Executive Summary

We propose to apply SIM observations to 4 distinct but related microlensing experiments:
1) Measure the mass function (MF) of the Galactic bulge down to $0.01\,M_\odot$ from ~ 200 microlensing mass measurements with separate identification of brown dwarfs (BDs), white dwarfs (WDs), neutron stars (NSs), and black holes (BHs),
2) Measure the mass and distance of 5 of the lenses currently being detected toward the LMC/SMC, and so determine whether they are halo objects (MACHOs) and hence a substantial component of the dark matter, or are ordinary stars related to the LMC/SMC,
3) Measure masses of 15 nearby stars to 1% accuracy, including 4 low-metallicity stars,
4) Measure the masses of $\sim 15\%$ of planets found in bulge microlensing events.
Three of the four experiments will require (~ 4 day) target-of-opportunity (ToO) capability.

Microlensing (the gravititional deflection of light) is a powerful tool for detecting astrophysical objects irrespective of whether they shine. More than 500 objects have been detected over the last decade in *photometric microlensing* experiments, where the lens betrays its presence by focusing (and so magnifying) the light from a more distance source. Unfortunately, the information photometric microlensing yields about the lens is usually highly ambiguous. For example, about 15 microlensing events have been detected toward the Large Magellanic Cloud (LMC) but, despite considerable effort, it is not yet known if these are due to a major, previously undiscovered, component of the Milky Way's dark halo, or to ordinary stars associated with the LMC itself. As another example, it is likely that about 150 of the 500 events detected toward the Milky Way bulge are dark (BDs, WDs, NSs, and BHs), but at present we have no idea as to which 150 these are. Even if we did, we would not be able to estimate their masses to better than a factor of 100 because only the timescale of the event is measured, and this is only one of the three parameters needed to determine the mass and distance of the lens.

However, microlensing also gives rise to *astrometric* deflections in the apparent positions of the source, which are typically of order $200\,\mu$as. These are far too small to have been detected to date, but SIM will be able to measure them accurate to a few percent. This provides a second parameter. In addition, because SIM works by *counting photons* as a function of fringe position, its *astrometric* measurements are at the same time *photometric* measurements. Since SIM will be in solar orbit, it will "see" a signficantly different event than is seen from the ground. By comparing SIM and ground-based photometry, it is possible to extract a third parameter. With these three parameters, the mass, distance, and transverse speed of the lens can all be determined, often with a precision of 5%.

SIM will therefore revolutionize microlensing. It will resolve the nature of the lenses being detected toward the LMC by measuring their distance and so distinguishing between the Milky Way halo and the LMC for their location. By measuring the masses of bulge lenses, it will permit the separate identification of BDs ($M < 0.08\,M_\odot$), NSs ($M \sim 1.35\,M_\odot$), and BHs ($M > 3\,M_\odot$). And, while it may not be possible to distinguish WDs on a star-by-star basis from main-sequence stars of the same mass, they can be recognized from the sharp peak they induce in the MF. That is, SIM will permit an accounting of the dark (or at least very dim) objects in our Galaxy, the currently mysterious lenses detected toward the LMC and the non-luminous objects among the ordinary stars. *Note that because these mass measurements rely on SIM's photometry from solar orbit, they* *cannot be duplicated by ground-based interferometers, even if these achieved an order of magnitude better precision than SIM.*

A variant on this technique will permit SIM to measure the masses of nearby stars very accurately ($< 1\%$). If a nearby star (e.g. $< 100\,\mathrm{pc}$) comes close to the line of sight to a more distant source, it generates a *purely astrometric* microlensing event, i.e., there is negligible magnification, while the astrometric deflection can be hundreds of μas. Under favorable circumstances, this deflection can be measured to 1% precision, which means that the mass can be measured to the same level. The main difficulty is just to find these favorable lens-source pairs that come close enough during the mission to permit such precision measurements. We have already identified 11 pairs that will have encounters between 2005 and 2015, and expect to be able to find 20 additional ones using existing catalogs or catalogs to be published well before launch. These require on average 12 hours of SIM mission time to achieve 1% precision in mass. Since the mission will last for 5 years, about half of these 30 potential measurements can actually be carried out by SIM. Approximately 25% of these candidates are low-metallicity, while to date there are no accurate mass measurements for low-metallicity stars.

Photometric microlensing is sensitive to planets, particularly those that are $\sim 4\,\mathrm{AU}$ from their parent star. None have been detected to date, but when they are, only the planet/star mass ratio will be measured photometrically, not the planet mass or planet/star projected separation. For $\sim 15\%$ of planetary events, SIM observations can start quickly enough to measure the parent star's mass and distance and so recover the planet's mass and projected separation. The technique is the same as to measure the bulge MF.

We request a total mission time of 1515 hours, including 1200 hours for the bulge MF, 100 hours to resolve the nature of the LMC lenses, 180 hours for the mass measurements of nearby stars, and 35 hours to measure the masses of planets. The great majority of the observations require only relative astrometry and will therefore be carried out in narrow-angle mode. However, there will be some supplementary (and less essential) absolute-astrometry observations that require wide-angle mode.

The detection of non-luminous objects, particularly black holes and planets, evokes extremely wide interest in the general public. We plan to take advantage of this interest by working with science education professionals to develop scientific inquiry-based units on light, optics, and astronomy for use in grades 4-8. These units will link our measurements of exotic objects with the broad scientific and technological goals of SIM.

Science Investigation and Technical Description

1. Introduction

We propose to apply SIM observations to 4 distinct but related microlensing experiments:
1) Measure the mass function (MF) of the Galactic bulge down to $0.01\,M_\odot$ from ~ 200 microlensing mass measurements with separate identification of brown dwarfs (BDs), white dwarfs (WDs), neutron stars (NSs), and black holes (BHs),
2) Measure the mass and distance of 5 of the lenses currently being detected toward the LMC/SMC, and so determine whether they are halo objects (MACHOs) and hence a substantial component of the dark matter, or are ordinary stars related to the LMC/SMC,
3) Measure masses of 15 nearby stars to 1% accuracy, including 5 low-metallicity stars,
4) Measure the masses of $\sim 15\%$ of planets found in bulge microlensing events.
Three of the 4 experiments will require ($\sim 4\,$day) target-of-opportunity (ToO) capability.

2. Scientific Objectives

2.1 Bulge Mass Function

The mass functions (MFs) of stellar populations are of fundamental importance to astronomy. First, if the distribution of all stellar objects, including non-luminous ones such as BDs were known, one could accurately determine the mass-to-light ratio and so derive important constraints on dark matter. Second, any structure in the MF (such as breaks in the power-law index) would provide an important clue to the process of star formation. Third, measurement of the remnant (WD, NS, BH) MF for an old population would provide important constraints on its initial mass function (IMF). The IMF plays a crucial role in the history of galaxies and in galaxy formation in general.

At present, the only way to measure the MF of a population is to measure its luminosity function (LF) and then convert to a MF using an empirical or theoretical mass-luminosity relation (MLR). There are two shortcomings to this approach. First, the calibration of MLRs is based on a very special class of stars (field binaries in the solar neighborhood, e.g., Henry & McCarthy 1993) and there may be unrecognized problems when applying this calibration to other populations. Second, and most fundamentally, the LF measurement is sensitive only to *luminous* objects. Any non-luminous objects (e.g., BHs, NSs, old WDs or BDs) will automatically be excluded from the MF derived in this way. It is true that for certain special populations one can partially overcome this problem. For example, in young open clusters the BDs are still luminous enough to see, and one can therefore measure the MF well into the BD regime (Hamby et al. 1999; Lucas & Roche 2000). However, analyses of young star clusters (ages $< 10\,$Myrs) rest on theoretical isochrones, while dynamical effects, such as mass segregation, may affect older clusters like the Pleiades. Moreover, these nearby open clusters may or may not be representative of star-formation processes in the disk as a whole. Similarly, one can hope to measure the WD LF (and hence learn about the IMF) by deep exposures of globular clusters, but again these systems are affected by dynamical processes.

One would really like to directly measure the *masses* of a population of objects, and to be sensitive to all objects that have mass, regardless of their luminosity. Microlensing surveys are sensitive only to mass (not light) and have detected hundreds of objects over the last 7 years (Alcock et al. 1993, MACHO; Aubourg et al. 1993, EROS; Udalski et al. 1993, OGLE). Unfortunately, normal microlensing observations do not permit measurements of the mass of the lenses. However, combined astrometry and photometry by SIM would. To resolve the structure of the WD MF (which may be quite strongly peaked) requires 5% mass-measurement precision, while mapping the much smoother expected structure of the stellar+BD MF requires only $\sim 15\%$ precision. We therefore propose in § 3.1.2 a strategy for making measurements that optimizes observation time to cover this entire range of precisions for ~ 200 events.

These observations will probe the MF down to $0.01\,M_\odot$, i.e., a factor ~ 8 below the hydrogen-burning limit. (The origin of this lower limit of sensitivity is given in § 3.1.4.) Thus, we will obtain an accurate census over an unprecedented mass range. For comparison, the LF route to the bulge stellar MF has only reached $\sim 0.3\,M_\odot$ in the optical (Holtzman et al. 1998) and $\sim 0.15\,M_\odot$ in the infrared (Zoccali et al. 2000). Of course, these studies did not probe the remnant MF at all. Since the Galactic bulge is generally considered a proxy for old spiral bulges and ellipticals, a detailed understanding of its MF (including both dark and luminous stellar objects) will provide essential insight into these systems which contain the majority of the stellar mass in the universe. The bulge MF cannot be measured over this mass range by any other proposed technique, whether gound-based or space-based. In particular, ground-based interferometry, *even if it achieved an order-of-magnitude improvement over SIM* would not be able to make this measurement.

