Modeling the retirement process for policy evaluation and research

A sizable minority of men who retire continue to work part time, although the duration of partial retirement tends to be brief; a small number actually increase hours of work after a period of retirement or semiretirement when personal or economic circumstances change

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The economics literature has generally conceived of the retirement process as a one-way flow from an "in the labor force" status to a "not in the labor force" status. However, evidence from recent studies suggests that the retirement transition is much more complex, involving both major flows from full-time work to full retirement, either directly or indirectly through partial retirement, and much smaller flows in the opposite direction. Information about these flows provides a richer description of the retirement process. It may also help in establishing values for parameters which are important to the retirement decision and, thereby, in understanding the nature of that decision.

This article presents an analytical framework for investigating transitions of white men among full-time work, partial retirement, and full retirement. Of special importance are flows to partial retirement, which usually are associated with a reduction in wage rates and frequently entail a change in employers as well. Various descriptive statistics related to the retirement process, including probabilities of older workers being in particular labor force states at given ages, transition rates among the various states, and continuation rates in the states, also are examined. A final section discusses implications of the descriptive statistics for the estimation of retirement models.

The analytical framework

The framework for this analysis reflects a number of relevant findings presented in our previous work. One such finding is that partial retirement is indeed a widespread phenomenon.[1] Between the ages of 65 and 69, partial retirement is as common as continued full-time work.[2] More than one-third of the older white men who were not self-employed in the Social Security Administration's Retirement History Survey indicated that, during at least 1 of the 4 sample years between 1969 and 1975, they were partially retired.[3] Moreover, the probability of partial retirement remained high even for those who were in good health, did not face mandatory retirement, and were not covered by a pension.

A second important finding is that partially retired workers had significantly lower wage rates than comparable full-time workers.[4] These lower wage rates may come about for at least two reasons. First, surveys of both workers and firms indicate that, in a majority of jobs, an individual is not permitted to cut back from full-time to part-time work.[5] Under such circumstances, if an older worker wishes to reduce his work effort below full time, he must quit his main job and find one that does permit part-time work,

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usually at a lower wage rate. In some cases, the worker can reduce his hours of work without changing jobs, albeit at a reduction in the wage. As would be expected, the relatively small fraction of individuals who are free to reduce hours of work on their main job are overrepresented among those who do partially retire.[6]

These findings may be incorporated into a formal life-cycle model as follows. An individual is presumed to choose a time path for consumption and labor supply so as to maximize lifetime utility:

$$(1) \qquad U = \int_O^T u[C(t), L(t), t; \beta]\, dt$$

where $C(t)$ is consumption at time t, $L(t)$ is leisure at time t, and β is a vector of parameters that determine the nature of the utility function u at any time t. The maximization of the utility function is subject to the lifetime budget constraint:[7]

$$(2) \qquad \int_O^T d(t)\, \{W_N[H_N(t), t] + W_P\, [H_P(t), t]\}\, dt + A_0 = \int_O^T d(t)\, C(t)\, dt$$

where $d(t)$ is the discount factor to time t; $W_N[H_N(t), t]$ is the total compensation, including changes in pension and social security asset values, from working $H_N(t)$ in the nonretirement job; $W_P[H_P(t), t]$ is the corresponding compensation for $H_P(t)$ hours in the partial retirement job; and A_0 is the discounted value of the individual's exogenous assets.[8] Further constraints limit the potential quantities of labor supply and relate labor supply to leisure:

$$(3) \qquad H_N(t)\, [h_N - H_N(t)] = 0$$

$$(4) \qquad 0 \leq H_P(t) \leq h_N$$

$$(5) \qquad H_N(t)\, H_P(t) = 0$$

$$(6) \qquad L(t) = 1 - H_N(t) - H_P(t) \geq 0$$

The first constraint specifies that the individual must work either full time (where full-time work is a fraction, h_N, of available time) or not at all in the nonretirement job, while the second specifies that the labor supplied to the partial retirement job can range between none and full time.[9] The third constraint specifies that the individual cannot work at both jobs simultaneously, and the last constraint defines leisure as the time not supplied as labor.

Within the context of this model, the paths of wages in the two types of jobs (that is, tenure dependence) will induce bunching of hours. Most people will spend the first part of their working lives in nonretirement jobs, where the wage rate is higher than in partial retirement jobs. With increasing age, however, the individual's utility function is likely to change in such a manner that full-time work generates increasing disutility to the point that he will quit the nonretirement job. Some people will find it advantageous to spend additional years in part-time employment, albeit at a lower wage rate, while others will elect to bypass the stage of partial retirement entirely and move directly to full retirement. Even for those who partially retire, the within-period utility function will continue to shift over time to make work increasingly onerous, so that these people, too, will eventually wish to retire fully. Hence, the sequence that we expect to find most often is nonretirement, possibly followed by partial retirement, followed by full retirement.

