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Probability Sampling and Estimation of the Oil Remaining in 2001 from the *Exxon Valdez* Oil Spill in Prince William Sound, Alaska

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J. Pella and J. Maselko

U.S. DEPARTMENT OF COMMERCE
National Oceanic and Atmospheric Administration
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J. Pella and J. Maselko

Alaska Fisheries Science Center
Auke Bay Laboratories
11305 Glacier Highway
Juneau, AK 99810-8626
www.afsc.noaa.gov

U.S. DEPARTMENT OF COMMERCE

Carlos M. Gutierrez, Secretary

National Oceanic and Atmospheric Administration

Vice Admiral Conrad C. Lautenbacher, Jr., U.S. Navy (ret.), Under Secretary and Administrator

National Marine Fisheries Service

William T. Hogarth, Assistant Administrator for Fisheries

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ABSTRACT

Disputes concerning the amount of spill oil remaining in Prince William Sound, Alaska, from the *Exxon Valdez* grounding in 1989 required revisiting the region in 2001. In contrast to earlier surveys which were based on purposeful selection of sampling locations, probability sampling was applied in order that unbiased estimates with measures of their precision could be computed. Beach segments and subsegments were stratified by their oiling histories and lengths, and random samples were selected for a visit. At each beach visited, the surface was grided into tidal elevation intervals and perpendicular columns, and every intersecting block was further subdivided into quadrats (0.25 m² in area) for random and adaptive sampling. Adaptive sampling pursued oil found in initial random quadrats in order to delimit entire patches. Subsurface oiled sediments were classified to a visual scale, and oil present in selected quadrats was extracted and weighed in a calibration study. The surface and subsurface oiled areas of the sediments at the visited beaches were estimated, together with the weight of oil in their subsurface sediments from the calibrated visual scale. Conservative estimates of oiled areas and weights for the visited beaches included only oil seen at random and adaptive quadrats, but unbiased estimates were computed by expansion for quadrats not sampled. The estimates at visited beaches were expanded for unsampled beach segments of strata, and summed for the total in Prince William Sound. Precision was evaluated by analytical formulas as well as by bootstrap resampling. Unbiased estimation of oiled areas and weights from oil found at the random and adaptive quadrats with precision evaluated by bootstrap resampling determined that until 2001, Prince

William Sound still had a total of 41,000 m² surface oiled area (95% interval, 20,700 – 70,500 m²), and a total of 71,000 m² subsurface oiled area (95% interval, 37,700 – 113,200 m²), having subsurface oil weighing 50,000 kilograms (50 metric tons (t)) (95% interval, 24.4 - 82.6 t).

Unbiased estimates based on the random quadrats only were 78,000 m² for subsurface oiled area (95% interval, 40,600 – 127,300 m²), and 56 t (95% interval, 26.1 - 94.4 t) for subsurface oil mass, agreeing well with the estimates based on combined random and adaptive quadrats when considered in light of sampling error.

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INTRODUCTION

On 24 March 1989, the oil tanker *Exxon Valdez* ran aground on Bligh Reef in Prince William Sound, Alaska, releasing 41 million liters (36,490 metric tons (t)) of Alaska North Slope crude oil into the sea (Fig. 1). Subsequent linear estimates for the oil's dispersal on beaches declined from 783 km in 1989 to 10 km in 1992 (Neff et al. 1996), while the remaining oil volume in the fall of 1991 was estimated at 40,882 liters (Koons and Jahns 1992). The traditional methodology to estimate volume of oil stranded on shorelines uses visual surveys together with average oil depths. The accuracy of both linear and volume estimates depends on the validity of an assumption that absence of surface oil on beaches indicates the same condition for subsurface oil (Finkelstein and Gundlach 1981, Owens 1987). This supposition is plausible shortly following the stranding of oil on the beaches, but as time passes after the spill, surface oil becomes a poor indicator of subsurface oil.

Neff et al. (1996) reported that in 1991, about 2 years after the spill, only 33% of pits with surface oil also had subsurface oil, but importantly, no information was available regarding the occurrence of subsurface oil without surface oil presence. When residents of Prince William Sound complained of oil persistence at a local beach used for subsistence harvest of shellfish, a shoreline cleanup in 1996 and 1997 about 8 years after the spill unearthed substantial deposits of subsurface oil that were not evident from surface observations (Brodersen et al. 1999). During the present survey in 2001, about 12 years after the spill, only 24 of 225 (11%) pits with surface oil also had subsurface oil, and more importantly, only 24 of 341 (7%) pits with subsurface oil also had surface oil.

Previous studies targeting subsurface oil have used purposeful selection of sampling locations (Gibeaut and Piper, 1998; Neff et al. 1996) in place of long-standing methods of

probability sampling (e.g., see Cochran 1963). Both Gibeaut and Piper (1998) and Neff et al. (1996) selected shoreline sampling locations in Prince William Sound based on their oiling histories and recommendations from the public. In contrast to probability sampling, purposeful selection does not guarantee unbiased estimates, nor does it allow an evaluation of accuracy and precision of estimates. Owens (1987), Neff et al. (1996), and Gibeaut et al. (1998) concede that sampling crew judgment has a large effect on such survey estimates, especially when applied to subsurface oil. Both Neff et al. (1996) and Gibeaut and Piper (1998) used a systematic sampling approach at the selected sites to delineate the extent of subsurface oil patches, similar to the adaptive sampling plan followed in the present Auke Bay Laboratory survey at randomly selected sites. Critical to probability-based estimation with adaptive sampling is the random selection of initial pits at a selected site, but both Neff et al. (1996) and Gibeaut and Piper (1998) failed to describe how locations of the initial subsurface pits at a site were chosen.

The use of purposeful selection of sampling units by the different surveys resulted in conflicting estimates of oil persistence. Whereas Neff et al. (1996) reported 12,000 m² of subsurface oil remaining in 1992, Gibeaut and Piper (1998) concluded that in 1993, the remaining subsurface oil area in Prince William Sound was closer to 33,749 m². The 1993 survey also provided a volume estimate for subsurface oiled sediments adjacent to only the surveyed shoreline, equal to 2,041 m³. Neff et al. (1996) estimated oil that persisted on 96 km of shoreline in 1991 and 10 km in 1992. Because probability sampling was not used previously by either Exxon or public sponsored (Exxon Valdez Oil Spill Trustee Council) scientists, of course, their accuracy or precision could not be appraised. The actual amounts of oil remaining was unknown. Therefore, the disagreements among scientists over the remaining amounts of oil required

another survey for oil in Prince William Sound, this time using probability sampling so that accuracy and precision of the estimates could be evaluated. The Exxon Valdez Trustees Oil Spill Council contracted with the ABL in October 2000 to perform the survey and estimation.

Here, we describe the ABL's survey sampling design, provide related estimation formulas, and report the point estimates and their precision which were computed for the dispersal area and quantity of the remaining oil. The dispersal area was determined from a stratified random sampling design of beaches in Prince William Sound, based on their oiling histories. The oiled surface and subsurface areas of selected beaches were estimated by the initial random sampling of smaller comprising units called quadrats, followed by delineation of discovered oil patches by systematic search. The quantity, or weight, of oil remaining was determined from visual observations of oiling at sampled quadrats in the survey, which were transformed to oil weights from data of a calibration study. Two approaches to evaluate the precision of estimation are compared—analytical formulas derived from sampling theory (Cochran 1963, Thompson 1992), and the computer-intensive method called bootstrap resampling (Efron 1982).

METHODS

Sampling Design

Segments and Subsegments

Since the 1989 *Exxon Valdez* oil spill, Prince William Sound beaches have been surveyed repeatedly for the prevalence of oil. The main surveys occurred in 1989, 1990, 1991, and 1993, with the goal to map the geographical extent of oil on beaches and to classify the levels of

contamination. The surveys measured subsurface contamination, or oiling, levels by a coarse ordered scale, whose decreasing values were termed “heavy”, “medium”, “light”, “oil film/trace” and “clean”. The surveys related uninterrupted horizontal distances of such contamination levels to a list of adjacent beach segments. Three separate oiling categories of these beach segments and their total shoreline lengths were identified for which our expectations of finding oil were highest:

- 1) Heavy oiling found during the 1990, 1991, and 1993 surveys, 24.4 km.
- 2) Medium oiling found during the 1990, 1991, and 1993 surveys, 49.1 km.
- 3) Heavy oiling found during the 1989 survey, but becoming less than heavy oiling more recently, 43.1 km.

Other shorelines of Prince William Sound that had medium and low oiling in 1989 and low oiling more recently were assumed to be clean without further measurement in 2001, and therefore were excluded from the estimation for total oil persisting in Prince William Sound.

Since the oiled shorelines were not contiguous, the total shoreline within any oiling category consisted of beach segments of varying lengths. These variable length segments were then divided into subsegments of 100 m in length or less. This division was necessary because the field crews could only sample a beach of about 100 m length between high tides each day. Since the segment lengths were not even multiples of 100, leftover subsegments less than 100 m resulted. Therefore, each oiling category contained two subcategories: 100 m and < 100 m beach subsegments. Overall, six strata of beach subsegments resulted, being the possible combinations of two subsegment length categories nested within each of the three oiling categories (Fig. 2).

Beach subsegments to be sampled were selected at random from the substrata. For 100 m subsegments, simple random selection without replacement was used. For < 100 m subsegments, random selection with probability proportional to length (ppl) was used. The ppl sampling requires replacement.

Within Subsegment Sampling

Because the oil was not deposited on the shorelines uniformly with respect to tidal elevation, the beach surface of each selected subsegment was divided into six 0.5 m tidal elevation intervals (0-0.5, 0.5-1.0, 1.0-1.5, 1.5-2.0, 2.0-2.5, 2.5-3.0 m, referring to elevation below mean high tide). These intervals covered the beach between the highest elevations that deposition could have occurred (+4.8 m above mean low tide) and the lowest elevation accessible diurnally to surface sampling (+1.8 m above mean low tide).

Stratified adaptive sampling (Thompson 1992) was used within subsegments because the oil was expected to be distributed in rare patches and mapping these patches would improve precision. Such delineation of the oil distribution was consistent with the earlier traditional survey methods, but at a much finer geographic scale. In order to distribute the sampling over the beach surface of a subsegment, the surface was divided into vertical, equal-width columns. In the case of 100 m subsegments, eight columns, 12.5 m wide, were formed. Shorter beach subsegments had fewer columns of about the same width. The intersections of horizontal elevation intervals and vertical columns on the beach surface resulted in blocks (e.g., 48 blocks per 100 m subsegment), each of which was subsampled for the presence of oil (Fig. 3).

The surface of each block consisted of sampling units of 0.5 by 0.5 m quadrats. Two sampling units per block were chosen at random without replacement. For a 100 m beach subsegment, a total of 96 quadrats were chosen. The surface of each randomly selected sampling unit was scrutinized for presence of oil, and next excavated to a maximum depth of 0.5 m, or (often) less if bedrock was present. The resulting pit was named the origin pit, to distinguish it from any succeeding pits, called adaptive pits. Any subsurface oil found in the origin pit was visually classified by increasing quantity: oil film (OF), light oil residue (LOR), medium oil residue (MOR), and heavy oil residue (HOR). Digging the origin pit could disturb the surface of nearby quadrats, so surface oil observations were restricted to the origin quadrats. For subsurface oil, no confusion occurred between oil present before sampling and that recently disturbed by digging pits. If subsurface oil was found in the origin pit, a neighborhood of four bordering adaptive pits, each of the same dimensions as the origin pit, were dug around it—above, below, to the right and left. When oil was present in one or more of these adaptive pits, additional neighborhoods of adaptive pits were dug around each. Eventually, the entire patch of subsurface oil was uncovered, which could extend even beyond the block of the origin pit. The record for each randomly selected sampling unit indicated the presence or absence of surface oil, as well as the visual index of amount of subsurface oil, and the associated numbers of succeeding adaptive pits (Fig. 4).

The information about oil from the pits was used to estimate the area occupied by subsurface oil per stratum by three modes—Stratified Adaptive (Adaptive), Stratified Random Sampling (SRS), and Observed. The Adaptive estimate for each stratum used both the randomly selected and adaptive pits, first expanding oiled area within sampled subsegments for the

unsampled portion, and then expanding estimated oiled areas in sampled subsegments for unsampled subsegments in the stratum. The SRS estimate differs from the Adaptive estimate in that only the randomly selected origin pits were used in the calculations, ignoring the adaptive pits, and the expansion within sampled subsegments differs as a result. This allowed us to compare the efficiency of the Adaptive and SRS estimators. Neither the Adaptive nor the SRS estimator of oiled area per stratum are theoretically biased. The Observed estimate used the oil patches found by the random and adaptive pits without expansion for the unsampled portion within subsegments, but with expansion for unsampled subsegments in the stratum. The Observed estimator of oiled area per stratum is biased low, and no variance is computed for uncertainty at the beach segment level. Because the adaptive pits were not scored for surface oil, neither an Adaptive estimate nor an Observed estimate was available for surface oil, only an SRS estimate based on the randomly selected origin pits. Because information about the weight of oil was derived from subsurface observations, and not surface observations, only subsurface oil weight could be estimated using the Adaptive, SRS, and Observed approaches.