2.2 Nature of the Dark Halo

Microlensing observations were initiated toward the LMC and SMC primarily to determine whether the dark halo is composed of massive compact halo objects (MACHOs) (Alcock et al. 1993; Aubourg et al. 1993). 5.7 years of observation by MACHO (Alcock et al. 2000b) seemed to prove that a substantial fraction of the dark halo is in MACHOs because the optical depth (probability that a given source star is microlensed at a given time) was $\tau = 1.2^{+0.4}_{-0.3}\times 10^{-7}$, about 1/4 the value expected for a MACHO dark halo, and 5 times higher than the theoretical upper limit of the "self-lensing" that could be generated by a virialized LMC disk (Gould 1995c). Nevertheless, the community has been extremely skeptical of this result, primarily because the estimated mean mass of the lenses (assuming they are in halo) was $M \sim 0.4\,M_\odot$. Hence they could not be made of hydrogen, else they would be luminous. Space considerations do not allow us to properly review all the ideas that have been advanced to explain this conundrum, nor all the counter-arguments challenging these suggestions. Very briefly, Sahu (1994) and Wu (1994) originally suggested the lenses could be in the LMC, which was countered by Gould's (1995c) virialized-disk argument. Zhou (1998) proposed that the lensing could be due to a random object travelling through the Galactic halo such as a dwarf galaxy or tidal debris from a disrupted dwarf, and Zaritsky & Lin (1997) claimed to detect such an object. A variety of counter-, counter^2-, and counter^3-arguments were given by Alcock et al. (1997b), Beaulieu & Sackett (1998), Ibata, Lewis, & Beaulieu (1998), Gallart (1998), Gould (1998, 1999), Johnston (1998), Bennett (1998), and Zaritsky et al. (1999). Another possibility is that the lenses are halo WDs, which would be consistent with the above mass estimates. However, WDs appear to be strongly ruled out by a variety of independent arguments (Graff et al. 1999 and references therein). Ibata et al. (1999) believe they may have detected such a halo WD population in the HDF, but Flynn et al. (2000) argue that this is unlikely. As we discuss in § 3.1.1, the mean mass estimate $\sim 0.4\,M_\odot$ actually depends on the details of the assumed velocity and distance distributions of the MACHOs. Gyuk, Evans, & Gates (1998) therefore investigated whether any such distribution (consistent with other constraints) could bring the mass estimate down to the BD regime, but found that none could.

A large number of ideas have been advanced to deduce the location of individual lenses for various special cases of events (Gould 1996; Han & Gould 1997; Zhao 1999a,b, 2000; Zhao, Graff, & Guhathakurta 2000). Nevertheless, to date there is only one Magellanic Cloud event (a caustic-crossing binary-lens) for which there is an unambigous determination of the lens location: a massive observing campaign by 5 microlensing collaborations operating from 8 observatories measured the relative lens-source proper motion $\mu_{\rm rel} = 1.30 \pm 0.08\,{\rm km\,s^{-1}\,kpc^{-1}}$ and therefore showed that the lens must be in the SMC (Afonso et al. 2000). Another (LMC) binary-lens event provides similar, but less convincing evidence of being self-lensing (Alcock et al. 1997a,2000a). In principle, more caustic-crossing binary lenses could have their locations determined. However, since $\lesssim 10\%$ of lenses are caustic-crossing binaries, and since one could (quite plausibly) argue that MACHOs do not form many binaries, one cannot rule out the MACHO-halo hypothesis by any number of such measurements.

In brief, after a decade of determined effort, the nature of the lenses being detected toward the LMC/SMC is not resolved, and there is no clear resolution on the horizon. This problem, which carries with it the nature of the dark matter, is of immense importance to astronomy. If the lenses are halo objects, then they represent more mass than all previously identified stars and gas in the Milky Way. So many dark halo objects in the half solar mass range would have striking implications for the structure and formation history of the Milky Way, and raise several confounding questions about galaxy formation in general.

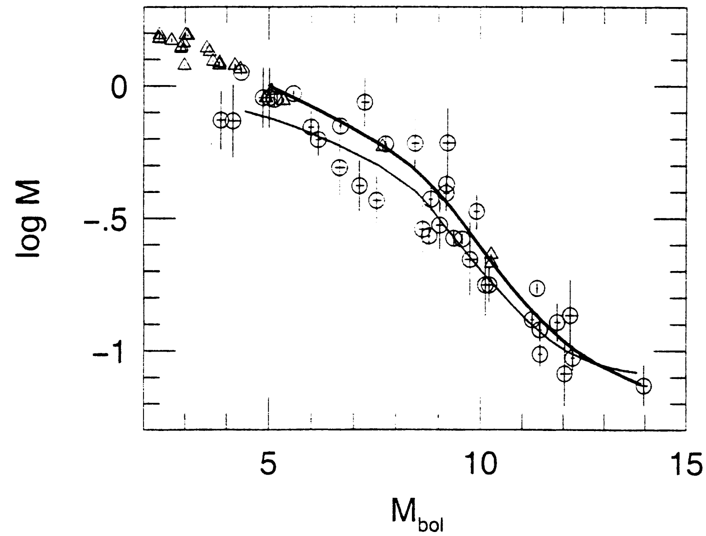
SIM can resolve the nature of these lenses unambiguously. The same type of measurements that we propose for measuring the mass of individual bulge lenses, can be applied to measure the mass, velocity, and distance of lenses towards the LMC/SMC. Because the LMC/SMC sources are much fainter than the bulge sources, the required SIM time is much greater, even to achieve substantially worse precision. However, to definitively resolve the nature of the lenses, only ~ 5 mass/distance measurements are required and only to a precision of $\sim 20\%$. This can be achieved in about 100 hours of SIM time.

Thus, it will be straight forward for SIM to determine the nature of these lenses, and hence determine whether the dark matter is composed in significant part of MACHOs.

2.3 Masses of Nearby Stars

Except for the Sun, all stars with measured masses are in eclipsing or visual binaries. These are essentially drawn respectively from magnitude- and volume-limited catalogs, and hence the former are overwhelmingly high-mass stars and the latter are mainly low-mass. In both cases, there is a strong bias toward metallicities near solar. The precision of the eclipsing-binary measurements can be better than 1%, but the visual-binaries are generally measured to no better than 5% and often much worse (Henry & McCarthy 1993), although in recent years there have been a few high-precision measurements of visuals using interferometry (Armstrong et al. 1992; Boden et al. 1999a,b, 2000; Hummel et al. 1994, 2000). See Figure 1. In principle, one would like to use these mass measurements to check theoretical predictions for mass as a function of luminosity and metallicity, and also to determine whether mass is a function of any additional parameters. In practice, the large

Fig. 1: Existing mass measurements as a function of bolometric magnitude, including eclipsing binaries (*solid circles*) and visual binaries (*open triangles*). The curves show various theoretical predictions for [Fe/H]= 0 (*bold*) and [Fe/H]= −1 (*solid*). Clearly the data are incapable of distinguishing between the models. SIM will produce about 15 mass measurements accurate to 1%, primarily at the faint end where the errors are now the largest. Of these 15, 4 will be low-metallicity stars. Presently there are no mass measurements at low metallicities.



errors and narrow range of metallicities explored mean that it has been possible only to derive a MLR at fixed (roughly solar) metallicity.

Astrometric microlensing of nearby stars provides an alternate route to masses with a number of appealing features. First, it works by a completely different physical principle than the standard methods. Second, it works for single stars (and components of binaries with much wider separations than visuals). Third, the selection bias is toward high proper-motion (hence low-metallicity, $-2 \lesssim$ [Fe/H] $\lesssim -1$) stars. Finally, 1% precisions are relatively easily achieved. About 15 such measurements could be made in about 180 hours of SIM time during a 5-year mission. We discuss these four advantages in turn.

Standard mass measurements are based on Keplerian orbits, while astrometric microlensing mass measurements are based on deflection of light. Hipparcos measurements of light deflection by the Sun on the relevant ($\sim$AU) scales confirm general relativity's prediction of the correspondence of the two methods to within a factor $\gamma = 0.997 \pm 0.003$ (Froeschle, Mignard, & Arenou 1997). Hence, comparison of the results of the two methods is an excellent test of any "hidden systematics" in either.

While there is no known reason for binary members and single stars to have different spectral properties at the same mass, neither is there any observational evidence to the contrary. Astrometric microlensing is the only known way to obtain accurate masses of single stars (except the Sun), and so provides the only check on this assumption.

The complete lack of mass data for low-metallicity stars means that stellar models in this regime are very poorly constrained by observations. This limits our ability to transform the LFs of globular clusters into MFs, and also creates uncertainty in estimates of the age of globulars from the brightness of the turnoff. Moreover, if models were compelled to reproduce observed masses at a range of metallicities, they would become more sensitive to the details of the internal structures of stars.

Finally, 1% mass measurements of all types of faint stars (dwarfs, subdwarfs, WDs, and BDs) would mark an order of magnitude improvement over the current situation and hence would challenge stellar models at a much higher level.

It is possible that some, but not all, aspects of what SIM can achieve could be accomplished by other means. For example, photometric surveys of globulars are revealing some eclipsing binaries (e.g., Kaluzny et al. 1999) that could ultimately be used for low-metallicity mass measurements, assuming the components are sufficiently separated to rule out interactions. However, precisions of better than 5% are unlikely and even these may be restricted to turn-off (i.e., evolved, not main-sequence [MS]) stars for the foreseeable future. Hummel (1999) predicts that the Navy Prototype Optical Interferometer (NPOI) will obtain 1% mass measurements of 164 stars. If so, two completely independent mass-measurement methods (SIM and NPOI) would both achieve 1% precision. Any discrepencies between masses at fixed spectral type would then imply undetected systematic errors in one of the techniques or a dependence of mass on stellar characteristics not easily detected in spectra. Either would be quite important for precision modeling of stars.

2.4 Masses of Planets Discovered in Microlensing Searches

Microlensing is a potentially powerful way to discover planets (Mao & Paczyński 1991; Gould & Loeb 1992). While several interesting candidates have been discovered, there are as yet no secure detections (Rhie et al. 1999; Bennett et al. 1999; Albrow et al. 2000a,b; Gaudi et al. 2000). The major advantages of microlensing relative to other techniques are: 1) it is sensitive to planets at larger radii, and 2) it is sensitive to lower mass planets. The major disadvantage is that typically one measures only the star/planet mass ratio, but measures neither the planet mass nor the star/planet separation in physical units. Hence, microlensing is useful mainly to characterize planets statistically rather than on a planet-by-planet basis. Another disadvantage is that no follow-up study of the planetary system is possible. This disadvantage is somewhat compensated for because, unlike most other techniques, microlensing can discover planetary systems throughout the Galaxy.