It is possible that some people will find it desirable or necessary to move in the reverse direction from the sequence indicated above. That is, they may work in a partial retirement job after being fully retired, or they may take a nonretirement job after being partially or fully retired. Such "reverse" flows may be generated by very substantial jumps in wage rates in an individual's later years, but this is not a very persuasive reason for many such flows. A more plausible explanation involves unexpected changes in a person's economic or social circumstances that induce him to change his mind and return to work for a period during which he had anticipated being partially or fully retired. For example, an individual might suffer large losses in the financial markets, and subsequently find he has fewer assets than anticipated. His spouse might suffer from a serious illness or injury which increases the household's need for income. Alternatively, he may retire and then find that he does not enjoy his new status. Any of these circumstances could lead the individual to recalculate the optimal path of labor supply over his remaining lifetime, causing him to move in the reverse direction from the typical nonretirement-to-retirement sequence.

Descriptive statistics on retirement flows

Statistical evidence bearing on the magnitudes of the labor force flows associated with the model described above is available from the Retirement History Survey. Survey data used in this study pertain only to white men who had not been self-employed before retirement, for the years 1969, 1971, 1973, and 1975. Respondents included in the sample were 58 to 63 years old in 1969.

So that a meaningful number of observations could be obtained for each cell, some of the tables in this article do not distinguish among different cohorts in survey years. In such cases, the entry for a particular cell mixes observations at different points in time and for different cohorts. There are two problems with this procedure which the reader should bear in mind. First, the unemployment rate differed widely among the four survey years, ranging from 3.5 percent in 1969 to 8.5 percent in 1975. Second, there have been downward trends in male labor force participation rates—since 1900 for those over 65, and since the late 1960's for those 55 to 64.[10] These trends could possibly be caused by secular changes in many of the explanatory variables included in the life-cycle model we describe, but might also reflect true cohort effects. When the observations are pooled, these differences are either hidden or, where the focus is on calendar age, may be correlated to some extent with the age variable.

State probabilities. First, the simple percentages of the sample who were not retired, were partially retired, or were fully retired were examined. Table 1 presents these percentages by survey year and by age of the respondent.[11] Three aspects of this table are particularly noteworthy.

First, departure from the nonretirement state is indeed pervasive between the ages of 58 and 68. The percentage of individuals reporting in a particular survey that they were not retired at all falls from 85 percent to 8 percent during this 10-year span. This is accompanied by a very large increase in the fraction of the sample who were fully retired, and a smaller rise in the portion who were partially retired.

Second, among those who worked at all, partial retirement was more common than nonretirement for individuals past the age of 65. For the 5-year span beginning at age 65 and ending at age 69, the fraction of individuals who were partially retired holds fairly steady between 15 and 20 percent. During the same period, nonretirement falls from around 20 percent to a little more than 5 percent. These figures again suggest that partial retirement is an important phenomenon, particularly in the older age ranges.

Finally, there appear to be unmistakable trends in the proportions of the men not retired and fully retired. Between 1969 and 1973, the percentage of 62- and 63-year-olds not retired dropped by 8 to 9 percentage points, and similar declines are observed for 64- and 65-year-olds between 1971 and 1975. The figures for full retirement display an equally large change in the opposite direction. What is not clear from the table is whether there are similarly strong changes for men under 62 or over 65, for whom the 1969–75 Retirement History Survey includes data from at most two adjacent surveys. These data give some hint of such changes for those cohorts, but they do not appear to be as strong as for the 62- to 65-year-old group.

Transition rates among retirement states. Table 2 presents the entry and exit rates among the various retirement states.[12] The top panel of the table indicates entry rates from specific states 2 years earlier. For example, of the people who were partially retired, 44.1 percent had been not retired 2 years earlier, 40.0 percent had been partially retired, and so on. Note that the rows of this panel sum to 1. The bottom panel of the table conveys the same kind of information on exit rates to specific states 2 years later. For instance, of the people who were not retired as of a particular survey, 10.5 percent were partially retired by the next survey, and 26.8 percent were fully retired. The columns of the bottom panel sum to 1.

