Moral Hazard, Mergers, and Market Power

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Abstract

Most analysis of market power assumes that managers act as perfect agents for the shareholders. This paper relaxes this assumption. When managers of a multiproduct firm must exert unobservable effort to improve product quality, there will be a tension between the optimal incentive scheme for eliminating price competition between the products and creating optimal effort incentives. This makes some intra-firm price competition inevitable. When quality improving effort generates positive spillovers, the optimal amount of price competition can be as great or greater than when the products are under separate ownership. Even when this is not the case, intra-firm price competition can be severe enough that quality adjusted price is lower when the products are under common ownership.

1 Introduction

Despite the fact that the last few decades of research on the theory of the firm has shown that managers will often *not* act as a perfect agent for the firm's shareholders, most analyses of mergers and market power continue to assume that any multi-product firm will choose its decision variables to maximize the joint profits of the products within the firm. This paper is one of the first to relax (at least in some respects) that assumption and examine the impact on the pricing, innovation, and incentive strategies of multi-product firms.[1] These impacts are particularly relevant when formulating merger policy since, in differentiated products industries, mergers often result in multi-product firms.

Consider the following hypothetical. There are two owner managed firms, A and B, that make software for some industry. While these products both meet the same need for the same group of firms, they are not identical. Maybe product A is somewhat better for smaller firms while B is better for larger ones. Or maybe A's product does some set of tasks better and others not as well, and the importance of these tasks differs among firms in this industry. Now, imagine that A and B merge. Since these products are far from perfect (they are software products, after all), and each manager knows her product far better than does the other manager, each must remain as part of the combined firm to improve her product. Just because they are part of one firm, however, does not mean they will now only act in their joint, as opposed to their individual, interest. Moreover, because A and B serve slightly different (though overlapping) markets, and each manager knows far more about her product and how potential customers perceive it, the manager of each product has private information about the demand curve for her product. As a result, each manager must have the sole authority to set the price for her product.[2] (This is consistent with empirical evidence that shows that merged firms are frequently organized in a multisubsidiary form and target management is often retained (Prechel, Boies, and Woods 1999; Zey and Swenson 1999). Time Warner, for example, operated each of its different divisions as independent profit centers with a great deal of intra-firm competition.)

Of course, since both managers now jointly own the two products, they will want to reduce or

[1] Alternatively, one could view this paper as a model of the optimal cartel contract.

[2] In any revelation mechanism that determines price, each manager would provide information that generates her preferred price unless she were penalized for excessive sales. Doing so would almost certainly introduce other distortions and limit a manager's incentive to respond to new information. Moreover, if each manager sells many different versions of her product at different prices, the number of sales may not be verifiable even if total revenues are. This presumably explains why we rarely observe managers being punished for excessive sales.

eliminate price competition between them. One way to accomplish this would be to for them to agree to equally share the profits from the sales of both products (I call this complete profit sharing). The division of profits, however, not only affects pricing incentives, but it also affects a manager's incentive to improve her product. If product quality is unverifiable, then product profit is the only available instrument for inducing the managers to engage in costly product improving effort. If the managers elect to share the profits from the two products equally, then each manager will bear all of the cost of product improvement (the effort required to fix bugs or add new features to the software) but receive only half the benefit (the increased profit earned by the two products together when one of them is better). To improve effort or innovation incentives, then, the managers will prefer that each one retains greater than half of the profit from its own product even though this generates some incentive for price competition between the two products.

For the same reason that the managers do not want to completely sacrifice innovation incentives to eliminate price competition, one might also think that they will not want to completely sacrifice pricing coordination to maximize innovation incentives. If innovative effort is of purely private value (that is, it has no effect on the quality of the other product), then this intuition is correct. If there are innovation spillovers (when one manager figures something out about how to fix a bug or add a feature it makes it easier for the other manager make her product better), however, then there can actually be a reduced incentive to share profit, so much so that it can be the case that no profit sharing is optimal.[3] The reason is that spillovers can create a positive externality associated with product improving effort, even if each manager keeps the entire profit from her product.[4] So long as the private value from this effort is greater than the external value, effort incentives will be stronger the less profit sharing there is. The closer in magnitude the spillover benefits are to the private benefits, however, the weaker the relationship between effort incentives and profit sharing becomes. So, when the spillover effect is so strong that product B is improved almost as much as product A is by manager A's product improving effort, there will be very little value, in terms of improved effort incentives, to intra-firm price competition, making a large degree of profit sharing optimal.

Determining the optimal level of profit sharing between the two products, of course, is critical to assessing the competitive impact of the merger of A and B. If innovation incentives are likely

[3] In fact, it can be optimal (at least if one ignores the possibility of sabotage) for manager A to give manager B an added bonus based on the profit generated by product B and vice versa.

[4] I say "can be" because even with though spillovers create a positive externality, there is also a negative externality associated with business stealing that could make the net external effect negative.

to be very important relative to the effect of intra-firm price competition, then one might expect very little profit sharing, and thus very little harmful impact on consumers. In fact, if there are spillover effects (that can only be realized when A and B are part of the same firm), the merger might even benefit consumers. This argument seems very similar to the claim often made by merging companies that improved efficiency from the merger will result in lower prices. The important difference, however, is that (as I demonstrate below) when product improving effort is non-contractible, spillover effects not only have the obvious direct effect on product quality but also have an indirect effect on prices through their effect on the optimal amount of profit sharing. By making effort incentives relatively more important, spillover effects (so long as they are not too large) can cause the merged firm to create substantial price competition between the two products (despite the common ownership), leading to lower prices. On the other hand, if spillover effects are very large, a firm that could contractually obligate the manager of each product to exert an efficient level of effort might have lower quality adjusted prices than an identical firm where quality improving effort is non-contractible. The second firm would have some intra-firm price competition (which the first would not), but it would be quite small since when spillovers are large limiting profit sharing only has a small effect on effort incentives. The first firm, however, would have a much greater level of quality improving effort since it could contractually internalize the positive externality. If, as is generally the case, the firm chose to benefit from this quality improvement through greater sales as well as through higher prices, consumers might be better off in the first situation. So, while the possibility that a multi-product firm may have to use the profit sharing instrument to influence both effort and pricing incentives can have an important impact on the welfare effects of mergers, this effect can be in either direction.