If the planetary perturbation occurs early enough in the event, then SIM observations (see §§ 2.1 & 3.1) can determine the mass and distance to the primary lens and thus the planet mass and star/planet projected separation. At present it is difficult to estimate the rate of planet detection during the mission because microlensing planetary searches are nowhere near their maximum potential for discovery. Our initial projection is that it will be possible to measure masses for only a small number ($\lesssim 5$) of events over the SIM mission. This sample would not be large enough to make a statistical statement about the planets being detected. [By the same token, it should require relatively little ($\lesssim 35\,\mathrm{hrs}$) of observation time.] However, it should be useful as a calibration of the (now purely theoretical) estimate of what part of planet parameter space microlensing is sensitive to. If, as we note above is possible, the true rate is significantly larger, we will expand our planet mass measurements, somewhat at the expense of our bulge MF program.

3. Science Measurements

3.1 Bulge Mass Function

3.1.1 Physical Principles

The quantities that can, in principle, be measured via microlensing are related in a complicated way to the mass M and the other physical parameters of interest:

$$\theta_\mathrm{E} \equiv \sqrt{\frac{4GM}{D_\mathrm{rel}c^2}}, \qquad \tilde{r}_\mathrm{E} \equiv \sqrt{\frac{4GMD_\mathrm{rel}}{c^2}}, \qquad t_\mathrm{E} = \frac{\theta_\mathrm{E}}{\mu_\mathrm{rel}}, \tag{1}$$

where

$$D_\mathrm{rel}^{-1} \equiv d_l^{-1} - d_s^{-1}, \qquad \boldsymbol{\mu}_\mathrm{rel} \equiv \boldsymbol{\mu}_l - \boldsymbol{\mu}_s, \tag{2}$$

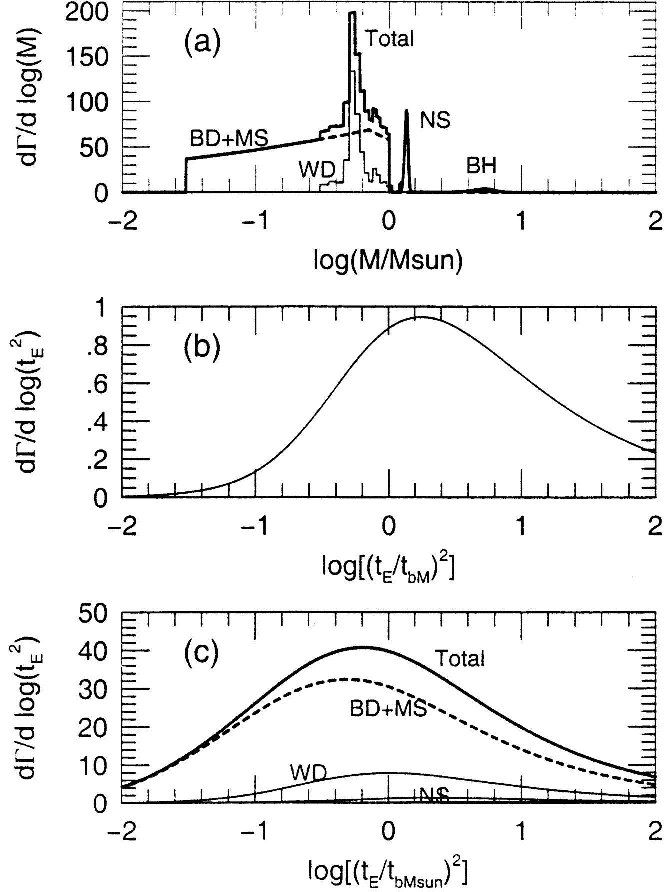


Fig. 2: Rates of microlensing events toward the bulge by mass (panel *a*) and time scale (panel *c*) for MS+BDs ($0.03\,M_\odot < M < 1\,M_\odot$) (*bold dashed curve*) and WD, NS, and BH remnants (*solid curves*). The total is shown by a *bold solid curve*. The mass model (*a*) is described in § 2 of Gould (2000b). The mass model is convolved with the time scale distribution at fixed mass (*b*) derived in § 2.2 of Gould (2000b), to produce the observable time scale distribution (*c*). The abscissae of panels (*b*) and (*c*) contain $\log t_{\rm E}^2$ rather than $\log t_{\rm E}$ so that they can be directly compared with panel (*a*), since $t_{\rm E}^2 \propto M$. All three classes of remnants are clearly identifiable in the mass distribution which could be extracted from SIM observations, but are utterly lost in the time scale distribution. The normalizations in panels (*a*) and (*c*) are for 100 events. Panel (*b*) is normalized to unity.

d_l and $\boldsymbol{\mu}_l$ are the lens distance and proper motion, and d_s and $\boldsymbol{\mu}_s$ are the corresponding quantities for the source. Note that $\mathrm{AU}/D_{\rm rel} \equiv \pi_{\rm rel}$, the lens-source relative parallax.

Here, $\theta_{\rm E}$ is the angular Einstein radius, $\tilde{r}_{\rm E}$ is the projected Einstein radius, and $t_{\rm E}$ is the Einstein crossing time. The reason that these are the 3 measurable quantities is that all microlensing effects scale with the Einstein ring radius, the effective range of action of the lens, while $\theta_{\rm E}$ is the Einstein ring projected onto the plane of the sky, $\tilde{r}_{\rm E}$ is the Einstein ring projected onto the plane of the observer, and $t_{\rm E}$ is the time it takes the source to traverse the Einstein ring.

In fact, the only one of these quantities that can be *routinely* measured by traditional (photometric) microlensing techniques is $t_{\rm E}$. Since $M \propto t_{\rm E}^2$, it seems at first sight possible to obtain a rough estimate of M from $t_{\rm E}$ and equation (1) by making statistical estimates of $D_{\rm rel}$ and $\mu_{\rm rel}$. However, from Figure 2, the FWHM of this mass estimate is a factor ~ 100, and the full-width quarter maximum is a factor ~ 1000, which imply that very little about the mass function apart from its mean and variance can be extracted from normal microlensing observations, even assuming that the distributions of $D_{\rm rel}$ and $\mu_{\rm rel}$ are known. In fact, these distributions are not known and are the subject of some controversy.

For a few, very atypical events, it has been possible to measure $\theta_{\rm E}$ by using the angular size of the source as a "standard angular ruler" (Alcock et al. 1997c,2000a; Afonso et al. 2000; Albrow et al. 2000b), and for a different few, it has been possible to measure $\tilde{r}_{\rm E}$ by using the Earth's orbit as a "standard ruler" in the plane of the observer (Alcock et al. 1995; Bennett et al. 1997; Mao 1999). However, to date there has not been a single event for which both $\theta_{\rm E}$ and $\tilde{r}_{\rm E}$ could be measured, and for which M and $D_{\rm rel}$ could consequently be unambiguously determined: $M = (c^2/4G)\tilde{r}_{\rm E}\theta_{\rm E}$, $D_{\rm rel} = \tilde{r}_{\rm E}/\theta_{\rm E}$.

3.1.2 SIM Implementation

To understand how SIM can make mass measurements *routinely*, even though no masses have been measured previously, it is helpful to write equation (1) in physical units,

$$\theta_{\rm E} = 280\,\mu{\rm as}\left(\frac{M}{0.3\,M_\odot}\right)^{1/2}\left(\frac{D_{\rm rel}}{30\,{\rm kpc}}\right)^{-1/2} \qquad \tilde{r}_{\rm E} = 8.5\,{\rm AU}\left(\frac{M}{0.3\,M_\odot}\right)^{1/2}\left(\frac{D_{\rm rel}}{30\,{\rm kpc}}\right)^{1/2}, \tag{3}$$

where we have scaled to typical bulge parameters.

The two images are separated by $\sim 2\theta_{\rm E}$ which is an order of magnitude smaller than can be resolved by SIM. However, SIM can determine $\theta_{\rm E}$ from the *deviation of the centroid of the combined images*, $\boldsymbol{\theta}_{\rm dev}$, relative to the expected position of the source in the absence of microlensing (Boden, Shao, & Van Buren 1998; Paczyński 1998),

$$\boldsymbol{\theta}_{\rm dev} = \frac{\mathbf{u}}{u^2+2}\theta_{\rm E}, \qquad \mathbf{u} \equiv \frac{\boldsymbol{\theta}_{\rm rel}}{\theta_{\rm E}}. \tag{4}$$

Here, $\boldsymbol{\theta}_{\rm rel}$ is the angular position of the lens relative to the source (as it would appear in the absence of light deflection), and $\mathbf{u}$ is the same quantity normalized to the Einstein ring. Note that $\theta_{\rm dev}$ achieves a maximum at $u = \sqrt{2}$, when $\theta_{\rm dev} = \theta_{\rm E}/\sqrt{8}$. Thus, measurement of this deviation of the centroid is well within SIM's capabilities. Although it is not obvious from equation (4), $\boldsymbol{\theta}_{\rm dev}$ traces out an ellipse (see Boden et al. 1998, Fig. 7). The axis ratio of the ellipse is determined by the impact parameter $\beta = u_{\rm min}$, which is much better determined from the photometry (see below) than the astrometry. Hence, a series of astrometric measurments can be fit to an ellipse of known axis ratio. The size of this fit ellipse gives $\theta_{\rm E}$, while its orientation gives the direction of relative proper motion, $\boldsymbol{\mu}_{\rm rel} = d\boldsymbol{\theta}_{\rm rel}/dt$.