Three features of this table are of particular interest. First, consider the exit rates from nonretirement. About 37.3 percent of the individuals who were not retired in one survey were either partially or fully retired by the time of the next survey 2 years later. Of those who left nonretirement and did not become unemployed, 28.2 percent (calculated as .105/.373) partially retired, and the remaining 71.8 percent fully retired. Again, these figures underscore the fact that partial retirement is a phenomenon that affects a significant fraction of the labor force during their later years.

The exit rates for partially retired workers also bear mention. There was only a 48.5-percent chance that an individual who was partially retired during one survey would still be partially retired 2 years later. If exit from partial retirement were a random process with a constant hazard rate, this would imply that the average duration of partial retirement is a little under 3 years.[13] The assumption of a constant hazard rate is undoubtedly an oversimplification, but the findings nevertheless suggest that the duration of partial retirement is quite short, particularly when compared to the duration of an individual's full-time work.

A third interesting feature of the table involves the flows against the normal retirement paths. We noted earlier that, in some cases with unusual wage paths, or in cases where the individual encounters unforseen events, it is possible that he might move in a direction counter to the predominant retirement path. Table 2 indicates that this does indeed occur. More specifically, of the people who entered partial retirement (and who were not unemployed), about 23.4 percent [calculated as .135/(.135 + .441)] had been fully

Table 1. Retirement status by age, selected years, 1969–75

Age	Nonretirement				Partial retirement				Full retirement			
	1969	1971	1973	1975	1969	1971	1973	1975	1969	1971	1973	1975
58	.85	—	—	—	.05	—	—	—	.09	—	—	—
59	.81	—	—	—	.06	—	—	—	.12	—	—	—
60	.79	.77	—	—	.06	.06	—	—	.12	.14	—	—
61	.72	.70	—	—	.08	.07	—	—	.18	.20	—	—
62	.64	.61	.56	—	.12	.09	.10	—	.23	.27	.33	—
63	.56	.49	.47	—	.16	.12	.09	—	.27	.36	.40	—
64	—	.44	.40	.35	—	.13	.12	.13	—	.41	.47	.50
65	—	.24	.17	.17	—	.19	.16	.15	—	.54	.66	.67
66	—	—	.13	.11	—	—	.17	.17	—	—	.69	.70
67	—	—	.13	.09	—	—	.15	.18	—	—	.71	.72
68	—	—	—	.08	—	—	—	.15	—	—	—	.77
69	—	—	—	.06	—	—	—	.17	—	—	—	.76

Table 2. Two-year transition rates between labor force states

Final status	Entry rates from— Nonretirement	Partial retirement	Full retirement	Unemployment
Nonretirement	.959	.024	.007	.010
Partial retirement	.441	.400	.135	.024
Full retirement	.303	.094	.588	.014
Unemployment	.711	.093	.069	.127
	Exit rates from— Nonretirement	**Partial retirement**	**Full retirement**	**Unemployment**
Nonretirement	.609	.077	.008	.226
Partial retirement	.105	.485	.057	.208
Full retirement	.268	.426	.932	.448
Unemployment	.018	.012	.003	.118

retired in the previous survey. Of the men who left partial retirement and did not become unemployed, 15.3 percent [calculated as .077/(.077 + .426)] were not retired in the next survey. The entry rate for the third "reverse" flow, that from full retirement to nonretirement, and the corresponding exit rate were both less than 1 percent.

Continuation rates by age. It is useful to examine in more depth the way these flows, and especially the continuation rates—the diagonal elements of the lower parts of table 2—vary with age. Table 3 reports, by respondent's age in the initial year, the proportions of individuals who continue in the same retirement category until the next survey 2 years later.

How should these continuation rates behave? We know that pension programs and mandatory retirement provisions boost the likelihood of retirement at ages 62 and 65, either by providing incentives for individuals to leave their jobs or by forcing them to retire at a specified age. Moreover, while there is controversy about the effects of social security payments at 62, we know that beyond age 64 the benefit adjustments for this cohort were not actuarially fair, providing further incentive for retirement.[14] In terms of labor supply, the effects of changing health and family structure and the increasing disutility of work should act to reduce continuation rates in nonretirement below the high levels typical of individuals in their prime working years.

There is indeed evidence of rapidly falling continuation rates for nonretirement up to age 64. These range from 87 percent at age 58 to 27 percent at age 64—the age when the strongest economic incentives to leave nonretirement are about to be encountered. The continuation rates for 65- to 67-year-olds lie above those for 64-year-olds, but well below the rates observed for those in their late 50's and early 60's.

