our heteroskedasticity-based estimator, in which the shift in the variance of the policy shock is so large that it dominates all other shocks. However, as we show, such a strong assumption is not needed to identify the impact of monetary policy. We instead estimate the relevant parameter by relying only on a shift in the relative importance of the shocks.

The results indicate that increases in the short-term interest rate have a negative impact on stock prices, with the largest effect on the Nasdaq index, and a significant positive effect on market interest rates, with the largest effect on rates with shorter maturities.

The estimated parameters are compared extensively to those found under the event-study method that is used in nearly all of the existing literature. The differences across the coefficients found under the two approaches suggest that there is some bias in the event-study estimates. In particular, the heteroskedasticity-based results find a larger negative impact of monetary policy on the stock market and a smaller positive impact on market interest rates, particularly at longer maturities. These differences are in the direction that one might have expected from the potential biases of the event-study estimates arising from endogeneity and omitted variables.

The differences across these estimators are used to statistically test whether the assumptions underlying the event-study approach are satisfied—something that has been absent in that literature. The biases in the event-study estimates are found to be statistically significant for stock prices and longer-term interest rates, indicating that the event-study assumptions do not strictly hold in those cases. In contrast, there appears to be little bias in the event-study estimates of the response of market interest rates with shorter maturities, perhaps because news about the path of policy is likely to be the predominant influence on those rates.

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A Properties of the Instrumental Variables Implementation

In this appendix we study the validity of the instruments introduced in the text, as well as their asymptotic properties. For convenience, we replicate the model used. Assume that the change in the interest rate and in the asset price are determined by the following equations:

$$\begin{aligned}\Delta i_t &= \beta\Delta s_t + \gamma z_t + \varepsilon_t \\ \Delta s_t &= \alpha\Delta i_t + z_t + \eta_t,\end{aligned}$$

where Δi_t and Δs_t are $T \times 1$ vectors and ε_t, η_t, and z_t are the structural shocks described in the text. As in the text, denote policy and non-policy dates as F and $\tilde{F}$, respectively, and define the following variables across all dates in the two subsample:

$$\begin{aligned}\Delta i &= \{\Delta i_t | t \in F\} \bigcup \{\Delta i_t | t+1 \in F\} \\ \Delta s &= \{\Delta s_t | t \in F\} \bigcup \{\Delta s_t | t+1 \in F\},\end{aligned}$$

where we are assuming that $\tilde{F}$ is the set of days that precede those days in F.

A.1 Validity of the instruments:

First we investigate the validity of the instruments.

Proposition 1 *If the variance of η_t and z_t are constant between the policy days (F) and the non-policy days $(\tilde{F})$, then the stochastic variables defined as*

$$\begin{aligned}\omega_i &= \{\Delta i_t | t \in F\} \bigcup \{-\Delta i_t | t+1 \in F\} \\ \omega_s &= \{\Delta s_t | t \in F\} \bigcup \{-\Delta s_t | t+1 \in F\} \;\; for\ all\ s_t \in S_t\end{aligned}$$

are all valid instruments to estimate α.

Proof. A valid instrument needs to be correlated with the explanatory variables from the regression but uncorrelated with the residuals. The reduced form of the above equations is given by

$$\begin{aligned}\Delta i_t &= \frac{1}{1-\alpha\beta}\left[\beta\eta_t + \varepsilon_t + (\beta+\gamma)\, z_t\right] \\ \Delta s_t &= \frac{1}{1-\alpha\beta}\left[\eta_t + \alpha\varepsilon_t + (1+\alpha\gamma)\, z_t\right],\end{aligned}$$

and the equation that we estimate is the following:

$$\Delta s = \alpha\Delta i + z + \eta,$$

where z and η are stacked across all dates in the same manner as Δi and Δs. Under the assumptions that

$$
\begin{aligned}
\sigma_\varepsilon^F &= \sigma_\varepsilon^{\tilde{F}} + \Delta\sigma_\varepsilon \\
\sigma_\eta^F &= \sigma_\eta^{\tilde{F}} \\
\sigma_z^F &= \sigma_z^{\tilde{F}},
\end{aligned}
$$