This paper is related to the extensive literature on the relationship between managerial incentives schemes and product market competition. Most closely related are the papers that extend this analysis to multiproduct firms. For example, Vickers (1985) showed that inducing competition between divisions can be profitable because of its effect on the behavior of competitors. Fauli-Oller and Giralt (1995) and Barcena-Ruiz and Espinosa (1999) explore in greater detail the relationship between the types of incentives given to managers of multi-product firms and product market competition. Huck, Konrad, and Müller (2001) analyze mergers between identical Cournot competitors. They assume that the merged firms are kept as separate entities and continue to maximize the profits of the individual entity. Because the merger gives the two entities more information about the production of the entity within the same firm, however, it allows one firm to act as a Stackelberg leader with respect to the other, increasing the combined profits of the firm and total quantity while reducing price. Thus, keeping the entities separate makes a horizontal merger profitable when it otherwise would not be. In none of these papers, however, is unobservable managerial effort an issue. So, in some sense, my model performs the opposite analysis of these papers. It looks at the effect of the need to motivate managers on product market competition rather than the effect of product market competition on how managers are compensated.[5]

The literature on research joint ventures (e.g., Kamien, Muller, and Zang 1992; Ziss 1994; Vonortas 1994; Cabral 2000) also shares some similarities with this paper. These papers, however, typically assume that the joint venture can contractually fix the level of research and development (R&D) done by participating firms. The one exception is Cabral (2000). He shows that R&D cooperation can be sustained even when R&D effort is unobservable. Unlike the current paper, however, his model assumes complete R&D spillovers between the participating firms and completely rules out any degree of profit sharing. So, his paper is not aimed at examining the interaction between effort incentives and profit sharing for varying degrees of spillovers.

Also related is the literature on revenue sharing in partnerships (Gaynor 1989; Gaynor 1990). In these papers, the partners each produce a heterogeneous product and, contrary to the assumption in my model, they set price collectively. As a result, there is no price competition within the firm, but there is non-price competition. This theory has been applied to medical and legal partnerships. Because price is set collectively, if the parnters are risk neutral, then the degree of revenue sharing can be set to induce the optimal amount of non-price competition (there is one target and one instrument). With risk aversion, of course, there will more revenue sharing and too little non-price competition (even from the perspective of the partnership).

In some ways, this paper is an application of Holmstrom's (1982) work on moral hazard in teams. In that paper, individual signals of effort were unavailable, whereas here they are available but distortionary. Holmstrom's insight that using only a joint signal will lead to suboptimal effort incentives is what drives the main result in this paper that it can pay to use individual signals of effort even when doing so reduces the total surplus available for any given effort level. This paper is also related to Fulghieri and Hodrick's (1997) paper on the interaction of synergies and agency conflicts for multi-divisional firms. Their focus, however, is conflicts between managers who are paid a fixed wage and a board of directors that acts in the best interests of the shareholders, not

[5]These papers do provide an additional, strategic, justification for my assumption of independent managerial decision making.

on the use of profit-sharing between divisions to minimize the distortions associated with team production problems and price competition.

The plan of the paper is as follows. The next section outlines the general model. Section 3 examines the linear demand case with 3.1 discussing optimal profit sharing, 3.2 discusses price effects, and 3.3 discusses how moral hazard changes the analysis of mergers. Section 4 concludes. All proofs, and some additional formulas, are in the Appendix.

2 Model

There are two managers, each of which has specialized expertise over one product. These two products compete with each other in the sense that the price and quality of one product affects the demand for the other product. Each manager can exert unobservable effort to improve the quality of her product. I assume the demand function for each product is symmetrical, thus the demand for product i (where the other product is $-i$) can be written as follows:

$$q^i = q(p^i, p^{-i}, e^i, e^{-i}) \tag{1}$$

Here e^i represents the quality improving effort made by manager i. Since I assume that each product is produced with a constant marginal cost, c, product i generates the following profit:

$$\pi^i = (p^i - c)q(p^i, p^{-i}, e^i, e^{-i}) \tag{2}$$

If these two managers decide to merge their products into one firm (where, I assume, they are also the owners), then they must decide how much profit to share with the other manager. Since demand, production costs, and effort costs are symmetric, I require the optimal degree of profit sharing to be identical for each product (this will be optimal so long as the products are not very close substitutes).[6] If α is the fraction of the profit from product i that accrues to manager i, and

[6] If the products are very close substitutes, then it may be optimal for the firm to give almost all of the profit from both products to one manager. This way, price competition can be eliminated and one manager will have very

exerting quality improving effort e^i imposes a private and unobservable cost $k(e^i)$ on manager i, then her utility is given by:

$$u^i(\alpha) = \alpha\pi^i + (1-\alpha)\pi^{-i} - k(e^i) \tag{3}$$

Since each manager has a symmetrical utility function, each wants to choose α to maximize $u(\alpha)$. At this stage there is no conflict among the managers since each manager's utility varies with α in the same way. If quality improving effort were contractible, then it would clearly be optimal to set $\alpha = 1/2$ so that neither manager would have any incentive to lower price to steal customers from the other. On the other hand, if prices could be set jointly, then α would only affect a manager's effort incentives. Both managers would agree to set prices at the joint profit-maximizing level and set $\alpha > 1/2$ so that they would not exert too little effort.[7] If, however, the manager of each product has private information about the demand for that product, then allowing the other manager control over the pricing decision may not be feasible. The error due to inadequate knowledge of the demand function could easily reduce joint profits far more than the incentive to cut prices resulting from $\alpha > 1/2$.

Of course, in determining the optimal degree of profit sharing, I assume that the demand function for each product is symmetric with the other, so there is no private information at this point. However, it will often be the case that when the managers are deciding on the optimal degree of profit sharing that they have common expectations about demand, but when the time comes to set price, each manager has acquired substantial further information about the actual state of demand for her product, requiring independent pricing decisions. To simplify the analysis, instead of specifying the distribution of demand functions, I just assume that the managers find the optimal α under the assumption that each manager will choose her effort and price based on demand given by q.

Thus, there are three periods in the model. In period 1, the two managers choose the degree of profit-sharing, $1-\alpha$. In period 2, manager i chooses her level of non-contractible effort e^i. I assume that, while not contractible, this is observable to manager $-i$ in period 3. In period 3,

strong effort incentives.

[7] $\alpha = 1$ would not be optimal since then there would be excessive incentives to exert effort so as to steal customers from the other product.

each manager sets the price of her product and profits are realized and distributed according to the level of profit-sharing determined in period 1.

The solution concept I use for this model is subgame perfection. I solve the model using backward induction. In period 3, each manager chooses price to maximize her utility given α, e^i, e^{-i}. By differentiating (3), this gives the following first order condition for p^i:

$$\alpha(q^i + (p^i - c)q_1^i) + (1-\alpha)(p^{-i} - c)q_2^{-i} = 0 \tag{4}$$

Throughout the paper, subscripts will be used to denote partial derivatives. Thus, q_1^i is the derivative of q^i with respect to its first argument (which is p^i) and q_2^{-i} is the derivative of q^{-i} with respect to its second argument (which is p^i). The term multiplied by α is the standard condition for profit maximization by a one product firm. The second term is the adjustment due to the fact that manager i also has a stake in the profits generated by product $-i$, whose sales are affected by the price of product i.