The projected Einstein radius $\tilde{r}_{\rm E}$ can be measured using the Earth-SIM separation (projected onto the plane of the sky), $\mathbf{d}_{\oplus-{\rm SIM}}$ as a "ruler". The event as seen from SIM will be displaced in the Einstein ring from the one seen from Earth by $\Delta\mathbf{u} = \mathbf{d}_{\oplus-{\rm SIM}}/\tilde{r}_{\rm E}$. Hence, the impact parameters as observed from the two stations will differ by $\Delta\beta$ and the times of closest approach will differ by Δt_0 according to

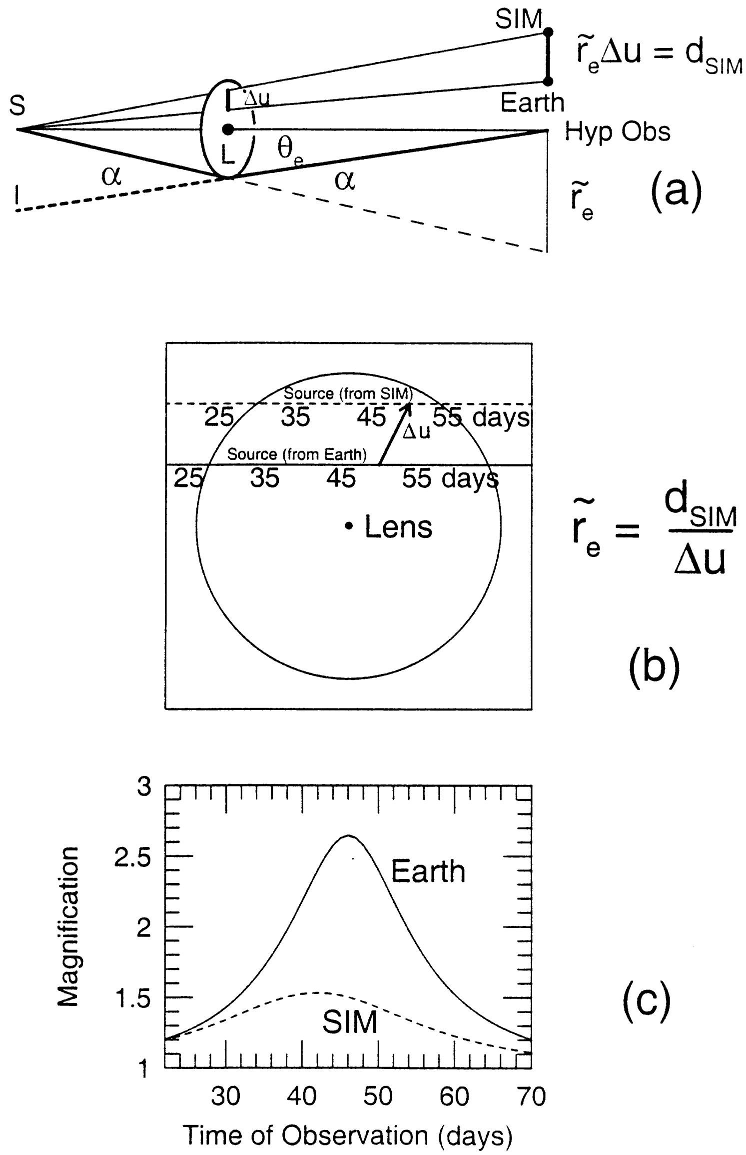
$$\frac{\Delta t_0}{t_{\rm E}} = \Delta u\,\cos\phi = \frac{d_{\oplus-{\rm SIM}}}{\tilde{r}_{\rm E}}\cos\phi, \qquad \Delta\beta = \Delta u \sin\phi = \frac{d_{\oplus-{\rm SIM}}}{\tilde{r}_{\rm E}}\sin\phi, \tag{5}$$

where ϕ is the angle between $\boldsymbol{\mu}_{\rm rel}$ and the Earth-SIM axis. These offsets can be measured by comparing the *photometric* microlensing events as seen from Earth and SIM. For each, the observed flux is given by

$$F(t) = F_s A(u[t;t_0,\beta,t_{\rm E}]) + F_b, \quad A(u) = \frac{u^2+2}{u\sqrt{u^2+4}}, \quad u(t) = \left[\left(\frac{t-t_0}{t_{\rm E}}\right)^2 + \beta^2\right]^{1/2}, \tag{6}$$

where F_s is the unmagnified source flux, and F_b is the sum of all background light (which does not participate in the event). Each light curve can be fit for the 5 parameters $t_0, \beta, t_{\rm E}, F_s$, and F_b. Then $\Delta\beta = \beta_{\rm SIM} - \beta_\oplus$, and $\Delta t_0 = t_{0,{\rm SIM}} - t_{0,\oplus}$. From these measurements one can therefore determine both $\tilde{r}_{\rm E}$ and ϕ (Refsdal 1966; Gould 1995b). This chain of deduction is illustrated in Figure 3. Note that the SIM orbit will drift away from Earth at $\sim 0.1\,{\rm AU\,yr^{-1}}$, so that at mid-mission $\Delta u \sim 3\%$ for typical bulge parameters.

Fig. 3: Geometry of microlensing parallax. Panel (a) is an elaboration of the lensing geometry for a hypothetical observer perfectly aligned with the source-lens axis: α is the bending angle and $\theta_{\rm E}$ is the angular Einstein radius. The physical Einstein ring is the circle centered on L. Its radius, $r_{\rm E}$, is not explicitly labeled. In addition, the diagram shows $\tilde{r}_{\rm E}$, the projection of $r_{\rm E}$ onto the plane of the observer. Panels (b) and (c) show how $\tilde{r}_{\rm E}$ is measured by parallax. Panel (b) shows the source position in the Einstein ring as a function of time (labeled in days) as seen from the Earth and SIM. The vector separation $\Delta\mathbf{u} = (\Delta t_0/t_{\rm E}, \Delta\beta)$ remains a constant during the event. The resulting light curves as seen from the Earth and SIM are shown in (c). By measuring the impact parameters β and β' and the times of maximum t_0 and t_0', one can reconstruct the geometry of (b) and so determine $\Delta\mathbf{u}$. From (a) it is clear that the magnitude of this vector, Δu, is the distance to SIM $d_{\rm SIM}$ as a fraction of $\tilde{r}_{\rm E}$. One therefore recovers $\tilde{r}_{\rm E} = d_{\rm SIM}/\Delta u$. The solution is actually degenerate because from the light curves (c) alone one cannot tell whether the source trajectories in (b) pass on the same or opposite sides of the lens. As discussed in the text, the degeneracy can be broken.



While SIM was not originally designed to do photometry, it makes astrometric measurements by centering the central fringe which in turn requires scanning (i.e., *counting*) photons over this fringe. These counts (if recorded and sent to the ground) constitute photometry. The Earth-based photometry need be only somewhat better than the SIM photometry, and therefore can be obtained easily on 1m class telescopes.

Gould & Salim (1999) have analyzed the full astrometric+photometric problem and found that for a range of typical bulge parameters, it is possible to measure both M and $D_{\rm rel}$ to $\sim 5\%$ precision with a total of 5 hours of SIM time for an $I = 15$ source star. For $I = 17$, the precision drops to 12% for the same 5 hours of observation time. That is, the precision scales simply as the inverse square root of the total number of photons.

From the measurements of $\theta_{\rm E}, \tilde{r}_{\rm E}, t_{\rm E}$, and ϕ and equation (1), one obtains $M, D_{\rm rel}$, and $\boldsymbol{\mu}_{\rm rel}$. From the late-time astrometric measurements of the source, one obtains d_s and $\boldsymbol{\mu}_s$. Combining these gives the mass, distance, and transverse velocity of the lens. [We note that getting d_s and $\boldsymbol{\mu}_s$ for ~ 200 bulge sources would be of considerable interest in its own right, although one would have to take account of the (microlensing) selection bias.]

3.1.2.1 Enhanced Sensitivity to Stellar-Mass Black Holes

The program of observation described in the previous section has relatively low sensitivity to stellar mass BHs, primarily because such objects are expected to be rare ($\lesssim 1\%$, Gould 2000b), but also because the ratio $d_{\oplus-\mathrm{SIM}}/\tilde{r}_\mathrm{E}$ is so small that it is hard to determine $\tilde{r}_\mathrm{E}$ (Gould & Salim 1999). Fortunately, BH events tend to last long enough that $\tilde{r}_\mathrm{E}$ can usually be measured from the ground due to the distortion of the light curve produced by the motion of the Earth about the Sun (Gould 1992; Alcock et al. 1995). Thus, it is only necessary for SIM to measure θ_E in order to determine the lens mass and so confirm that it is a BH. Since θ_E tends to be exceptionally large (see eq. 1), this measurement is exceptionally easy. It can be done for fainter sources because lower astrometric precision is required. It can even be done for *events that ended (photometrically) before SIM was launched.* For example, MACHO 98-BLG-6 peaked in July 1998 and has a timescale of $t_\mathrm{E} = 225$ days, so the photometric event is already over. However, if the lens is a BH of mass $M \sim 6\,M_\odot$, then $\theta_\mathrm{E} = 4.8\,\mathrm{mas}$. Hence, between 2006 and 2011, the proper motion of the image centroid will change by $\Delta\mu = 28\,\mu\mathrm{as\,yr}^{-1}$, which would be measurable by SIM.

We expect that of order 10 long-timescale events will take place either before or during the SIM mission for which ground-based parallaxes will be obtained, and which therefore can easily have their masses measured by SIM. In this way, we will dramatically enhance our sensitivity to BHs.

3.1.3 Resolution of Degeneracies

A potential complication is that $\Delta\beta$ cannot in fact be determined just from $\beta_\oplus$ and β_SIM. Rather, there is a four-fold degeneracy $\pm\Delta\beta_\pm = \pm|\beta_\oplus \pm \beta_\mathrm{SIM}|$ which arises because the two trajectories in Figure 3b can be on the same or opposite side of the center, and because the entire diagram can equally well be reflected about the x-axis. The latter two-fold degeneracy would affect only the direction of motion, but not the mass or distance, and hence would not seriously degrade the science. However, the former would have a major adverse impact because it would affect both M and d_l.

In fact, these degeneracies are not a serious problem for $M \gtrsim 0.01\,M_\odot$. First, they can usually be broken by two independent methods (photometric and astrometric), provided $M \gtrsim 0.01\,M_\odot$ (Gould & Salim 1999; Gaudi & Gould 1997a). Second, for the specific SIM orbit (and for the same mass range), almost all events have β_- as their true solution. The false (β_+) solution then corresponds to a lens far in the foreground, which would be obviously anomalous since real such events are very rare. Hence, there will be multiple independent methods for breaking the $\beta_\pm$ degeneracy. All these methods break down for $M \lesssim 0.01\,M_\odot$, and this sets the lower mass limit on the sensitivity of the experiment.