Gravimetric Samples

Gravimetric samples of oiled beach material were collected in order to calibrate the visual oiling classification to the physical amounts of oil present in the pits. A total of 100 pits were sampled on a schedule of 0-5 pits per day throughout the field season in order to obtain broad coverage of Prince William Sound beaches for the four oiling categories (OF, LOR, MOR, HOR). These samples were not chosen at random, but rather on systematic basis due to the uncertainty of obtaining sufficient replicates of each oiling category. Each gravimetric sample

was a subsample of the thoroughly mixed material from an oiled pit. The weight of oil present in each subsample was determined by chemical extraction and gravimetric measurement, and the total weight of oil in the pit was estimated by simple expansion for subsampling. The estimated total weight of oil per unit surface area for the pit was obtained from the pit surface area of 0.25 m². The oiling category OF was subsequently dropped from the analysis as only two gravimetric samples were taken in the field. Weight of oil for the OF category was assumed to equal zero.

Estimation Formulas

Oiled-area for a Selected Beach Subsegment

A stratified estimator of modified Horvitz-Thompson type (Chapter 26, in Thompson 1992) provides each subsegment Adaptive estimate of oiled area, \hat{y} , and its variance,

$$\hat{y} = \sum_{i=1}^H \frac{y_i I_i}{p_i}, \quad \text{var}(\hat{y}) = \sum_{i=1}^H \sum_{j=1}^H \frac{y_i y_j I_i I_j}{p_{ij}} \left(\frac{p_{ij}}{p_i p_j} - 1 \right),$$

where H oil patches occur in the entire subsegment,

y_i is the area of the i th patch,

I_i is a 0-1 indicator for the intersection of the i th patch by the initial pairs of random quadrats from the L blocks comprising the surface of the beach subsegment,

p_i is the probability of this intersection, and

p_{ij} is the probability of intersection of both the i th and j th patches by the initial pairs of random quadrats.

The probability of inclusion of the i th patch in the initial random quadrats is

$$p_i = 1 - \prod_{k=1}^L \binom{N_k - x_{ki}}{2} / \binom{N_k}{2},$$

where

N_k quadrats comprise the surface of the k th block, and

x_{ki} quadrats in the k th block intersect the i th patch.

The probability that the initial random quadrats intersect both the i th and j th patches is

$$p_{ij} = 1 - (1 - p_i) - (1 - p_j) + \prod_{k=1}^L \binom{N_k - x_{ki} - x_{kj}}{2} / \binom{N_k}{2}.$$

A stratified random sample (SRS) estimates of oiled areas in each subsegment, \hat{y} and its

variance $\text{var}(\hat{y})$ were computed as follows:

$$\hat{y} = \sum_{k=1}^L N_k \hat{p}_k, \quad \text{var}(\hat{y}) = \sum_{k=1}^L N_k (N_k - n_k) \frac{(\hat{p}_k (1 - \hat{p}_k))}{n_k - 1},$$

where N_k is the total number of quadrats in block k , $n_k (= 2)$ is the number of random quadrats

sampled in block k , \hat{p}_k is the observed proportion (0, 0.5, or 1) of random quadrats in block k

that were oiled.

Later, stratum and region totals for oiled area and weight were estimated using the Adaptive and SRS subsegment area estimates as well as the Observed subsegment areas.

Oiled-area Estimation for Strata

Prince William Sound beach subsegments were partitioned into three oiling categories and two length categories. Consider an oiling category in which N_1 subsegments were 100 m and N_2 , shorter lengths, denoted by L_1, L_2, \dots, L_{N_2} . Let n_1 , the 100 m subsegments to be sampled be drawn at random without replacement from the N_1 available, and n_2 subsegments of the second category be drawn with ppl sampling from the N_2 available. To do ppl sampling, n_2 units were drawn with replacement and probabilities $p_i = L_i / (L_1 + \dots + L_{N_2}) = L_i / L$. Adaptive stratified sampling (Thompson 1992) was applied to each selected beach subsegment for estimation of oiled area and oil weight. Let $y_{11}, y_{12}, \dots, y_{1n_1}$ be the unobserved oiled areas in the selected beach subsegments of the first stratum, and let $\hat{y}_{11}, \hat{y}_{12}, \dots, \hat{y}_{1n_1}$ be the corresponding estimated oiled areas. Let $y_{21}, y_{22}, \dots, y_{2n_2}$ and $\hat{y}_{21}, \hat{y}_{22}, \dots, \hat{y}_{2n_2}$ be the unobserved and estimated oiled areas for the ppl sample from the second stratum.

The unobserved total oiled area from the first stratum is $\sum_{i=1}^{N_1} y_{1i} = T_1$. If the y_{1i} were observable, an appropriate estimate of total oiled area would be $\hat{T}_1 = N_1 \bar{y}_1$, where \bar{y}_1 is the sample average of the y_{1i} , and its estimated variance would be $\text{var}(\hat{T}_1) = N_1 (N_1 - n_1) \frac{s_1^2}{n_1}$, where s_1^2 is the sample variance of the y_{1i} (Thompson 1992; p. 103). Because the y_{1i} are not observable and must be estimated, the estimate becomes

$$\hat{T}_1 = N_1 \tilde{y} = N_1 \left(\frac{\sum_{i=1}^{n_1} \hat{y}_{1i}}{n_1} \right). \quad (1)$$

The corresponding estimated variance of \hat{T}_1 is

$$\hat{V}(\hat{T}_1) = N_1(N_1 - n_1) \frac{s_1^2}{n_1} + \frac{N_1}{n_1} \left(\sum_{i=1}^{n_1} \text{var}(\hat{y}_{1i}) \right), \quad (2)$$

where s_1^2 is the sample variance of the \hat{y}_{1i} , and $\text{var}(\hat{y}_{1i})$ is the estimated error variance for the i th beach subsegment from stratified adaptive sampling. The first term of the estimated variance represents the variation among 100 m beach subsegments, and the second term is the contribution due to estimation error of the selected subsegments. The variance estimator is obtained from Equation 6 in Thompson (1992; p. 129), but with $\text{var}(\hat{y}_{1i})$ from stratified adaptive sampling substituted for the variance of the estimated total oiled area had simple random sampling of quadrats within the beach subsegment been used. Thompson's Equation 6 is an unbiased estimate of the variance for a population total obtained by simple random sampling at each of two stages, primary units and secondary subunits (see also Appendix I).

The unobserved total area from the second stratum is $\sum_{i=1}^{N_2} y_{2i} = T_2$. Were the y_{2i} observable and drawn with replacement and probability equal to L_i/L ($L = \sum L_i$), the Hansen-Hurwitz estimator in Thompson (1992; p. 47) for T_2 would be $\hat{T}_2 = \frac{L}{n_2} \sum_{i=1}^{n_2} \frac{y_{2i}}{L_i}$ with each value, y_{2i} , used in the sum as often as the i th beach subsegment was selected. The estimated variance of the Hansen-Hurwitz estimator would be

$$\hat{V}(\hat{T}_2) = \frac{L^2}{n_2(n_2 - 1)} \sum_{i=1}^{n_2} \left(\frac{y_{2i}}{L_i} - \frac{\hat{T}_2}{L} \right)^2.$$

Because the y_{2i} are not observable and must be estimated,

$$\hat{T}_2 = \frac{L}{n_2} \sum_{i=1}^{n_2} \frac{\hat{y}_{2i}}{L_i}. \quad (3)$$

The corresponding estimated variance of \hat{T}_2 is

$$\hat{V}(\hat{T}_2) = \frac{L^2}{n_2(n_2 - 1)} \sum_{i=1}^{n_2} \left(\frac{\hat{y}_{2i}}{L_i} - \frac{\hat{T}_2}{L} \right)^2, \quad (4)$$

where values of the \hat{y}_{2i} correspond to the estimated total in the i th beach subsegment selected (repeated selections of a particular subsegment result in differing estimates included in the sum).

This variance formula is described in Thompson (1992; pg. 132).

Sampling for oiled-area estimation is done independently between subsegment-length strata and among the three oiling categories. Therefore, estimates of total oiled area in Prince William Sound is estimated as the sum of the six oiled-area estimates (three oiling categories \times two subsegment-length categories) and the variance of this sum is the sum of the six individual variances. Estimates of oil weight are not done independently among subsegment-length strata and oiling categories, as we see next.

Oil-weight Estimation

In order to estimate the weight of oil on the beaches, a discrete visual scale having M (= 4) levels of oil contamination was defined. Consider a particular subsegment-length stratum of an oiling category. Denote its total oiled areas belonging to the M visual categories of oil

contamination by $\mathbf{A}' = (A_1, \dots, A_M)$, and let μ_{wm} be the average weight of oil per unit area of A_m . Then the weight of oil on its beaches is the total of visual-category products of the unobserved oiled areas and average weights of oil per unit area,

$$W = \sum_{m=1}^M A_m \mu_{wm} .$$

Estimates of areas in a stratum belonging to these visual categories are derived from the field sampling data. The field sampling for each subsegment allowed estimation of both the total oiled subsurface area and the proportions of that area belonging to the visual categories. Recall that in regular field sampling, the beach surface of each selected 100 m subsegment was divided into 48 blocks, and each member of a pair of random sampling units from each block was examined for surface and subsurface oil. If the subsurface was oiled, the origin pit was classified to a visual category of contamination. The beach surface of shorter (< 100 m) subsegments was divided into fewer blocks, but again each member of a pair of random origin pits from each block was classified to a visual category of subsurface oiling. The number of origin pits in the oiled area of a sampled beach subsegment was random, depended on the size of the oiled area, and actually ranged from 1 to 31, out of a possible 96, for 100 m subsegments, and 1 to 35 for < 100 m subsegments. Later, these subsegment counts that classified to the M visual categories are modeled with the multinomial probability function when estimating the visual-category proportions that compose the subsurface oiled area of a sampled beach subsegment.

Estimates of the average weight of oil per unit area within visual oiling categories were obtained from the gravimetric samples of material in oiled origin pits. Denote by $n_{w1}, n_{w2}, \dots,$

n_{wM} the numbers of pits from each of the M visual categories that were processed for weight of oil. The underlying means and variances for the weight of oil per unit area in the M visual categories, $\mu_{w1}, \mu_{w2}, \dots, \mu_{wM}$, and $\sigma_{w1}^2, \sigma_{w2}^2, \dots, \sigma_{wM}^2$ are estimated from the gravimetric samples by sample averages of their weights per unit area, $\bar{w}_1, \bar{w}_2, \dots, \bar{w}_M$, and associated sample variances, $s_{w1}^2, s_{w2}^2, \dots, s_{wM}^2$, respectively.

The oil weight estimation will be described in reverse order for the two categories of beach subsegments, beginning with <100 m subsegments and finishing with 100 m subsegments.

< 100 m Beach Subsegments

The total weight of spill oil in the second stratum comprised of < 100 m beach subsegments within an oiling category will be denoted by W_2 . This weight can be written as

$$W_2 = \sum_{m=1}^M A_m \mu_{wm},$$

where A_m is the oiled area comprised of visual category m , and μ_{wm} is the average weight of oil per unit area of A_m . Each sampled beach subsegment provides an estimate of $\mathbf{A}'=(A_1, \dots, A_M)$, the visual-category areas of the entire stratum. If a single beach subsegment, say the i th subsegment, were drawn, the Hansen-Hurwitz estimate of \mathbf{A} would be

$$\hat{\mathbf{A}}_i = \begin{pmatrix} \frac{L}{L_i} \hat{y}_i \hat{p}_{1i} \\ \frac{L}{L_i} \hat{y}_i \hat{p}_{2i} \\ \vdots \\ \frac{L}{L_i} \hat{y}_i \hat{p}_{Mi} \end{pmatrix},$$

where \hat{y}_i is the estimate of the oiled area from adaptive sampling, and $\hat{\mathbf{p}}_i = (\hat{p}_{i1}, \dots, \hat{p}_{iM})'$ is the vector of estimated proportions of the oiled area belonging to the M visual categories in the beach subsegment. The beach subsegments were drawn independently with replacement, and if a beach subsegment was drawn more than once, its oiled area and area proportions of the M visual levels were estimated repeatedly and independently of previous draws. As a result, the n_2 estimates, $\hat{\mathbf{A}}_i$, $i = 1, \dots, n_2$, are independent and identically distributed random vectors from an underlying multivariate sampling probability distribution for estimated stratum totals of the M visual categories of oiled areas. Their average,

$$\bar{\mathbf{A}} = \begin{pmatrix} \bar{A}_1 \\ \bar{A}_2 \\ \vdots \\ \bar{A}_M \end{pmatrix} = \frac{1}{n_2} \sum_{i=1}^{n_2} \hat{\mathbf{A}}_i = \frac{1}{n_2} \sum_{i=1}^{n_2} \frac{L\hat{y}_i}{L_i} \begin{pmatrix} \hat{p}_{i1} \\ \hat{p}_{i2} \\ \vdots \\ \hat{p}_{iM} \end{pmatrix}, \quad (5)$$

is an unbiased estimate of the visual oiled areas in the stratum, and it has unbiased variance-covariance matrix estimate,

$$S_{\bar{\mathbf{A}}} = (s_{\bar{A}_i, \bar{A}_j}) = \begin{pmatrix} s_{\bar{A}_1}^2 & s_{\bar{A}_1, \bar{A}_2} & \dots & s_{\bar{A}_1, \bar{A}_M} \\ - & s_{\bar{A}_2}^2 & \dots & s_{\bar{A}_2, \bar{A}_M} \\ - & - & \ddots & \vdots \\ - & - & - & s_{\bar{A}_M}^2 \end{pmatrix} = \frac{1}{n_2(n_2 - 1)} \cdot \begin{pmatrix} \sum_{i=1}^{n_2} (\hat{A}_{i1} - \bar{A}_1)^2 & \sum_{i=1}^{n_2} (\hat{A}_{i1} - \bar{A}_1)(\hat{A}_{i2} - \bar{A}_2) & \dots & \sum_{i=1}^{n_2} (\hat{A}_{i1} - \bar{A}_1)(\hat{A}_{iM} - \bar{A}_M) \\ \text{-----} & \sum_{i=1}^{n_2} (\hat{A}_{i2} - \bar{A}_2)^2 & \dots & \sum_{i=1}^{n_2} (\hat{A}_{i2} - \bar{A}_2)(\hat{A}_{iM} - \bar{A}_M) \\ \vdots & \vdots & \vdots & \vdots \\ \text{-----} & \text{-----} & \dots & \sum_{i=1}^{n_2} (\hat{A}_{iM} - \bar{A}_M)^2 \end{pmatrix}. \quad (6)$$