each instrument is correlated with the explanatory variable Δi:

$$
\begin{aligned}
\text{plim}\,\frac{1}{T}\omega_i'\Delta i &= \frac{1}{T}\sum_{t\in F}\Delta i_t^2 - \frac{1}{T}\sum_{t+1\in F}\Delta i_t^2 \\
&\longrightarrow Var\left(\Delta i_t|F\right) - Var\left(\Delta i_t|\tilde{F}\right) \\
&= \frac{1}{(1-\alpha\beta)^2}\Delta\sigma_\varepsilon \neq 0 \\
\text{plim}\,\frac{1}{T}\omega_s'\Delta i &= \frac{1}{T}\sum_{t\in F}\Delta s_t\Delta i_t - \frac{1}{T}\sum_{t+1\in F}\Delta s_t\Delta i_t \\
&\longrightarrow Cov\left(\Delta s_t,\Delta i_t|F\right) - Cov\left(\Delta s_t,\Delta i_t|\tilde{F}\right) \\
&= \frac{\alpha}{(1-\alpha\beta)^2}\Delta\sigma_\varepsilon \neq 0.
\end{aligned}
$$

The second step is to show that the instruments are uncorrelated with the residuals. To do so, we show that the instrument is uncorrelated with each of the structural shocks in the estimated equation, z and η:

$$
\begin{aligned}
\text{plim}\,\frac{1}{T}\omega_i'z &= \frac{1}{T}\sum_{t\in F}\Delta i_t z_t - \frac{1}{T}\sum_{t+1\in F}\Delta i_t z_t \longrightarrow \frac{\beta+\gamma}{1-\alpha\beta}\sigma_z^F - \frac{\beta+\gamma}{1-\alpha\beta}\sigma_z^{\tilde{F}} = 0 \\
\text{plim}\,\frac{1}{T}\omega_i'\eta &= \frac{1}{T}\sum_{t\in F}\Delta i_t \eta_t - \frac{1}{T}\sum_{t+1\in F}\Delta i_t \eta_t \longrightarrow \frac{1}{1-\alpha\beta}\sigma_\eta^F - \frac{1}{1-\alpha\beta}\sigma_\eta^{\tilde{F}} = 0.
\end{aligned}
$$

Similar equations are obtained for the other instrument:

$$
\begin{aligned}
\text{plim}\,\frac{1}{T}\omega_s'z &= \frac{1}{T}\sum_{t\in F}\Delta s_t z_t - \frac{1}{T}\sum_{t+1\in F}\Delta s_t z_t \longrightarrow \frac{1+\alpha\gamma}{1-\alpha\beta}\sigma_z^F - \frac{1+\alpha\gamma}{1-\alpha\beta}\sigma_z^{\tilde{F}} = 0 \\
\text{plim}\,\frac{1}{T}\omega_s'\eta &= \frac{1}{T}\sum_{t\in F}\Delta s_t \eta_t - \frac{1}{T}\sum_{t+1\in F}\Delta s_t \eta_t \longrightarrow \frac{\alpha}{1-\alpha\beta}\sigma_\eta^F - \frac{\alpha}{1-\alpha\beta}\sigma_\eta^{\tilde{F}} = 0.
\end{aligned}
$$

■

Note that the above proof demonstrates that the instrument ω_s defined using any $s \in S$ is valid for estimating the response of any asset price to the interest rate. As a result, the instrument set defined by equation (15), which is a linear combination of all the asset prices included, is also valid. Moreover, that instrument will be valid as long as any of the included asset prices are correlated with the interest rate, which makes it a more robust instrument.