For the partially retired, continuation rates hold relatively steady in the 45- to 50-percent range up to age 64. The continuation rates for full retirement are very high at all ages, ranging from 91.8 to 94.4 percent. Among the unemployed, there are too few observations to permit generalizations about the pattern of the continuation rates.

Duration dependence of continuation rates. A related issue, particularly for partial retirement, is whether the continuation rate depends on how long the individual has been partially retired—that is, the duration dependence of continuation rates.

To investigate this issue, we examined data for those individuals who were not retired in 1969 but who were partially retired in 1971. This avoids our having to deal with periods of partial retirement already in progress. Moreover, the requirement that individuals had been working full time in 1969 ensures that we are looking at persons who are following the normal retirement sequence and who perhaps are not quite as likely to be responding to unusual or unexpected circumstances. Of this group, 292 were still in the sample by 1973, and of that number 122, or 41.8 percent, were still partially retired in 1973. Hence, a person partially retired for the first time in 1971 had a 41.8-percent initial 2-year continuation rate.

Some 112 of the 122 individuals who were partially retired in both 1971 and 1973 were in the sample in 1975, and of those men 75, or 67.7 percent, were still partially retired. Hence, the 2-year continuation rate is considerably higher for individuals with durations in partial retirement of between 2 and 4 years than for individuals with durations of less than 2 years.[15] It should be kept in mind that these individuals were also growing older with each successive survey, and the evidence from table 3 indicates that this could be part of the explanation as to why the individuals exhibited higher continuation rates between 1973 and 1975 than between 1971 and 1973. Even so, the magnitude of the increase in the continuation rate from 41.8 percent to 67.7 percent is relatively large compared to changes in the continuation rates caused by an additional 2 years of age, as indicated in table 3. It would appear that there is some duration dependence, in that the continuation rate for partial retirement increases with the length of time the individual has been partially retired.

Table 3. Two-year labor force status continuation rates, by age of respondent

Age in initial period	Labor force status: Nonretirement	Partial retirement	Full retirement	Unemployment
58	.873	.409	.926	[1].222
59	.831	.472	.939	[1].267
60	.713	.467	.934	.242
61	.652	.450	.921	.070
62	.632	.458	.918	.105
63	.327	.446	.931	.114
64	.267	.475	.933	[1].000
65	.415	.510	.934	[1].053
66	.515	.597	.944	[1].000
67	.432	.604	.932	[1].000

[1]Sample size less than 25.

Detailed flows for the partially retired. Table 4 looks at the flows of partially retired men in somewhat greater detail to shed some light on the mechanism of partial retirement. These individuals are separated into three categories according to the relationship between their nonretirement and partial retirement jobs. The top line of both sections of the table considers individuals who have partially retired in jobs in which they reported themselves not retired in a previous survey or, if the observation is for the first survey, in jobs which they started before age 55. The second line refers to individuals who have partially retired to jobs which are different from any jobs in which they reported themselves not retired in prior surveys. The third line indicates individuals who are partially retired, but for whom the relationship of the nonretirement and partial retirement jobs could not be classified in one of the other two categories.[16]

The information in the table has a couple of interesting implications. First, it suggests that partial retirement in a job previously reported as a nonretirement job and partial retirement at a different job are relatively distinct paths. Of the individuals leaving partial retirement in a job previously reported as a nonretirement job, only 7.3 percent (calculated as .039/.531) were found to be partially retired in a different job, and of the individuals entering partial retirement in a job not previously reported as a nonretirement job, only 4.4 percent (calculated as .032/.727) were entering from partial retirement in a job previously reported as a nonretirement job or in a different job.

Second, a comparison of the exit rates of individuals partially retired in jobs previously reported as nonretirement jobs and individuals partially retired in different jobs indicates that the behavior of these two groups is generally similar. A man partially retired in a job previously reported as a nonretirement job was a couple of percentage points (9.1 percent vs. 6.9 percent) more likely to return to nonretirement, while one partially retired in a different job was 3 percentage points (35.4 percent vs. 32.4 percent) more likely to retire completely. Both groups were about equally likely (57.9 percent vs. 56.8 percent) to continue partial retirement in some form. Individuals who were partially retired but who could not be assigned to either one of the above categories appear to be somewhat different, with substantially lower probabilities of continuing partial retirement and substantially higher probabilities of complete retirement. It is likely that these unclassified individuals in fact are partially retired in jobs they did not previously hold as nonretirement jobs, but it is not possible to be entirely sure of this.