3.1.4 Real-time Identification of Candidates

To measure the mass of a microlens, SIM observations (and simultaneous intensive ground-based observations) must begin well before the peak of the event. The simulations of Gould & Salim (1999) specifically assumed they begin at $A = 1.5$, i.e., at about a time $0.75\,t_\mathrm{E}$ before the peak. Real-time alerts will be provided by the OGLE collaboration of which one of us (BP) is a member. OGLE has substantial experience providing microlensing alerts (http://www.astrouw.edu.pl/~ftp/ogle/ogle2/ews/ews.html) including 48 during 1999 and 23 in the first 3.6 months of 2000. OGLE expects to upgrade its microlensing search observations during 2000, primarily by increasing the size of its camera (16 times more pixels, 6.25 times larger angular area), but also by introducing improved (image-subtraction) processing algorithms. Because of the high S/N requirements, SIM observations will be restricted to lensing of relatively bright ($I < 17$) sources. At present, we are anticipating that OGLE will be able to alert on these at $A = 1.25$, or $\sim 1.1\,t_{\rm E}$ before peak. This implies that SIM observations can begin by $A = 1.5$ for events with $t_{\rm E} > t_{\rm E,thresh} = 6$ days, assuming an optimistic ToO warning time of 2 days. We adopt this timescale as the threshold for SIM observations and then discuss various contingencies.

The fraction of lensing events with $t_{\rm E} > t_{\rm E,thresh}$ for lenses of mass M observed in a field with Galactocentric impact parameter b is $F[(t_{\rm E,thresh}/t_{bM})^2]$ where (Gould 2000b),

$$t_{bM} = \frac{\sqrt{2GMb}}{\sigma c} = 6\,{\rm days}\biggl(\frac{b}{550\,{\rm pc}}\biggr)^{1/2}\biggl(\frac{M}{0.065\,M_\odot}\biggr)^{1/2}\biggl(\frac{\sigma}{110\,{\rm km\,s^{-1}}}\biggr)^{-1}, \tag{7}$$

σ is the bulge velocity dispersion, and where we have normalized to the approximate characteristics of Baade's Window. $F(x)$ is the cumulative distribution function whose differential form is shown in Figure 2b. Specifically, $F(1) = 0.73$, $F(2) = 0.61$, $F(5) = 0.41$, and $F(10) = 0.30$. That is, there is a very long tail toward higher timescales at fixed mass. This means that for $t_{\rm E,thresh} = 6$ days we would be 41% complete at $M = 0.013\,M_\odot$ and 30% complete at $0.0065\,M_\odot$. The estimate of a 2-day ToO warning time may prove too optimistic. However, this will not critically impact the science. If the ToO time is doubled to 4 days, we have several options that would improve our sensitivity to short timescales (and hence low masses): we can work with OGLE to achieve earlier alerts, we can somewhat increase the observing time for short events to compensate for starting at $A > 1.5$. Finally, even in a pessimisitic scenario where we were forced to adopt $t_{\rm E,thresh} = 10$ days, we would still be 30% complete at $M = 0.018\,M_\odot$, i.e., well into the BD regime.

Two factors will combine to degrade our sensitivity for $M \lesssim 0.01\,M_\odot$. First, our timescale threshold cannot be pushed much below 6 days, and so below $0.01\,M_\odot$ we capture an increasingly smaller fraction of events. Second, the degeneracies discussed in § 3.1.3 become increasingly difficult to break. Hence, $0.01\,M_\odot$ marks the approximate lower limit of the mass sensitivity of the experiment.

3.1.5 Ground-based Follow-up Observations

Ground-based photometry is essential to measuring $\tilde r_{\rm E}$ and so obtaining masses. In principle, SIM astrometry+photometry alone could measure $\tilde r_{\rm E}$ (Boden et al. 1998; Paczyński 1998), but they would require $\gtrsim 10^4$ times more SIM time to achieve the same precision (Gould & Salim 1999). To ensure that the ground-based observations are not the limiting factor, the observing time on a 1m ground-based telescopes should be approximately equal to the SIM time. Accurate determinations of $\Delta\beta$ and Δt_0 require good coverage over the peak. To ensure that these measurements are not often compromised by weather, there should be observatories on at least two continents that cede observing time to SIM targets whenever necessary. We are in the process of negotiating agreements with several such observatories and do not foresee any fundamental difficulties in obtaining coverage. Three of us (DB, DD, SR) have substantial experience in organizing and analyzing high-precision microlensing follow-up observations.

3.1.6 Extracting a Mass Function

How accurately will the bulge MF be measured? There are really three questions here: 1) how reliably can we reconstruct a histogram of observed masses? 2) how well do we understand the selection function that relates this histogram to the underlying MF?, and 3) how will statistics limit the interpretation of the selection-corrected histogram?

1) We have argued in § 3.1.3 that for masses $M \gtrsim 0.01\,M_\odot$, discrete degeneracies will not play a significant role, i.e., each mass measurement can be intrepreted almost unambiguously up to its 5%–15% statistical error.

2) The observed event rate at fixed M and $t_{\rm E}$ (both of which are observables), $\Gamma(M, t_{\rm E})$, is related to the MF, $\Phi(M)$, by

$$\Gamma_\Omega(M, t_{\rm E}) \propto \epsilon(t_{\rm E}) f_\Omega(t_{\rm E}^2/t_{bM}^2) M^{1/2}\Phi(M) \tag{8}$$

where $\epsilon(t_{\rm E})$ is the detection efficiency of finding events of timescale $t_{\rm E}$ in the OGLE survey, and $f_\Omega(t_{\rm E}^2/t_{bM}^2)$ is the differential distribution of $t_{\rm E}$ at fixed mass M and sky position Ω. If ϵ and f are known perfectly, the observed mass measurements can be translated into a MF histogram which still has statistical, but no systematic errors. ϵ can be calculated directly from the OGLE survey data. Two of us (DB and KG) have substantial experience doing this for the MACHO survey, and we do not expect any additional difficulties applying previously developed methods to the OGLE survey. In a simplified (isothermal) model of the Galactic bulge, f is a universal function which is shown in Figure 2b. This function has 1 adjustable parameter (σ) which can be directly determined from the mass measurements. More accurate models of the bulge will lead to slight variations of f_Ω as a function of Ω, but once σ is determined, these will not significantly affect the survey-area averaged value of f. Hence, the transformation from observed masses to MF-histogram should not be seriously affected by systematics. The MF determination will therefore be statistics limited.

To characterize the sensitivity of the experiment to the MF in the BD regime, we ask how well it could measure a hypothetical break in the power-law index at the hydrogen-burning limit from $\alpha_{\rm MS} = -1.3$ measured by Zocalli et al. (2000) in the range $0.15\,M_\odot < M < 0.7\,M_\odot$ (and extended arbitrarily by us to $M = 0.08\,M_\odot$) to $\alpha_{\rm BD} = \alpha_{\rm MS} + \Delta\alpha$ for $0.01\,M_\odot < M < 0.08\,M_\odot$. Following Gould (2000b), we assume that 20% of the lenses detected toward the bulge are disk stars, and 20% of the bulge lenses are remnants. For finding the break in the slope, we restrict consideration to masses $0.01\,M_\odot < M < 0.4\,M_\odot$ so as not to be affected by WDs on the one hand and the regime of mass degeneracies on the other. For simplicity we assume $\epsilon(t_{\rm E}) = \text{const}$. We apply the method of Gould (1995a) and find that for $t_{\rm E,thresh} = 6$ days

$$\sigma(\Delta\alpha) = 0.44\left(\frac{N}{200}\right)^{-1/2}. \tag{9}$$

This rises only to 0.46 for $t_{\rm E,thresh} = 10$ days, confirming the qualitative analysis in § 3.1.4. Thus, to detect a significant ($\Delta\alpha \sim 1$) break at the $2\,\sigma$ level requires ~ 200 events. If the slope in the stellar range ($0.08\,M_\odot < M < 0.4\,M_\odot$) is regarded as known (say from future star counts), then the coefficient in equation (9) is reduced to 0.20 for both thresholds.

Gould (2000b) estimates that a fraction $\sim 15\%$ of bulge microlensing events are due to WDs centered at $M = 0.525\,M_\odot$ of which $\sim 1/3$ lie in one $0.05\,M_\odot$ bin and $\sim 60\%$ lie within 3 contiguous bins. The MS "background" is about half of the WD density in the central bin. These numbers imply that for the $N = 100$ events with $\sim 5\%$ mass measurements, the central WD bin is expected to have 4 WDs and 2 MS stars, and each of the neighboring bins 1.5 WDs and 2 MS stars. The appearance of 6 stars in the central bin where 2 "background" were expected could occur by chance with probability only $\sim 1.5\%$ and hence would by itself imply detection with good confidence. For the full sample of $N = 200$ stars (including 100 with $\sim 15\%$ precision) there would be 14 WDs in the 3 central bins together with 12 background MS stars. Together these would allow a measurement of the frequency of WDs to $\sim 30\%$ at the $1\,\sigma$ level. Note that it may be possible to remove the "background" entirely by resolving the $\sim 0.6\,M_\odot$ MS lenses using ground-based interferometers several years after the event. In this case the WD frequency could be measured to $\sim 18\%$. Another $\sim 5\%$ (or 10 events) would be due to NSs and BHs. Gould (2000b) showed that for the better-precision half of the events, these would be recognizable as such, but perhaps not for the worse half.

