Growth Effects of Progressive Taxes*

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Abstract

Criticisms of endogenous growth models with flat rate taxes have highlighted two features that are not substantiated by the data. These models generally imply: (1) that economic growth must fall with the share of government expenditures in output across countries, and (2) that one-time shifts in marginal tax rates should instantenously lead to similar shifts in output growth. In contrast, we show that allowing for heterogeneous households and progressive taxes into otherwise conventional linear growth models radically changes these predictions. In particular, economic growth does not have to fall, and may even increase, with the share of government expenditures in output across countries. Moreover, discrete permanent shifts in tax policy now lead to protracted transitions between balanced growth paths. Both of these findings hold whether or not government expenditures are thought to be productive and better conform to available empirical evidence.

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The evidence that tax rates matter for growth is disturbingly fragile. This empirical fragility contrasts sharply with the robustness of the theoretical predictions: most models predict that income and investment taxes are detrimental to growth.

William Easterly and Sergio Rebelo, (1993)

1 Introduction

A central tenet of early endogenous growth models was that cross-country differences in average growth rates could be explained by variations in government policy. In contrast to the older neoclassical framework, where long-run growth was exogenously determined by the rate of technical progress, these models predicted that permanent variations in tax rates would give rise to different steady-state growth rates. However, many of the cross-country studies that followed were unable to confirm this prediction and shed doubt on the validity of the endogenous growth framework.[1] Jones (1995), and Stokey and Rebelo (1995) further argued that U.S. time series data was at odds with the implications of linear growth models. On the basis of these models, the dramatic increase in income taxation which took place in the early 1940s would have been expected to contemporaneously decrease the U.S. per capita growth rate. This did not appear to be the case. Thus, both cross-country studies and time series empirical evidence have suggested that long-run growth is independent of fiscal policy. In this paper, we present a case against this conclusion. Specifically, we argue that relaxing two important assumptions of the early endogenous growth framework, namely flat rate taxes and a representative agent, helps reconcile theory and data.

One of the more pressing problems intrinsic to any cross-country investigation of the growth effects of taxation is that marginal tax rates are not easily observable. Since the data on individual income and taxes required to compute these rates is only available for a small set of economies, one is forced to choose an appropriate proxy. Under the assumption that taxes are proportional, these proxies typically involve using some measure of tax revenue (or government expenditures) as a fraction of GDP, or regressing this revenue on its tax base (see Koester and Kormendi [1989]). If the government does not play a productive role, then endogenous growth models directly imply that per capita output growth should be decreasing in these ratios across economies. In contrast, cross-country data suggest that economic growth either does not change much or even increases with the share of tax revenue in GDP. In this paper, we show that this cross-sectional finding is consistent with standard endogenous growth models once progressive taxes are allowed and the income and wealth distributions are

[1] See, for instance, Levine and Renelt (1992), Levine and Zervos (1993).

nondegenerate. Furthermore, this result holds whether or not public expenditures contribute to private production.

There is little disagreement that tax policy is not only nonlinear but also varies substantially across economies (see Sicat and Virmani [1988]). The extent to which country-specific tax systems are considered progressive, however, is generally sensitive to the choice or progressivity index.[2] That being said, as a tax code becomes more progressive, one expects marginal tax rates for the wealthy to increase relative to the poor. This notion has two important consequences that help explain why economic growth may increase, rather than fall, with the ratio of tax revenue to GDP.

First, richer agents have less incentive to accumulate wealth, both physical and human, and may ultimately have lower *pre-tax* earnings in equilibrium. In developing economies, it is also common for these agents to spend substantial resources escaping taxation. In the end, as the degree of tax progressivity increases, it is not clear that the share of tax revenue in GDP should rise and, in fact, it may even decline. Second, because more progressive tax systems are generally more distortional (see Sarte [1997], and Castañeda, Diaz-Gimenez, and Rios-Rull [1999]), they are also more likely to be associated with lower economic growth *ceteris paribus*. Put together, these two observations imply that variations in progressivity across economies cause output growth and the share of tax revenue (or public expenditures) in GDP either to have little correlation or to move together.

The explicit modeling of non-linear taxes also has important dynamic implications. Consider, for instance, a closed economy where all factors of production are reproducible and the production technology is linear. This is the well-known Ak framework. When the marginal tax rate increases with income, the after-tax rate of interest is no longer invariant to changes in the composite capital good . Therefore, contrary to the original framework, a change in tax policy will induce some transitional dynamics as the economy moves from one balanced growth path to another (see Yamarik [2000]). Interestingly, in Rebelo's (1991) original linear growth model, transitional dynamics stemming from progressive taxation play very little role as the growth effects of a tax reform occur largely on impact. However, when government spending is allowed a productive role as in Barro (1990), the initial adverse effects of a tax increase on growth are small, and may even be reversed. In addition, the transition to a new balanced growth path may now be quite protracted which, consistent with Jones (1995), would make it difficult to identify the growth effects of tax changes in time series data.

This paper is organized as follows. In section 2, we briefly review previous cross-country

[2] Silber (1994) works out theoretical cases in which popular indices, such as Kakwani's as well as Musgrave and Thin's, rank tax system differently. Bishop, Formby, and Zheng (1998) show that these cases are empirically relevant.

evidence on fiscal policy and economic growth. Section 3 introduces our modeling of progressive taxes which stays constant across the different frameworks we consider. Section 4 revisits Rebelo's (1991) original endogenous growth model with progressive taxes and heterogenous households. In section 5, we allow government expenditures to play a productive role along the lines of Barro (1990), and Glomm and Ravikumar (1994, 1997). Section 6 offers concluding remarks.

2 Fiscal Policy and Economic Growth in the Cross-Section

Under the assumptions of proportional taxes and a representative agent, endogenous growth models typically predict a negative correlation between growth, γ, and the ratio of public spending to GDP, G/Y. This negative correlation reflects the distortional effects of taxation in that , with proportional taxes, $G/Y = \tau$. This prediction has long been a hallmark of the endogenous growth literature. However, empirical cross-country growth studies, notably by Levine and Renelt (1992), and Levine and Zervos (1993), have not been able to confirm this negative correlation.

Figures (1a) and (1b) illustrate this notion. Figure 1, panel (a) plots average per-capita growth rates versus taxes on income, profits, and capital gains as a fraction of GDP across 74 countries over the period 1976-1997. Figure 1, panel (b) illustrates the link between per-capita growth rates and the ratio of government expenditures to GDP for the same set of countries. The data is obtained from the World Development Indicators published by the World Bank in 2000. If anything, the link between per-capita growth rates and the relative size of public expenditures is *increasing*. To the extent that issues such as tax evasion are an important consideration in developing economies, Figures 1, panels (c) and (d) illustrate the same relationships as in Figures (1a) and (1b) for OECD countries only. In both cases, the data fail to establish a negative link between the relative size of government and per-capita output growth.[3]

On a less equivocal note, Tanzi and Zee (2000) argue that the relative size of government is actually higher for richer countries. They find "that for the period 1985-1987, the average total tax level in developing countries was about 17.5 percent of GDP. ... In contrast, the average total tax level in OECD countries in the same period was more than twice as high (36.6 percent of GDP), although there was significant variance across the OECD subcountry

[3]Levine and Renelt (1992) argue that this result continues to hold even when a wide range of conditioning variables are taken into account, including intial income.

groups. Essentially all of the foregoing comparative observations are equally applicable to the tax revenue data for the period 1995-1997."

To account for these cross-sectional relationships, the next sections explore the growth effects of progressive taxes in two prototypical endogenous growth models augmented to include a non degenerate distribution of income. These models, one first formulated by Barro (1990), and the other by Rebelo (1991), account for two polar assumptions regarding the use of public expenditures. At one extreme, in Rebelo's (1991) two-sector framework, government spending does not play a productive role. At the other extreme, in the environment envisioned by Barro (1990), all tax revenue serves to finance public services that enter as an input into private production. In both cases, we show that long-run growth can increase with the ratio of tax revenue to GDP as in the cross-section. In essence, the fact that taxes are progressive drives a wedge between the average marginal tax rate and the ratio of tax revenue to GDP; the distortional effects of higher marginal tax rates remain, but cannot be captured empirically with the latter ratio. Contrary to the original models, we also show that changes in tax policy now induce distinct short-run and long-run effects on economic growth. In Barro's model, the adverse effects of an increase in taxes are reversed initially, and there exists a long transition from one balanced growth path to another. This finding helps explain why the growth effects of changes in tax policy may be difficult to identify in time series data.

3 Progressive Taxation

We begin by describing the modeling of tax policy, which is common across the frameworks we consider. The government balances its budget at each point in time and chooses a tax code summarized by the tax rate, $\tau(y/Y)$, where y denotes household income and Y is aggregate income. Thus, the tax rate which applies to a given household depends only on its standing in the economy. This modeling assumption ensures that not all households eventually face the highest marginal tax rate simply as a result of economic growth. In other words, for the purpose of this paper, we abstract from tax drift considerations.[4] In the analysis below, we further assume that the government sets $\tau(y/Y)$ according to the following tax schedule,

$$\tau(\frac{y}{Y}) = \zeta \left(\frac{y}{Y}\right)^{\phi}, \text{ with } 0 \leq \zeta < 1, \ \phi > 0, \tag{1}$$

[4] This phenomenon is also known as "bracket creep". As part of the Cato Institute's policy recommendations to the 106th U.S. Congress, Moore (1999) suggests that "real income bracket creep should be ended by indexing tax brackets for inflation plus real income growth. ... In 1998, for example, worker incomes rose by a respectable 6 percent, but tax receipts were up 10 percent. The primary culprit is real bracket creep."

where, similarly to Lansing and Guo (1998), the parameters ζ and ϕ determine the level and the slope of the tax schedule respectively. When $\phi > 0$, households with higher taxable income are subject to higher tax rates, and the more common case of proportional taxes corresponds to $\phi = 0$, $\tau(y/Y) = \zeta$. In making decisions about how much to consume and invest, households will take into account the particular way in which the tax schedule affects their earnings.