Table 4. Detailed 2-year transition rates for partially retired individuals

Final status	Entry rates from— Nonretirement	Partial retirement— In the nonretirement job	Partial retirement— In a different job	Partial retirement— In a nonclassified job	Full retirement	Unemployment
Partial retirement in:						
The nonretirement job	.566	.434	.000	.000	.000	.000
A different job	.422	.032	.273	.094	.144	.034
A nonclassified job	.360	.051	.039	.285	.231	.034

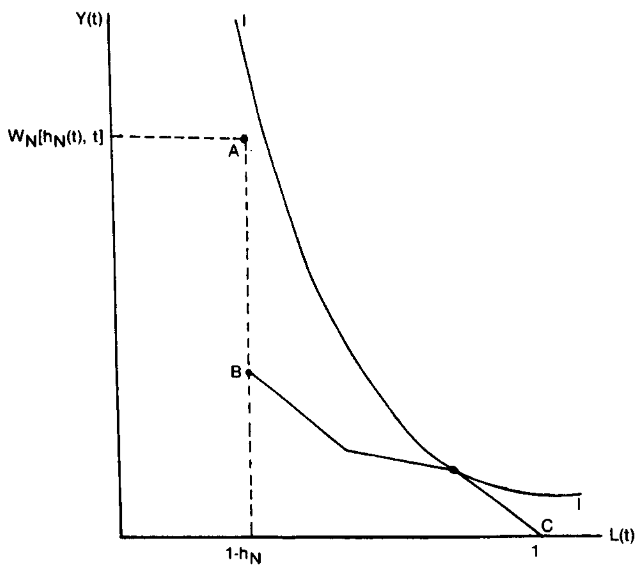
Final status	Exit rates from partial retirement in— The nonretirement job	A different job	A nonclassified job
Nonretirement	.091	.069	.071
Partial retirement in:			
The nonretirement job	.469	.000	.000
A different job	.039	.487	.082
A nonclassified job	.071	.081	.293
Full retirement	.324	.354	.536
Unemployment	.006	.009	.018

Implications for retirement models

One aspect of the descriptive statistics from the Retirement History Survey has particular importance for retirement models of the type presented earlier in this article. Namely, the data indicate that, although a substantial minority of older men pass through a stage of partial retirement, the spells of such retirement typically are very short. More than half of these spells appear to last less than 2 years, and it seems likely that few individuals would be partially retired for a significant fraction of their working lives. This fact, when considered together with the observed incidence of partial retirement, provides a powerful clue to the nature of the utility function on which individuals are basing their retirement decisions.

Exhibit 1 illustrates this maximization problem facing the individual at time t. (See appendix.) The indifference curve **I—I** is one of a set of such curves, all of which are vertical displacements of one another, or equivalently, all having the same slope along any vertical line.[17] The budget constraint for the individual at time t consists of point A plus the line segments between B and C. Point A corresponds to the earnings and leisure available if the person chooses to work on the nonretirement job. The series of line segments between B and C represent potential income opportunities if he works at a partial retirement job, allowing for effects such as the reduction in social security benefits after a disregard amount.[18] The individual chooses the point along this constraint which enables him to reach the highest indifference curve. This may occur at point A, in which case the individual is nonretired, or at some point between B and C, which corresponds to partial retirement, or at C, which represents full retirement. (Notice that a value of zero at time t, which is associated with point C, does not mean that consumption, or income from social security, pensions, or other programs would be zero should the outcome associated with point C be chosen.)

Exhibit 1. The earnings-leisure choice at time *t*



Over time, this diagram changes in some important respects. First, the indifference curves will rotate clockwise—that is, other things equal, they will become steeper as an individual ages, reflecting the fact that work is likely to be less attractive with increasing age. Point *A* may shift downward as well, because past a certain age the availability of both social security and private pensions may reduce effective compensation for work. The budget line between *B* and *C* may also be affected, but here we would not expect the effects to be too great, particularly for the part of the constraint that lies below the social security disregard amount. In this range, social security will not change the effective compensation for employment, and partial retirement jobs are unlikely to involve pension plans that alter the effective compensation.