Of course, the assumptions of the model were maintained in the above proof. If those assumptions are not met, then the validity of the instruments breaks down, as summarized in the following remarks:

Remark 2 *If the parameters are unstable* ($\{\alpha, \beta, \gamma\}_F \neq \{\alpha, \beta, \gamma\}_{\neg F}$), *then the instruments are not valid.*

There are two possible cases in which the stability of the parameters is violated. First, the underlying parameters may simply shift on policy dates. Second, the true model may be non-linear. One advantage of our method is that if the model is non-linear or if the parameters shift, then the overidentifying assumptions should be rejected.

Remark 3 *Note that if the assumptions on the heteroskedasticity of the policy shocks* $\left(\sigma_\varepsilon^F > \sigma_\varepsilon^{\neg F}\right)$ *and the homoskedasticity of the stock market shocks* $\left(\sigma_\eta^F = \sigma_\eta^{\neg F}\right)$ *and of the common shocks* $\left(\sigma_z^F = \sigma_z^{\neg F}\right)$ *are not satisfied, then the instruments are not valid.*

These two remarks are immediately apparent from the proof of Proposition 1.

A.2 Asymptotic properties.

Next we study the asymptotic properties of the instrumental variables estimates. The estimates are

$$\begin{aligned}\widehat{\alpha}_{het}^i &= \left(\omega_i{}' \Delta i\right)^{-1}\left(\omega_i{}' \Delta s\right) \\ \widehat{\alpha}_{het}^s &= \left(\omega_s{}' \Delta i\right)^{-1}\left(\omega_s{}' \Delta s\right).\end{aligned}$$

Substituting, the estimates are

$$\begin{aligned}\widehat{\alpha}_{het}^i &= \alpha + \left(\omega_i{}' \Delta i\right)^{-1}\left(\omega_i{}' \left(z_t + \eta_t\right)\right) \\ \widehat{\alpha}_{het}^s &= \alpha + \left(\omega_s{}' \Delta i\right)^{-1}\left(\omega_s{}' \left(z_t + \eta_t\right)\right).\end{aligned}$$

As we showed above,

$$\begin{aligned}\operatorname{plim} \frac{1}{T}\omega_i{}' \left(z_t + \eta_t\right) &\longrightarrow 0 \\ \operatorname{plim} \frac{1}{T}\omega_s{}' \left(z_t + \eta_t\right) &\longrightarrow 0\end{aligned}$$

and

$$\begin{aligned}\operatorname{plim} \frac{1}{T}\omega_i{}' \Delta i &\longrightarrow \frac{1}{(1-\alpha\beta)^2} M_\varepsilon \\ \operatorname{plim} \frac{1}{T}\omega_s{}' \Delta i &\longrightarrow \frac{\alpha}{(1-\alpha\beta)^2} M_\varepsilon,\end{aligned}$$

where

$$\operatorname{plim} \frac{1}{T}\varepsilon_t{}' \varepsilon_t | t \in F - \operatorname{plim} \frac{1}{T}\varepsilon_t{}' \varepsilon_t | t+1 \in F \longrightarrow M_\varepsilon = \Delta\sigma_\varepsilon.$$

Therefore,

$$\begin{aligned} \operatorname{plim} \widehat{\alpha}_{het}^{i} &\longrightarrow \alpha \\ \operatorname{plim} \widehat{\alpha}_{het}^{s} &\longrightarrow \alpha. \end{aligned}$$

This shows that consistency is obtained as the result of the validity of the instruments.

We now demonstrate that the estimates are asymptotically distributed as follows:

$$\begin{aligned} \sqrt{T}\left(\widehat{\alpha}_{het}^{i}-\alpha\right) &\underset{d}{\longrightarrow} N\left(0,\Omega_i\right) \\ \sqrt{T}\left(\widehat{\alpha}_{het}^{s}-\alpha\right) &\underset{d}{\longrightarrow} N\left(0,\Omega_s\right). \end{aligned}$$

Without loss of generality, the arguments about asymptotic normality are only developed for the first estimate. Substituting the definitions of the instrument as above, we obtain

$$\sqrt{T}\left(\widehat{\alpha}_{het}^{i}-\alpha\right) = \left(\frac{1}{T}\omega_i{}'\,\Delta i\right)^{-1}\left(\frac{1}{\sqrt{T}}\omega_i{}'\,(z+\eta)\right).$$