Given the tax rate in (1), $\tau(y/Y)y$ represents the total amount of taxes paid by a household with income y. Because we wish to draw the implications of progressivity on economic growth, it is helpful to distinguish between average and marginal tax rates. In this case, as taxable income changes, total taxes paid evolve according to

$$\frac{\partial\left[\tau(y/Y)y\right]}{\partial y} = \tau_m(\frac{y}{Y}) = (1+\phi)\zeta\left(\frac{y}{Y}\right)^{\phi}, \tag{2}$$

where $\tau_m(y/Y)$ is the tax rate applied to the last dollar earned. The average tax rate is simply $\tau(y/Y)$.

While there exists no single "appropriate" way to define the degree of progressivity of a tax schedule, one of the more widely used definitions is expressed in terms of the ratio of the marginal to the average tax rate. Specifically, a tax schedule is said to be progressive whenever the marginal rate exceeds the average rate at all levels of income.[5] In our set-up, equations (1) and (2) imply that $\tau_m(y/Y)/\tau(y/Y)$ is simply $1 + \phi$, so that the parameter ϕ captures the degree of progressivity in the tax code. In the limit, where $\phi = 0$, the tax schedule is "flat" and $\tau_m(y/Y) = \tau(y/Y)$. Other methods of measuring progressivity involve the use of indices and attempt in part to capture the degree of tax burden borne by households at different income levels.[6] A potential problem here is that the distribution of pre-tax income is endogenous and, therefore, expected to vary in response to changes in the tax schedule. To the degree that one is concerned with effective progressivity, Creedy (1999) writes that "the tax structure alone is insufficient to judge progressivity because the overall effect of a tax structure on the distribution of tax payments and the inequality of net income cannot be assessed independently of the form of the distribution of pre-tax income." In the models below, we shall see that as ϕ increases, Kakwani's index suggests a less than perfect correlation between statutory and effective progressivity.

While we have chosen to summarize the tax code by the specification in (1) for simplicity, it can be difficult in practice to gauge the degree of statutory progressivity of a given tax schedule. Even absent tax drift, such calculations would involve sifting through the tax code

[5] See Musgrave and Musgrave (1989). Another way to define progressivity is to require that the average tax rate be increasing over all income ranges, which is also satisfied in our framework.

[6] See, for instance, Kakwani (1977) and Suits (1977).

and accounting for various deductions to be netted out of gross income, determining the income tax rate which applies to net income, and computing the credits deductible from the resultant tax liability. Sicat and Virmani (1988) manage to work out marginal statutory tax rates at discrete income levels for a number of low-income and middle-income countries. Their results show that marginal tax rates on the highest bracket vary anywhere from 30 percent (Burkina Faso) to 95 percent (Tanzania) among the low-income countries alone. In contrast, the marginal tax rate on the lowest bracket computed for the same set of countries varies only from 2 percent to 20 percent. Although the authors do not publish estimated average tax schedules, their findings are nevertheless suggestive of significant differences in statutory progressivity across economies.

In the U.S., the income tax has undergone dramatic changes over the past two decades as the result of several important pieces of legislation, notably the Economic Recovery Act of 1981 and the Tax Reform Act of 1986. These changes in law, affecting tax deductions, credit phase-outs, and the bounds on statutory tax brackets, have made it difficult to measure true marginal tax rates. Studies on the subject are limited and probably the best data can be found in a Congressional Budget Office (2001) report which documents changes in the effective, or average, federal tax rate faced by households over the period 1979 to 1997, and includes estimates that embody the 2001 tax law. Interestingly, the study finds that "progressivity has increased over the past two decades, primarily because the rate faced by households with low incomes fell by nearly a third with the expansion, in the 1990s, of the earned income tax credit." In the context of our model, we can approximate this statutory change with an increase in the curvature parameter ϕ. We should point out that effective rates overall have also declined, or at most remained constant, between 1979 and 1997. Therefore a simultaneous reduction in the shift parameter ζ is likely to have occurred during this period.

4 Long-Run Policy and Long-Run Growth Revisited

This section modifies Rebelo's (1991) original linear endogenous growth model to take account of progressive taxes. In order for progressivity to play a redistributive role, we further introduce a nondegenerate distribution of income and wealth into the environment by assuming that households differ in their rates of impatience. We adopt this method of inducing income heterogeneity mainly for tractability. There exists, however, substantial empirical work which links differences in earnings and wealth to diverse rates of time preference.[7] More

[7] See Hausman (1979), Lawrance (1991) for work using the Panel Study of Income Dynamics, Samwick (1998) for an analysis using the Survey of Consumer Finances, as well as Warner and Pleeter (2001) for an

importantly, the results in this paper hinge on the fact that relatively wealthier agents may have an incentive to reduce their pre-tax earnings relative to aggregate income as the tax schedule becomes more progressive. In principle, this channel would remain operative in models where heterogeneity arises from other considerations, such as borrowing constraints as in Hugget (1993), Ayagari (1994), and Rios-Rull (1995) among others, or permanent differences in productivity as in Caucutt, Imrohoroglu, and Kumar (2001).

Consider a closed economy populated by a large number of households uniformly distributed on $[0,1]$. There are N types of households and each household type is indexed by a discount factor β_j where $0 < \beta_1 \leq \beta_2 ... \leq \beta_N < 1$. Thus, the most patient households have discount factor β_N. Within each group, the measure of households is given by $1/N$. Each household receives income from previous savings and from 1 unit of an inelastically supplied nonreproducible factor (which we can think of as land or labor). Income is either saved or used to purchase consumption goods. Households are allowed to borrow and finance their debt out of wage income.

On the supply side, the economy remains exactly as in Rebelo (1991) and consists of two production sectors. The first sector produces investment goods, I_t, using a fraction $1-\phi_t$ of the available capital stock, Z_t, according to the linear technology $I_t = A(1-\phi_t)Z_t$. Here, Z_t is to be interpreted as a reproducible composite capital good, which includes both human and physical capital, and which can be accumulated over time. Specifically, $Z_{t+1} = I_t + (1-\delta)Z_t$, where $0 < \delta < 1$, is the capital depreciation rate. The second sector combines the remaining capital stock, $\phi_t Z_t$, with nonreproducible factors to produce consumption goods, C_t. The quantity of nonreproducible factors is denoted by T and available in fixed supply in each time period. Consumption goods are produced according to the Cobb-Douglas technology, $C_t = B(\phi_t Z_t)^\alpha T^{1-\alpha}$.

Each household of type j chooses paths of consumption, $\{c_{jt}\}_{t=0}^{\infty}$, and capital, $\{z_{jt}\}_{t=0}^{\infty}$, to solve

$$\max_{c_{jt},\, z_{jt+1}} \sum_{t=0}^{\infty} \beta_j^t \frac{c_{jt}^{1-\sigma}-1}{1-\sigma},\ \sigma > 0, \tag{P1}$$

$$\text{subject to } q_t c_{jt} + z_{jt+1} - (1-\delta) z_{jt} = y_{jt}\left[1-\zeta\left(\frac{y_{jt}}{Y_t}\right)^{\phi}\right], \tag{3}$$

$$\text{where } y_{jt} = r_t z_{jt} + u_t T,\ Y_t = \sum_{j=1}^{N} y_{jt}\frac{1}{N}, \tag{4}$$

$$\text{and } c_{jt},\ z_{jt} \geq 0 \text{ for all } j \text{ and } t,\ z_{j0} > 0 \text{ given for all } j.$$

article based on the military drawdown program of the early 1990s.

We denote the price of consumption in terms of the composite capital good by q_t. The variables r_t and u_t denote the rates of return to capital and nonreproducible factors respectively. In solving their optimal consumption-investment allocation problem, households take the sequence of prices $\{r_t\}_{t=0}^{\infty}$ and $\{u_t\}_{t=0}^{\infty}$ as given. Thus, the following Euler equation is obtained for each household of type j,

$$\frac{q_{t+1}}{q_t}\left(\frac{c_{jt+1}}{c_{jt}}\right)^{\sigma} = \beta_j \left\{\left[1-(1+\phi)\,\zeta\left(\frac{y_{jt+1}}{Y_{t+1}}\right)^{\phi}\right] r_{t+1} + 1 - \delta\right\},\ j = 1, ..., N. \tag{5}$$

In addition, a transversality condition must hold for each household type, $\lim_{t\to\infty} \beta_j^t \left(c_{tj}^{-\sigma}/q_t\right) z_{jt} = 0$.

Firms make their production decisions to maximize profits and solve, $\max_{\phi,\, Z,\, T} A(1-\phi_t)Z_t + q_t B(\phi_t Z_t)^{\alpha} T^{1-\alpha} - r_t Z_t - u_t T$, where $Z_t = \sum_{j=1}^{N} z_{jt}(1/N)$, $0 \le \phi_t \le 1$, $Z_t \ge 0$ for all t. Optimal firm behavior implies that

$$r_t = A(1-\phi_t) + q_t \alpha B \phi_t^{\alpha} Z_t^{\alpha-1} T^{1-\alpha}, \tag{6}$$

$$u_t = q_t(1-\alpha) B(\phi_t Z_t)^{\alpha} T^{-\alpha}, \tag{7}$$

and

$$q_t \alpha B(\phi_t Z_t)^{\alpha-1} T^{1-\alpha} = A. \tag{8}$$

Total tax revenues are simply used to finance government expenditures on goods and services. The government budget constraint is then given by

$$G_t = \sum_{j=1}^{N} \zeta \left(\frac{y_{jt}}{Y_t}\right)^{\phi} y_{jt} \frac{1}{N}. \tag{9}$$

It can be easily shown that the above set-up implicitly defines an economy-wide resource constraint.