Note that binaries do not pose a significant problem for the MF determination (as they do when it is derived from a LF) because the fraction of binaries that are far enough inside the Einstein ring to appear combined as single masses is $\lesssim 10\%$ (Gaudi & Gould 1997b; Chang & Han 1999; Han, Chun, & Chang 1999)

3.1.7 Mid-mission Reality Check

Clearly, this experiment is quite complicated. Not only does it seek to make use of SIM's previously unrecognized photometric capabilities, but it requires real time alerts of microlensing events for SIM to observe as well as simultaneous follow-up observations from the ground. While all these aspects have been simulated by Gould & Salim (1999), and while such simulations will be updated and refined as the characterization of SIM's capabilities evolves, the very complexity of the project creates the possibility that some unrecognized interplay among its elements will critically degrade its performance. If such a problem were not discovered until after the mission, a significant fraction of SIM time would be wasted on a promising but unproven idea.

We therefore point out it should be possible to use intermediate SIM data to test whether this complex experiment is yielding its expected precision one year after the first microlensing observation. While the full results (in particular d_l and $\boldsymbol{\mu}_l$) depend on accurate measurements of d_s and $\boldsymbol{\mu}_s$, and therefore on an accurate grid solution, the measurements of $\boldsymbol{\theta}_{\rm dev}$ (and therefore $\theta_{\rm E}$, and so M and $D_{\rm rel}$) do not require a grid solution. Rather, $\boldsymbol{\theta}_{\rm dev}$ need be measured only relative to the source position expected from its parallax and proper motion *relative* to the reference stars. Gould & Salim (1999) show that this can be obtained one year after the peak of the event. Thus, if the experiment contains a hidden flaw, it will be possible to find it long before the mission is over.

3.2 Nature of LMC/SMC Lenses

The techniques to measure the mass, distance, velocity of the LMC/SMC lenses are essentially identical to those used to measure the bulge lenses. Hence, almost all questions relevant to this experiment are covered in § 3.1. A few specific points call for some clarification. First, OGLE will again provide alerts for these events. Second, since the events are relatively long $\langle t_E \rangle \sim 45$ days, ToO requirements are less stringent. Third, LMC/SMC events that are accessible to SIM ($V \lesssim 20$) are quite rare, essentially all such events should be followed. We expect ~ 5 events, and our simulations show that even 4 would decisively determine whether the LMC/SMC events are due to halo lenses or self-lensing. Finally, we note that for halo lenses we will be able to measure both θ_E and $\tilde{r}_E$, and so determine M, d_l, and $\boldsymbol{\mu}_l$, all of which would provide extremely interesting clues to the nature of the MACHO halo. On the other hand, if the lenses are in the Clouds, the measurements of θ_E and $\tilde{r}_E^{-1}$ will both be consistent with 0, so there will be no quantitative information on M, d_l, and $\boldsymbol{\mu}_l$ (Gould & Salim 1999). However, the combination of these *two indpendent* null results will provide unambiguous confirmation that the lenses are indeed in the Clouds.

3.3 Masses of Nearby Stars

3.3.1 Physical Principles

For $\theta_{\rm rel} \gg \theta_E$ (i.e. $u \gg 1$), the astrometric effect falls $\propto u^{-1}$. From equation (4),

$$\boldsymbol{\theta}_{\rm dev} = \frac{\theta_E^2}{\theta_{\rm rel}^2}\boldsymbol{\theta}_{\rm rel} = 800\,\mu{\rm as}\,\frac{M}{M_\odot}\left(\frac{\theta_{\rm rel}}{1''}\right)^{-1}\frac{\pi_{\rm rel}}{0''.1}\,\hat{\boldsymbol{\theta}}_{\rm rel}. \tag{10}$$

By contrast, the photometric effect falls $\propto u^{-4}$ (see eq. 6). Hence, for nearby lenses, there can be a major astrometric effect without any detectable photometric effect. This is the regime of "pure astrometric microlensing". Since $\boldsymbol{\theta}_{\rm dev}, \pi_{\rm rel}$, and $\boldsymbol{\theta}_{\rm rel}$ can all be determined from a time series of relative astrometric measurements of the lens and source, it is possible to extract the lens mass from these measurements and equation (10) (Refsdal 1964).

3.3.2 SIM Implementation

Paczyński (1995, 1998) and Miralda-Escudé (1996) advocated applying this idea to modern astrometry missions, especially SIM. Paczyński (1998) noted in particular that Hipparcos stars alone might give rise to several dozen such events over a few years.

The probability that a given nearby star will deflect the light of some more distant source by enough to measure its mass to some fixed precision (say 1%) in a given amount of SIM time (say 10 hours) scales as

$$P \propto M\pi\mu N \tag{11}$$

where M, π, and μ are the mass, parallax, and proper motion of the lens, and N is the number density of background stars. Hence, the best place to look is a catalog of nearby, high proper-motion stars with good coverage of the Galactic plane. We draw our candidates for mass measurement from two existing catalogs (Hipparcos and Luyten), plus one catalog that we expect to become available well before the SIM mission (either Guide Star Catalog II or USNO-B). To understand the role of these catalogs and the importance of supplementary ground-based observations in choosing candidates, we first note that the total amount of SIM time required for a 1% mass measurement is given by (Gould 2000a),

$$T_{SIM} = 4\,{\rm hours}\left(\frac{\beta}{1''}\right)^{-2}\left(\frac{D_{\rm rel}}{10\,{\rm pc}}\right)^{-2}\left(\frac{M}{M_\odot}\right)^{2}\frac{\gamma}{10}10^{0.4(V-17)} \tag{12}$$

where β is the angular impact parameter of the event, V is the apparent magnitude of the source, and γ depends on the geometry of the event. (γ achieves its minimum, $\gamma_{\rm min} = 10$, for most events that we select.) For all catalogs, the errors in estimating M and V do not introduce major uncertainties in T_{SIM}. However, for some catalogs estimates of β and $D_{\rm rel}$ can be off by a factor 3 or more, leading to uncertainties in T_{SIM} of a factor 100, which is unacceptably high. Nevertheless, the uncertainty in T_{SIM} can be reduced to a factor < 2 by ground-based observations, as we describe below. This means that if SIM observations are planned to achieve 1% mass errors, they will actually achieve 0.7%–1.4%.

3.3.2.1 Hipparcos

Salim & Gould (2000a) compared the future predicted positions of Hipparcos stars with the (roughly fixed) position of background stars in the all-sky position catalog USNO-A2.0 (Monet 1998) and found 11 events between 2005 and 2015 that require SIM time $T_{SIM} < 14$ hours to yield 1% mass measurements. Given equation (11) it is perhaps not surprising that these 11 include Proxima Cen, 61 Cyg A (2 events), and 61 Cyg B, and that Barnard's star yields a 2% measurement. Hipparcos astrometry gives excellent measurements of β and $D_{\rm rel}$, so supplemental observations are not required to confirm Hipparcos candidates (although they will be needed to fine tune the SIM observing strategy).

3.3.2.2 Luyten (NLTT)

Luyten's (1980) NLTT catalogue provides proper motions and photographic photometry for $\sim 42,000$ stars with $\mu > 0''.18$. The proper motions are only accurate enough to predict 2010 positions to $1''.2$, and the NLTT stars do not in general have trigonometric parallaxes. However, Salim & Gould (2000a,b) have identified 181 proto-candidates. CCD imaging (Salim & Gould 2000b) provides photometric parallaxes and astrometry accurate enough to predict SIM times to within a factor of 2. From a statistical analysis they predict that the full sample will yield 10 good candidates (2005–2015) with SIM times < 15 hours, including 4 metal poor stars (thick disk or halo) and 1 WD.

3.3.2.3 GSC II or *USNO-B*

In addition to problems in the quality of its proper motions and photometry, NLTT also suffers from shortcomings in its coverage, especially for $\delta < -20°$ and for $|b| < 10°$. In the regions where it is nominally complete for $V \lesssim 18.5$, it is actually 85–90% complete. Finally, for 10% of NLTT stars, poor NLTT astrometry compromised Salim & Gould's (2000a) automated identification. Most of these shortcomings will be made good when either of two groups release their planned all-sky position and proper-motion (PPM) catalogs based on photographic surveys at two or more epochs, USNO-B (D. Monet 1999, private communication) and Guide Star Catalog II (Baruffolo, Benacchio, & Benfante 1999). In addition, these catalogs should reach about 1 mag fainter than Luyten. Combining all these effects, we expect to recover an additional ~ 10 candidates (2005–2015) from the new catalogs. This implies a total of ~ 15 candidates for a 5 year SIM mission, with an average SIM time of 12 hours to achieve 1% mass measurements.

3.3.2.4 Other Objects

There are potentially other types of objects that we could add to our list such as BDs (which are currently being discovered in large numbers by 2MASS, DENIS, and SDSS), nearby NSs, or possibly others. At present, we expect $\lesssim 1$ such mass measurement to be feasible, which is why we have not highlighted this possibility in the proposal. However, by the time SIM is launched new observational results could force an upward revision of this number. So we will be on the lookout for other interesting candidates.

3.4 Masses of Planets

The techniques to measure the star-lens mass and Einstein ring size and thus (use the photometrically determined star/planet mass ratio and separation in the Einstein ring to measure) the planet mass and star/planet projected separation are essentially identical to those used to measure the bulge lenses. Two points require additional clarification. First, mass measurements are possible only when SIM observations begin before the peak (see § 3.14). Thus, it is only planets that are found on the rising part of the light curve, at least a week before peak, for which mass measurements are possible. Second, since even a $\sim 20\%$ mass measurement would be quite adequate, the source stars need not be extremely bright to obtain useful results in 5–10 hours of mission time.

Program Requirements

Program	Targets (number)	$\langle T_{\rm mission}\rangle$ (hours)	Total time (hours)	ToO?	SIM Phot?
Bulge MF	200	6	1200	Yes	Yes
LMC/SMC	5	20	100	Yes	Yes
Nearby Stars	15	12	180	No	No
Planets	5	7	35	Yes	Yes

4. Technical Matters

4.1 Observing Time Calculations

All astrometric microlensing observations require a customized observing sequence. For example, as detailed in § 4.1 of Gould & Salim (1999), the bulge MF and LMC/SMC mass measurements require a fit to a 15-parameter model which is much more complex than the standard 5-parameter astrometric model that will be used for most SIM observations. Therefore, all observing time estimates reported in this proposal are based ultimately on the customized observing models of Gould & Salim (1999), Gould (2000a), and Salim & Gould (2000a) (hereafter Gould/Salim). These calculations were in turn based on a set of simplified assumptions made in advance of estimates published by the SIM project at http://sim.jpl.nasa.gov/ao_support/grid.html. Nevertheless, our revised calculations based on these latest project estimates lead only to small changes.