These two first order conditions (one each for $i = 1, 2$) implicitly determine p^i given α, e^i, e^{-i}. Because of the symmetry in the model, one can write $p^i = p(\alpha, e^i, e^{-i})$ for $i = 1, 2$. Thus, in period 2 manager i's first order condition for e^i can be obtained by differentiating (3) with respect e^i where $p^i = p(\alpha, e^i, e^{-i})$. This gives the following condition:

$$\alpha(p^i - c)(q_3^i + p_3 q_2^i) + (1-\alpha)(p_3 q^{-i} + (p^{-i} - c)(q_4^{-i} + p_3 q_1^{-i})) - k'(e^i) = 0 \tag{5}$$

The term multiplied by α is the marginal increase in profit for product i from product improving effort. There is a direct effect due to increased demand and an indirect effect due to how product improving effort by i affects the price charged by $-i$ and how that change in price affects the demand for product i. (Note, there is no term from the effect of e^i on the p^i since p^i is set optimally by (4) already.) The term multiplied by $1 - \alpha$ is the effect of increasing e^i on the profitability of product $-i$. The first term, $p_3 q^{-i}$, is from the effect of increasing e^i on p^{-i}. The next two terms are quantity effects, directly from a larger e^i and indirectly from the price effect. The last term is the marginal cost of increasing e^i. Using the first order condition for p^{-i}, equation (5) can be rewritten as follows:

$$\alpha q_3^i(p^i - c) + (1-\alpha)q_4^{-i}(p^{-i} - c) + \frac{2\alpha - 1}{\alpha} p_3 q_2^i(p^{-i} - c) - k'(e^i) = 0 \tag{6}$$

These two first order conditions (one each for $i = 1, 2$) implicitly determine e^i given α. Thus, one can write $e^i = e(\alpha)$ (again, the function is identical for $i = 1, 2$ because of symmetry).

The optimal degree of profit sharing in period 1 can then be determined by differentiating (3) with respect to α where $e^i = e(\alpha)$ and $p^i = p(\alpha, e^i, e^{-i})$ for $i = 1, 2$. Doing so, using the pricing and effort first order conditions and rearranging terms, gives the following first order condition for α:

$$(p_1 + e' p_2)(q + (p - c)(q_1 + q_2)) + e'(p - c)(\alpha q_4 + (1 - \alpha)q_3) = 0 \tag{7}$$

Increasing α affects a manager's utility through its effects on price and quality improving effort. Of course, a manager knows that she will set her price and effort level to maximize her utility, so her concern is how α affects the other manager's pricing and effort incentives. The first term represents the price effect. Profit sharing has a direct effect on prices (the p_1 term): the less profit sharing (greater α) the more incentive there is to cut price. There is also an indirect effect via the effect on effort incentives (the $e'p_2$ term). If $e' > 0$, then less profit sharing (greater α) increases the other manager's product improving effort causing her to increase her price. (There is no corresponding term for how α affects the first manager's effort incentives and how that affects the other manager's price since this effect is already taken into account in the first manager's effort decision.) The marginal benefit from increasing prices is given by $(q + (p - c)(q_1 + q_2))$. It is easy to see, from (4) that this is the left hand side of (4)when $\alpha = 1/2$, that is, when there is complete profit sharing. Thus, for any $\alpha > 1/2$ we know that $q + (p - c)(q_1 + q_2) > 0$; a manager benefits if the other manager raises her price.

The second term represents the effort effect on the other manager. When she increases her effort, the effect on her sales (profit) is given by q_3 ($(p - c)q_3$), and the effect on the first manager's sales (profit) is given by q_4 ($(p - c)q_4$). Since the first manager receives $(1 - \alpha)$ of the profit from the other product and α of the profit from her own product, the effects are weighted accordingly.

From (7), one can see that if $e'(1/2) > 0$, then each manager will retain more than half of the profit from her product, $\alpha > 1/2$. To see this, evaluate the left hand side of (7) at $\alpha = 1/2$:

$$(p_1 + e' p_2)(q + (p - c)(q_1 + q_2)) + \frac{1}{2} e'(p - c)(q_4 + q_3) \tag{8}$$

The first term in (8) is zero by first order condition for price, (4): at $\alpha = 1/2$ (4) is:

$$(q^i + (p^i - c)q_1^i) + (p^{-i} - c)q_2^{-i} = 0 \tag{9}$$

Imposing symmetry, this becomes:

$$q + (p - c)(q_1 + q_2) = 0 \tag{10}$$

Thus, (8) can now be written as:

$$\frac{1}{2} e'(p - c)(q_4 + q_3) \tag{11}$$

Since $p - c > 0$ (the firms make a profit) and $q_4 + q_3 > 0$ (improving the quality of one product does not decrease total demand for both products), this is positive. If $e'(1/2) > 0$, then the managers want less than complete profit sharing.

Intuitively, this result is not surprising, it follows from the envelope theorem. With complete profit sharing, the pricing distortion from increasing α (less profit sharing) is of second order while the effort effects are of first order. Since the incentive for product improving effort is sub-optimal when a manager bears all the cost but only gets half the benefit, increasing α above 1/2 makes the managers better off.

Unfortunately, with a fully general demand function the conditions necessary to guarantee that effort will increase when moving away from complete profit sharing are quite complicated and not very illuminating. While the direct affect of increasing α is positive (keeping more of one's profits increases the incentive to make the good more profitable) there are also two different indirect effects. Increasing α causes the other manager to cut price, reducing the first manager's sales which reduces the incentive to increase quality. This price reduction can also affect the magnitude of the effect of effort on demand. Lastly, increasing α increases the impact of the effect of other manager's price response to the first manager's increase in effort. So long as effort spillovers are small, the other manager will cut price, reducing the first manager's demand.

If the general demand function can be written as only a function of the quality adjusted price for each product, however, then it is straight forward to show that decreasing profit sharing from the complete profit sharing level will increase effort. Thus, in this case, the following proposition holds.

Proposition 1 *If demand can be written as* $q^i = q(p^i - (e^i + \gamma e^{-i}), p^{-i} - (e^{-i} + \gamma e^i))$ *(with* $0 \leq \gamma < 1$*), then complete profit sharing (*$\alpha = 1/2$*) is never optimal whenever* $q_{11} + q_{22} - q_{12} < 0$.

In the demand function in the Proposition, γ measures the magnitude of effort spillovers. So, whenever effort by one manager improves the quality of her own product more than it improves the quality of the other product, there will never be complete profit sharing. Thus, when it is important to motivate managers to make product improving effort and demand depends only on the quality adjusted price of both products, a multi-product monopolist will not price at the fully collusive level. Even when both products are under common ownership, there will still be some price competition between them.

3 Linear Demand Case

In order to more fully explore the effect of unobservable effort on the degree of competition between managers in the same firm, I fix a demand and effort cost function. I assume the effort cost function is $k(e) = ke^2/2$ and that the demand curve takes the following form:

$$q^i = a - b_1(p^i - e^i - \gamma e^{-i}) + b_2(p^{-i} - e^{-i} - \gamma e^i) \tag{12}$$

The demand for product i is a linear function of the quality-adjusted price for both products. Quality enhancing effort by the manager is measured in units of quality improvement. Moreover, I allow for effort spillovers. When manager i exerts effort e^i this not only improves her product by e^i, it also improves product $-i$ by γe^i. The existence of such spillovers is a common, pro-competitive, justification for the merger of two competing products. When these spillovers result from unobservable effort, however, they may indirectly affect the quality-adjusted price paid by consumers through an effect on the optimal degree of intra-firm price competition, in addition to having a direct effect on product quality.