The unknown weight of oil in the stratum, W_2 , is estimated as

$$\hat{W}_2 = \sum_{m=1}^M \bar{A}_m \bar{w}_m, \quad (7)$$

and it has variance (See Appendix II)

$$V(\hat{W}_2) = \sum_{i=1}^M \left\{ \bar{A}_i^2 \frac{\sigma_{wi}^2}{n_{wi}} + \mu_{wi}^2 V(\bar{A}_i) + V(\bar{A}_i) \frac{\sigma_{wi}^2}{n_{wi}} \right\} + 2 \sum_{i=1}^M \sum_{j<i} \mu_{wi} \mu_{wj} Cov(\bar{A}_i, \bar{A}_j).$$

This variance is estimated unbiasedly by

$$\hat{V}(\hat{W}_2) = \sum_{i=1}^M \left\{ \bar{A}_i^2 \frac{s_{wi}^2}{n_{wi}} + \bar{w}_i^2 s_{\bar{A}_i}^2 + s_{\bar{A}_i}^2 \frac{s_{wi}^2}{n_{wi}} \right\} + 2 \sum_{i=1}^M \sum_{j<i} \bar{w}_i \bar{w}_j s_{\bar{A}_i, \bar{A}_j}, \quad (8)$$

where $s_{\bar{A}_i}^2$ the estimated variance of \bar{A}_i , and $s_{\bar{A}_i, \bar{A}_j}$ is the estimated covariance between \bar{A}_i and

\bar{A}_j .

100 m Beach Subsegments

The total weight of spill oil in the first subsegment-length stratum, comprised of 100 m beach subsegments, will be denoted by W_1 . Let $\mathbf{a}_i = (a_{i1}, a_{i2}, \dots, a_{iM})'$ denote the oiled areas composing the visual categories in beach subsegment i , and $\mathbf{A}' = (A_1, \dots, A_M)$ denote the total areas by visual category in the entire stratum. For a sample of n_1 beach subsegments, the M -dimensional visual area arrays, $\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_{n_1}$ are estimated by adaptive sampling. The visual area

estimates are denoted by the vectors, $\hat{\mathbf{a}}_1, \hat{\mathbf{a}}_2, \dots, \hat{\mathbf{a}}_{n_1}$, where $\hat{\mathbf{a}}_i = \hat{y}_i \hat{\mathbf{p}}_i$. Recall that \hat{y}_i is the estimated total oiled area from adaptive sampling in the i th beach subsegment, and $\hat{\mathbf{p}}_i$ is the associated vector of estimated proportions of the oiled area belonging to the M visual categories. The visual-category areas in the stratum are estimated by expanding the sample average as

$$\hat{\mathbf{A}} = N_1 \cdot \left(\frac{\sum_{i=1}^{n_1} \hat{a}_{i1}}{n_1}, \frac{\sum_{i=1}^{n_1} \hat{a}_{i2}}{n_1}, \dots, \frac{\sum_{i=1}^{n_1} \hat{a}_{iM}}{n_1} \right)' = N_1 \tilde{\mathbf{a}}' \quad (9)$$

If the areas within each beach subsegment could have been directly observed rather than estimated as was necessary, the variance-covariance matrix of these area estimates would be

$$V_{\hat{\mathbf{A}}} = N_1(N_1 - n_1) \frac{V_{\mathbf{a}}}{n_1},$$

where $V_{\mathbf{a}}$ is the population variance-covariance matrix of the arrays, $\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_{N_1}$. The estimate of this matrix would be

$$\hat{V}_{\hat{\mathbf{a}}} = N_1(N_1 - n_1) \frac{S_{\mathbf{a}}}{n_1},$$

where $S_{\mathbf{a}}$ is the sample variance-covariance matrix with elements

$$s_{\mathbf{a},i,j} = \frac{1}{n_1 - 1} \sum_{k=1}^{n_1} (a_{ki} - \bar{a}_i)(a_{kj} - \bar{a}_j).$$

Here $\bar{a}_i = \frac{1}{n_1} \sum_{k=1}^{n_1} a_{ki}$ is the sample average area of visual category i.

Because the \mathbf{a}_i were not observed and had to be estimated, the variance-covariance matrix of $\hat{\mathbf{A}}$ includes a term for this estimation error and is

$$V_{\hat{\mathbf{A}}} = N_1(N_1 - n_1) \frac{V_{\mathbf{a}}}{n_1} + \frac{N_1}{n_1} \sum_{k=1}^{N_1} V_{\hat{\mathbf{a}}_k - \mathbf{a}_k}.$$

Here $V_{\hat{\mathbf{a}}_k - \mathbf{a}_k}$ is the variance-covariance matrix of estimation errors in beach subsegment k (see

Appendix III),

$$\mathbf{V}_{\hat{\mathbf{a}}_k - \mathbf{a}_k} = \left(y_k^2 + \sigma_{\hat{y}_k}^2 \right) \cdot V_{\hat{\mathbf{p}}_k - \mathbf{p}_k} + \sigma_{\hat{y}_k}^2 \left(\mathbf{p}_k \mathbf{p}_k' \right).$$

The matrix, $V_{\hat{\mathbf{p}}_k - \mathbf{p}_k}$, is the variance-covariance matrix of the estimated visual category proportions in the kth beach subsegment from multinomial sampling of origin pits in the oiled area.

The jth diagonal element of the matrix, $V_{\hat{\mathbf{a}}_k - \mathbf{a}_k}$, can be written as

$$v_{\hat{\mathbf{a}}_k - \mathbf{a}_k, j, j} = \left(y_k^2 + \sigma_{\hat{y}_k}^2 \right) \cdot \frac{p_{kj}(1 - p_{kj})}{h_k} + \sigma_{\hat{y}_k}^2 p_{kj}^2,$$

where

p_{kj} is the proportion of the oiled area in the k th beach subsegment belonging to the j th visual category of oiling,

y_k is the oiled area of subsegment k ,

$\sigma_{\hat{y}_k}^2$ is the variance of the estimated oiled area in subsegment k , and

h_k is the number of origin pits that were found to be oiled and were classified to the several visual categories.

The off-diagonal element in the i th row and j th column ($i \neq j$) is

$$V_{\hat{a}_k - a_k, i, j} = -\left(y_k^2 + \sigma_{\hat{y}_k}^2\right) \cdot \frac{p_{ki} p_{kj}}{h_k} + \sigma_{\hat{y}_k}^2 p_{ki} p_{kj} \quad .$$

An unbiased estimate of $V_{\hat{a}_k - a_k}$ is denoted by $S_{\hat{a}_k - a_k}$ with diagonal elements

$$S_{\hat{a}_k - a_k, j, j} = \left[\frac{\hat{y}_k^2}{h_k} \right] \left[\left(1 - \frac{1}{h_k}\right)^{-1} \hat{p}_{kj} (1 - \hat{p}_{kj}) \right] + \hat{\sigma}_{\hat{y}_k}^2 \left[\hat{p}_{kj} - \left(1 - \frac{1}{h_k}\right)^{-1} \hat{p}_{kj} (1 - \hat{p}_{kj}) \right],$$

and off-diagonal elements,

$$S_{\hat{a}_k - a_k, i, j} = - \left[\frac{\hat{y}_k^2}{h_k} + \hat{\sigma}_{\hat{y}_k}^2 \right] \left[\left(1 - \frac{1}{h_k}\right)^{-1} \hat{p}_{ki} \hat{p}_{kj} \right].$$

The estimate of $V_{\hat{A}}$ is

$$S_{\hat{A}} = N_1(N_1 - n_1) \frac{S_{\hat{a}}}{n_1} + \frac{N_1}{n_1} \sum_{k=1}^{n_1} S_{\hat{a}_k - a_k}, \quad (10)$$

where

$S_{\hat{a}}$ is the sample variance-covariance matrix of the $\hat{\mathbf{a}}_k$, $k = 1, \dots, n_1$, and

$S_{\hat{a}_k - a_k}$ is the estimated variance-covariance matrix of estimation errors in the k th sampled subsegment.

The unknown weight of oil in the stratum, W_1 , is estimated as

$$\hat{W}_1 = \sum_{m=1}^M \hat{A}_m \bar{w}_m, \quad (11)$$

and it has a variance of

$$V(\hat{W}_1) = \sum_{i=1}^M \left\{ A_i^2 \frac{\sigma_{wi}^2}{n_{wi}} + \mu_{wi}^2 V(\bar{A}_i) + V(\bar{A}_i) \frac{\sigma_{wi}^2}{n_{wi}} \right\} + 2 \sum_{i=1}^M \sum_{j < i} \mu_{wi} \mu_{wj} \text{Cov}(\bar{A}_i, \bar{A}_j).$$

This variance is estimated unbiasedly by

$$\hat{V}(\hat{W}_1) = \sum_{i=1}^M \left\{ \hat{A}_i^2 \frac{S_{wi}^2}{n_{wi}} + \left(\bar{w}_i^2 - \frac{S_{wi}^2}{n_{wi}} \right) S_{\bar{A}_i}^2 \right\} + 2 \sum_{i=1}^M \sum_{j < i} \bar{w}_i \bar{w}_j S_{\bar{A}_i, \bar{A}_j}, \quad (12)$$

where $s_{\hat{A}_i}^2$ is the estimated variance of \hat{A}_i , and $s_{\hat{A}_i, \hat{A}_j}$ is the estimated covariance between \hat{A}_i and \hat{A}_j .

Total Weight of Spill Oil Remaining in Prince William Sound

The total subsurface oiled area in Prince William Sound by visual category equals the sum of individual components,

$$A_{\bullet\bullet m} = \sum_{i=1}^3 \sum_{j=1}^2 A_{ijm}, \quad m = 1, 2, \dots, M,$$

where A_{ijm} is the total oiled area of visual category m in the j th subsegment-length category ($j = 1$, or 100 m, $j = 2$, or < 100 m) of the i th oiling category ($i = 1, 2$, or 3).