As was argued before, $\operatorname{plim} \frac{1}{T}\omega_i{}'\,\Delta i \longrightarrow \frac{1}{(1-\alpha\beta)^2}M_\varepsilon$. By the central limit theorem, the variable $\frac{1}{\sqrt{T}}\omega_i{}'\,(z+\eta)$ is asymptotically normal with asymptotic variance $M \equiv \frac{1}{T}\omega_i{}'\,(\sigma_z+\sigma_\eta)\,\omega_i$, where σ_z and σ_η are the variance of the structural shocks across both the F and $\tilde{}F$ dates, which under the null hypothesis are equal.

Thus, the estimator has the asymptotic covariance matrix $(1-\alpha\beta)^4\,M_\varepsilon^{-1}MM_\varepsilon^{-1\,\prime}$. The estimator is not asymptotically efficient, as suggested by the fact that the asymptotic covariance matrix does not collapse into a single matrix.

Lastly, note that the previous proofs are based on the assumption that the number of policy days and non-policy days go to infinity. In small samples the following adjustment to the instruments is needed to ensure that the estimators are unbiased:

$$\begin{aligned} \omega_i &= \left\{\frac{\Delta i_t}{T_F-L}|t\in F\right\}\bigcup\left\{-\frac{\Delta i_t}{T_{\tilde{}F}-L}|t+1\in F\right\} \\ \omega_s &= \left\{\frac{\Delta s_t}{T_F-L}|t\in F\right\}\bigcup\left\{-\frac{\Delta s_t}{T_{\tilde{}F}-L}|t+1\in F\right\}, \end{aligned}$$

where L is the number of parameters estimated.

B Identification under More General Conditions

In the main text, we assumed that the shocks across different FOMC days have the same variance, but this is unlikely to be the case. In this section we show that our estimator is still consistent even if we allow the variables to be heteroskedastic over time.

For convenience, we replicate the model used. Assume that the change in the interest rate and in the asset price are determined by the following equations:

$$\begin{aligned}\Delta i_t &= \beta \Delta s_t + \gamma z_t + \varepsilon_t \\ \Delta s_t &= \alpha \Delta i_t + z_t + \eta_t.\end{aligned}$$

The structural shocks are assumed to be uncorrelated and to have conditional moments given by

$$\begin{aligned}E\left[\varepsilon_t | I_{t-1}\right] &= 0,\ E\left[\varepsilon_t^2 | I_{t-1}\right] = \sigma_{\varepsilon,t} \\ E\left[\eta_t | I_{t-1}\right] &= 0,\ E\left[\eta_t^2 | I_{t-1}\right] = \sigma_{\eta,t} \\ E\left[z_t | I_{t-1}\right] &= 0,\ E\left[z_t^2 | I_{t-1}\right] = \sigma_{z,t},\end{aligned}$$

where I_t is the information set at time t. The reduced form of the above equations is as follows:

$$\begin{aligned}\Delta i &= \frac{1}{1-\alpha\beta}\left[\beta\eta_t + \varepsilon_t + (\beta+\gamma) z_t\right] \\ \Delta s &= \frac{1}{1-\alpha\beta}\left[\eta_t + \alpha\varepsilon_t + (\beta+\gamma) z_t\right].\end{aligned}$$