When demand is given by (12), the first order condition for p^i $(i = 1, 2)$ becomes the following:

$$\alpha(a - b_1(2p^i - e^i - \gamma e^{-i} - c)) + b_2(p^{-i} - \alpha(e^{-i} + \gamma e^i) - (1-\alpha)c) = 0 \quad (13)$$

Solving these first order conditions for p^i gives the following equation for price:

$$p^i = \frac{\begin{array}{c} 2\alpha^2 b_1(a + b_1(c + e^i + \gamma e^{-i})) + \alpha b_2(a + b_1(2\alpha - 1)(c - e^{-i} - \gamma e^i)) \\ -b_2^2((1-\alpha)c + \alpha(e^i + \gamma e^{-i})) \end{array}}{(2\alpha b_1 - b_2)(2\alpha b_1 + b_2)} \quad (14)$$

Using this expression for price and (12) in the first order condition for effort, (6), it is possible to explicitly solve for effort, e^i, as a function of α, γ, and the demand and effort parameters a, b_1, b_2, and k. Because the expression is even more complicated than the one for p^i, I relegate it to the appendix. Note, however, that the second order condition for the effort equation is satisfied whenever $k \geq \frac{b_1}{4}$. Henceforth, I will only consider values of k that satisfy this condition.[8]

Using these explicit solutions for price and effort, one can get a reduced form expression for utility that can be optimized over α. Unfortunately, the resulting first order condition for α is an 8-degree polynomial in α (also given in the appendix), so it is not possible to get an explicit analytical solution for α. Even analytic comparative statics are complicated by the fact that the reduced form utility function is not everywhere concave in α. By expressing b_2 and k in terms of their ratio to b_1(i.e., doing the change of variables $b_2 = \tilde{b}_2 b_1$ and $k = \tilde{k} b_1$), however, the first order condition for α becomes independent of a, b_1 and c. As a result, it is possible to learn a great deal about the optimal degree of intra-firm competition via numerical simulation. To ensure that the characteristics of one's own product affects its demand more than the characteristics of the competing product, I assume $0 < \tilde{b}_2 < 1$. To ensure that the effort first order condition is a necessary and sufficient condition for the optimal effort level, I also require that $\tilde{k} \geq 1/4$.

To obtain numerical solutions for the optimal level of α, I first numerically solve the first order condition. I then discard all solutions that are not between one-half and one. (It will never be optimal for a manager to retain less than half of the profit from her product. I rule out a manager receiving more than hundred percent of the profit from her product because the other manager may not have the funds to make such a transfer, and because it gives her too great an incentive to sabotage production. Whenever I find that the optimal $\alpha = 1$, however, this will mean that if $\alpha > 1$ were feasible, then intra-firm price competition would exceed inter-firm price competition.)

[8]The proof of this claim is omitted but is available from the author upon request.

The optimal α must either be one of the solutions to the first order conditions, or it must be at an endpoint. Since Proposition 1 demonstrates that the first order condition is always positive at $\alpha = 1/2$, I only need to consider the odd (non-discarded) solutions (arranged in ascending order) to the first order condition and $\alpha = 1$.[9] I then feed these candidate solutions back into the reduced form utility function to see which gives the largest utility (this comparison is also independent of a, b_1 and c since these enter the reduced form utility function only through a multiplicative term that does not contain α).

3.1 Optimal Profit Sharing

Before proceeding to the results from the simulations, however, the linear demand model does permit one analytic result on the magnitude of the optimal α. This is given in the following proposition.

Proposition 2 *When the demand curve is given by (12) and effort costs are given by $ke^2/2$, if there are no spillovers and $\tilde{k} \geq 1/4$, then $\alpha < 1$, i.e. intra-firm price competition is strictly less than inter-firm price competition.*

If a manager keeps all her own profit, then her private benefit from effort is the full amount of the increase in profit her product generates as a result. When the products compete, however, part of this increase in profit comes from stealing customers away from the other product. Thus, if there are no spillovers, a manager who keeps all her own profit has an excessive incentive to exert quality improving effort (from the standpoint of the firm). Thus, reducing α below one will improve both effort incentives and pricing incentives. This argument doesn't necessarily hold when there are spillover effects from quality improving effort. In this case, while part of the private benefit from increased effort is from business stealing, there is a countervailing positive externality. Greater effort makes the other product better which may increase its sales. Thus, it is possible that effort incentives are insufficient even when $\alpha = 1$. Of course, as the spillover effect gets stronger the distortion in effort incentives generated by large degrees of profit sharing becomes smaller. So,

[9] Only the odd solutions can be local maxima since the first order condition is positive at $\alpha = 1/2$. This follows since the marginal benefit of increasing α is positive when α is less than the first solution. So long as the second derivative at the odd solution is not zero, this means that in between the first and second solutions the marginal benefit must be negative, making the second solution a local minimum. The reasoning is the same for more solutions. In every simulation I do check that the second order conditions at the solutions guarantee that only odd solutions can be maxima.

it is not necessarily the case, as the simulations below demonstrate, that greater spillovers always induce larger α (more intra-firm competition). Before providing some graphs that demonstrate both the range of possible magnitudes for α and how the optimal α varies with various parameters, I will summarize the key findings from the simulations.

Simulation Results–Optimal α: *(A) The optimal α is: (1) (almost everywhere) increasing in γ up to some critical level and then decreasing; (2) decreasing in $\tilde{b}_2$; (3) decreasing in $\tilde{k}$.*

(B) When $\tilde{k}$ is small enough the optimal α can be one even when $\tilde{b}_2 > .6$.

(C) When $\tilde{b}_2 = 1/6$, the optimal α can be one even when $\tilde{k} > 7$ and is always greater than $.89$ for $\tilde{k} \leq 1, \gamma \leq .95$.

(D) For some parameter values the optimal α can be very sensitive to small changes in γ, $\tilde{k}$, and $\tilde{b}_2$.

When the spillover effects from product improving effort are very small, increasing the spillover effect will increase the positive externality associated with product improving effort. Thus, the importance of reducing the effort distortion becomes more important. As the spillover benefit from effort becomes closer to the size of the direct benefit, however, the distortion associated with large degrees of profit-sharing becomes smaller. In the extreme case, where effort by one manager improves the quality of both her product and the other product by the same amount, the effort distortion is unaffected by the degree of profit sharing. So when γ is very large, minimizing price competition becomes relatively more important than minimizing the effort distortion. This effect can be seen in the following figures.