Equilibrium

An equilibrium for this economy is a set of prices $\{r_t, u_t, q_t\}$, $t = 0, ..., \infty$, household allocations $\{c_{jt}, z_{jt}\}$, $t = 0, ..., \infty$, $j = 1, ..., N$, and firms' decision rules $\{\phi_t, Z_t, T\}$, $t = 0, ..., \infty$, such that given prices and the tax schedule $\tau(.)$, i) households' allocation decisions maximize their lifetime utility, ii) firms' decision rules maximize profits, and iii) all markets clear.

We now turn to the description of a balanced growth equilibrium in which all individual and aggregate variables, expressed in units of the composite capital good, eventually grow at the same constant rate.[8] In what follows,we denote the growth rate of variable X by $\gamma_X = X_{t+1}/X_t$.

[8] Note that if individual variables grow at some constant rate, and their aggregate also grows at a constant rate, then these rates must all be equal.

Along a balanced-growth path, ϕ_t is constant and the relative price of consumption increases at rate $\gamma_q = \gamma_Z^{1-\alpha}$ by equation (8). Given the production technology in the consumption sector, we have that $\gamma_C = \gamma_Z^{\alpha}$. Hence, when measured in units of the composite capital good, aggregate consumption, $q_t C_t$, grows at rate $\gamma_{qC} = \gamma_Z$.

From (4), we have that $Y_t = \sum_{j=1}^{N} y_{jt}(1/N) = r_t \sum_{j=1}^{N} z_{jt}(1/N) + u_t T = r_t Z_t + u_t T$. Therefore, $Y_t = A(1-\phi_t)Z_t + q_t B(\phi_t Z_t)^{\alpha} T^{1-\alpha}$ by equations (6) and (7), and substituting for q_t in this last expression yields $Y_t = A(1-\phi)Z_t + (A/\alpha)\phi Z_t$. It follows that $\gamma_Y = \gamma_Z$ in the steady state.

Observe also that $r_t = A$ using equations (6) and (8). The law of motion for capital further implies that $\gamma_I = \gamma_Z$. Thus, we ultimately have that $\gamma_{qC} = \gamma_I = \gamma_Y = \gamma_Z$, and it remains only for us to describe how the growth rate of the composite capital good, γ_Z, is determined in equilibrium.

Because individual and aggregate variables grow at the same rate in the long run, y_j/Y in equation (5) is constant in the steady state. The left-hand side of this equation can be written as $\gamma_q \gamma_{c_j}^{\sigma} = \gamma_Z^{1-\alpha}\gamma_{cj}^{\sigma}$ and, since γ_{c_j} is the same for all j, individual consumption increases at rate γ_Z^{α} in the long run. In this model, therefore, the balanced growth rate, γ_Z, and the relative distribution of income, as summarized by y_j/Y for each j, are jointly determined as a set of $N+1$ equations in $N+1$ unknowns,

$$\gamma_Z^{1-\alpha(1-\sigma)} = \beta_j \left\{ \left[1 - \underbrace{(1+\phi)\,\zeta\left(\frac{y_j}{Y}\right)^{\phi}}_{\text{marginal tax rate}} \right] A + 1 - \delta \right\}, \; j = 1, ..., N, \tag{10}$$

and

$$\sum_{j=1}^{N} \left(\frac{y_j}{Y}\right) \frac{1}{N} = 1. \tag{11}$$

In the appendix, we discuss the conditions under which a solution to this set of equations exists and is unique. A crucial difference between this model and Rebelo's (1991) original framework is that tax reforms affect both economic growth, γ_Z, and the distribution of relative pre-tax earnings, y_j/Y, simultaneously. Furthermore, this implies that effective marginal tax rates, $(1+\phi)\,\zeta\left(\frac{y_j}{Y}\right)^{\phi}$, are ultimately endogenous. In the original single agent set-up with proportional taxes, the formula for long-run growth,

$$\gamma_Z^{1-\alpha(1-\sigma)} = \beta\left\{(1-\tau)A + 1 - \delta\right\}, \tag{12}$$

with τ being the constant marginal tax rate, could not possibly capture any feedback effects from economic growth to effective tax rates. Easterly and Rebelo (1993) observed that this feature of represented a serious caveat in the interpretation of their results.

Because households' relative income respond to changes in progressivity in the environment we consider, the direction in which the share of tax revenue in output adjusts is not immediately clear. Therefore, whatever the growth response, it may have been misleading to look for evidence of a robust negative relationship between the size of government, as measured by the ratio of government expenditures to GDP, and economic growth. In contrast, more conventional endogenous growth models with flat rate taxes, whereby $G = \tau Y$, necessarily predicted a rise in G/Y as τ increased, and this rise was unambiguously accompanied by a fall in the rate of growth by equation (12).

4.1 The Steady State Effects of Changes in Progressivity

To understand the importance of progressivity for the cross-sectional relationship linking growth and taxes, consider the effects of a rise in ϕ. For simplicity, let us focus on the case where there are only two household groups, impatient households indexed by β_1 and patient households with discount rate $\beta_2 > \beta_1$.

Figure 2, panel (a) illustrates a typical equilibrium where y_1/Y and y_2/Y solve equation (10), reproduced below as

$$\underbrace{(1+\phi)\zeta\left(\frac{y_j}{Y}\right)^{\phi}}_{\tau_m\left(\frac{y_j}{Y}\right)} = 1 - \frac{1}{A}\left[\frac{\gamma_Z^{1-\alpha(1-\sigma)}}{\beta_j} - (1-\delta)\right], \tag{13}$$

for impatient and patient households respectively. As expected, impatient households are relatively poorer in the long run. At the initial equilibrium growth rate, γ_Z, equation (11) must also hold so that $(1/2)\sum_{j=1}^{2}(y_j/Y) = 1$.

Suppose that ϕ increases to ϕ' in Figure 2, panel (b), so that the marginal tax rate increases at all levels of income. Because taxes are progressive, this upward shift entails a heavier tax burden for the patient households at the initial solutions for y_1/Y and y_2/Y. Furthermore, as a result of this higher marginal tax rate, all households have an incentive to lower their relative pre-tax earnings to y_1'/Y and y_2'/Y, and this change is particularly pronounced for the more patient households. However, at the initial growth rate, γ_Z, y_1'/Y and y_2'/Y no longer represent an equilibrium distribution of relative income since $(1/2)\sum_{j=1}^{2}(y_j'/Y) < 1$. Hence, in order to reach the new steady state, the growth rate must fall to γ_Z' in Figure 2, panel (b), which induces the new distribution y_1''/Y and y_2''/Y. Ultimately, an increase in ϕ has led to a fall in economic growth, slightly higher pre-tax relative income for impatient households, and lower pre-tax income for patient households so that $(1/2)\sum_{j=1}^{2}(y_j''/Y) = 1$.

The effects of the adjustment mechanism we have just described are less straightforward for the steady state share of government expenditures in GDP. In our framework,

G/Y is initially given by $(1/2)\ \sum_{j=1}^{2} \zeta(y_j/Y)^{1+\phi}$. In response to the change in tax policy, and given Figure (2b), the relative size of government expenditures is ultimately $(1/2)$ $\sum_{j=1}^{2} \zeta(y_j''/Y)^{1+\phi'}$ where $\phi' > \phi$, $y_1''/Y > y_1/Y$, and $y_2''/Y < y_2/Y$. The end result of an increase in progressivity, therefore, is ambiguous as patient and impatient households' relative earnings move in different directions. It follows that, while γ_Z unambiguously falls in Figure 2b), the relationship between γ_Z and G/Y may be much flatter than originally suggested by the early growth literature. In fact, if G/Y also falls as the tax schedule becomes more progressive, than differences in progressivity across economies would lead to an *increasing* relationship between economic growth and the relative size of government. As in Stokey and Rebelo (1995), the strength of this relationship would depend importantly on the elasticity of intertemporal substitution and depreciation rates.

At this point, we find it helpful to introduce a simple calibrated example in order to make matters more concrete. In addition, this numerical example will also help us in computing the dynamic effects of tax reforms. We shall think of our benchmark as that of an economy resembling the United States. We set α to 0.25 so as to generate a long-run investment share of 21 percent. The capital depreciation rate is set to 5 percent annually, and we choose the technology shift parameter, A, to yield a 2 percent per capita output growth rate per year. Households are assumed to have log utility, $\sigma = 1$. We set the number of households, N, to 15 and assume that discount rates are equally spaced between β_1 and β_N. We then choose β_1 and β_N to generate a Gini coefficient of income of 0.40, and a net rate of return to capital of 5.4 percent. This procedure yields $\beta_1 = 0.95$ and $\beta_N = 0.99$. The idea that the range of discount rates help shape the distribution of relative income should be clear. To see why time preference parameters also affect the return to capital, observe that in our model, the average net rate of return to the composite capital good is given by $(1/N)\sum_{j=1}^{N}\left(\left[1-(1+\phi)\,\zeta\left(\frac{y_j}{Y}\right)^{\phi}\right]A+1-\delta\right)$ which, by equation (10), is simply $\gamma_Z^{1-\alpha(1-\sigma)}(1/N)\sum_{j=1}^{N}\left(1/\beta_j\right)$. The parameter ϕ is chosen so as to generate a value of 0.1 for Kakwani's index.[9] Finally, we set ζ to 0.20 which yields a steady state share of government expenditures in GDP of 22 percent.

