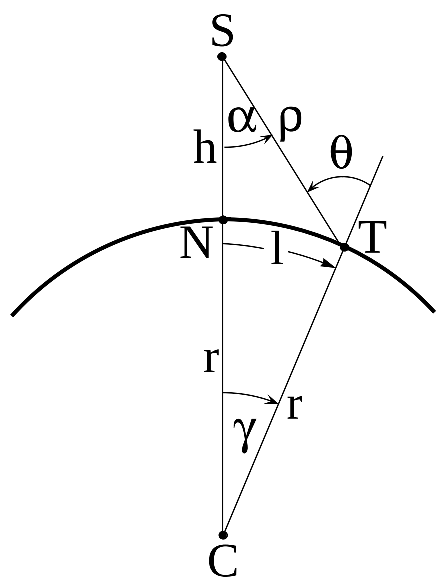
# CERES SB 95-08: Solution to Orbital Sampling Triangle

**Richard Green** (green@green.larc.nasa.gov)
**Beth Flug** (beth@flug.larc.nasa.gov)

## Purpose:

The purpose of this bulletin is to document for easy reference the solution to the orbital sampling triangle in the figure. There are three angles involved. Given one angle we can solve for the other two angles. These three solutions and a numerical example are given. Numerical accuracy of the solutions is a concern. These three solutions are the basis for solving many CERES sampling problems.

$\alpha$ = cone angle
$\theta$ = viewing zenith
$\gamma$ = earth central angle
$\rho$ = slant range
r = radius of surface
h = satellite altitude
l = surface length




S = spacecraft
C = center of Earth
T = target point
N = nadir

Since angles of a triangle sum to $180°$:

$$\alpha + \gamma + (180 - \theta) = 180$$
$$\alpha + \gamma - \theta = 0$$

The surface length is $l = r\gamma$.

I. Given $\theta$, find $\alpha$ and $\gamma$:

From Law of Sines:

$$\frac{\sin(180 - \theta)}{(r + h)} = \frac{\sin\alpha}{r}$$

$$\alpha = \sin^{-1}\left[\frac{r}{r + h}\sin\theta\right] \qquad 0 \le \alpha \le 90°$$

$$\gamma = \theta - \alpha$$

II. Given $\gamma$, find $\alpha$ and $\theta$:

From Law of Cosines

$$\rho = \left[ (r+h)^2 + r^2 - 2\,(r+h)\, r \cos\gamma \right]^{1/2}$$

Project ST and TC onto SC

$$\rho \cos\alpha + r\cos\gamma = r + h$$

$$\alpha = \cos^{-1}\left[ \frac{(r+h) - r\cos\gamma}{\rho} \right] \qquad 0 \le \alpha \le 90°$$

$$\theta = \alpha + \gamma$$

III. Given $\alpha$, find $\gamma$ and $\theta$:

From Law of Cosines

$$r^2 = (r+h)^2 + \rho^2 - 2\,(r+h)\,\rho\cos\alpha$$

$$\rho^2 - [2\,(r+h)\cos\alpha]\,\rho + \left[ (r+h)^2 - r^2 \right] = 0$$

* Choose smallest solution to quadratic solution or negative sign

$$\rho = (r+h)\cos\alpha - \left[ r^2 - (r+h)^2 \sin^2\alpha \right]^{1/2}$$

Project ST and TC onto SC

$$\rho\cos\alpha + r\cos\gamma = (r+h)$$

$$\gamma = \cos^{-1}\left[ \frac{(r+h) - \rho\cos\alpha}{r} \right] \qquad 0 \le \gamma \le 90°$$

$$\theta = \alpha + \gamma$$

EXAMPLE: r=6367, h=350 (ATBD: Table 4.4-1, 4.4-2)

I. $\theta = 70°$

$$\alpha = \sin^{-1}\left[\frac{6367}{6367 + 350}\sin 70\right] = 62.96$$

$$\gamma = 70 - 62.96 = 7.04$$

II. $\gamma = 7.04$

$$\rho = \left[(6367 + 350)^2 + 6367^2 - 2\,(6367 + 350)\,(6367)\,\cos 7.04\right]^{1/2} = 875.48$$

$$\alpha = \cos^{-1}\left[\frac{(6367 + 350) - 6367\cos 7.04}{875.48}\right] = 62.96$$

$$\theta = 62.96 + 7.04 = 70.00$$

III. $\alpha = 62.96$

$$\rho = (6367 + 350)\cos 62.96 - \left[(6367)^2 - (6367 + 350)^2\sin^2 62.96\right]^{1/2} = 875.48$$

$$\gamma = \cos^{-1}\left[\frac{(6367 + 350) - 875.48\cos 62.96}{6367}\right] = 7.04$$

$$\theta = 62.96 + 7.04 = 70.00$$

[Warning: Double precision is needed to compute these angles to 2 decimals using Fortran 77 on Unix. Seven decimal digits of precision is needed to compute these angles to 2 decimals using Fortran 90 on Unix. Note that the above calculations were made using the actual values. The results are shown in the “f#.2” format.]