The same procedure that produced consistent estimates of the responsiveness of asset prices under assumptions (6) to (8) also identifies the parameter α under more general conditions for the variances of the shocks. In particular, we now allow the variances of the structural shocks to change over time, or $\sigma^2_{\varepsilon,i} \neq \sigma^2_{\varepsilon,j}$, $\sigma^2_{\eta,i} \neq \sigma^2_{\eta,j}$, and $\sigma^2_{z,i} \neq \sigma^2_{z,j}$ for $i \neq j$ and both $i, j \in F$. This allows the variances of the shocks can change across different policy dates. However, we still assume that the variances around each policy date satisfy the following conditions:

$$\sigma^2_{\varepsilon,t-1} + \Delta\sigma^2_{\varepsilon,t} = \sigma^2_{\varepsilon,t} \tag{17}$$

$$\sigma^2_{\eta,t-1} = \sigma^2_{\eta,t} \tag{18}$$

$$\sigma^2_{z,t-1} = \sigma^2_{z,t}, \tag{19}$$

where we have taken the set of non-policy dates to be the days preceeding each policy date. In other words, each shock can exhibit heteroskedasticity through the sample, but the change in the variances around each policy day must be explained entirely by the shift in the volatility of the policy shocks.

Under conditions (17) to (19), the procedure described in the text provides a consistent estimate for α under the following assumption:

Assumption 4 *Assume ε_t, η_t, and z_t have finite fourth moments, and that there exists an*

$M < \infty$ *such that*

$$E\left[\varepsilon_t^4 | I_{t-1}\right] < M, \;\; E\left[\eta_t^4 | I_{t-1}\right] < M, \;\; E\left[z_t^4 | I_{t-1}\right] < M.$$

Under that assumption, the following proposition holds:

Proposition 5 *The reduced form residuals are uniformly integrable.*

Proof. The reduced form residuals are a linear combination of three uncorrelated finite fourth moment shocks. Hamilton [1994] (proposition 7.7, page 191) shows that a linear combination of stochastic variables with at least finite second moments is uniformly integrable. ■

Define, as in the main text, the sequence of changes in the interest rate and asset price during policy days and prior days as follows:

$$\begin{aligned} X_F &\equiv \{(\Delta i_t, \Delta s_t) \, | t \in F\} \\ X_{\tilde{\ }F} &\equiv \{(\Delta i_t, \Delta s_t) \, | t+1 \in F\}. \end{aligned}$$

The following proposition then holds:

Proposition 6 *Define the martingale difference sequence for the policy dates and the dates immediately preceding the policy dates as follows:*

$$\tilde{X}_F \equiv \frac{1}{T_F} \sum X_F, \quad \tilde{X}_{\tilde{\ }F} \equiv \frac{1}{T_{\tilde{\ }F}} \sum X_{\tilde{\ }F},$$

where T_F *and* $T_{\tilde{\ }F}$ *are the number of observations in* F *and* $\tilde{\ }F$*, respectively. Then,* $\sqrt{T_F}\tilde{X}_F$ *and* $\sqrt{T_{\tilde{\ }F}}\tilde{X}_{\tilde{\ }F}$ *are normally distributed with mean zero and variances*

$$\begin{aligned} \frac{1}{T_F} \sum \sigma_{F,t} &\longrightarrow \tilde{\sigma}_F \\ \frac{1}{T_{\tilde{\ }F}} \sum \sigma_{\tilde{\ }F,t} &\longrightarrow \tilde{\sigma}_{\tilde{\ }F}. \end{aligned}$$

Proof. The reduced form variables have mean zero, given that the structural shocks are assumed to have mean zero. Therefore, $\tilde{X}_F$ and $\tilde{X}_{\tilde{\ }F}$ are martingale difference sequences. White [1984] (corollary 5.25, page 130) shows that if $\tilde{X}_i$ is a scalar martingale difference sequence, with finite fourth moments, then $\sqrt{T_i}\tilde{X}_i$ is asymptotically normal with limiting variance $\tilde{\sigma}_i$. ■