The total weight of oil remaining in Prince William Sound is

$$W = \sum_{m=1}^M A_{\bullet\bullet m} \mu_{wm}.$$

The visual category totals, $A_{\bullet\bullet m}$, are estimated by replacing each A_{ijm} by its estimate (Equations 5 and 9) in this equation, and the total weight of oil remaining in Prince William Sound is estimated by

$$\hat{W} = \sum_{m=1}^M \hat{A}_{\bullet\bullet m} \bar{w}_m. \quad (13)$$

The variance of the estimated total weight of oil remaining is

$$V(\hat{W}) = \sum_{m=1}^M \left\{ A_{\bullet\bullet m}^2 \frac{\sigma_{wm}^2}{n_{wm}} + \mu_{wm}^2 V(\hat{A}_{\bullet\bullet m}) + V(\hat{A}_{\bullet\bullet m}) \frac{\sigma_{wm}^2}{n_{wm}} \right\} + 2 \sum_{m=1}^M \sum_{j<m} \mu_{wm} \mu_{wj} \text{Cov}(\hat{A}_{\bullet\bullet m}, \hat{A}_{\bullet\bullet j}).$$

This variance is estimated unbiasedly by

$$\hat{V}(\hat{W}) = \sum_{m=1}^M \left\{ \hat{A}_{\bullet\bullet m}^2 \frac{S_{wm}^2}{n_{wm}} + \bar{w}_m^2 \tilde{S}_{\hat{A}_{\bullet\bullet m}}^2 + \tilde{S}_{\hat{A}_{\bullet\bullet m}}^2 \frac{S_{wm}^2}{n_{wm}} \right\} + 2 \sum_{m=1}^M \sum_{j<m} \bar{w}_m \bar{w}_j \tilde{S}_{\hat{A}_{\bullet\bullet m}, \hat{A}_{\bullet\bullet j}}, \quad (14)$$

where

$$\tilde{S}_{\hat{A}} = \begin{pmatrix} \tilde{S}_{\hat{A}_{\bullet\bullet 1}}^2 & \tilde{S}_{\hat{A}_{\bullet\bullet 1}, \hat{A}_{\bullet\bullet 2}} & \cdots & \tilde{S}_{\hat{A}_{\bullet\bullet 1}, \hat{A}_{\bullet\bullet M}} \\ \tilde{S}_{\hat{A}_{\bullet\bullet 2}, \hat{A}_{\bullet\bullet 1}} & \tilde{S}_{\hat{A}_{\bullet\bullet 2}}^2 & \cdots & \tilde{S}_{\hat{A}_{\bullet\bullet 2}, \hat{A}_{\bullet\bullet M}} \\ \vdots & & \ddots & \\ \tilde{S}_{\hat{A}_{\bullet\bullet M}, \hat{A}_{\bullet\bullet 1}} & \cdots & \cdots & \tilde{S}_{\hat{A}_{\bullet\bullet M}}^2 \end{pmatrix}$$

is the estimated variance-covariance matrix for the visual area array comprised of elements,

$\hat{A}_{\bullet\bullet m}$, $m = 1, \dots, M$. The elements of this covariance array are obtained by summing

corresponding elements of the estimated variance-covariance matrices of visual areas,

$S_{\bar{A}} \text{ or } S_{\hat{A}}$ (Equations 6 or 10), of the six combinations of subsegment-length and oiling categories.

The formulas for estimates and their standard errors (square root of variances) for oiled areas and weight of oil remaining in Prince William Sound are complete. Approximate 95% confidence intervals can be computed for any estimate by subtracting (lower limit) and adding (upper limit) 2 standard errors to the point estimate. If the distribution of the estimator is not approximately normal, confidence intervals are better computed by the bootstrap method. These bootstrap estimates of standard errors and confidence intervals were computed as described the following section.

Bootstrapping for Determination of Precision of Estimates

Adaptive, SRS, and Observed estimates of oiled areas and weights are computed by bootstrapping methods. As before, the estimation is described in reverse order for the two categories of beach subsegments, beginning with < 100 m subsegments and finishing with 100 m subsegments. The average, standard deviation, and lower 2.5 percentile and upper 97.5 percentile of the empirical distributions are tabulated and reported. The percentiles provide a 95% confidence interval.

Oiled Area, <100 m Subsegments

The total oiled area in each of the three oiling categories (heavy oiling, 1990-1993; medium oiling, 1990-1993; and heavy oiling, 1989) has been estimated by the Hansen-Hurwitz formula from oiled areas, Observed, SRS, or Adaptive found in n_2 ppl randomly selected beach subsegments of N_2 available (n_2 and N_2 vary among oiling categories). Let

$$z_i = \frac{L}{L_i} \hat{y}_i, \quad i = 1, 2, \dots, n_2,$$

denote the estimate of the stratum total oiled-area from the i th subsegment. The Hansen-Hurwitz estimate is the average of the z_i . The z_i of each stratum are independent and identically distributed random variables, so a standard bootstrapping method is used to draw samples by which to estimate precision of the estimated total oiled area for a stratum: 1,000 resamples of size n_2 are drawn with replacement from the n_2 original z -values of the sample of beach subsegments. Denote these values for the b th resample as $z_{b1}^*, z_{b2}^*, \dots, z_{bn_2}^*$. Notice that each resample is composed of the original z -values of the sampled subsegments, each repeated a random number of times from 0 to n_2 . The stratum total oiled-area estimate computed by the Hansen-Hurwitz formula from subsegments of the b th resample, say T_{2b}^* , $b = 1, 2, \dots, 1,000$, equals the b th resample average of its z^* -values. The average, standard deviation, and lower 2.5 percentile and upper 97.5 percentile of the empirical distribution of the 1000 T_{2b}^* are recorded for Adaptive, SRS, and Observed oiled areas.

Oiled Area, 100 m Subsegments

The total oiled area in each of three strata (heavy oiling, 1990-1993; medium oiling, 1990-1993; and heavy oiling, 1989) has been estimated by simple expansion of average oiled areas among the n_1 sampled subsegments, either Observed, SRS, or Adaptive. To evaluate the precision of each stratum estimate of total oiled area, 1,000 bootstrap resamples of n_1 subsegments are drawn. The populations of subsegments and sampling method for generating these bootstrap samples depends on the stratum sampling fraction, n_1/N_1 . If n_1/N_1 is small

(< 0.05), dependence caused by sampling without replacement from the finite population of size N_1 is ignored, and standard bootstrapping methods for independent and identically distributed random variables apply. The bootstrap samples are drawn with replacement from the n_1 subsegments originally sampled. If n_1/N_1 is larger (≥ 0.05), the dependence is not ignored, and bootstrapping methods for finite populations apply. This method used approximations suggested by Gross (1980), as described by Booth et al. (1994). The bootstrap samples are drawn without replacement from a population of $[N_1/n_1] + 1$ copies of the original n_1 subsegments, where the expression, $[\cdot]$, denotes the integer part of the argument. Notice that any resample is composed of the estimated oiled areas of the original sampled subsegments, each occurring between 0 and n_1 times if n_1/N_1 is small, or between 0 and $[N_1/n_1] + 1$ times if n_1/N_1 is large. Denote the estimated oiled areas of the subsegments in the b th bootstrap sample as $\hat{y}_{b1}^*, \hat{y}_{b2}^*, \dots, \hat{y}_{bn_1}^*$. Let T_{1b}^* , $b = 1, 2, \dots, 1,000$, be the stratum total oiled-area estimates computed by expansion of the average oiled areas among subsegments for the i th resample,

$$T_{1b}^* = N_1 \bar{y}_b^* = N_1 \left(\frac{\sum_{i=1}^{n_1} \hat{y}_{bi}^*}{n_1} \right).$$

The average, standard deviation, and lower 2.5 percentile and upper 97.5 percentile of the empirical distribution of the 1,000 T_{1b}^* are recorded for Adaptive, SRS, and Observed oiled areas.

Visual-category Composition of Oiled Areas, < 100 m or 100 m Subsegments

Each subsegment sampled and found oiled during the survey has an estimated visual-category composition, $\hat{\mathbf{p}}$, from the h random quadrats in its oiled area. Specifically, if the counts

of quadrats in the four visual categories were $\mathbf{h} = (h_1, h_2, h_3, h_4)$, the visual-category composition of the oiled area is estimated as

$$\hat{\mathbf{p}} = (h_1 / h, \dots, h_4 / h), \quad h = \sum_{i=1}^4 h_i \quad .$$

A total of 1,000 bootstrap samples of $\mathbf{h} = (h_1, h_2, h_3, h_4)$, are obtained by one of two procedures, depending on the apparent sampling fraction. If the sampling fraction, h/\hat{y} , is less than 0.05, a sample of h quadrats is drawn with replacement from the original h quadrats observed. If the sampling fraction exceeds 0.05, a sample of size h is drawn without replacement from a population composed of $[\hat{y}/h] + 1$ copies of the original h random quadrats in the oiled area, where $[\cdot]$ truncates the number to its integer part. In either case, denote the bootstrap sample by $\mathbf{h}^* = (h_1^*, \dots, h_4^*)$. Then the corresponding estimated visual-category composition is

$$\hat{\mathbf{p}}^* = (\hat{p}_1^*, \hat{p}_2^*, \dots, \hat{p}_4^*) = (h_1^* / h, \dots, h_4^* / h).$$

Average Weight of Oil per Quadrat of the Visual Categories

The n gravimetric-extraction samples of visual category m are sampled with replacement n times to provide averages, \bar{w}_{bm}^* , $b = 1, 2, \dots, 1,000$; $m = 1, 2, 3, 4$.

Oil Weight for Strata with Subsegments < 100 m

The algorithm for computing bootstrapped values of the total oil weight for an oiled category of <100 m subsegments is as follows:

1. Set $b=1$
2. Let $z_{b1}^*, z_{b2}^*, \dots, z_{bn_2}^*$ denote the b th bootstrap resample of previously calculated z -values for a stratum of < 100 m subsegments. Corresponding to these z^* -values are the particular beach subsegments from which they were computed. For these subsegments, generate bootstrap samples for the visual-category compositions, $\hat{\mathbf{p}}_{b1}^*, \dots, \hat{\mathbf{p}}_{bn_2}^*$ as calculated above.
3. Compute the estimated visual-category oiled areas in the stratum by

$$\bar{A}_b^* = \begin{pmatrix} \bar{A}_{b,1}^* \\ \vdots \\ \bar{A}_{b,4}^* \end{pmatrix} = \frac{1}{n_2} \sum_{i=1}^{n_2} z_{bi}^* \begin{pmatrix} \hat{p}_{bi,1}^* \\ \vdots \\ \hat{p}_{bi,4}^* \end{pmatrix}.$$

4. Compute the total weight of oil in the stratum by $\hat{W}_{2,b}^* = \sum_{m=1}^4 \bar{A}_{b,m}^* \bar{w}_{b,m}^*$ using the bootstrapped samples of average weight per quadrat as calculated above.
5. If $b < 1,000$, set $b = b + 1$ and go to step 2. Otherwise, stop bootstrap sampling.

Oil Weight for Strata with Subsegments of 100 m

The algorithm for computing bootstrapped values of the total oil weight for an oiling category of 100 m subsegments is as follows:

1. Set $b=1$.
2. Let $\hat{y}_{b1}^*, \hat{y}_{b2}^*, \dots, \hat{y}_{bn_1}^*$ denote the b th bootstrap resample of oiled-area estimates for a stratum of 100 m subsegments previously calculated above. Corresponding to these oiled-area estimates are the particular beach subsegments from which they were computed. For these subsegments,

generate bootstrap samples for the visual-category compositions $\hat{\mathbf{p}}_{b1}^*, \dots, \hat{\mathbf{p}}_{bn_1}^*$ as described above.

3. Compute the visual-category areas of each beach subsegment in the b th bootstrap resample as

$$\hat{\mathbf{a}}_{bi}^* = \begin{pmatrix} \hat{a}_{bi1}^* \\ \vdots \\ \hat{a}_{bi4}^* \end{pmatrix} = \hat{y}_{bi}^* \begin{pmatrix} \hat{p}_{bi1}^* \\ \vdots \\ \hat{p}_{bi4}^* \end{pmatrix}, \quad i=1,2,\dots,n_l.$$

4. Compute the estimate of total oiled area by visual-category in the stratum for the b th bootstrap sample as

$$\hat{A}_b^* = \begin{pmatrix} \hat{A}_{b1}^* \\ \vdots \\ \hat{A}_{b4}^* \end{pmatrix} = \frac{N_1}{n_1} \begin{pmatrix} \sum_{i=1}^{n_1} \hat{a}_{bi1}^* \\ \vdots \\ \sum_{i=1}^{n_1} \hat{a}_{bi4}^* \end{pmatrix}.$$

5. Compute the total weight of oil in the stratum by $\hat{W}_{1,b}^* = \sum_{m=1}^4 \hat{A}_{b,m}^* \bar{w}_{b,m}^*$ using the bootstrapped samples of average weight per quadrat as previously calculated.

6. If $b < 1,000$, set $b = b + 1$ and go to step 2. Otherwise, stop bootstrap sampling.

The algorithm above for total oiled area and oil weight apply to a single stratum. For Prince William Sound totals of area estimates, the b th area estimates, T_{1b}^* and T_{2b}^* , are summed over the strata to provide the b th bootstrap estimate of the Prince William Sound total oiled area. For Prince William Sound totals of weight estimates, the b th estimates, $\hat{W}_{1,b}^*$ and $\hat{W}_{2,b}^*$, are summed

over the strata to provide the b th bootstrap estimate of the Prince William Sound total oil resulting in a distribution of 1,000 bootstrap estimates.

RESULTS AND DISCUSSION

Sampling was concentrated (17% - 20% sampling fraction, by number of subsegments, or 20% - 27% by length) on beaches for which the most recent surveys found heavy oiling (lines 1 and 2, Table 1). Lower intensity (3% - 5% sampling fraction, by number of subsegments, or 3% - 8%, by length) was directed to beaches found to have medium oiling during these surveys (lines 3 and 4, Table 2), and yet lower intensity (2% sampling fraction, by number of subsegments, and 2%, by length) was applied to beaches found with heavy oiling in earlier surveys, but less than medium oiling more recently.