[9] Kakwani's index is defined as $TCI - Gini$, where TCI is the Tax Concentration Index and $Gini$ denotes the Gini coefficient of income. The Tax Concentration Index is a measure of the relative tax burden born by households at different income levels and, in our framework, is defined as $TCI = 1-(2/N^2)\sum_{j=1}^{N}\sum_{i=1}^{j}\{[\tau(y_i/Y)y_i]/T\}$, where T is total tax revenue, $(1/N)\sum_{i=1}^{N}\tau(y_i/Y)y_i$. In the case of flat rate taxes, Kakwani's index is zero and, given a fixed distribution of income, this index increases with the degree of progressivity in the tax code. By subtracting the Gini coefficient from the Tax Concentration Index, Kakwani's index attempts to neutralize the effects of a change in tax policy on the distribution of pre-tax earnings. For developed economies, calculations of Kakwani's index range from 0.1 to 0.17 in Bishop, Formby and Zheng (1998).

Figure 3 illustrates the steady state effects generated by varying the degree of statutory progressivity. As expected, output growth decreases as taxes become more progressive. This fall in economic growth is associated with a rise in the average marginal tax rate, $(1/N)\sum_{j=1}^{N}(1+\phi)\zeta(y_j/Y)^{\phi}$, in Figure 3, panel (b). There is no question, therefore, that the distortional effects of higher marginal taxes remain.[10] Additionally, because of the endogenous adjustment in the distribution of relative pre-tax income, Figure (3b) also shows that the share of government expenditures falls. The end result, therefore, is a slightly increasing relationship between output growth and the size of government as shown in Figure 3, panel (c). This relationship stands as the model analog to Figure 1, panels (a) through (d). Evidently, Figure (3c) also shows that if accurate cross-country data on effective marginal tax rates were obtained, one would continue to expect a negative correlation between per capita output growth rates and average marginal tax rates. Finally, Figure 3, panel (d) emphasizes the point made by Creedy (1999), namely that more progressive statutory rates do not always translate into more progressive effective rates. In particular, Kakwani's index does not increase monotonically with the ratio of the marginal to the average tax rate. Put alternatively, one cannot judge of the progressivity effects of a statutory reform independently of the induced changes in the distribution of pre-tax earnings.

4.2 Tax Reform and the Dynamics of Economic Growth

The original environment described in Rebelo (1991) did not allow for transitional dynamics. Consequently, any change in the marginal tax rate would have been instantaneously reflected in a new steady state growth rate. On the basis of this model, therefore, one expects that evidence of tax reforms would be detectable in time series data, in the U.S. and elsewhere. However, once progressive taxes are introduced into the environment, equation (3) shows explicitly that after-tax output exhibits diminishing returns to the composite capital good. For that reason, changes in tax policy may now induce a long transitional period as the economy moves from one balanced growth path to another. In addition, if the initial growth effects of the tax reforms are small, it is not clear that such changes can be easily identified in time series data.

To study the dynamics induced by a change in tax policy, we must first transform our economy's variables so as to make them constant in the steady state. This can simply be achieved by normalizing each variable by the composite capital good, Z_t, except for consumption variables, which we divide by Z_t^{α}, and their relative price which we normalize by $Z_t^{1-\alpha}$. This transformation defines a new set of state-like variables, z_{jt}/Z_t, $j = 1, ..., N$.

[10] Caucutt, Imrohoroglu, and Kumar (2000) also emphasize this point in a different environment.

Since our state space is quite large, we choose to linearize the dynamics of our transformed system around its stationary equilibrium. The resulting set of linearized equations possesses a continuum of solutions but only one of these is consistent with the transversality condition for each household type. It is useful to think of the law of motion for the state variables as

$$S_{t+1} = MS_t + H\varepsilon_t, \tag{14}$$

where $S_t = (\widehat{z_t^1/Z_t}, ..., \widehat{z_t^N/Z_t}, \widehat{\phi}_t)'$, M is the state transition matrix evaluated at the steady state, ε_t captures the innovation to policy, summarized by ϕ_t, and the 'hat' notation stands for percent deviations from steady state values. The linearized equations also establish a relationship between the transformed state and control variables.

Figure 4 illustrates the effects of a one-time permanent increase in tax progressivity calibrated to generate a 1.5 percent fall in output growth in the long run. As conjectured, the introduction of progressive taxation does imply some transitional dynamics. However, the striking aspect of the transition from the old to the new balanced growth path is that most of the adjustment occurs contemporaneously. At the time of the shock, the balanced growth rate decreases by 1.45 percent. The dashed line depicts the growth effects of a permanent rise in the marginal tax rate in Rebelo's (1991) framework. As the inside panel makes plain, relative to the initial steady state growth rate (i.e. relative to zero percent deviation from steady state), there is little difference between the original representative agent formulation with flat rate taxes and our model with heterogeneous households and progressive taxes. Consequently, the notion that significant changes in tax policy should be contemporaneously reflected in output growth rates continues to hold. In particular, Stokey and Rebelo (1995) stress that, while U.S. per capita growth rates have shown substantial variation at time, they have never displayed a clear break in their average value.

5 Government Spending in a Simple Model of Endogenous Growth: New Implications

In the analysis thus far, all tax proceeds were spent in a way that affected neither the marginal utility of private consumption nor the production possibilities of the private sector. We now explore an alternative formulation, first suggested by Barro (1990), in which tax revenue is used to finance public services that contribute to private production.[11] There are two reasons that lead us to reexamine this case with progressive taxes and heterogenous households.

[11] See also Glomm and Ravikumar (1994), (1997).

First, because public spending played a productive role in Barro's initial set-up, the relationship between growth and taxes tended to be that of an inverted U, reflecting the fact that higher taxes financed a higher level of productive expenditures on the one hand, and their distortional effects on the other. In our framework, however, the relative size of government expenditures tends to *fall* with changes in progressivity in the long-run (Figure 3, panel c). Therefore, as the marginal tax rate rises relative to the average rate, we expect growth to fall not only because of the distortional effects of taxes but also because government contributions to private output are lower. Consequently, unlike in Barro's (1990) article, both per capita output growth and G/Y would unambiguously decrease in equilibrium so that, in plotting one variable against the other, the cross-sectional relationships in Figure 1 would continue to hold.

Second, while more progressive taxes ultimately lower output growth, the idea that taxes finance valuable public services opens up the possibility that the initial adverse effects of a progressivity increase may be small, or even reversed. Note that in Barro's (1990) original framework, there are no transitional dynamics. We have seen that an increase in marginal rates motivated by higher progressivity leads to a gradual endogenous adjustment in households' pre-tax income. In the long run, this mechanism can lower productive government expenditures by reducing effective tax revenue. The endogenous adjustment in pre-tax income, however, may be limited at the beginning of the transition, and an increase in marginal tax rates would simply raise productive public expenditures during this phase. A direct implication is that the short and long-run growth effects of an increase in progressivity may go in opposite directions; and identifying the impact of tax reforms on economic growth in time series data may be more subtle than previously anticipated. To illustrate these ideas, we now turn to a more detailed description of the economic environment.[12]

The production technology is given by

$$Y_t = AK_t^{\alpha} G_t^{1-\alpha} L_t^{1-\alpha}, \text{ with } 0 < \alpha < 1, \tag{15}$$

where K_t and L_t stand for aggregate capital and aggregate labor input respectively. As much as possible, we have attempted to keep the notation in this and the previous section as in the original papers. We continue to think of K_t as a composite capital good which includes both human and physical components. It follows that the definition of labor in this context

[12] In models where the underlying technology for output is of the type $AK + H(K)$, where $H(K)$ satisfies the properties of a neoclassical production function (see Jones and Manuelli [1990] for example), permanent changes in tax rates will also induce transition dynamics between balanced growth paths. However, the steady state implications of these models with a representative agent are typically inconsistent with the cross-sectional data we reviewed earlier.

is that of raw labor and is separate from that which allows for investment in human capital. Total government purchases at date t are represented by G_t. For the purpose of this analysis, we shall think of G_t as nonrival and nonexcludable and, therefore, abstract from congestion considerations.[13]

As in the section above, we assume that there exists a large number of profit-maximizing firms that solve, $\max_{K_t,\, L_t} AK_t^{\alpha}G_t^{1-\alpha}L_t^{1-\alpha} - r_tK_t - w_tL_t$, where r_t and w_t denote the rental rate on capital and wages respectively. Profit maximization yields

$$r_t = \alpha A \left(\frac{G_t}{K_t}\right)^{1-\alpha} L_t^{1-\alpha}, \tag{16}$$

and

$$w_t = (1-\alpha)A\left(\frac{K_t}{L_t}\right)^{\alpha} G_t^{1-\alpha}. \tag{17}$$

The household side of the economy remains essentially as in section 4. We assume that each household supplies one unit of labor inelastically so that, in equilibrium, $L_t = (1/N)\sum_{j=1}^{N} 1 = 1$. We describe the problem of a type j household as,

$$\max_{c_{jt},\, k_{jt+1}} \sum_{t=0}^{\infty} \beta_j^t \frac{c_{jt}^{1-\sigma} - 1}{1-\sigma},\ \sigma > 0 \tag{P2}$$

$$\text{subject to } c_{jt} + k_{jt+1} - (1-\delta)k_{jt} = y_{jt}\left[1 - \zeta\left(\frac{y_{jt}}{Y_t}\right)^{\phi}\right], \tag{18}$$

$$\text{where } y_{jt} = r_tk_{jt} + w_t,\ Y_t = \sum_{j=1}^{N} y_{jt}\frac{1}{N}, \tag{19}$$

and $c_{jt},\ k_{jt} \geq 0$ for all j and t, $k_{j0} > 0$ given for all j.