Now consider the implications of the two facts noted above: first, that a substantial minority of older workers go through a phase of partial retirement and, second, that for most of them the period in partial retirement is fairly short. According to exhibit 1, there are two ways in which an individual might find it optimal to retire partially for a short time. One possibility is that he has a set of indifference curves with just the right degree of curvature so that when he leaves point *A*, the tangency with the budget segment *BC* will already be very close to *C*. In this case, only a slight rotation of the curve or a slight decline in the partial retirement wage rate would be sufficient to induce him to retire fully after a brief period of partial retirement. This might be a satisfactory explanation for the behavior of some individuals, but it seems unlikely that most partial retirees have indifference curves so shaped that they retire for only a short period despite the fact that they face a wide range of wages in the nonretirement and partial retirement jobs.

This leaves a second explanation for short periods of partial retirement, specifically, that the indifference curves are rotating fairly rapidly. For example, when an individual leaves the nonretirement job and partially retires, his indifference curve may have a tangency at any point along *BC*. If the indifference curves are rotating rapidly, the point of tangency will travel along *BC* toward *C* rather quickly, and the individual will fully retire after a fairly brief spell of partial retirement.

The descriptive statistics cited earlier thus suggest that the indifference curves in exhibit 1 have enough curvature that at least some persons partially retire, and that the curves are rotating fairly rapidly, becoming significantly steeper as the individual ages. But what do these results imply about the utility function in the original structural model, namely $u[C(t), L(t), t; \beta]$? To examine this issue, consider the specific function

$$(7)\quad u[C(t), L(t), t; \beta] = [\theta(t)/\rho]\cdot[C(t)^{\rho} + b(\beta, t)L(t)^{\rho}],\ \rho<1$$

where $\theta(t)$ is the time preference discount factor and $\sigma = 1/(1-\rho)$ is within-period elasticity of substitution between consumption and leisure.[19] The indifference curves implied by this utility function have slope $S_u = -b(\beta,t)\,[L(t)/C(t)]^{\rho-1}$. The corresponding indifference curves in exhibit 1 have slope $S_Z = -b(\beta,t)\ \theta(t)\ L(t)^{\rho-1}/[\lambda_Y\ d(t)]$.[20] For a given point in exhibit 1, S_Z changes over time with the quantity $b(\beta,\ t)\ \theta(t)/d(t)$, while for a given point in the consumption-leisure space of u, S_u changes according to $b(\beta,t)$. Unless the rate of time preference exceeds the discount rate by a considerable amount, both sets of indifference curves will be rotating rapidly if either is. Thus, the fact that few individuals who partially retire do so for long, which suggests that the indifference curves in exhibit 1 are rotating rapidly, implies that the indifference curves corresponding to the utility function in the structural model also are rotating rapidly as the individual ages.

Models for policy evaluation and research

The descriptive statistics presented in this article impose some important requirements for a good structural retirement model. First, the model should be able to explain the behavior of labor force status continuation rates, especially the sharp dip in these rates as workers approach age 65. It seems likely that the explanation for this dip lies in the effect of pension and social security benefit formulae, mandatory retirement policies, and other factors affecting the individual's consumption-leisure budget line. Certainly, models that explain these continuation rates in terms of *ad hoc*, discrete, age-related changes in slopes of the indifference curves should be interpreted cautiously, particularly if they are intended to predict the effects of hypothetical changes in social security or pension rules.[21] Second, a good structural model must deal with the minority of observations for which the flows appear to run counter to the normal retirement sequence. In particular, it is necessary to determine whether these reverse flows are the result of expected but unusual paths of the wages in the full-time or partial retirement jobs, or if, as seems more likely, they signify responses to unforeseen events or to miscalculations. In the latter case, the proper model may be a stochastic model in which the individual recalculates the optimal labor supply path in each period conditional on his past decisions, taking into account current or expected future changes which were not foreseen when he made his previous calculations.

The statistics also suggest an important characteristic of the lifetime utility function that individuals are attempting to maximize. Specifically, the fact that a significant number partially retire but that few of them remain in the state for very long implies that, whether the tradeoff is in terms of earnings vs. leisure or consumption vs. leisure, the indifference curves of the individuals are relatively convex but rotating fairly rapidly with age. If confirmed by further studies, this would be an important finding, for the speed with which these indifference curves rotate is a major factor in estimating the effects of potential changes in such programs as social security and private pensions on the amount of labor individuals wish to supply to the market. □

——*FOOTNOTES*——

[1] Alan L. Gustman and Thomas L. Steinmeier, "Partial Retirement and the Analysis of Retirement Behavior," *Industrial and Labor Relations Review*, April 1984.

[2] For that analysis, the main job was defined as the job held by the individual at age 55.