For narrow-angle observations, the web page gives

$$\sigma^2_{\rm sma} = \sigma^2_{\rm system} + \sigma^2_{\rm photon} 10^{(m-m_0)/2.5}\frac{t}{t_0} \qquad (13)$$

where

$$\sigma_{\rm photon} = 4\,\mu{\rm as}, \quad m_0 = 20, \quad t_0 = 14.1\,{\rm hrs}, \quad \sigma_{\rm system} = 1.718\,\mu{\rm as} \qquad \text{(SIM Web Page)}.$$

For V band, Gould/Salim used the same equation (13) but with,

$$\sigma_{\rm photon} = 4\,\mu{\rm as}, \quad V_0 = 20, \quad t_0 = 26.4\,{\rm hrs}, \quad \sigma_{\rm system} = 0. \qquad \text{(Gould/Salim)}.$$

The underestimate ($\sigma_{\rm system} = 0$) has no practical impact since all of our simulated measurements have $\sigma_{\rm sma} \gg \sigma_{\rm system}$. Our value of t_0 is a factor $\sim 15/8$ *too long.* After reviewing the Narrow-Angle Timeline (Figure 1), we believe that this overestimate of t_0 is approximately compensated by the overhead. For example, we find that in typical mock observation scenarios, we have 41 minutes of on-target science observations during a 1 hour Performance Specificiation Period, plus 11 minute slew and settling, i.e., a science efficiency of $\sim 4/7$. Hence, repeating the calculations of Gould/Salim, we find that the observing time requirements are reduced by a factor $7/4 \times 8/15 = 14/15$. Since this factor is extremely close to unity, we ignore it.

For stars selected in I band, Gould/Salim used a similar formula except with $I_0 = 19$. A better estimate would be $I_0 = 20 + 2.5\log(4/15) = 18.6$ since one gets $\sim 3/5$ as many photons in I as V at $V - I = 0$, and since the photons are 3/2 times longer and so $(2/3)^2$ less efficient in terms of finding a centroid. On the other hand, we expect to be able to develop substantially more efficient observing strategies compared to the simple algorithm of Gould & Salim (1999), and so to be able to approximately make up this difference. We therefore use the Gould/Salim figures with only the minor adjustments described below.

4.2 Narrow-Angle vs. Wide-Angle

As we have detailed in the separate sections of this proposal, the great majority of the information that we extract from the observations depends only on *relative astrometry.* Hence we will be using primarily the narrow-angle mode (and so use reference field stars unless grid stars are conveniently available). However, as we discussed in § 3.1.7, narrow-angle astrometry + photometry gives us (in addition to the mass M), only the *relative* lens-source parallax and proper motion, $\pi_{\rm rel}$ and $\boldsymbol{\mu}_{\rm rel}$. While our main concern is the mass, we would also like to recover the lens parallax $\pi_l = \pi_{\rm rel} + \pi_s$ and lens proper motion $\boldsymbol{\mu}_l = \boldsymbol{\mu}_{\rm rel} + \boldsymbol{\mu}_s$, which requires knowing the *absolute* source parallax and proper motion. To reach the limit of precision set by the rest of the observations (see Table 1 of Gould & Salim 1999), we find that this requires measuring the absolute parallax of at least one of the reference stars to $\sim 35\,$mas, which (from Table 2 of the Wide-Angle web site) requires 0.6 hours. That is, (for the cases where our reference star is not a grid star) about 10% additional time is required to secure these additional absolute parallaxes beyond what was calculated in Gould & Salim (1999). We allow a further 10% increase to compensate for the small loss of precision that comes from including fits of the relative parallaxes and proper motions of the reference star relative to the source star.

4.3 Crowded Field Astrometry

Three of our four projects (1, 2, and 4) require precision astrometric measurements in the crowded fields where microlensing events are found (Galactic bulge and LMC/SMC). How well can we recover the size and orientation of the astrometric ellipse given by equation (4) in the presence of unwanted "background" sources? We find from simulations that if the background source lies up to a few hundred mas from the microlensed source, it can signficantly affect the measurement if the centroiding is done from the "white light" fringe and if the background is equal in brightness to the microlensed source. However, we also find that these effects can be identified and removed if the wavelength-dispersed fringe is available for a small subset of the observations. In any event, since (for S/N reasons) we will be measuring some of the brightest stars in the field (e.g. $I < 17$ in the bulge), and the density of such bright stars is only $\sim 25\,\mathrm{arcmin}^{-2}$, we expect that random background sources will not pose a significant problem.

Binary companions to the source can be expected to lie within the central fringe ($\sim 80\,\mathrm{AU}$) for $\sim 40\%$ of events. However, the vast majority of these will be $\mathcal{O}(100)$ times fainter than the source, and so will not significantly affect the measurement. Moreover, we find from simulations that the companions that are bright enough to affect the centroid significantly can usually be identified and their effects removed.

4.4 Descopes

How will this project be affected if SIM fails to meet its performance goals? Of course, the full answer to this question depends on the details of the performance characteristics achieved, but we can convey the general nature of the effect by considering three descopes:

1) **Mission time estimates are doubled to achieve the same astrometric precision**. In this case, the number of nearby-star masses we could measure in the same 180 hours of mission time would fall by a factor $2^{-1/3}$ (Gould 2000a) from ~ 15 to ~ 12. We would have to double the time allocated to LMC/SMC events from 100 to 200 hours because the scientific goals of this experiment depend critically both on the event number and measurement precision that we have assumed. We would cut back the number of bulge MF and planet mass measurements by approximately 1/3. These would be the events with the faintest sources, so that the remaining 1135 hours of time devoted to these experiments would achieve approximately the same precision as the fullscope experiment, but with a reduced number of events. That is, such a descope would significantly, but not critically, degrade our science achievements.

2) **Target of Opportunity times are doubled to 8 days**. This would not affect the nearby-star mass measurements at all, since they do not require ToO capability. It would probably not seriously affect the LMC/SMC events, since these are long and hence can tolerate longer response times. However, our ability to measure the bulge BD MF would be critically impacted. We probably could not successfully monitor events with timescales $t_{\rm E} \lesssim 20\,\mathrm{days}$, which means that we would recover only 30% of the events from marginal BDs, $M \sim 0.07\,M_\odot$. We could still recover the remnant MF, but a key element of our project would be effectively lost. We would probably refocus some of our efforts monitoring the disk source/disk lens events which are very long ($t_{\rm E} \sim 80\,\mathrm{days}$) and hence do not stress the ToO capabilities. To date, however, these events are relatively rare, so we could not use them to measure the disk MF to anywhere near the precision we hope to measure the bulge MF in the current experiment.

3) **Photometric Capability is lost or seriously degraded**. Again, the nearby stars are not affected. We would still get useful information on the location of the lenses toward the LMC from astrometry, but it would be less definitive. The information recovered on most bulge lenses would be more-or-less useless, except for the long events for which we could measure $\tilde{r}_{\rm E}$ from the ground and so circumvent the need for SIM photometry. We would therefore focus on these, but as stated above, they are quite rare. That is, much of what we are trying to accomplish would be critically degraded.

References

Afonso, C., et al. 2000, ApJ, 532, 340 "Combined Analysis of the Binary-Lens Caustic-Crossing Event MACHO 98-SMC-1"

Albrow, M.D., et al. 2000a, ApJ, 535, 000 (astro-ph/9909325), Limits on Stellar and Planetary Companions in Microlensing Event OGLE-1998-BUL-14

Albrow, M.D., et al. 2000b, ApJ, 534, 000 (astro-ph/9910307), "Detection of Rotation in a Binary Microlens: PLANET Photometry of MACHO 97-BLG-41"

Alcock, C., et al. 1993, Nature, 365, 621, "Possible Gravitational Microlensing of a Star in the Large Magellanic Cloud"

Alcock, C., et al. 1995 ApJ, 454, L125, "First Observation of Parallax in a Gravitational Microlensing Event"

Alcock, C., et al. 1997a, ApJ, 486, 697 "The MACHO Project Large Magellanic Cloud Microlensing Results from the First Two Years and the Nature of the Galactic Dark Halo"

Alcock, C., et al. 1997b, ApJ, 490, 59, "Is the Large Magellanic Cloud Microlensing Due to AN Intervening Dwarf Galaxy?"

Alcock, C., et al. 1997c, ApJ, 491, 436, "MACHO Alert 95-30: First Real-Time Observation of Extended Source Effects in Graviational Microlensing"

Alcock, C., et al. 2000a, ApJ, submitted (astro-ph/9907369), "Binary Microlensing Events from the MACHO Project"

Alcock, C., et al. 2000b, ApJ, in press (astro-ph/0001272), "The MACHO Project: Microlensing Results from 5.7 Years of LMC Observations"

Armstrong, J.T., et al. 1992, AJ, 104, 2217 'T'he orbit of Phi Cygni measured with long-baseline optical interferometry -Component masses and absolute magnitudes"

Aubourg, E., et al. 1993, Nature, 365, 623, "Evidence for Gravitational Microlensing by Dark Objects in the Galactic Halo"

Baruffolo, A., Benacchio, L., & Benfante, L. 1999, Astronomical Data Analysis Software and Systems VIII, ed. D.M. Mehringer, ASP Conference Series, 172, 237 (San Francisco: ASP), "A System for On-line Access to the GSC II"

Beaulieu, J.-P., & Sackett, P.D. 1998, AJ, 116, 209, "Red Clump Morphology as Evidence against a New Intervening Stellar Population as the Primary Source of Microlensing toward the Large Magellanic Cloud"

Bennett, D.B. 1997, BAAS, 191, 8303, "The MACHO Project: Parallax and Binary Source Microlensing Events"

Bennett, D. 1998, ApJ, 493, L79, "Red Clump Stars as a Tracer of Microlensing Optical Depth"

Bennett, D.P., et al. 1999, Nature, 402, 57, "Discovery of a planet orbiting a binary star system from gravitational microlensing"