Figure 1 depicts the optimal α (the height of the surface) for relatively large values of $\tilde{k}$. Because I do not allow $\alpha > 1$, there are plateaus in the surface at one (where $\alpha = 1$ provides more utility than any smaller α). For smaller values of $\tilde{k}$, when $\tilde{b}_2 = 1/6$, the optimal α is one over such a large range of γ that it is harder to see the relationship between the degree of spillovers and the optimal degree of intra-firm competition. The next figures show the same relationship for a larger value of $\tilde{b}_2$ and smaller values of $\tilde{k}$. The relationship looks quite similar.

One point that can be seen a little more clearly in Figure 2 than in Figure 1 is that once the combination of $\tilde{b}_2$ and $\tilde{k}$ is large enough that zero profit sharing is never optimal, the relationship between α and γ quickly becomes very flat, even for γ in the middle range. This effect is even more obvious in the Figure 3 where the products are even closer substitutes.

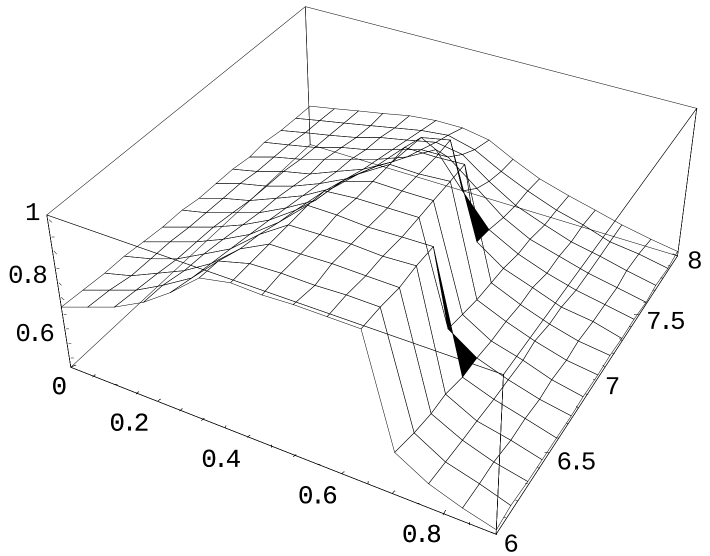

Figure 1: $\tilde{b}_2 = 1/6$; (x, y, z) axes=(γ, $\tilde{k}$, α)

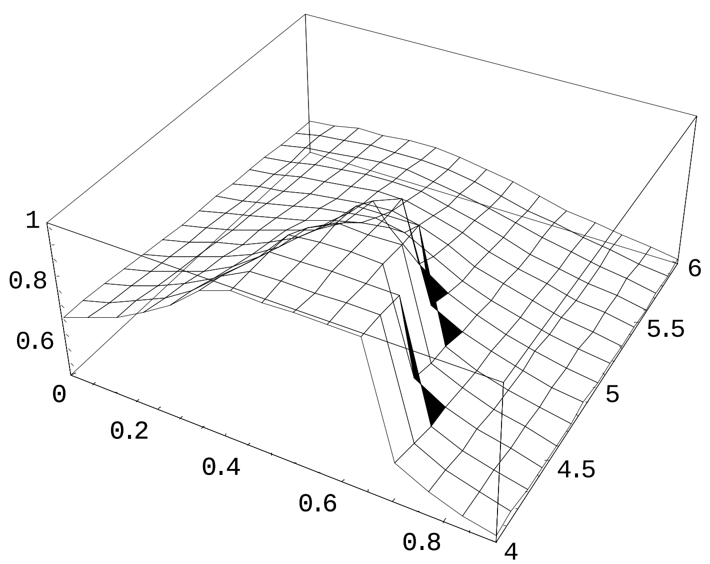

Figure 2: $\tilde{b}_2 = 1/5$; (x, y, z) axes=(γ, $\tilde{k}$, α)

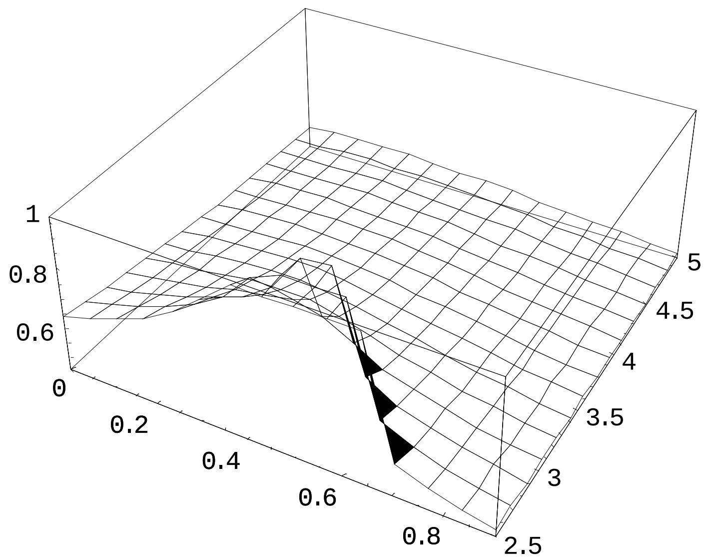

Figure 3: $\tilde{b}_2 = 1/4$; (x, y, z) axes=(γ, $\tilde{k}$, α)

Not surprisingly, as the products become closer substitutes ($\tilde{b}_2$ becomes larger) the more important it is to reduce the incentive for intra-firm price competition. Similarly, as the cost of effort increases the level of product improving effort falls. When this effort level is low, the impact of distorting effort incentives is also low. Even when α is chosen to maximize effort incentives, there will still be very little effort if the cost is high enough. As a result, it is not worth inducing much price competition to get a very small improvement in effort incentives (this is due to the linear impact of effort on demand). The first three figures provide ample evidence of this second point. The following figures demonstrate the effect of the products becoming closer substitutes.

The relationship depicted in Figure 4 is very similar to the relationship for other values of $\tilde{k}$. The main difference is only that the point at which there is the steep decline in α occurs for larger values of $\tilde{b}_2$ the smaller is $\tilde{k}$. Because of the very sharp decline in α for intermediate values of γ, however, it is difficult to get a sense of the effect of $\tilde{b}_2$ on α for smaller spillovers from Figure 4. To get a better idea of this relationship, the next figure restricts the domain of spillovers to $\gamma \leq .3$.