These assumptions and propositions indicate that the interest rates and the vector of asset prices are asymptotically joint normal for any subset of dates in the sample. In particular,

$$\begin{aligned} \frac{1}{T_F} \sum_{t \in F} (\Delta i_t, \Delta s_t)' (\Delta i_t, \Delta s_t) &\longrightarrow \Omega_F \\ \frac{1}{T_{\tilde{\ }F}} \sum_{t+1 \in F} (\Delta i_t, \Delta s_t)' (\Delta i_t, \Delta s_t) &\longrightarrow \Omega_{\tilde{\ }F} \end{aligned}$$

where

$$
\begin{aligned}
\Omega_F &= \frac{1}{(1-\alpha\beta)^2}\begin{bmatrix} (\beta+\gamma)^2\,\Sigma_z^F+\beta^2\Sigma_\eta^F+\Sigma_\varepsilon^F & (\beta+\gamma)(1+\alpha\gamma)\,\Sigma_z^F+\beta\Sigma_\eta^F+\alpha\Sigma_\varepsilon^F \\ & (1+\alpha\gamma)^2\,\Sigma_z^F+\Sigma_\eta^F+\alpha^2\Sigma_\varepsilon^F \end{bmatrix} \\
\Omega_{\tilde{F}} &= \frac{1}{(1-\alpha\beta)^2}\begin{bmatrix} (\beta+\gamma)^2\,\Sigma_z^{\tilde{F}}+\beta^2\Sigma_\eta^{\tilde{F}}+\Sigma_\varepsilon^{\tilde{F}} & (\beta+\gamma)(1+\alpha\gamma)\,\Sigma_z^{\tilde{F}}+\beta\Sigma_\eta^{\tilde{F}}+\alpha\Sigma_\varepsilon^{\tilde{F}} \\ & (1+\alpha\gamma)^2\,\Sigma_z^{\tilde{F}}+\Sigma_\eta^{\tilde{F}}+\alpha^2\Sigma_\varepsilon^{\tilde{F}} \end{bmatrix}
\end{aligned}
$$

and the variances of the shocks in the elements of the matrix are given by

$$
\begin{aligned}
\frac{1}{T_F}\sum_{t\in F}\varepsilon_t^2 &\longrightarrow \Sigma_\varepsilon^F, \\
\frac{1}{T_F}\sum_{t\in F}\eta_t^2 &\longrightarrow \Sigma_\eta^F, \\
\frac{1}{T_F}\sum_{t\in F}z_t^2 &\longrightarrow \Sigma_z^F,
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{1}{T_{\tilde{F}}}\sum_{t+1\in F}\varepsilon_t^2 &\longrightarrow \Sigma_\varepsilon^{\tilde{F}}, \\
\frac{1}{T_{\tilde{F}}}\sum_{t+1\in F}\eta_t^2 &\longrightarrow \Sigma_\eta^{\tilde{F}}, \\
\frac{1}{T_{\tilde{F}}}\sum_{t+1\in F}z_t^2 &\longrightarrow \Sigma_z^{\tilde{F}}.
\end{aligned}
$$

The result then follows under the same assumption about the homoskedasticity of the asset price shocks and the common shock:

Assumption 7 *As before, assume that the asset prices shocks and the common shock have the same variance before and during the policy day. More specifically,*

$$\Sigma_\eta^{\tilde{F}} = \Sigma_\eta^F, \quad \textit{and} \quad \Sigma_z^{\tilde{F}} = \Sigma_z^F.$$

Under assumptions 4 and 7, the change in the covariance matrix between non-policy and policy dates is as follows:

$$\Delta\Omega \equiv \Omega_F - \Omega_{\tilde{F}} = \frac{\Sigma_\eta^F - \Sigma_\eta^{\tilde{F}}}{(1-\alpha\beta)^2}\begin{bmatrix} 1 & \alpha \\ & \alpha^2 \end{bmatrix},$$

which implies that the coefficient is identified in the same manner as in the text.