[3] The Retirement History Survey is a 10-year longitudinal survey of a national sample of 11,153 persons age 58 to 63 in 1969. The survey reports on the individual's work history, health, financial status, and other information relevant for studying retirement. For a description of this survey, see U.S. Department of Health, Education and Welfare, Social Security Administration, Office of Research and Statistics, *Almost 65: Baseline Data From the Retirement History Study* (Washington, U.S. Government Printing Office, 1976).

[4] Alan L. Gustman and Thomas L. Steinmeier, *Partial Retirement and Wage Profiles for Older Workers*, NBER Working Paper No. 1000 (Cambridge, Mass., National Bureau of Economic Research, October 1982).

[5] Alan L. Gustman and Thomas L. Steinmeier, "Minimum Hours Constraints and Retirement Behavior," *Contemporary Policy Issues*, April 1983, pp. 77–91.

[6] Counting each observation for a given employer only one time, we found that for a sample of older white men who were not self-employed, 53 percent of the partially retired were in jobs at which they had previously worked full time, and the remainder were in jobs at which they had not previously reported working full time. See Gustman and Steinmeier, *Partial Retirement and Wage Profiles*. The proportion who partially retire on jobs they held at age 55 is considerably smaller than the proportion who partially retire on jobs they held while not retired. See Gustman and Steinmeier, "Partial Retirement and Retirement Behavior."

[7] Including a bequest motive in the budget constraint would leave the discussion unchanged.

[8] In this formal model, the "partial retirement" job may refer to a job distinct from the nonretirement job, or it may refer to the opportunity to remain in the nonretirement job and work less than full time at a reduced wage. Separation from a job may also be involuntary. Note, however, that it would be difficult to interpret the meaning of the reason for separation. For example, an employer with an unemployment insurance tax rate that was outside the range of experience rating might have agreed to lay off some workers before they retired to allow them to collect unemployment insurance benefits. The period covered by our data predates changes in unemployment insurance regulations which were designed to deal with such problems.

[9] A closely related model could be developed with the assumption that labor supplied to the partial retirement job must fall within a more restricted range.

[10] See, for example, Henry Aaron, *Economic Effects of Social Security* (Washington, The Brookings Institution, 1982).

[11] A fourth category, not included in table 1, consists of anyone who reported that his major activity during the survey week was looking for work. With the exception of one cell (61 year-olds in 1971), the percentage in this category never exceeded 2 percent. People were classified as not retired, partially retired, or fully retired on the basis of their answers to the question "Do you consider yourself to be completely retired, partially retired, or not retired at all?"

[12] The figures in table 2 exclude data for individuals who dropped out of the sample in the subsequent survey (for exit rates) or who were not in the sample in the previous survey (for entry rates). The principal reasons for being out of the sample were death and nonresponse. The percentages of individuals who dropped out of the sample by the next survey were 10.1 for nonretired workers, 11.4 for partially retired workers, 15.0 for fully retired workers, and 10.2 for unemployed workers. Only 2.2 percent of the individuals who dropped out of the sample subsequently reentered, and most of those who did so reported themselves as fully retired.

[13] With a constant hazard rate, durations are distributed with the exponential density function $f(t) = \gamma exp(-\gamma t)$. If 51.5 percent of this distribution lies between zero and 2, γ may be calculated as .362. The mean of the distribution is then calculated as 2.76 years.

[14] There is little reliable information on the incentive effects for partial retirement. For some discussion, see Gustman and Steinmeier, "Minimum Hours Constraints," and "Partial Retirement and the Analysis of Retirement Behavior." Reduced-form retirement equations which include partial retirement as an outcome are reported in the first paper.

[15] Given the sizes of the two samples, the difference between 41.8 percent and 67.7 percent is statistically significant at better than a 1-percent confidence level.

[16] Note especially that this group includes anyone who was partially retired during the initial survey but whose current job began after age 55.

[17] This may be shown by examining the slope of an indifference curve at any point in the diagram. This slope is given by $S_L = Z_L/Z_Y$. From appendix equation (12), Z_L does not depend on $Y(t)$, so that it may be written $Z_L\ [L(t), t; \beta, \lambda_Y d(t)]$, and $Z_Y = \lambda_Y d(t)$. Thus:

$$S_Z = \frac{Z_L\ [L(t), t; \beta, \lambda_Y d(t)]}{[\lambda_Y d(t)]}$$

Because $Y(t)$ does not appear either directly or indirectly in this expression, the slope of the indifference curve at time t depends only on $L(t)$ and all the curves thus must have the same slope.