Comparison between subsurface oiled area estimates within sampled subsegments (strata combined) for the SRS and Adaptive modes (Fig. 5) shows good agreement as expected, but with several material discrepancies for subsegments with larger oiled areas. In particular, SRS underestimated the known minimum oiled areas on four subsegments (Fig. 6) in contrast to adaptive estimation (Fig. 7). In general, the estimated standard errors of oiled areas in the sampled subsegments by the two modes reflect the reduced uncertainty by including the adaptive pits (Fig. 8).

Estimates of the total subsurface oiled area per stratum and measures of their precision were computed for Adaptive, SRS, and Observed modes, either from analytical formulas or by bootstrap resampling (Table 2). As expected, Observed estimates of oiled quadrats within

subsegments, when extrapolated to stratum totals, provide minimal, biased values. Adaptive and SRS estimates are in good agreement for the more heavily sampled strata 1 and 2, and in fair agreement as sampling intensity declines in strata 3, 4, and 5. Analytical and bootstrap computations are generally in good agreement, but only the bootstrap confidence intervals appropriately shift to accommodate the skewed distributions. Considering the bootstrap computations for the Adaptive estimates as most trustworthy, strata 1 and 2 had considerable oiled area, 13,000 m² (4,400 - 24,600 m²), and 22,000 m² (10,300 - 36,000 m²), respectively. Stratum 4 may also have had substantial oiled area, estimated imprecisely at 27,000 m² (2,400 - 63,900 m²). The other strata, 3, 5, and 6, had less oiled area, also imprecisely determined as 6,000 m² (900 - 13,800 m²), 3,000 m² (0 - 9,700 m²), and 0 m² (none observed), respectively. Although the separate strata provide relatively imprecise subsurface oiled area estimates, the grand sum for Prince William Sound is reasonably precise (Table 3), with Adaptive estimates from the bootstrap computations equaling 71,000 m² (37,700 - 113,200 m²). The differences in grand sum of subsurface oiled area for Prince William Sound, whether Adaptive or SRS estimates, and analytical or bootstrap computations, are relatively less concerning as well (Table 3).

Estimates of the total surface oiled area per stratum and measures of their precision were computed for stratified random sampling (SRS), either from analytical formulas or by bootstrap resampling (Table 4). Analytical and bootstrap computations are generally in good agreement, but only the bootstrap confidence intervals accommodate the skew distributions reasonably.

Considering the bootstrap computations as most trustworthy, the total surface oiled area was estimated for strata 1, 2, and 3 as 8,000 m² (3,300 - 12,700 m²), 12,000 m² (6,300 - 19,000 m²), and 3,000 m² (800 - 4,800 m²), respectively. The corresponding estimate for stratum 4 was very

imprecise at 19,000 m² (1,600 - 46,500 m²). No surface oil was found in strata 5 and 6. The grand sum for Prince William Sound is 41,000 m² (20,700 - 70,500 m²) (Table 5).

Estimates of the total subsurface oil weight (metric tons (t), or 1,000s kg) per stratum and measures of their precision were computed for Adaptive, SRS, and Observed modes, either from analytical formulas or by bootstrap resampling (Table 6). Observed estimates of oil weight, when extrapolated to stratum totals, provide minimal, biased values. Considering Adaptive estimates and bootstrap computations as most reliable, total weights of subsurface oil for strata 1 through 6 were estimated at 8 t (2.7 - 13.8 t), 18 t (6.3 - 34.7 t), 4 t (0.3 - 10.9 t), 17 t (0.5 - 42.9 t), 4 t (0 - 11.3 t), and 0 t (none observed), respectively. The grand sum of subsurface oil weight for Prince William Sound was 50 t (24.4 - 82.6 t) (Table 7).

Although the added effort for adaptive pits reduced uncertainty for the oiled areas of sampled subsegments, the variation among subsegments within strata greatly increased the variances for stratum totals. For example, the variance equation for stratum total oiled area in the 100 m subsegments (see Equation 2) comprises two terms, the first representing variation among subsegments, and the second, estimation error within subsegments. In applying Equation 2 to Adaptive estimates of strata 2 and 4, the contribution to variance of stratum total oiled area from variation among - subsegments represented more than 99% of total variance. Because the within - subsegment variation was negligible as compared to between - subsegment variation, the adaptive sampling would have been better implemented by allocating sampling effort among strata from oil found in a preliminary exploration, instead of delimiting the oil patches within an oiled beach subsegment. For example, stratum 4 contained roughly 40% of all estimated subsurface oil (Tables 2 and 3) whereas only 3% of the segments were sampled in that stratum (Table 1). By

implementing an adaptive program on the strata with additional beach subsegments sampled from strata where more oil was found, precision in estimation of the amount of *Exxon Valdez* spill oil remaining in Prince William Sound would have been enhanced. Unfortunately, the specific nature of the beach sampling permit requirements prevented such adaptation to discovery.

The bootstrap procedure, although computer intensive, provided more realistic measures of precision evident from the asymmetric 95% confidence intervals from skewed distributions of the bootstrap estimates. The minimal oil estimate should never be less than zero, as occurred with the analytical mode, nor less than the amount of oil actually found. The bootstrap calculations were more consistent with this principle than the analytical calculations. None of the lower bounds from the bootstrap were negative, and only when the oil was very scarce did bootstrap resampling fail to produce a lower bound at least as great as the found oil. In implementing the bootstrap resampling, the negligible estimation error within 100 m subsegments, but not < 100 m subsegments, was omitted from the computations. If sampling effort were better distributed among oiling strata in future surveys, the relative importance of estimation error within subsegments could increase and necessitate its inclusion in bootstrap calculations.

Although statistically unbiased within the areas sampled, the estimates of oiled area and weight for Prince William Sound are likely biased low nonetheless. The sampling scheme was constrained by the tidal nature of Prince William Sound and allowed sampling only at lower tidal stages. However, on a number of occasions, our adaptive pits extended below the sampling grid which was predetermined at 3.0 m below high tide line. Since the sampling design did not include the oil found in this lower intertidal zone, the reported values are minimum estimates.

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Table 1.-- Strata sizes, sample sizes, and sampling fractions of the beach subsegments in Prince William Sound classified by oiling category, in terms of number and length (m).

Stratum (h)	Segments (N _h)	Sampled Segments (n _h)	Sampling Fraction (%)	Total Length (m)	Sampled Length (m)	Sampling Fraction of Length (%)
1. 1990-93, heavy, <100 m	133	23	17	5935	1576	27
2. 1990-93, heavy, 100 m	184	37	20	18458	3708	20
3. 1990-93, med, < 100 m	310	16	5	12827	978	8
4. 1990-93, med, 100 m	362	10	3	36303	1000	3
5. 1989, heavy, < 100 m	221	5	2	11294	253	2
6. 1989, heavy, 100 m	317	5	2	31775	500	2
Totals	1,527	96	6	116,592	8,015	7

Table 2.-- Estimated subsurface oiled area (1,000s m²) per stratum as determined by three modes of expansion for unexamined beach area - Adaptive, Stratified random sampling (SRS), and Observed - and precision from analytical and bootstrap calculations. Note that no oil was found in stratum 6. The column “estimate”, refers to point estimate if calculation is analytical, or mean estimate, if bootstrap. Lower and upper limits refer to bounds of the 95% confidence interval (CI).

Stratum (h)	Computation Method	Estimate	Lower 95% CI	Upper 95% CI	Standard Error	Coefficient of Variation
1. 1990-93, heavy, <100 m	Analytical Adaptive	13.4	3	23.9	5.35	0.4
	Bootstrap Adaptive	13.3	4.4	24.6	5.18	0.39
	Analytical SRS	12.8	3.7	21.9	4.64	0.36
	Bootstrap SRS	12.9	4.9	23.2	4.63	0.36
	Analytical Observed	10.4	0.4	20.3	5.09	0.49
	Bootstrap Observed	10.6	2.5	21.3	4.95	0.47
2. 1990-93, heavy, 100 m	Analytical Adaptive	21.8	9.2	34.5	6.46	0.3
	Bootstrap Adaptive	21.6	10.3	36	6.61	0.31
	Analytical SRS	22.3	10.4	34.2	6.05	0.27
	Bootstrap SRS	22.6	11.4	36.7	6.48	0.29
	Analytical Observed	15.5	3.8	27.2	5.97	0.38
	Bootstrap Observed	15.6	4.8	29.3	6.31	0.41
3. 1990-93, med, < 100 m	Analytical Adaptive	5.8	-1.6	13.2	3.78	0.65

	Bootstrap Adaptive	5.8	0.9	13.8	3.73	0.64
	Analytical SRS	6.9	-1.5	15.4	4.31	0.62
	Bootstrap SRS	7.1	0.9	16.6	4.2	0.59
	Analytical Observed	1.8	-0.6	4.1	1.2	0.67
	Bootstrap Observed	1.7	0.07	4.2	1.14	0.65
4. 1990-93, med, 100 m	Analytical Adaptive	26.6	-7.9	61.1	17.6	0.66
	Bootstrap Adaptive	27.3	2.4	63.9	17.1	0.63
	Analytical SRS	29.8	-10.9	70.4	20.7	0.7
	Bootstrap SRS	29	2.4	71.4	20.4	0.7
	Analytical Observed	18.3	-16.5	53.1	17.8	0.97
	Bootstrap Observed	18	0.05	54.4	16.8	0.94
5. 1989, heavy, < 100 m	Analytical Adaptive	3.2	-3.1	9.5	3.22	1
	Bootstrap Adaptive	3.2	0	9.7	2.91	0.91
	Analytical SRS	6.6	-6.4	19.7	6.65	1
	Bootstrap SRS	6.4	0	19.9	6.25	0.98
	Analytical Observed	2.6	-2.5	7.7	0.87	1
	Bootstrap Observed	3.2	0	9.4	0.955	0.89

Table 3.-- Estimated total subsurface oiled area (1,000s m²) remaining in Prince William Sound in 2001 as determined by three modes of expansion for unexamined beach area - Adaptive, Stratified random sampling (SRS), and Observed - and precision from analytical and bootstrap calculations. The column "estimate," refers to point estimate if calculation is analytical, or mean estimate if bootstrap. Lower and upper limits refers to bounds of the 95% confidence interval (CI).

Computation Method	Estimate	Lower 95% CI	Upper 95% CI	Standard Error	Coefficient of Variation
Analytical Adaptive	71	31.5	110.4	20.1	0.28
Bootstrap Adaptive	71.2	37.7	113.2	19.7	0.28
Analytical SRS	78.4	32.4	124.4	23.5	0.3
Bootstrap SRS	78	40.6	127.3	22.8	0.29
Analytical Observed	48.6	10.2	87.1	19.6	0.4
Bootstrap Observed	49.1	19.9	92.7	18.7	0.38

Table 4.-- Estimated surface oiled area (1,000s m²) per stratum as determined by Stratified random sampling (SRS) mode of expansion for unexamined beach area and precision from analytical and bootstrap calculations. Note that no surface oil was found in strata 5 and 6. The column “estimate,” refers to point estimate if calculation is analytical, or mean estimate, if bootstrap. Lower and upper limits refer to bounds of the 95% confidence interval (CI).

Stratum (h)	Computation Method	Estimate	Lower 95% CI	Upper 95% CI	Standard Error	Coefficient of Variation
1. 1990-93, heavy, <100 m	Analytical SRS	8.8	3.9	13.7	2.49	0.28
	Bootstrap SRS	7.6	3.3	12.7	2.44	0.32
2. 1990-93, heavy, 100 m	Analytical SRS	12	5.9	18	3.1	0.26
	Bootstrap SRS	12.1	6.3	19	3.29	0.27
3. 1990-93, med, < 100 m	Analytical SRS	2.7	0.6	4.7	1.05	0.39
	Bootstrap SRS	2.6	0.8	4.8	1.02	0.39
4. 1990-93, med, 100 m	Analytical SRS	19.4	-6.1	45	13	0.67
	Bootstrap SRS	19	1.6	46.5	12.5	0.66

Table 5.-- Estimated total surface oiled area (1,000s m²) remaining in Prince William Sound in 2001. The Stratified random sampling (SRS) mode was used to expand for unexamined beach area and precision was obtained from analytical and bootstrap calculations. The column "Estimate" refers to point estimate if calculation is analytical, or mean estimate, if bootstrap. Lower and upper limits refer to bounds of the 95% confidence interval (CI).

Computation Method	Estimate	Lower 95% CI	Upper 95% CI	Standard Error	Coefficient of Variation
Analytical SRS	42.9	16.1	69.7	13.7	0.32
Bootstrap SRS	41.3	20.7	70.5	13.2	0.32

Table 6.-- Estimated subsurface oil weight (metric tons) per stratum as determined by three modes of expansion for unexamined beach area - Adaptive, Stratified random sampling (SRS), and Observed - and precision from analytical and bootstrap calculations. The column “estimate,” refers to point estimate if calculation is analytical, or mean estimate, if bootstrap. Lower and upper limits refer to bounds of the 95% confidence interval (CI).