Figure 1
Joint Determination of Interest Rates and Asset Prices

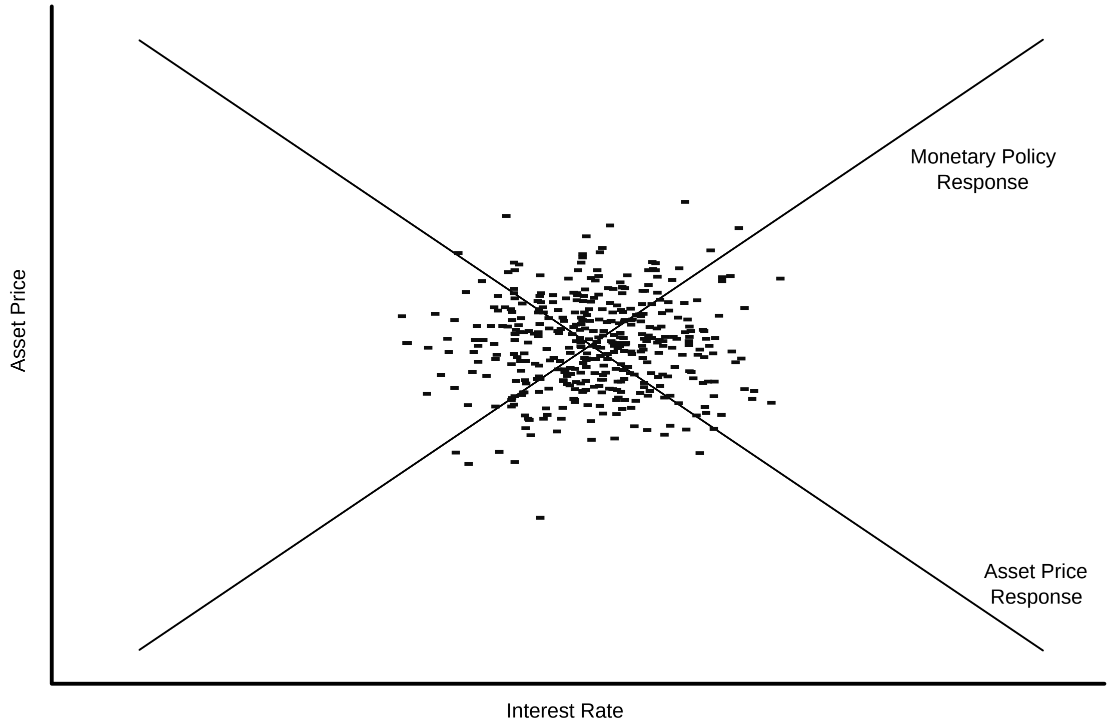

Figure 2
Policy Dates

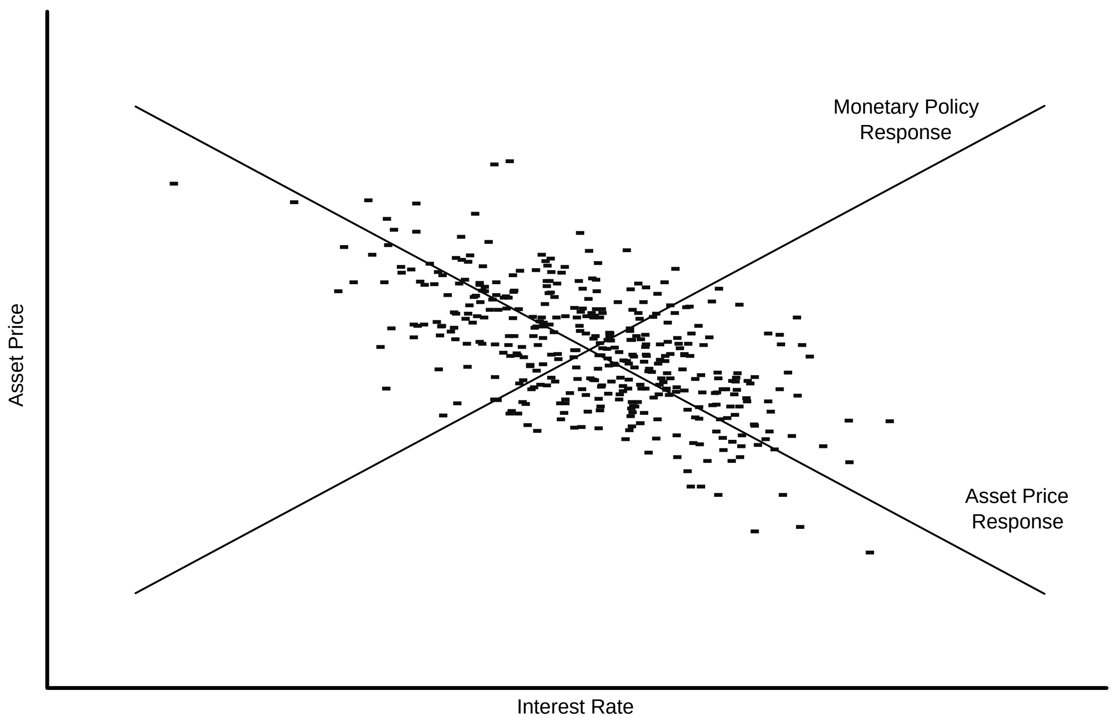