[18] For a related discussion, see Alan S. Blinder, *Private Pensions and Public Pensions: Theory and Fact*, NBER Working Paper No. 902 (Cambridge, Mass., National Bureau of Economic Research, June 1982).

[19] This is similar to the utility function presented in Roger H. Gordon and Alan S. Blinder, "Market Wages, Reservation Wages and Retirement," *Journal of Public Economics*, Vol. 14, 1980, pp. 277–308. Note that it would not make sense to choose u to be linear homogeneous, for the resulting indifference curves in the exhibit would necessarily be straight lines. This may be shown as follows. In a linear homogenous function, both u_C and u_L are strict functions of the ratio $C(t)/L(t)$, and hence of each other. By appendix equation (10), u_C is equal to $\lambda_Y d(t)$, which is independent of earnings and leisure at time t, giving the result. The Gordon-Blinder function does satisfy the criterion that the degree of homogeneity should be less than 1, yielding convex indifference curves as in exhibit 1.

[20] If ρ is close to 1, the indifference curves associated with both u and Z have little curvature. Hence the existence of a substantial amount of

partial retirement at reduced compensation rates would suggest that ρ cannot be close to 1 for all individuals. This reasoning is contrary to Gordon and Blinder's empirical finding that $\rho = 0.9$, relatively close to 1.

[21] It might be argued that a more elaborate model than ours is appropriate because discontinuities at particular ages may result from the influence of some socially acceptable retirement age, which in turn is influenced by program parameters. But to analyze the effects of changes in retirement policy, the role of a socially acceptable retirement age should be modeled explicitly, because the effects of these age terms may be altered by the policy change.

APPENDIX: Converting from consumption to earnings

To derive the tradeoffs between earnings and leisure shown in text exhibit 1, it is necessary to derive the relationship between consumption and earnings along the solution path of the model. At any point in time, the solution path maximizes the quantity

$$(8) \quad Z(t) = u[C(t), L(t), t; \beta] + \lambda_Y d(t) S(t)$$

where $S(t) = Y(t) - C(t)$ is the amount saved in period t and $Y(t) = W_N[H_N(t), t] + W_P[H_P(t), t]$ is the net compensation for labor supplied in period t. λ_Y may be interpreted as the marginal utility of discounted lifetime income—that is, the marginal utility of relaxing the lifetime budget constraint by \$1.[1] It is chosen so that when this optimization is implemented for all periods, the lifetime budget constraint $\int_O^T d(t)\, S(t)\, dt + A_o = 0$ is just satisfied. The maximization is subject to the constraints of equations (3) through (6), which describe the hours limitations on the two types of employment.

If we substitute for $S(t)$ in equation (8), the maximand in this problem becomes

$$(9) \quad Z(t) = u[C(t), L(t), t; \beta] + \lambda_Y d(t) [Y(t) - C(t)]$$

Because $C(t)$ appears neither in the definition of $Y(t)$ nor in any of the constraints in text equations (3) through (6), the value of $C(t)$ which maximizes equation (9) may be found simply by differentiating the equation and setting the result equal to zero:

$$(10) \quad \frac{dZ(t)}{dC(t)} = u_C[C(t), L(t), t; \beta] - \lambda_Y d(t) = 0$$

where u_C indicates the partial derivative with respect to the first argument. This equation may then be solved for the optimal $C^*(t)$ as a function of $L(t)$ and $\lambda_Y d(t)$:

$$(11) \quad C^*(t) = C^* [L(t), t; \beta, \lambda_Y d(t)]$$

This may in turn be substituted into equation (9) to yield:

$$(12) \quad \begin{aligned} Z(t) &= u\{C^* [L(t), t; \beta, \lambda_Y d(t)], L(t), t; \beta\} \\ &\quad + \lambda_Y d(t) \{Y(t) - C^* [L(t), t; \beta, \lambda_Y d(t)]\} \\ &= Z[Y(t), L(t), t; \beta, \lambda_Y d(t)] \end{aligned}$$

At a particular time, this means that the individual may be viewed as maximizing a utility function involving only income and leisure,[2] instead of consumption and leisure as in equation (8). The maximization is done subject to the definition of $Y(t)$ and the constraints of equations (3) through (6).

FOOTNOTES

[1] See Thomas L. MaCurdy, "An Empirical Model of Labor Supply In a Life-Cycle Setting," *Journal of Political Economy*, December 1981, pp. 1059–85.

[2] The fact that λ_Y appears in $z[\cdot]$ means that the function cannot be viewed as constant from individual to individual, because λ_Y depends on earnings opportunities in other years.