While the comparative statics results about the effects of the cross-elasticity of the products and the magnitude of effort costs are not surprising, it is important to realize that even when the products are quite close substitutes, a firm owning both of them may still want to induce as much price competition between them as would occur if they were in separate firms. For example, when k is as small as one-fourth the size of b_1 there will be no profit sharing for some values of γ (between

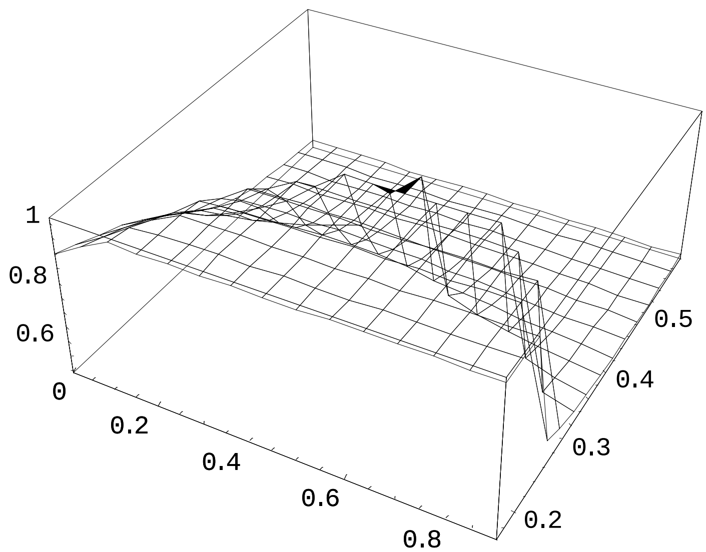

Figure 4: $\tilde{k} = 1$; (x, y, z) axes=(γ, $\tilde{b}_2$, α)

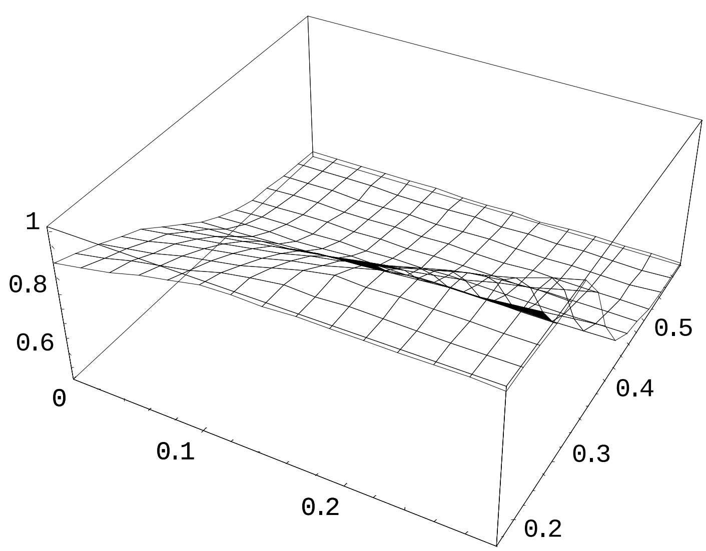

Figure 5: $\tilde{k} = 1$; (x, y, z) axes=(γ, $\tilde{b}_2$, α)

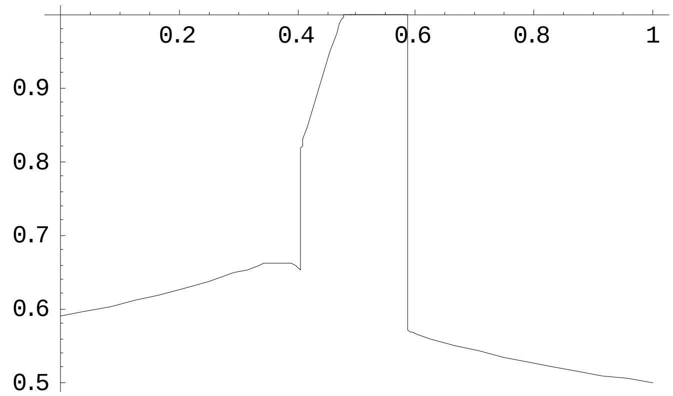

Figure 6: $\tilde{k} = 1/4, \tilde{b}_2 = 3/5$; (x, y) axes $= (\gamma, \alpha)$

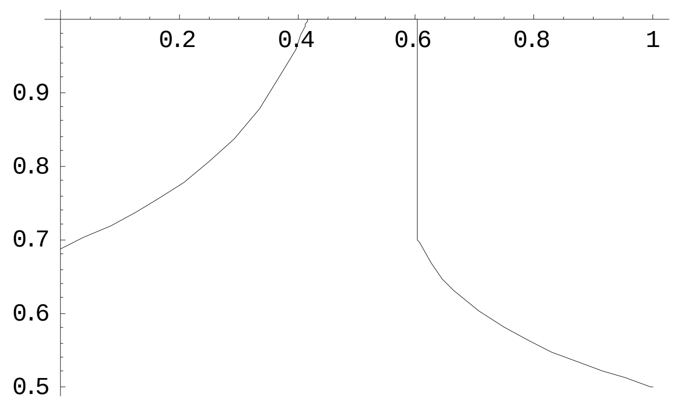

Figure 7: $\tilde{k} = 7, \tilde{b}_2 = 1/6$; (x, y) axes $= (\gamma, \alpha)$

about .48 and .58) when the cross price derivative is as large as 60 percent the size of the magnitude of the own price derivative.

On the other hand, if the products are not as close substitutes, say the cross price derivative is only one-sixth the magnitude of the own price derivative, then there will be no profit sharing for some values of γ (between about .42 and .60) even when the effort cost parameter is seven times the size of the magnitude of the own price derivative.

Of greater relevance is the fact that when the relative size of the cross price derivative is this small, and the effort cost parameter is equal in magnitude to the own price derivative, each manager will share less than 11 percent of the profit from her product even when there are no spillovers. For these parameters, there will be no profit sharing whenever γ is greater than .1 and less than

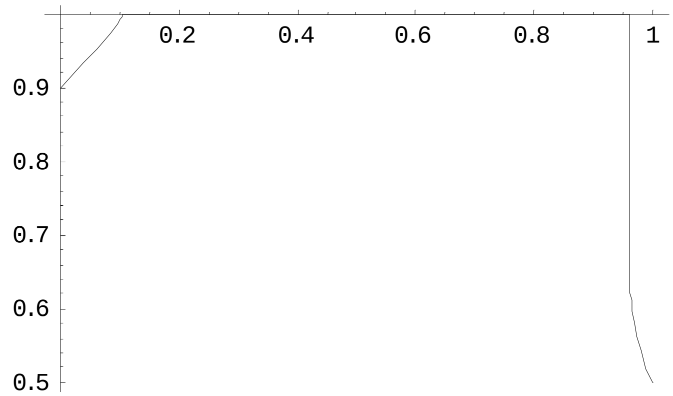

Figure 8: $\tilde{k} = 1, \tilde{b}_2 = 1/6$; (x, y) axes $= (\gamma, \alpha)$

.95.

This value of one-sixth for the cross price derivative has not been picked at random, but was chosen based on standard antitrust market definition practices. According to the U.S. Department of Justice and the Federal Trade Commission's Horizontal Merger Guidelines (1997), the two closest substitutes will be a product market by themselves if a hypothetical monopolist would find it profitable to impose a small but significant price increase. This price increase is normally taken to be five or ten percent. As the following Lemma shows, so long as the cross price derivative is at least one-sixth (and marginal costs are not too great), this method of product market definition will always find that a merger between these two products is a merger to monopoly.