All households take the sequence of prices $\{r_t\}_{t=0}^{\infty}$ and $\{w_t\}_{t=0}^{\infty}$ as given, and the following Euler equation obtains,

$$\left(\frac{c_{jt+1}}{c_{jt}}\right)^{\sigma} = \beta_j\left\{\left[1 - (1+\phi)\,\zeta\left(\frac{y_{jt+1}}{Y_{t+1}}\right)^{\phi}\right] r_{t+1} + 1 - \delta\right\},\ j = 1, ..., N. \tag{20}$$

In addition, the usual transversality condition must also hold for each j, $\lim_{t\to\infty} \beta_j^t c_{tj}^{-\sigma} k_{jt} = 0$. Productive government purchases are financed by tax revenue as in equation (9), which we reproduce below,

$$G_t = \sum_{j=1}^{N} \zeta\left(\frac{y_{jt}}{Y_t}\right)^{\phi} y_{jt}\frac{1}{N}. \tag{21}$$

[13] See Barro and Sala-i-Martin (1992) for a discussion of how congestion in public services can eliminate scale effects in economic growth.

The definition of equilibrium is analogous to that in section 4. In a manner comparable to that in the previous section, we now turn to the description of a balanced growth equilibrium in which all individual and aggregate variables grow at the same constant rate. We denote this growth rate by γ.

Along the balanced growth path, y_j/Y is constant for each j. Equation (21) implies that the relative size of government expenditures is given by $G/Y = (1/N)\sum_{j=1}^{N} \zeta\,(y_j/Y)^{1+\phi}$ in the steady state. Furthermore, in this steady state, wages grow at rate γ while the return to capital, r, is constant. Specifically, $r = \alpha A\,(G/K)^{1-\alpha}$ by equation (16) and, given the production technology in (15), $(G/K)^{\alpha} = A\,(G/Y)$. Combining these two expressions for r and G/K yields

$$r = \alpha A^{\frac{1}{\alpha}} \left(\frac{G}{Y}\right)^{\frac{1-\alpha}{\alpha}}. \tag{22}$$

In this model, therefore, an increase in public services relative to GDP unambiguously raises the marginal product of capital.

In the end, we can think of long-run growth, γ, the long-run distribution of relative pre-tax earnings, y_j/Y, and the size of government in the steady state, G/Y, as being simultaneously determined as a set of $N+2$ equations in $N+2$ unknowns,

$$\gamma^{\sigma} = \beta_j \left\{ \left[1 - (1+\phi)\,\zeta\left(\frac{y_j}{Y}\right)^{\phi}\right] \alpha A^{\frac{1}{\alpha}} \left(\frac{G}{Y}\right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta \right\},\ j = 1, ..., N, \tag{23}$$

$$\frac{G}{Y} = \sum_{j=1}^{N} \zeta\left(\frac{y_j}{Y}\right)^{1+\phi} \frac{1}{N}, \tag{24}$$

and

$$\sum_{j=1}^{N} \left(\frac{y_j}{Y}\right) \frac{1}{N} = 1. \tag{25}$$

In this framework with public expenditures contributing to private production, the long-run effects of changes in tax progressivity cannot be worked out in terms of a simple diagram. The endogenous adjustment in the distribution of pre-tax earnings now affects economic growth not only directly, through the income tax rate, but also indirectly through its impact on the relative size of public infrastructures. Furthermore, unlike the case with flat rate taxes originally explored by Barro (1990), and Glomm and Ravikumar (1994, 1997), these two channels no longer have to necessarily offset each other.

5.1 Productive Government Services and the Steady State Implications of Changes in Progressivity

To explore the long-run effects induced by changes in progressivity in this new environment, we introduce once more a calibrated example similar to the one used in the previous section. We shall also use this example in computing the transition between different balanced growth paths. All the parameters are chosen as in section 4, except for α, which we now set to 0.45 in order to continue matching a U.S. investment share of approximately 20 percent.

Figure 5 depicts the steady state effects generated by changes in the degree of statutory progressivity when government expenditures contribute to private output. Contrary to most other growth models with productive public spending, an upward shift in the tax function caused by a rise in progressivity, ϕ, does not lead to an inverted U shape for output growth, γ. Because more patient households choose to lower their pre-tax earnings relative to aggregate income when faced with higher marginal tax rates, an increase in progressivity reduces the share of government expenditures in output, G/Y, as shown in Figure 5, panel (b). By equation (22), this effect directly contributes to reducing the return to investment. In addition, the distortional effects of higher marginal tax rates remain and, in equilibrium, output growth falls with increases in progressivity. Note in Figure 5, panel (b) that the average effective marginal tax rate still increases with ϕ. Because both output growth and the relative size of government expenditures fall as the marginal tax rate rises relative to the average rate, the implied relationship between growth and G/Y is slightly increasing in Figure 5, panel (c). Hence, as in section 4, this relationship is once more consistent with Figure 1, panels (a) through (d).

In the end, if there exist important variations in tax progressivity across economies, whether or not government expenditures contribute to private production is immaterial for the cross-sectional correlation between economic growth and the ratio of public expenditures to output. In either case, the upward shift in higher marginal tax rates implied by higher values of ϕ lowers output growth and G/Y simultaneously. Should government services play a productive role, the downward adjustment in G/Y simply further decreases economic growth.

5.2 Productive Government Services, Tax Reform, and the Dynamics of Economic Growth

We have already remarked that variations in progressivity, while they can explain the cross-sectional correlation between output growth and the relative size of government, should also be reflected in time series data. In Figure 4, a rise in the marginal tax rate implied by more progressive taxes was shown to have a large contemporaneous impact on economic growth. Interestingly, per capita output growth rates in the U.S., and in most OECD countries, do not show any breaks in their average value.

With government services entering as an input into private production, Figure 6 shows the dynamic transition between balanced growth paths resulting from a one-time permanent increase in tax progressivity. As before, the rise in ϕ is calibrated to generate a 1.5 percent fall in economic growth in the long run. Note that output growth increases on impact, even if it is eventually lower in the long run. To see why, observe that as the economy moves to its new steady state only gradually, the distribution of relative pre-tax income, y_j/Y, does not adjust instantaneously to a change in progressivity. Therefore, as shown in the inside panel of Figure 6, the immediate effect of an increase in ϕ is to finance a higher ratio of public spending to output, G/Y. Given the model's assumptions, this increase in the relative size of government expenditures initially raises the marginal product of capital and, as a result, economic growth.

In the long run, the more affluent households eventually have lower pre-tax earnings in relative terms. As shown in Figure 6, this eventual adjustment implies lower tax revenue relative to output and, consequently, less public infrastructures and lower economic growth. The key point here is that the growth effects of tax reform may go in opposite directions over time. Contrary to standard linear growth frameworks with flat rate taxes, a permanent increase in the marginal tax rate does not necessarily imply a corresponding and permanent fall in economic growth contemporaneously. In particular, Figure 6 illustrates that the transition to the new lower balanced growth path may be rather slow, with a half-life of approximately 40 years.

In order to identify the growth effects of tax policy in the U.S., Stokey and Rebelo (1995) test for breaks in the average value of per capita output growth. The transitional dynamics in Figure 6, however, suggest that this strategy may be inappropriate. In our example, changes in growth rates resulting from the increase in progressivity average close to zero over the first sixty years. Clearly, this does not mean that economic growth is unaffected by tax policy. Moreover, the mean change in the share of government spending is 1.65 percent for the same period. Thus, following a tax reform, it is conceivable for the average ratio of government expenditures to output to increase while the average growth rate displays little change. Over the post-war era, U.S. data show exactly this scenario if only in a more drastic fashion. While the mean rate of economic growth has stayed mainly constant since the early 1940s, the mean size of government relative to GDP has increased on the order of 15 percent. In our framework, such a dramatic outcome likely requires a simultaneous increase in the scaling parameter ζ in addition to a change in progressivity. This suggests that the successive wave of tax reforms in the U.S. since the end of World War II have not always affected the rich disproportionately relative to the poor.

5.3 Additional Considerations

It is important to emphasize that a discussion of developing economies in terms of the model in this section, as well as section 4, would have to involve several additional considerations. In poorer countries, the lack of proper monitoring infrastructure often makes it less costly for households to underreport income. For instance, a 2000 report from the Associated Press states that "only a small fraction of Russians filed tax returns in 1999, and many of them likely fudged their declarations to underreport income." In and of itself, this phenomenon only enhances one of the main threads in this paper, namely that some households may find it worthwhile to try and lower their pre-tax income in the face of higher taxes. According to the same Associated Press report, the "cash-strapped government's attempt to wrench out a few more rubles every year through high taxes has backfired, causing rampant evasion." This feature of developing economies often calls into question the relevance of taxing income rather than consumption or imports.

In fact, the composition of tax revenue does differ considerably between developed and developing countries. In particular, the income-consumption tax mix is generally biased towards consumption in the latter economies. That being said, there are at least two reasons that tend to insure a non-trivial role for income taxes in poorer countries. First, consumption taxes are commonly regarded as more regressive than income taxes. This is often a concern in poorer economies and, according to Tanzi and Zee (2000), India and Sri Lanka which experimented with a graduated tax on consumption 40 years ago soon abandoned it because of severe difficulties in implementing it. Second, Tanzi and Zee (2000) also write that "the rate structure of the personal income tax is often the most convenient and visible policy instrument for most governments in developing countries to underscore their commitments to social justice, and hence to gain political support for their policies. It is, therefore, not surprising to find that many developing countries attach great importance to maintaining some degree of nominal personal income tax rate progressivity by applying many rate brackets, and are reluctant to undertake personal income tax reforms that would suggest any lessening of such commitments."