Stratum (h)	Computation Method	Estimate	Lower 95% CI	Upper 95% CI	Standard Error	Coefficient of Variation
1. 1990-93, heavy, <100 m	Analytical Adaptive	7.7	1.9	13.5	2.95	0.38
	Bootstrap Adaptive	7.6	2.7	13.8	2.88	0.38
	Bootstrap SRS	7.5	3	13.8	2.71	0.36
	Bootstrap Observed	5.9	1.6	12.2	2.71	0.46
2. 1990-93, heavy, 100 m	Analytical Adaptive	18.3	4.1	32.5	7.25	0.4
	Bootstrap Adaptive	18	6.3	34.7	7.49	0.42
	Bootstrap SRS	18.7	6.7	36.6	7.59	0.41
	Bootstrap Observed	14.5	3.6	31	7	0.48
3. 1990-93, med, < 100 m	Analytical Adaptive	4	-1.7	9.6	2.88	0.73
	Bootstrap Adaptive	4	0.3	10.9	2.92	0.74
	Bootstrap SRS	4.9	0.3	12.4	3.27	0.66
	Bootstrap Observed	1.3	0.02	3.3	0.88	0.7
4. 1990-93, med, 100 m	Analytical Adaptive	16	-6.6	38.6	11.5	0.72

	Bootstrap Adaptive	16.5	0.5	42.9	11.4	0.69
	Bootstrap SRS	17.5	0.5	48.8	13.6	0.78
	Bootstrap Observed	11.5	0.01	37	11	0.96
5. 1989, heavy, < 100 m	Analytical Adaptive	3.5	-3.4	10.4	3.53	1
	Bootstrap Adaptive	3.5	0	11.3	3.3	0.94
	Bootstrap SRS	7	0	24.1	7.03	1
	Bootstrap Observed	3.5	0	10.7	3.21	0.91

Table 7.-- Estimated total subsurface oil weight (metric tons) remaining in Prince William Sound in 2001 as determined by three modes of expansion for unexamined beach area - Adaptive, Stratified random sampling (SRS), and Observed - and precision from analytical and bootstrap calculations. The column "estimate," refers to point estimate if calculation is analytical, or mean estimate, if bootstrap. Lower and upper limits refer to bounds of the 95% confidence interval (CI).

Computation Method	Estimate	Lower 95% CI	Upper 95% CI	Standard Error	Coefficient of Variation
Analytical Adaptive	49.5	22.4	76.7	13.8	0.28
Bootstrap Adaptive	49.6	24.4	82.6	15.2	0.31
Bootstrap SRS	55.6	26.1	94.4	17.9	0.32
Bootstrap Observed	36.7	14.5	66.4	14	0.38

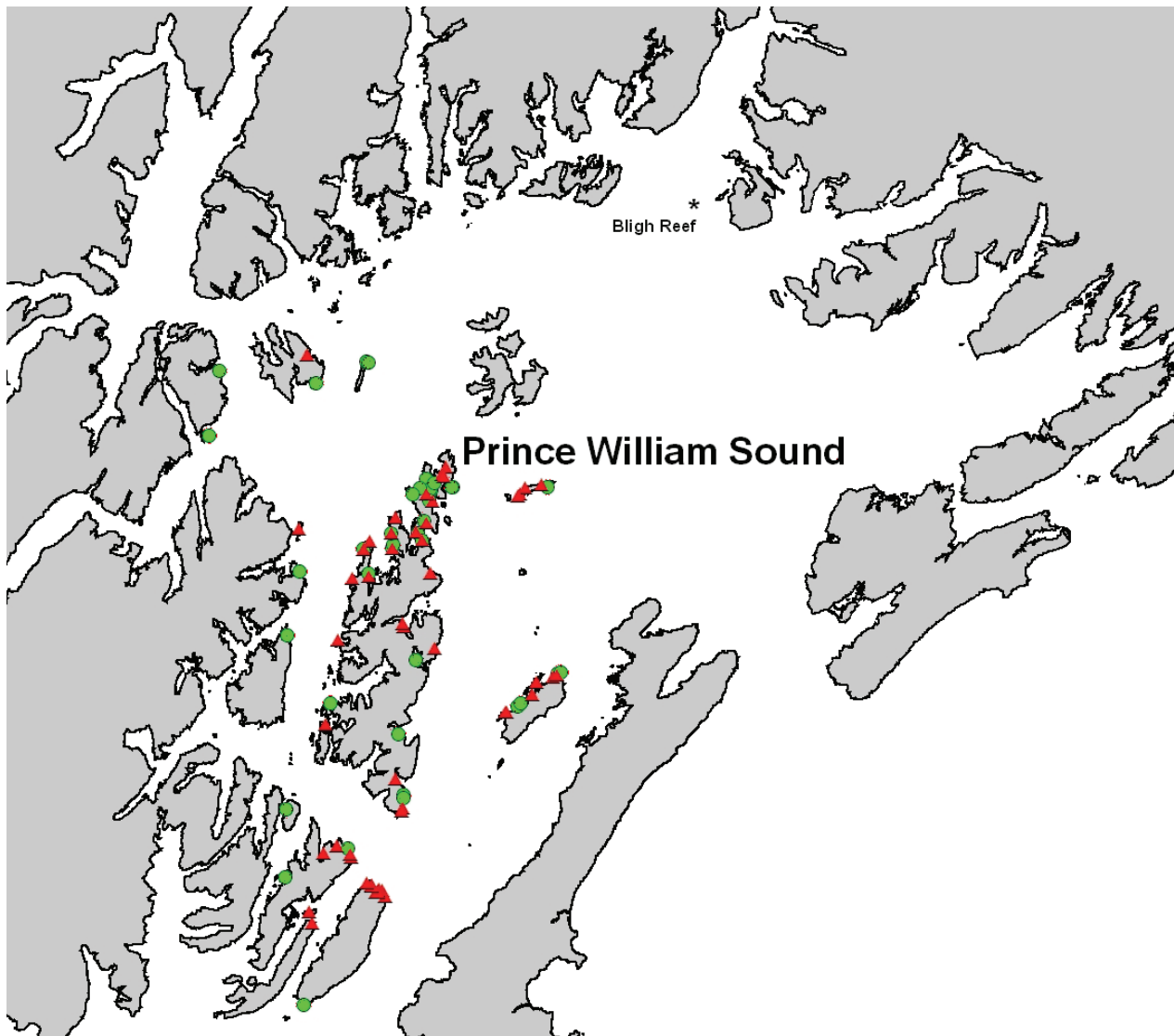


Figure 1.-- Prince William Sound with Bligh Reef (*), the grounding site of the *Exxon Valdez*, and the beaches affected by the spill. Symbols indicate stations where oil was detected (triangles) or not (circles).

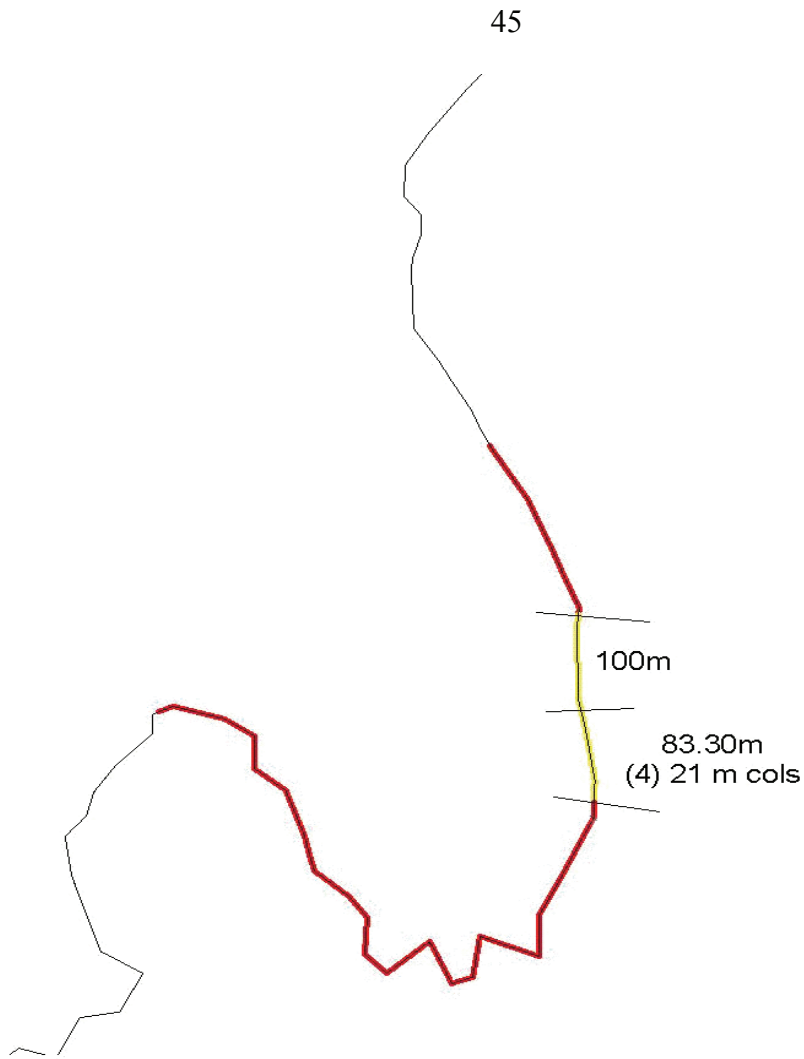


Figure 2. -- The total shoreline within any oiling category consisted of discontinuous beach segments of varying lengths scattered throughout the spill area. These variable length segments were then divided into subsegments of 100 m length or less. In the case above, a 183 m segment was divided into a 100 m and a 83 m subsegment.

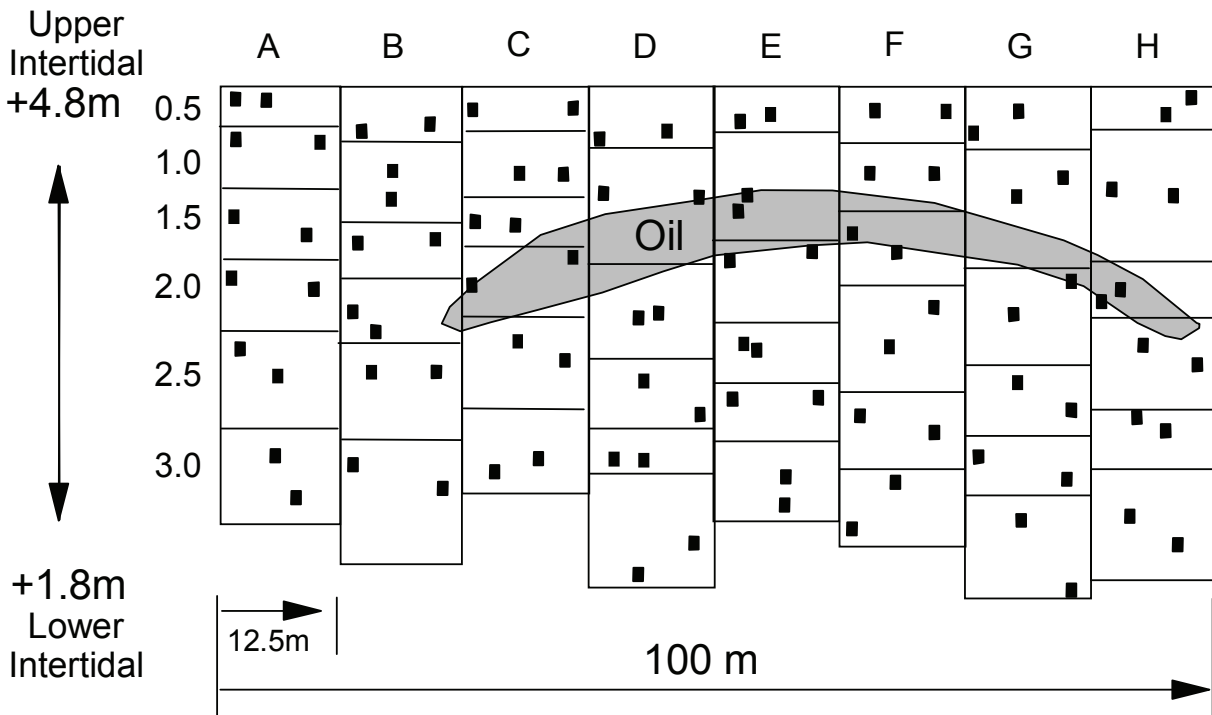


Figure 3. -- Sampling scheme for a 100 m beach subsegment. The 100 m length beach is divided into 8 columns, 12.5 m wide which are then surveyed for 3 vertical meter drops in 0.5 m intervals. Two points are chosen at random within each of the resulting 48 blocks and a 0.5 by 0.5 m area is dug down 0.5 m depth and closely observed for oil. Therefore 96 pits are excavated in a 100 m beach, barring cliffs or bedrock. Fewer columns are sampled if the beach is less than 100 m in length. Within each subsegment, column widths are always kept the same width. We used the following calculation to determine the number of columns for beaches <100 m in length: # of columns = integer (length of beach/12) with a minimum of 1 column (if length is less than 12 m). The column width was calculated by dividing the beach length by the number of columns determined above and rounding to the nearest 0.5 m.