Lemma 1. *With a demand curve defined by (12), a monopolist of the two products, when product quality is taken as given (and $e^i \geq c$), will find it optimal (not just profitable) to impose a quality adjusted price increase of ten percent over the duopoly outcome whenever $\tilde{b}_2 \geq 1/6$.*

This result, in fact, gives a stronger condition for the degree of substitutability than will typically be required for antitrust authorities to determine that these products constitute a market in and of themselves. First, the price increase used is ten percent, while the five percent standard is often used by the antitrust agencies. Second, the Lemma shows that such a price increase will be *optimal* whenever $\tilde{b}_2 \geq 1/6$, while the standard only requires that this price increase be *profitable.* For smaller $\tilde{b}_2$, a ten percent price increase could be profitable even if a smaller price increase is optimal. Third, this demand curve takes as given the prices of other products, but since rivals will typically increase price when this firm does (prices are strategic complements), this demand curve

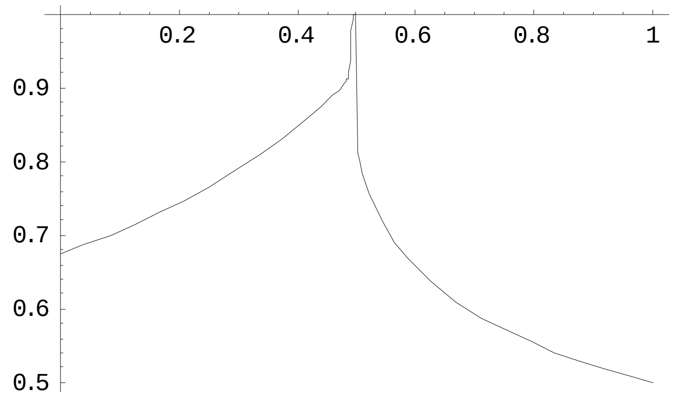

Figure 9: $\tilde{k} = 7.446, \tilde{b}_2 = 1/6$; (x, y) axes $= (\gamma, \alpha)$

underestimates the profit gain from a price increase. So, when demand is described by (12), $e^i \geq c$, and $\tilde{b}_2 = 1/6$, if inducing effort were not an issue, a merger of these two products would almost certainly be deemed anti-competitive. When managerial moral hazard is important, however, the simulation results reveal that there will be substantial intra-firm competition even if there are no spillovers. And, if there are even small effort spillovers, such a merger might not reduce the degree of price competition between the two products at all provided the cost of effort is not too high. Thus, the question of whether or not profit sharing must be limited to motivate managers can be of critical importance in analyzing the competitive effects of some mergers.

The condition that $e^i \geq c$ warrants some further discussion. Note that this will always hold whenever marginal cost is sufficiently small, e.g., in the software example discussed in the introduction. It also holds whenever the cost of effort is sufficiently small. Of course, if the condition does not hold and the market is very small (so that demand is very small even with marginal cost pricing), then the necessary magnitude of the cross derivative for a monopolist to optimally choose a ten percent price increase will be substantially larger.

As part (D) of the summary of the simulation results suggests, there are circumstances where estimating the value of the parameters accurately can be critical to predicting the optimal degree of intra-firm competition. The reason for this is that, in some cases, the reduced form utility function can have local maxima that are either very similar in value or very similar in value to the value at $\alpha = 1$. This is less pronounced for smaller values of $\tilde{b}_2$ than for larger values as the next two figures demonstrate.

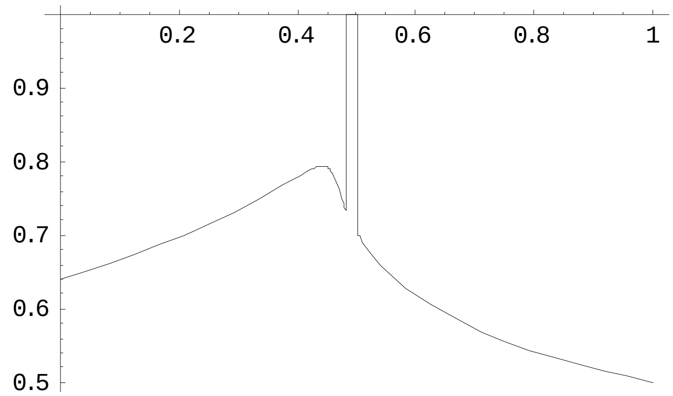

Figure 10: $\tilde{k} = 1.45825, \tilde{b}_2 = 1/3$; (x, y) axes $= (\gamma, \alpha)$

Figure 9 shows that when γ is near one-half a small shift in the size of the effort spillovers (for these parameter values) can change the optimal α by anywhere from .1 to .2. Of course, since the value of this critical γ will change with $\tilde{k}$ and $\tilde{b}_2$, a small change in either of these parameters can have dramatic effects on the optimal α as well. This effect is even more pronounced for larger $\tilde{b}_2$, as the next figure demonstrates. It also gives an example of where the optimal α is not always related to γ in the manner described in part (A) of the summary of the simulation results.

What is going on in Figure 10 is that for γ between around .45 and .5, the utility at the interior local maximum (where there is less price competition and less efficient effort incentives) is very close to the utility at $\alpha = 1$ (where there is much more price competition but more efficient effort incentives). For α in between the utility is much less. So, while the utility of the managers is not sensitive to small changes in parameters in this region, the way they maximize their utility is. Because the welfare implications of the outcome is also very sensitive to this choice of α, in these regions getting accurate estimates of the parameters is especially important. Of course, as the prior figures (and many other that are not shown) demonstrate, situations like these (especially the one in Figure 10) are not common.

3.2 Price Effects

This section looks at the effect on quality adjusted prices of a potential merger between two products in the linear demand model. Quality adjusted price is defined as price minus the increase in product quality that occurs as a result of managerial effort. That is, the quality adjusted price for product

i is: $P^i = p^i - e^i - I\gamma e^{-i}$ where I is an indicator function that takes the value one if the two products are in one firm and zero otherwise. Computing the pre-merger prices is straightforward since the pre-merger prices and efforts are given by the reduced form equations for these variables with $\alpha = 1$ (no profit sharing) and $\gamma = 0$ (no spillovers).[10] Computing the post-merger prices and efforts must be done numerically because finding the optimal α after the merger can only be done numerically. Fortunately, as was the case for finding the optimal α, it turns out that the direction of the post-merger quality adjusted price change is independent of a, b_1, and c. Thus, to determine whether a merger will cause an increase or a decrease in quality adjusted prices, I only need to know the magnitude of the cross price derivative and the effort cost parameter (in units of the own price derivative, b_1) and the magnitude of the effort spillovers. It should be noted, however, that the magnitude of any price effect will depend on these other parameters. In the following table, I give the minimum degree of spillovers necessary for a merger to result in a decrease in quality adjusted price ($\Delta P^i < 0$) for various parameter values. Of course, as the above figures suggest, there is also a maximum value since for very large degrees of spillovers the optimal degree of profit sharing increases dramatically. This maximum is not shown, but for all entries in the table it is greater than $\gamma = .6$ (and is larger where the minimum value, given in the table, is smaller).