6 Conclusion

With the advent of the endogenous growth framework, it became theoretically possible to address some of the cross-country dispersion in average growth rates in terms of differences in public policy. Unfortunately, early endogenous growth models, of the type posited by Jones and Manuelli (1990), or Rebelo (1991), were later shown to be at odds with the data both in the cross section and in the time series. Above all, these models implied that economic growth should fall with the size of government spending relative to GDP in the cross section. They also suggested that one-time permanent shifts in tax policy would be associated with an instantaneous and permanent change in economic growth in the time series.

In this paper, we have attempted to show that allowing for progressive taxes and household heterogeneity in these models considerably changes their predictions. In the economies presented above, an increase in tax progressivity did lead to lower growth. However, the endogenous adjustment in the distribution of pre-tax earnings ultimately prevented this policy change from yielding higher tax revenues as a fraction of GDP. When plotted against each other, both of these results seemed to match well with available cross-country evidence. Second, the explicit modeling of non-linear taxes meant that a change in tax policy would induce a gradual adjustment from one balanced growth path to another. Furthermore, in the case where public spending served as an input into private production, we found that the short and long-run effects of tax reforms could go in opposite directions. In particular, within the context of a standard endogenous growth model, a one-time permanent shift in tax policy did not lead to an instantaneous shift in the average rate of economic growth.

Appendix

If $(1+\phi)\zeta < 1$, and $[A+(1-\delta)-A(1+\phi)\zeta] < \left(\frac{\beta_1}{\beta_N}\right)[A+(1-\delta)]$, then a solution to the set of equations (10) and (11) exists and is unique.

The first restriction places an upper bound on the scale of the marginal tax schedule, $\tau_m(y/Y) = (1+\phi)\zeta(y/Y)^{\phi}$, and insures that households have sufficient incentive to invest. The second restriction on parameters holds when the degree of progressivity in the tax schedule, as measured by ϕ, is high enough relative to the spread in discount factors $\beta_1/\beta_N < 1$.

Equation (10) can be re-written as

$$\frac{y_j}{Y} = \left[\frac{1}{(1+\phi)\zeta} + \frac{1-\delta}{A(1+\phi)\zeta} - \frac{\gamma_Z^{1-\alpha(1-\sigma)}}{\beta_j A(1+\phi)\zeta}\right]^{\frac{1}{\phi}}, \; j = 1, ..., N. \tag{A1}$$

and, from equation (11), it follows that a solution for $\gamma_Z \geq 0$ must solve

$$\sum_{j=1}^{N}\left[\frac{1}{(1+\phi)\zeta} + \frac{1-\delta}{A(1+\phi)\zeta} - \frac{\gamma_Z^{1-\alpha(1-\sigma)}}{\beta_j A(1+\phi)\zeta}\right]^{\frac{1}{\phi}}\left(\frac{1}{N}\right) = 1. \tag{A2}$$

Define the left-hand side of (A2) as $F(\gamma_Z)$. There are two cases to consider, namely $1-\alpha(1-\sigma) \geq 0$ and $1-\alpha(1-\sigma) < 0$.

Suppose that $1-\alpha(1-\sigma) \geq 0$. First, since ϕ can be greater than 1, the expression inside the square brackets of equation (A1) cannot be negative. In particular, $F(\gamma_Z)$ is always well defined as long as $\gamma_Z \leq \overline{\gamma}_Z$, where $\overline{\gamma}_Z = \{\beta_1[A+(1-\delta)]\}^{\frac{1}{1-\alpha(1-\sigma)}}$. Recall that β_1 is the discount rate of the most impatient households.

Now, when $1-\alpha(1-\sigma) \geq 0$, $F(0) = \sum_{j=1}^{N}\left[\frac{1}{(1+\phi)\zeta} + \frac{1-\delta}{A(1+\phi)\zeta}\right]^{\frac{1}{\phi}}\left(\frac{1}{N}\right)$. Therefore, if $(1+\phi)\zeta < 1$, then $\sum_{j=1}^{N}\left[\frac{1}{(1+\phi)\zeta}\right]^{\frac{1}{\phi}}\left(\frac{1}{N}\right) > 1$ and, for any $A \geq 0$, $F(0) > 1$.

Define $\underline{\gamma}_Z = \{\beta_N[A+(1-\delta)-A(1+\phi)\zeta]\}^{\frac{1}{1-\alpha(1-\sigma)}}$. Because β_N is the discount rate of the most patient households, when $\gamma_Z = \underline{\gamma}_Z$, $y_j/Y < 1$ for $j = 1, ..., N-1$ in equation (A1). Hence, $F(\underline{\gamma}_Z) < 1$.

Since $F(\gamma_Z)$ is continuous, by the Intermediate Value Theorem, there exists $0 < \gamma_Z < \underline{\gamma}_Z$ such that $F(\gamma_Z) = 1$. Furthermore, because $F(\cdot)$ falls monotonically with γ_Z, this solution is unique. The restriction that $[A+(1-\delta)-A(1+\phi)\zeta] < \left(\frac{\beta_1}{\beta_N}\right)[A+(1-\delta)]$ above simply insures that $\beta_N[A+(1-\delta)-A(1+\phi)\zeta]\}^{\frac{1}{1-\alpha(1-\sigma)}}$ lies in the admissible domain for γ_Z, in particular $\{\beta_N[A+(1-\delta)-A(1+\phi)\zeta]\}^{\frac{1}{1-\alpha(1-\sigma)}} = \underline{\gamma}_Z < \{\beta_1[A+(1-\delta)]\}^{\frac{1}{1-\alpha(1-\sigma)}} = \overline{\gamma}_Z$ (see Figure 7).

[Insert Figure 7]

One can then solve for the distribution of relative income, y_j/Y for each j, by simply using equation (A1). The case where $1 - \alpha(1 - \sigma) < 0$ can be worked out in a similar fashion.$\square$

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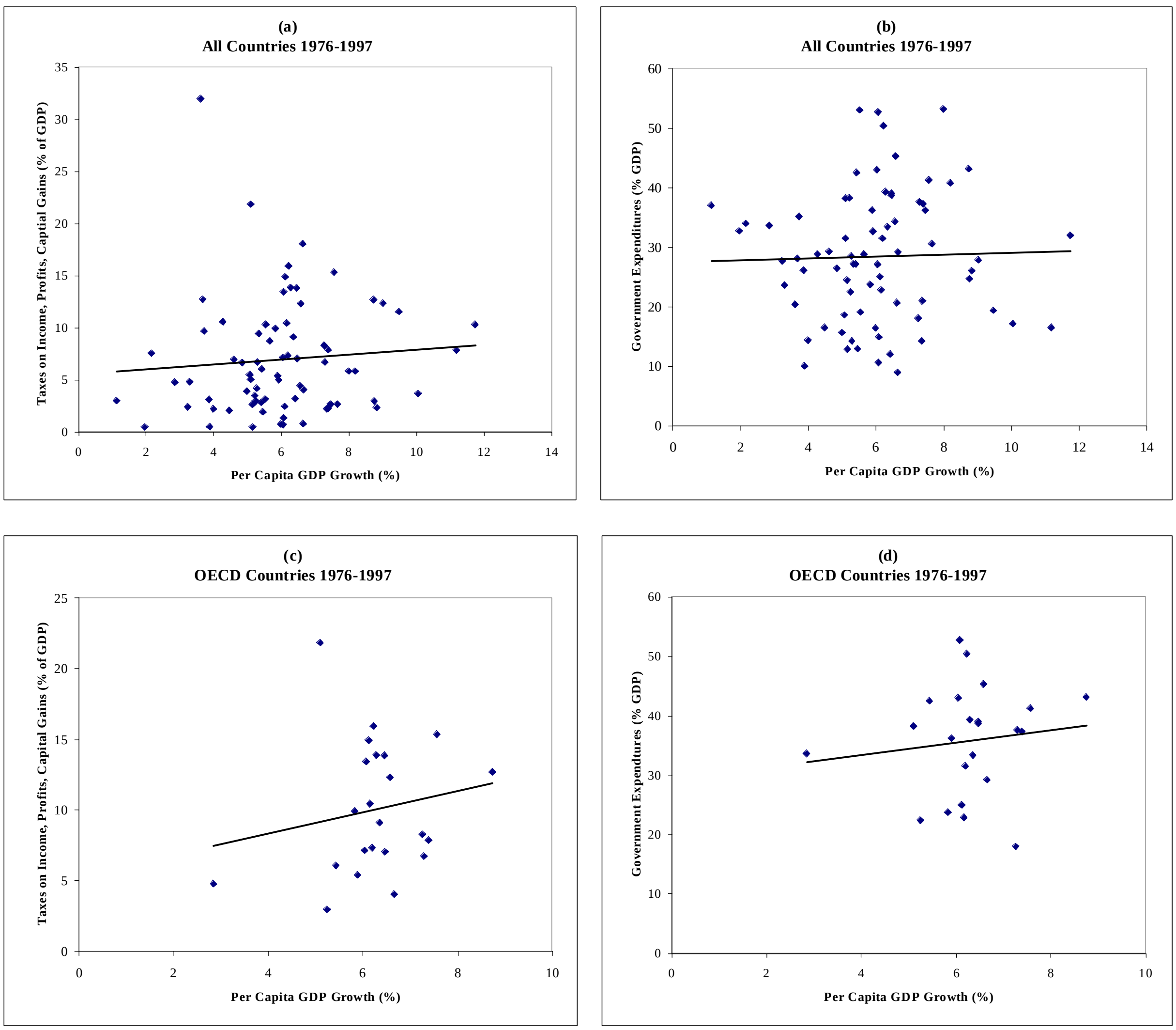
Figure 1.