Figure 3
Responsiveness of Treasury Rates

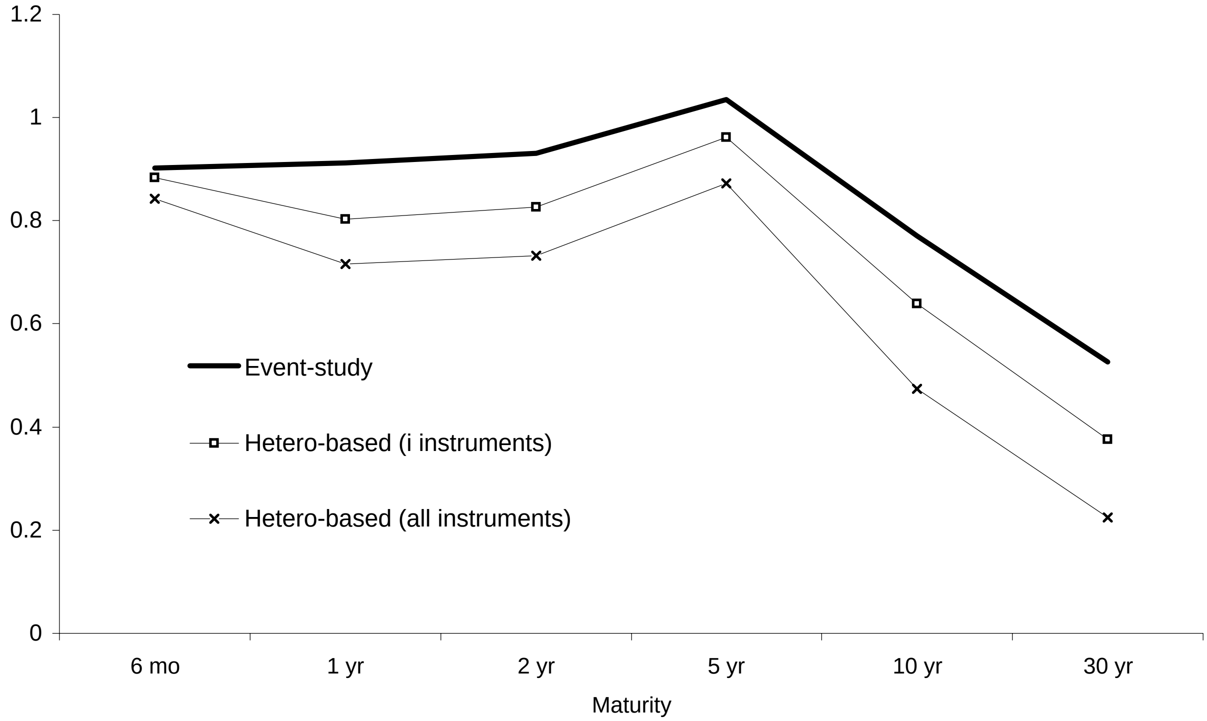

Figure 4
Responsiveness of Eurodollar Futures Rates