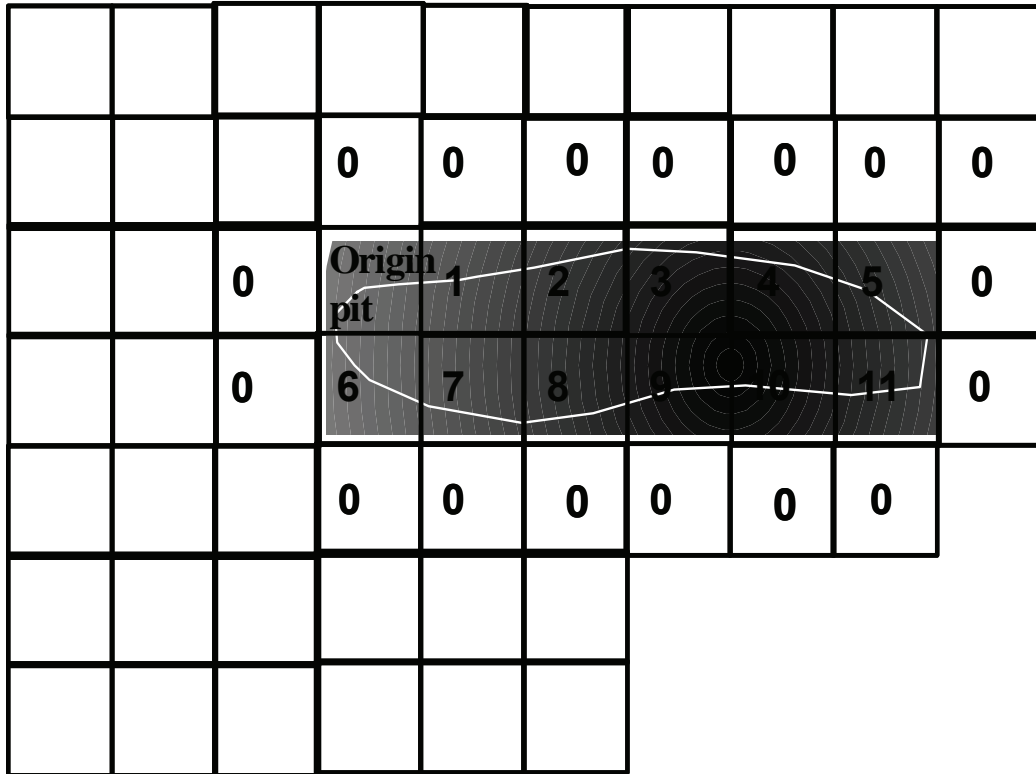


Figure 4. -- An example of the adaptive sampling methodology. Each square in the grid represents a 0.5 by 0.5 m area of a potential pit. The origin pit's location is chosen at random within each block (2 pits per block). If oil is observed in the origin pit, four quadrats are excavated to a 0.5 m depth, directly above, below, to the right, and the left of the origin pit. If oil is discovered in any of the adaptive pits, more pits are dug in the same fashion. Eventually the total area of the oil patch is delineated unless the patch extends outside the chosen beach boundaries. In the figure above, the numbered squares represent dug pits with 0s being adaptive pits with no oil found.

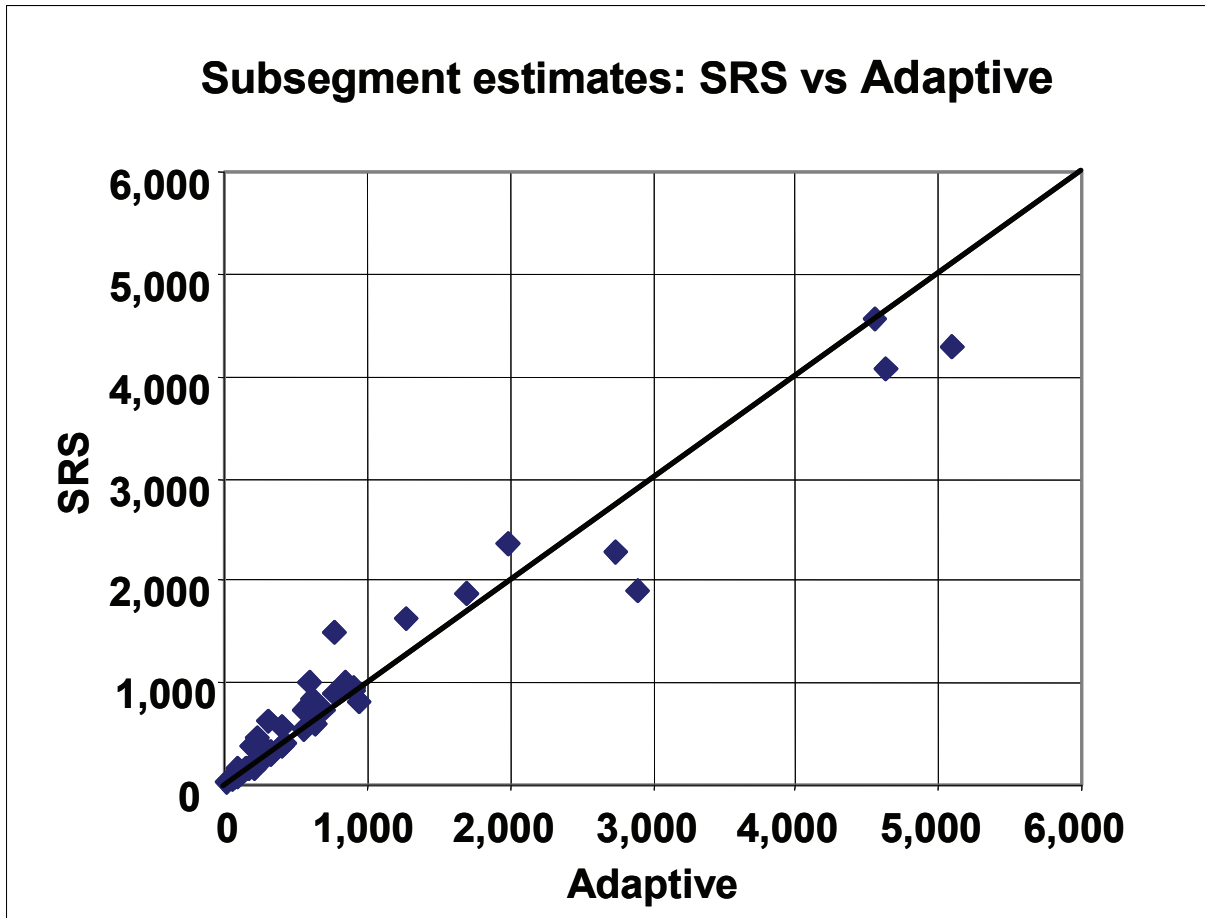


Figure 5.-- Estimated subsurface oiled areas (quadrats) in sampled subsegments by Stratified andom sampling (SRS) and Adaptive modes, and the line of equality between modes.

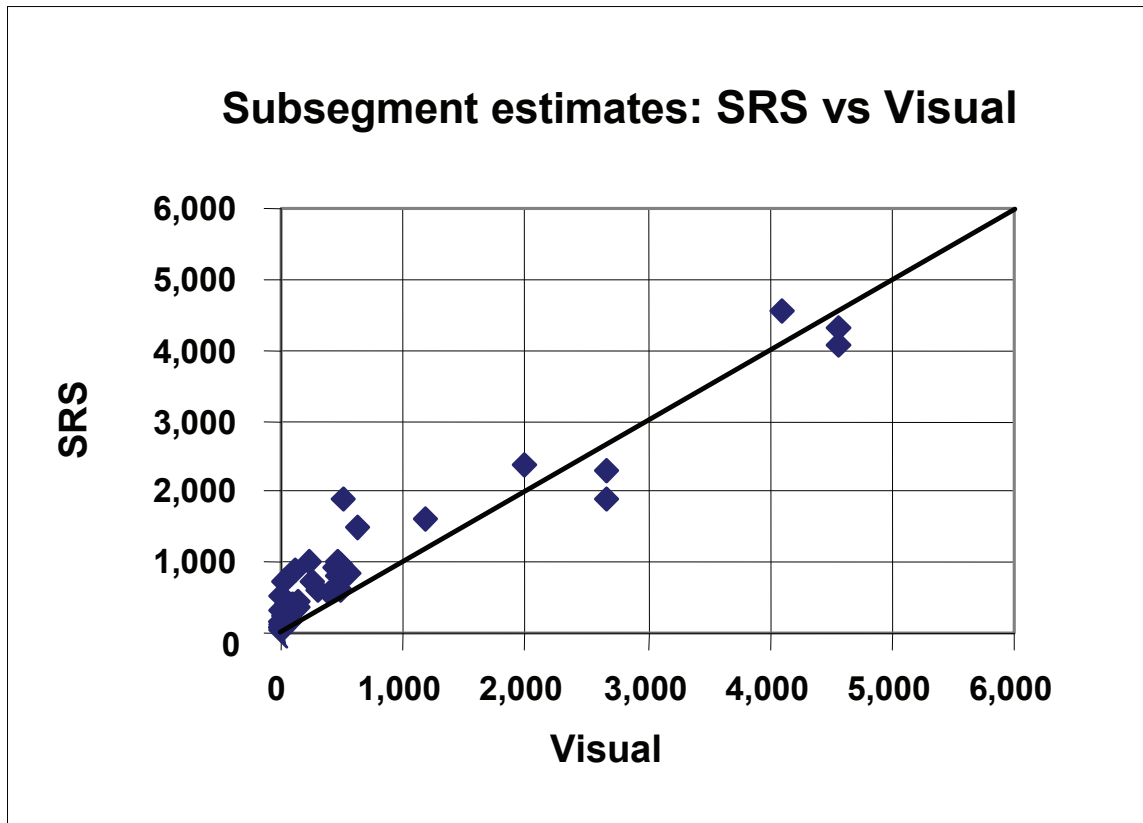


Figure 6.-- Estimated subsurface oiled areas (quadrats) in sampled subsegments by Stratified random sampling (SRS) compared to the numbers of sampled quadrats with oil present (Visual). Line of equality separates known underestimates (below line) from feasible estimates (above line).

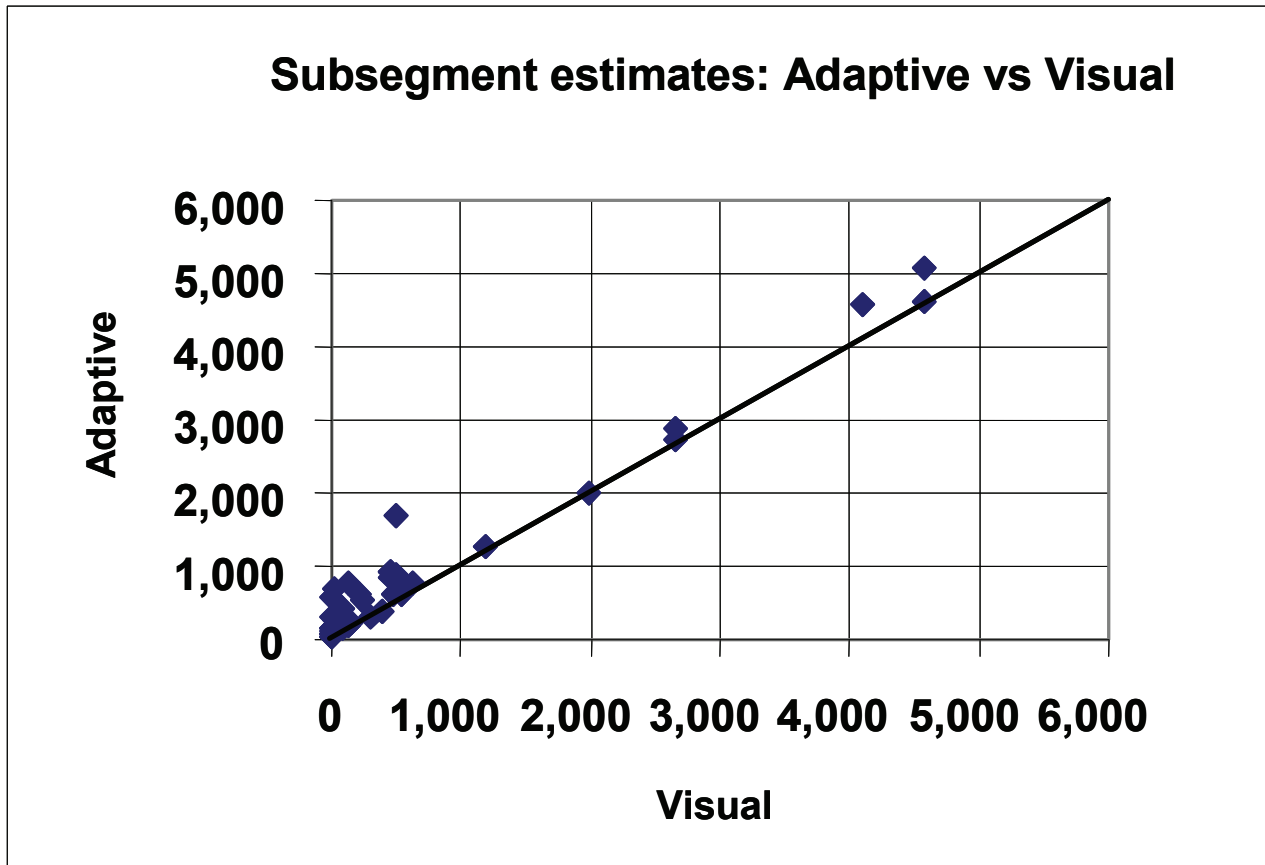


Figure 7.-- Estimated subsurface oiled areas (quadrats) in sampled subsegments by Adaptive sampling compared to the numbers of sampled quadrats with oil present (Visual), and line of equality.