(a)
All Countries 1976-1997
Taxes on Income, Profits, Captial Gains (% of GDP)
Per Capita GDP Growth (%)
(b)
All Countries 1976-1997
Government Expenditures (% GDP)
Per Capita GDP Growth (%)
(c)
OECD Countries 1976-1997
Taxes on Income, Profits, Capital Gains (% of GDP)
Per Capita GDP Growth (%)
(d)
OECD Countries 1976-1997
Government Expenditures (% GDP)
Per Capita GDP Growth (%)

Figure 2.

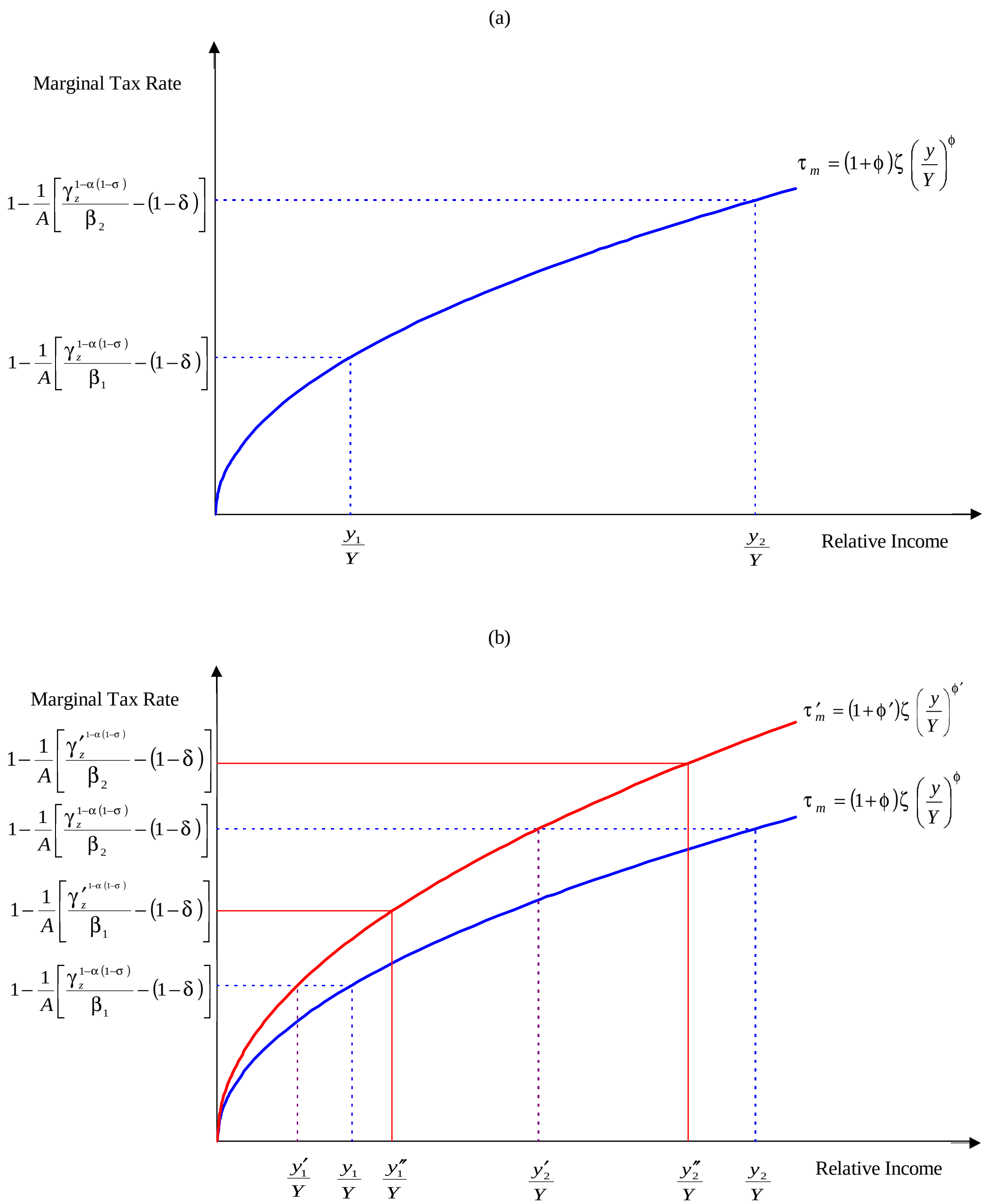

(a)
Marginal Tax Rate
$1-\frac{1}{A}\left[\frac{\gamma_z^{1-\alpha(1-\sigma)}}{\beta_2}-(1-\delta)\right]$
$1-\frac{1}{A}\left[\frac{\gamma_z^{1-\alpha(1-\sigma)}}{\beta_1}-(1-\delta)\right]$
$\tau_m=(1+\phi)\zeta\left(\frac{y}{Y}\right)^{\phi}$
$\frac{y_1}{Y}$
$\frac{y_2}{Y}$
Relative Income
(b)
Marginal Tax Rate
$1-\frac{1}{A}\left[\frac{\gamma_z'^{1-\alpha(1-\sigma)}}{\beta_2}-(1-\delta)\right]$
$1-\frac{1}{A}\left[\frac{\gamma_z^{1-\alpha(1-\sigma)}}{\beta_2}-(1-\delta)\right]$
$1-\frac{1}{A}\left[\frac{\gamma_z'^{1-\alpha(1-\sigma)}}{\beta_1}-(1-\delta)\right]$
$1-\frac{1}{A}\left[\frac{\gamma_z^{1-\alpha(1-\sigma)}}{\beta_1}-(1-\delta)\right]$
$\tau_m'=(1+\phi')\zeta\left(\frac{y}{Y}\right)^{\phi'}$
$\tau_m=(1+\phi)\zeta\left(\frac{y}{Y}\right)^{\phi}$
$\frac{y_1'}{Y}$
$\frac{y_1}{Y}$
$\frac{y_1''}{Y}$
$\frac{y_2'}{Y}$
$\frac{y_2''}{Y}$
$\frac{y_2}{Y}$
Relative Income

Figure 3.

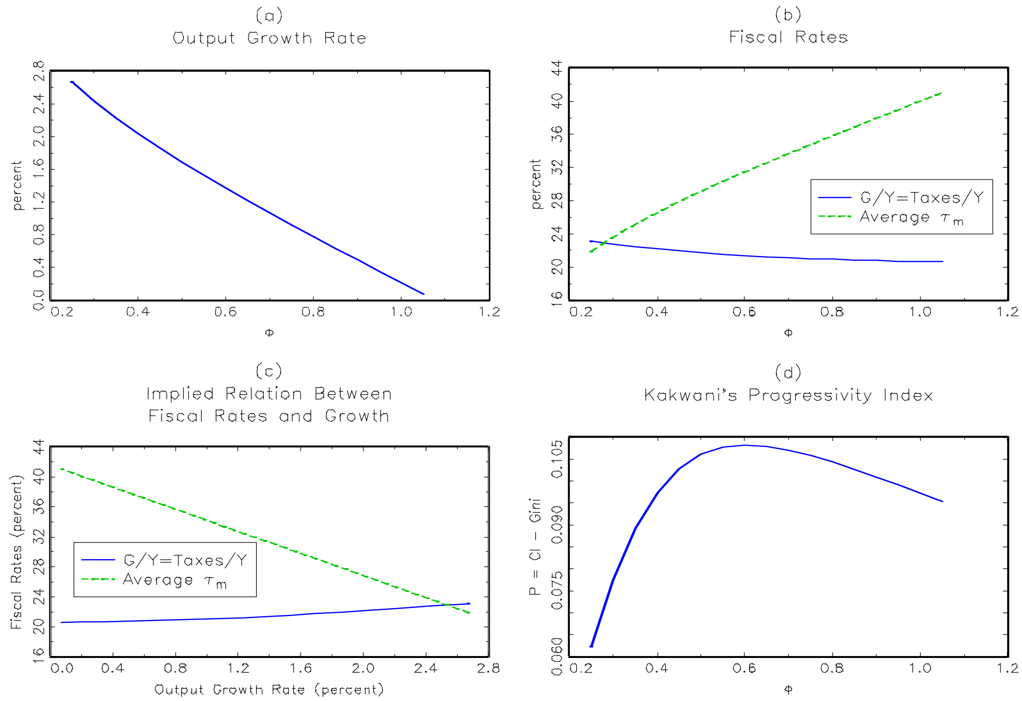


Figure 4.