Boden, A.F., Creech-Eakman, M.J., & Queloz, D. 2000, ApJ, 534, 000, (astro-ph/9910245) "The Visual Orbit and Evolutionary State of 12 Bootes"

Boden, A.F., Shao, M.,& Van Buren, D. 1998 ApJ, 502, 538, "Astrometric Observation of MACHO Gravitational Microlensing"

Boden, A.F., et al. 1999a, ApJ, 515, 356, "The Visual Orbit of IOTA Pegasi"

Boden, A.F., et al. 1999b, ApJ, 527, 360, "The Visual Orbit of 64 Piscium"

Chang, K. & Han, C. 1999, ApJ, 525, 434, "Improved Detection Rates for Close Binaries Via Astrometric Observations of Gravitational Microlensing Events"

Flynn, C., Sommer-Larsen, J., Fuchs, B., Graff, D.S., & Salim, S. 2000 MNRAS, submitted (astro-ph/9912264) "No nearby counterparts to the moving objects in the Hubble Deep Field"

Froeschle, M., Mignard, F., & Arenou, F. 1997, Proceedings of the ESA Symposium 'Hipparcos – Venice '97', p. 49-52, ESA SP-402, "Determination of the PPN Parameter gamma with the HIPPARCOS Data"

Gallart, C. 1998, ApJ, 495, L43, "Observational Discovery of the Asymptotic Giant Branch Bump in Densely Populated Color-Magnitude Diagrams of Galaxies and Star Clusters"

Gaudi, B.S., et al. 2000, in Microlensing 2000, A New Era of Microlensing Astrophysics, eds., J.W. Menzies and P.D. Sackett, ASP Conference Series, (astro-ph/0004269), "Microlensing Constraints on the Frequency of Jupiter Mass Planets"

Gaudi, B. S., & Gould, A. 1997a, ApJ, 477, 152, "Satellite Parallaxes of Lensing Events toward the Galactic Bulge"

Gaudi, B. S., & Gould, A. 1997b, ApJ, 482, 83 "Detection Rates For Close Binaries via

Microlensing"
477, 152, "Satellite Parallaxes of Lensing Events toward the Galactic Bulge"

Gould, A. 1992, ApJ, 392, 442, "Extending the MACHO Search to $\sim 10^6\, M_\odot$"

Gould, A. 1995a, ApJ, 440, 510, "Analytic Error Estimates"

Gould, A. 1995b, ApJ, 441, L21, "MACHO Parallaxes From A Single Satellite"

Gould, A. 1995c ApJ, 441, 77, "Self-Lensing By A Stellar Disk"

Gould, A. 1996, PASP, 108, 465, "Microlensing and the Stellar Mass Function"

Gould, A. 1998, ApJ 499, 728, "Large Magellanic Cloud Microlenses: Dark or Luminous?"

Gould, A. 1999, ApJ, 525, 734, "A New Argument Against An Intervening Stellar Population Toward the Large Magellanic Cloud"

Gould, A. 2000a ApJ, 532, 936 "Selection of Nearby Microlensing Candidates for Observation by the Space Interferometry Mission"

Gould, A. 2000b ApJ, 535, 000 (astro-ph/9906472), "Measuring the Remnant Mass Function of the Galactic Bulge"

Gould, A., & Loeb, A. 1992, ApJ, 392, 104, "Discovering Planetary Systems Through Gravitational Microlenses"

Gould, A., & Salim, S. 1999, ApJ, 524, 794, "Photometric Microlens Parallaxes with the Space Interferometry Mission"

Graff, D.S., Freese, K., Walker, T.P., & Pinsonneault. M.H. 1999, ApJ, 523, 77, "Constraining the Cosmic Abundance of Stellar Remnants with Multi-TEV Gamma Rays"

Gyuk, G., Evans, N.W., & Gates, E. 1998, ApJ, 502, L29, "Brown Dwarfs, White Knights, and Demons"

Han, C., Chun, M.S., & Chang, K. 1999, ApJ, 526, 405 "Astrometric Properties of Gravitational Binary-Microlens Events and Their Applications"

Han, C., & Gould, A. 1997, ApJ, 480, 196, "Einstein Radii From Binary-Source Lensing Events"

Hambly, N.C., Hodgkin, S.T., Cossburn, M.R., & Jameson, R.F. 1999, MNRAS 303, 835, "Brown dwarfs in the Pleiades and the initial mass function across the stellar/substellar boundary"

Henry, T.J., & McCarthy, D.W.Jr., 1993, AJ, 106, 773, "The mass-luminosity relation for stars of mass 1.0 to 0.08 solar mass"

Holtzman, J.A., Watson, A.M., Baum, W.A., Grillmair, C.J., Groth, E.J., Light, R.M., Lynds, R., & O'Neil Jr., E.J. 1998, AJ, 113, 1948, "The Luminosity Function and Initial Mass Function in the Galactic Bulge"

Hummel, C.A. 2000, Proceedings of 'Working on the Fringe: An International Conference on Optical and IR Interferometry from Ground and Space', (S. Unwin and R. Stachnik, editors), p. 17 (ASP: San Francisco) "Imaging and Modeling of Binaries with NPOI"

Hummel, C.A., Armstrong, J.T., Quirrenbach, A., Buscher, D.F., Mozurkewich, D., Elias, N.M., II, Wilson, R.E. 1994, AJ, 107, 1859, "Very high precision orbit of Capella by long baseline interferometry"

Hummel, C.A., et al. 2000, in preparation, "The Three-Dimensional Orbit of Omicron Leonis"

Ibata, R.A., Lewis, G.F., & Beaulieu, J.-P. 1998, ApJ, 509, L29, "Reexamination of the Possible Tidal Stream in Front of the Large Magellanic Cloud"

Ibata, R.A., Richer, H.B., Gilliland, R.L., & Scott, D. 1999, ApJ 524, L95 "Faint, Moving Objects in the Hubble Deep Field: Components of the Dark Halo?"

Johnston, K.V. 1998, ApJ, 495, 297, "A Prescription for Building the Milky Way's Halo from Disrupted Satellites"

Kaluzny, J., Thompson, I., Krzeminski, W., & Pych, W. 1999, A&A, 350, 469, "A photometric survey for variable stars in the globular cluster M5"

Lucas, P.W., & Roche, P.F. 2000, MNRAS, in press (astro-ph/0003061), "A Population of very young Brown Dwarfs and free-floating Planets in Orion"

Luyten, W. J. 1979, 1980, "New Luyten Catalogue of Stars with Proper Motions Larger than Two Tenths of an Arcsecond (NLTT)" (Minneapolis: University of Minnesota Press)

Mao, S. 1999 A&A, 350, L19, "An Ongoing OGLE Parallax Microlensing Event OGLE-1999-CAR-1 toward Carina"

Mao, S., & Paczyński, B. 1991, ApJ, 374, L37, "Gravitational microlensing by double stars and planetary systems"

Miralda-Escudé, J. 1996, ApJ, 470, L113, "Microlensing Events from Measurements of the Deflection"

Monet, D. 1998, BAAS, 193, 120.03, "The 526,280,881 Objects In The USNO-A2.0 Catalog"

Paczyński, B. 1995, Acta Astron., 45, 345, "The Masses of Nearby Dwarfs Can Be Determined with Gravitational Microlensing"

Paczyński, B. 1998, ApJ, 494, L23, "Gravitational Microlensing with the Space Interferometry Mission"

Refsdal, S. 1964, MNRAS, 128, 295, "The Gravitational Lens Effect"

Refsdal, S. 1966, MNRAS, 134, 315, "On the Possibility of Obtaining the Distances and Masses of Stars From The Gravitational Lens Effect"

Rhie, S.H., Becker, A.C., Bennett, D.P., Fragile, P.C., Johnson, B.R., King, L.J., Peterson, B.A., & Quinn, J. 1999, ApJ, 522, 1037, "Observations of the Binary Microlens Event MACHO 98-SMC-1 by the Microlensing Planet Search Collaboration"

Sahu, K.C. 1994, Nature, 370, 275, "Stars Within the Large Magellanic Cloud as Potential Lenses for Observed Microlensing Events"

Salim, S., & Gould, A. 2000a, ApJ, 539, 000 (astro-ph/9909455), "Nearby Microlensing Events - Identification of the Candidates for the Space Interferometry Mission"

Salim, S., & Gould, A. 2000b, ApJ, in preparation "Nearby Microlensing Events - The Candidates From NLTT"

Udalski, A., et al. 1993, Acta Astron., 43, 289, "The optical gravitational lensing experiment. Discovery of the first candidate microlensing event in the direction of the Galactic Bulge"

Wu, X.-P. 1994, ApJ, 435, 66, "Gravitational microlensing by the MACHOs of the Large Magellanic Cloud"

Zaritsky, D., & Lin, D.N.C. 1997, AJ, 114, 1545, "Evidence for an Intervening Stellar Population Toward the Large Magellanic Cloud"

Zaritsky, D., Shectman, S.A., Thompson, I., Harris, J., & Lin, D.N.C. 1999, AJ, 117, 2268, "Constraints on Intervening Stellar Populations toward the Large Magellanic Cloud"

Zhao, H. 1998, MNRAS, 294, 139, "Microlensing of tidal debris on the Magellanic great circle"

Zhao, H. 1999a, ApJ, 526, 141, "A Radial Velocity Survey for Large Magellanic Cloud Microlensed Sources"

Zhao, H. 1999b, ApJ, 527, 167, "Reddening of microlensed LMC stars vs. the location of the lenses"

Zhao, H. 2000, ApJ, 530, 299, "Extinction bias of microlensed LMC stars towards LMC and fraction of Machos in the halo"

Zhao, H., Graff, D., & Guhathakurta, P. 1999, ApJ, 532, 37, "Magnitude bias of microlensed sources towards the LMC"

Zoccali, M., S., Cassisi, S., Frogel, J.A., Gould, A., Ortolani, S., Renzini, A., Rich, R.M. 1999, & Stephens, A. 2000, ApJ, 530, 418, "The Initial Mass Function of the Galactic Bulge Down to $\sim 0.15\,M_\odot$"