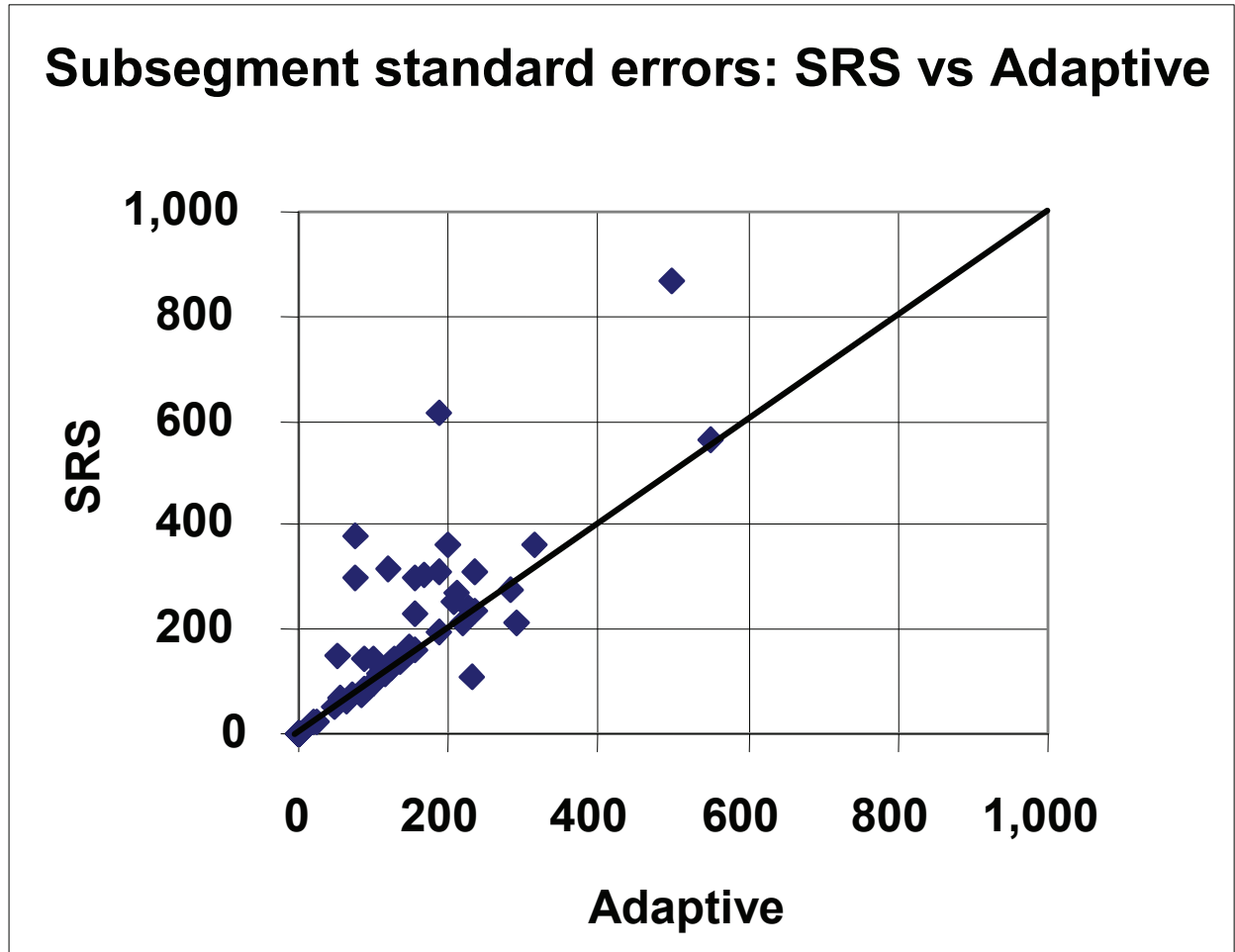


Figure 8.-- Estimated standard errors (quadrats) of subsurface oiled area estimates for sampled subsegments, by Stratified random sampling (SRS) and Adaptive modes, and line of equality.

Appendices

Appendix I

The estimation equation is $\hat{T}_1 = N_1 \left(\frac{\sum_{i=1}^{n_1} \hat{y}_{1i}}{n_1} \right)$, and its variance is given by the general

formula (see Equation 10 on page 134 of Thompson 1992),

$$Var(\hat{T}_1) = \text{var}[E(\hat{T}_1|s_1)] + E[\text{var}(\hat{T}_1|s_1)] .$$

Here s_1 denotes one of the $\binom{N_1}{n_1}$ possible selections of n_1 100 m beach subsegments from the N_1 available. The conditional expectation and variance of \hat{T}_1 , given a particular selection of 100 m beach subsegments, s_1 , are taken over the possible samples of quadrats obtained by adaptive sampling in s_1 . The unconditional variance and expectation are taken over all possible selections of s_1 . For any 100 m beach subsegment included in sampling (say the i th subsegment), the expected value of \hat{y}_i is y_i because the adaptive stratified sampling is unbiased.

Therefore,

$$E[(\hat{T}_1|s_1)] = N_1 \left(\frac{\sum_{i=1}^{n_1} E(y_i|s_1)}{n_1} \right) = N_1 \left(\frac{\sum_{i=1}^{n_1} y_i}{n_1} \right) = N_1 \bar{y}_1 .$$

Because of simple random sampling of the 100 m subsegments, the corresponding unconditional variance is

$$\text{var}\left[E(\hat{T}_1|s_1)\right] = \text{var}\left[N_1\bar{y}_1|s_1\right] = N_1^2 \text{var}(\bar{y}_1) = N_1(N_1 - n_1) \frac{\sigma_y^2}{n_1}$$

where σ_y^2 is the population variance of the unobservable y_i , $i=1, \dots, N_1$.

The conditional variance of \hat{T}_1 , given the selection of 100 m beach subsegments s_1 , depends on the estimation variances for the oiled areas of the individual beach subsegments. Let the estimation variance for the i th 100 m beach subsegment be denoted as

$$\sigma_{\hat{y}_i}^2 = E(\hat{y}_i - y_i)^2.$$

Because the estimation errors are independent among beach subsegments,

$$\text{var}(\hat{T}_1|s_1) = N_1^2 \text{var}\left(\frac{\sum_{i=1}^{n_1} \hat{y}_i}{n_1}\right) = \left(\frac{N_1}{n_1}\right)^2 \sum_{i=1}^{n_1} \sigma_{\hat{y}_i}^2$$

The unconditional expected value of this conditional variance over all possible selections of 100 m beach subsegments is obtained by setting z_i to be an indicator variable for presence of the i th beach subsegment in the random sample ($z_i = 1$ if present, and $z_i = 0$ otherwise) and rewriting the preceding conditional variance (noting that the expected value of $z_i = \frac{n_1}{N_1}$) as

$$E[\text{var}(\hat{T}_1|s_1)] = \sum_{i=1}^{N_1} \left(\frac{N_1}{n_1}\right)^2 E(z_i) \sigma_{\hat{y}_i}^2 = \left(\frac{N_1}{n_1}\right)^2 \sum_{i=1}^{N_1} \sigma_{\hat{y}_i}^2.$$

Appendix II

Notice that the variance,

$$V(\hat{W}_2) = \sum_{j=1}^M V(\bar{A}_j \bar{w}_j) + 2 \sum_j \sum_{k < j} \text{Cov}(\bar{A}_j \bar{w}_j, \bar{A}_k \bar{w}_k).$$

We assume reasonably that \bar{A}_j and \bar{w}_j are independent, so that (Goodman 1960)

$$V(\bar{A}_j \bar{w}_j) = A_j^2 V(\bar{w}_j) + \mu_{wj}^2 V(\bar{A}_j) + V(\bar{A}_j) V(\bar{w}_j).$$

Also, for different visual categories, the average weights, \bar{w}_j and \bar{w}_k , are independent and so it

can be shown by using the definition for covariance that

$$\text{Cov}(\bar{A}_j \bar{w}_j, \bar{A}_k \bar{w}_k) = \mu_{wj} \mu_{wk} \text{Cov}(\bar{A}_j, \bar{A}_k) \quad .$$

The variance of the visual category means, \bar{w}_j , is $V(\bar{w}_j) = \sigma_{wj}^2 / n_{wj}$. Therefore, the variance

of \hat{W}_2 is obtained by summing terms,

$$V(\hat{W}_2) = \sum_{i=1}^M \left\{ A_i^2 \frac{\sigma_{wi}^2}{n_{wi}} + \mu_{wi}^2 V(\bar{A}_i) + V(\bar{A}_i) \frac{\sigma_{wi}^2}{n_{wi}} \right\} + 2 \sum_{i=1}^M \sum_{j < i} \mu_{wi} \mu_{wj} \text{Cov}(\bar{A}_i, \bar{A}_j).$$

Appendix III

The estimate of visual areas in the k th subsegment is $\hat{\mathbf{a}}_k = \hat{y}_k \cdot \begin{pmatrix} \hat{p}_{k1} \\ \vdots \\ \hat{p}_{kM} \end{pmatrix} = \hat{y}_k \cdot \hat{\mathbf{p}}_k$, where \hat{y}_k is the estimated oiled area in beach subsegment k (a scalar) and $\hat{\mathbf{p}}_k$ is the estimated visual area composition of subsegment k (a vector). The two terms of $\hat{\mathbf{a}}_k$ are reasonably considered independent. The j th diagonal element of the variance-covariance matrix of estimation errors for the k th beach subsegment is defined as

$$v_{\hat{a}_k - a_k, j, j} = E\left[(\hat{y}_k \hat{p}_{kj})^2\right] - \left[E(\hat{y}_k \hat{p}_{kj})\right]^2.$$

A new expression is obtained by adding and subtracting the term, $[E(\hat{y}_k)]^2 E(\hat{p}_{kj}^2)$, and simplifying the result under the assumption that \hat{y}_k and \hat{p}_{kj} are unbiased as well as independent,

$$\begin{aligned} v_{\hat{a}_k - a_k, j, j} &= E(\hat{y}_k^2)E(\hat{p}_{kj}^2) - [E(\hat{y}_k)]^2 E(\hat{p}_{kj}^2) + [E(\hat{y}_k)]^2 E(\hat{p}_{kj}^2) - [E(\hat{y}_k)]^2 [E(\hat{p}_{kj})]^2 \\ &= E(\hat{p}_{kj}^2)\sigma_{\hat{y}_k}^2 + [E(\hat{y}_k)]^2 \frac{p_{kj}(1-p_{kj})}{h_k} = (y_k^2 + \sigma_{\hat{y}_k}^2) \cdot \frac{p_{kj}(1-p_{kj})}{h_k} + \sigma_{\hat{y}_k}^2 p_{kj}^2. \end{aligned}$$

The off-diagonal element at the i th row and j th column is obtained by similar expansion of the definition,

$$\begin{aligned}
v_{\hat{a}_k - a_k, i, j} &= E(\hat{y}_k \hat{p}_{ki} \hat{y}_k \hat{p}_{kj}) - E(\hat{y}_k \hat{p}_{ki}) E(\hat{y}_k \hat{p}_{kj}) = E(\hat{y}_k^2) E(\hat{p}_{ki} \hat{p}_{kj}) \\
&- E(\hat{y}_k^2) E(\hat{p}_{ki}) E(\hat{p}_{kj}) + E(\hat{y}_k^2) E(\hat{p}_{ki}) E(\hat{p}_{kj}) - [E(\hat{y}_k)]^2 E(\hat{p}_{ki}) E(\hat{p}_{kj}) \\
&= -\left(y_k^2 + \sigma_{\hat{y}_k}^2\right) \frac{P_{ki} P_{kj}}{h_k} + \sigma_{\hat{y}_k}^2 p_{ki} p_{kj}.
\end{aligned}$$

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