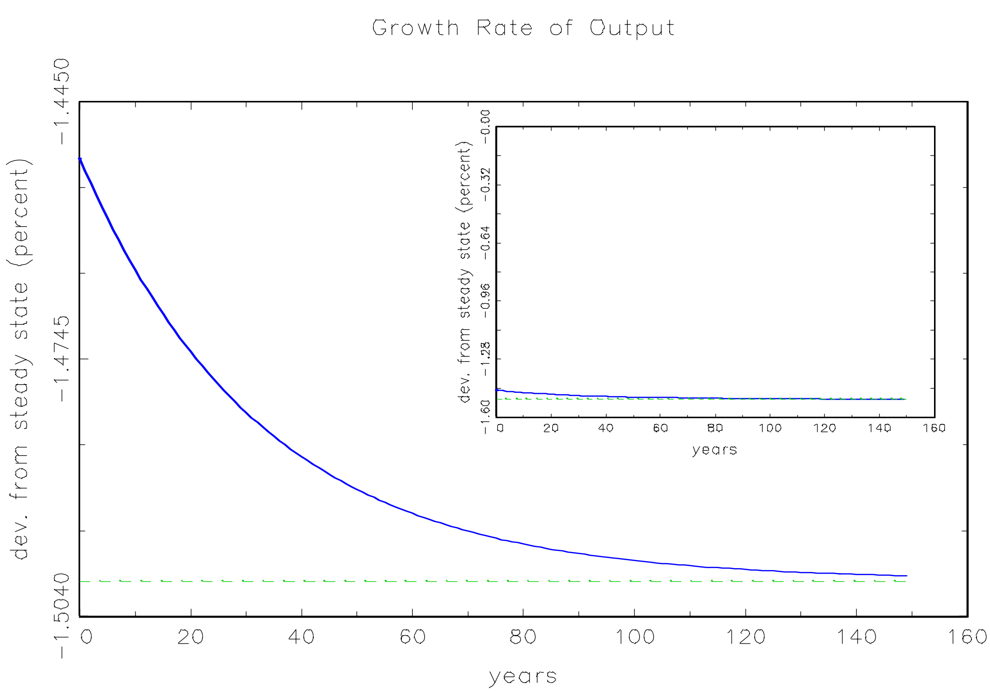


Figure 5.

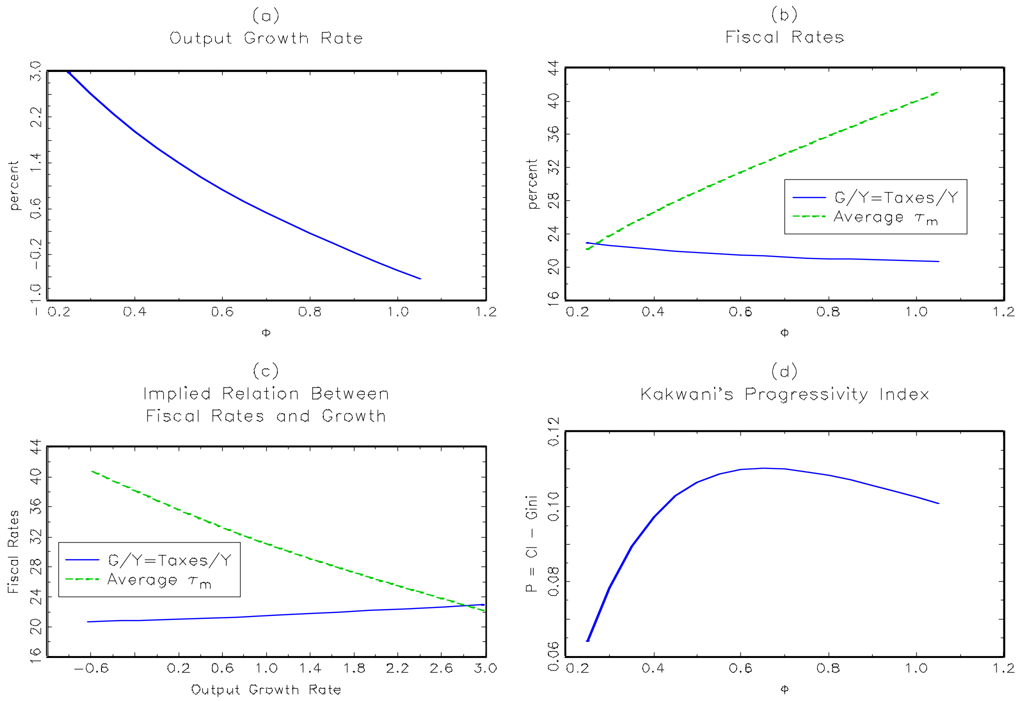


Figure 6.

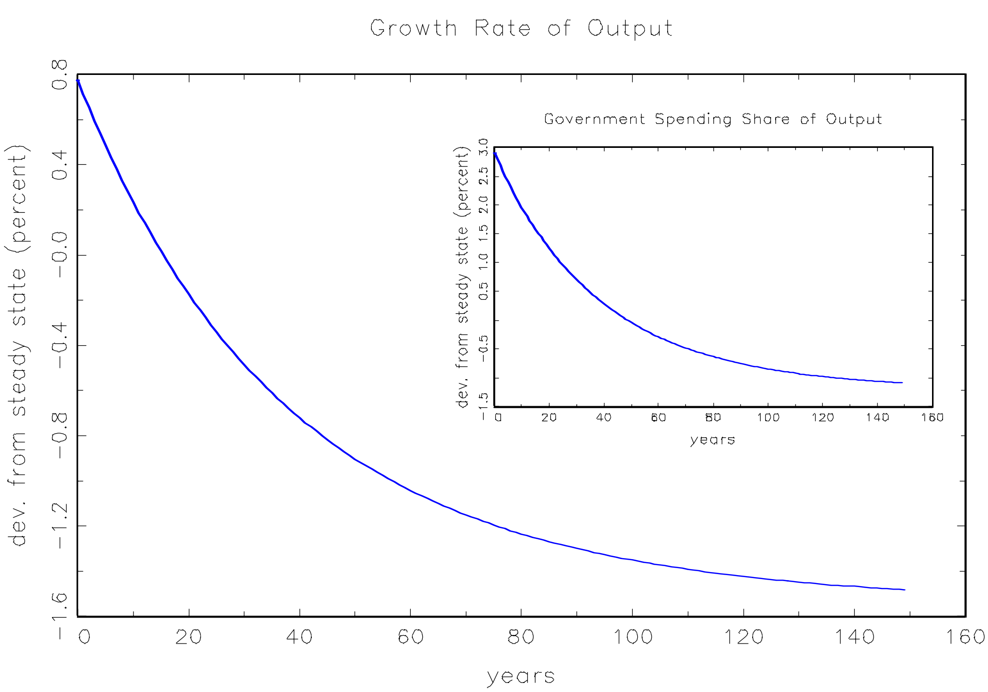
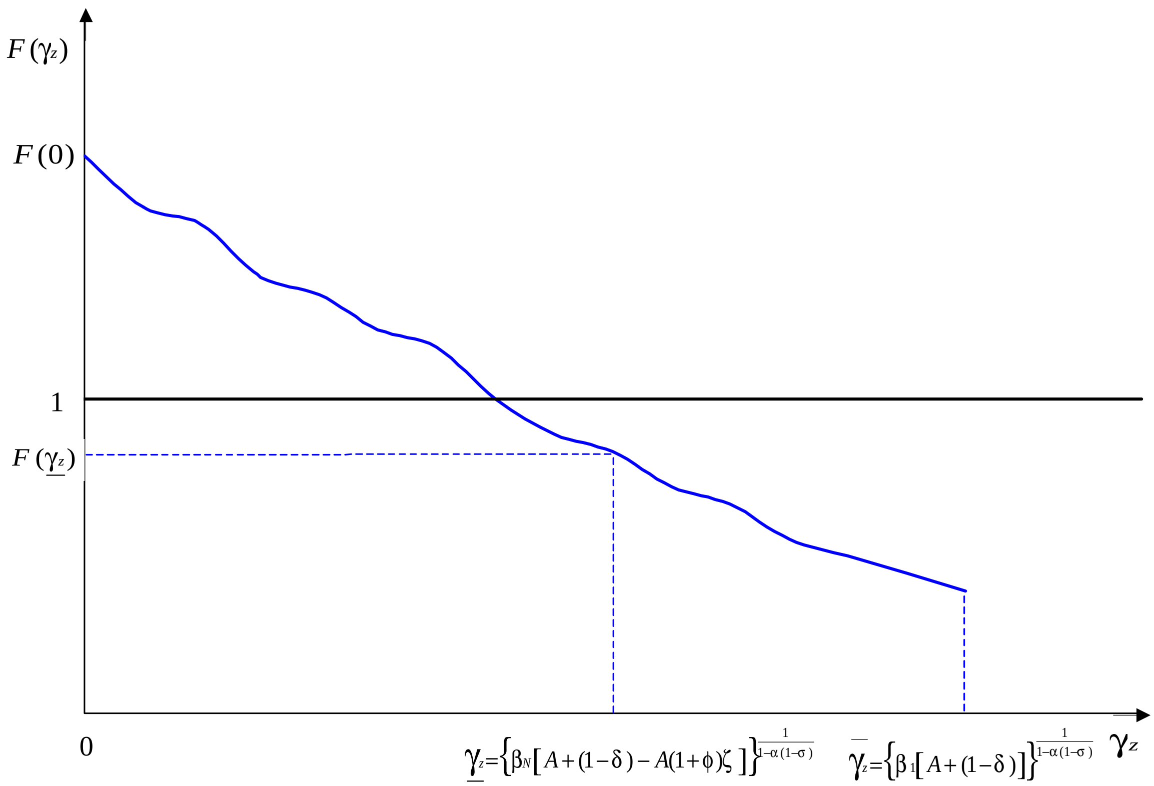


Figure 7.



$F(\gamma_z)$
$F(0)$
1
$F(\underline{\gamma}_z)$
0
$\underline{\gamma}_z = \left\{\beta_N\left[A + (1-\delta) - A(1+\phi)\zeta\right]\right\}^{\frac{1}{1-\alpha(1-\sigma)}}$
$\overline{\gamma}_z = \left\{\beta_1\left[A + (1-\delta)\right]\right\}^{\frac{1}{1-\alpha(1-\sigma)}}$
γ_z