Table 1[11]

Minimum value of γ for which $\Delta P^i < 0$

	$\tilde{k} = \frac{1}{4}$	$\tilde{k} = \frac{1}{2}$	$\tilde{k} = \frac{3}{4}$	$\tilde{k} = 1$	$\tilde{k} = \frac{5}{4}$	$\tilde{k} = \frac{3}{2}$	$\tilde{k} = 2$	$\tilde{k} = \frac{5}{2}$	$\tilde{k} = 4$
$\tilde{b}_2 = \frac{1}{6}$	.0454	.0516	.0581	.0648	.0718	.0791	.0944	.1109	.1681
$\tilde{b}_2 = \frac{1}{5}$	.0558	.0652	.0752	.0857	.0968	.1085	.1338	.1622	.2763
$\tilde{b}_2 = \frac{1}{4}$	.0728	.0889	.1064	.1256	.1465	.1696	.2241	.2986	DNE
$\tilde{b}_2 = \frac{1}{3}$	.1061	.1411	.1831	.2355	.3086	DNE	DNE	DNE	DNE

In Table 1, DNE (which stands for "does not exist") means that there is no value of γ for which the merger results in a decrease in the quality adjusted price. As one would expect from the above results, reductions in the cost of quality improving effort and the degree of substitutability decrease the minimum degree of spillovers necessary for the merger to result in a decrease in quality adjusted prices. When $\tilde{b}_2 = 1/6$, despite the fact that standard antitrust analysis would often find

[10]The reduced form price equation is (14). The reduced form effort equation is in the appendix.

[11]The values in this table were generated by numerically solving for the value of γ for which $\Delta P^i = 0$ using Newton's Method. This method cannot be used to determine the maximum value of γ for which $\Delta P^i = 0$ since at this maximum there is a discrete jump from $\Delta P^i < 0$ to $\Delta P^i > 0$.

this merger to be a merger to monopoly, even quite small spillovers will result in a decrease in quality adjusted prices provided the cost of effort is not too great relative to the magnitude of the own price derivative. When intra-firm competition may be necessary to motivate managerial effort, standard merger analysis may be far too strict. Of course, whether or not this is the case (for any given degree of product substitutability) will depend upon estimates of the degree of effort spillovers and the cost of quality improving effort. In order to accurately assess the competitive impact of a merger (where managers must be given profit based incentives to induce product specific, quality enhancing effort), one must estimate not only the demand parameters and the synergies from the combination but also the innovation cost parameters. Of course, if estimating the demand parameters shows that the products are very close substitutes, then (as Table 1 indicates) it is very unlikely that the merger will result in a quality adjusted price decrease no matter what the value of the other parameters.

3.3 Moral Hazard Effects

In the last section, I showed how spillovers affect whether a merger results in an increase or decrease in quality adjusted price. Of course, positive spillover effects are often cited as a reason why an otherwise anti-competitive merger might be pro-competitive. In this section, however, I show that the effects of spillovers in this model are, in fact, substantially different than they are in a similar model without managerial moral hazard. To show this, I determine the effect of a merger on the quality adjusted price when managerial effort is observable and contractible as follows. First, observe that in such a model it is optimal for there to be complete profit sharing; neither manager has an incentive to steal business from the other. Post-merger prices will be set at the fully collusive level, that is, prices are given by (14) with $\alpha = 1/2$. Given those equations for prices, the optimal effort level is given by the e^i that maximizes $u^i(1/2) + u^{-i}(1/2)$. Setting this first order condition equal to zero, imposing symmetry, and solving for e^i gives the following:

$$e^{i,NMH} = \frac{(a - (1 - \tilde{b}_2)b_1 c)(1 + \gamma)}{4k - (1 + \gamma)^2 (1 - \tilde{b}_2) b_1} \tag{15}$$

The formula for post-merger quality adjusted prices then is just $p^{i,NMH} - (1+\gamma)e^{i,NMH}$ where $e^{i,NMH}$ is given by (15) and $p^{i,NMH}$ is given by (14) with $\alpha = 1/2$ and $e^{i,NMH}$ as above. Pre-merger quality adjusted prices are unaffected by the observability of effort since when each product

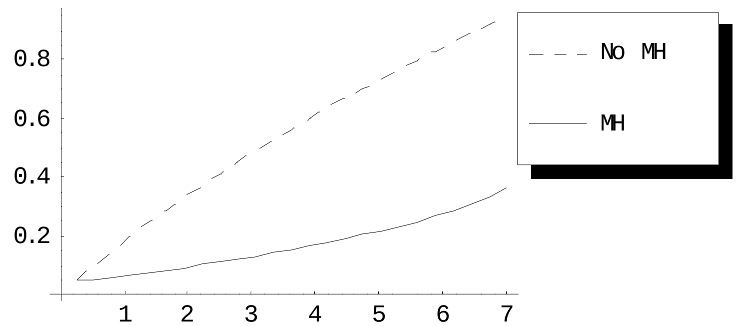

Figure 11: $\tilde{b}_2 = 1/6$; (x, y) axes $= (\tilde{k}, \gamma)$

is owned separately there is no one with whom to contract.

Since both effort and price can be explicitly solved for when there is no moral hazard, there is an explicit formula (given in the appendix) for the quality adjusted price change as a result of a merger in the no moral hazard case. Of course, since determining the effect of managerial moral hazard requires a comparison of the merger induced, quality adjusted price change in the moral hazard case (which must be found numerically, as discussed in the prior section) with the analogous change in the no moral hazard case, the effect can only be found numerically.

It is easy to show, however, that in the no moral hazard case the merger induced change in quality adjusted price is monotonically decreasing in the magnitude of the spillovers (see appendix). This is not surprising given that larger spillovers increase the benefits of quality improving effort when there is a merger but have no effect otherwise. Since it is not optimal to increase price one for one with an improvement in quality, greater improvements in quality (as a result of the merger) result in smaller increases or greater reductions in the quality adjusted price.

In the following figure, I compare the minimum magnitude of effort spillovers necessary for a merger to result in a decrease in quality adjusted prices as a function of the effort cost parameter in the moral hazard and no moral hazard cases.

While the figure considers the case of $\tilde{b}_2 = 1/6$, the picture is qualitatively similar for larger values of $\tilde{b}_2$. The main differences are that both lines are shifted up and, as is clear from Table 1, they end at $\tilde{k} < 7$. What Figure 11 makes clear is that there is a substantial region of spillover magnitudes and effort costs where a merger will result in a decrease in quality adjusted price if product improving effort is non-contractible but results in a price increase if this effort level is contractible. Since we also know (though it is not depicted in the figure) that for very high levels of spillovers, quality adjusted price can increase as a result of a merger when it would decrease for intermediate spillover levels, the reverse is also true. The region above the dashed line in Figure 11 in the right part of the figure (approximately $\tilde{k} > 5$) is where a merger with contractible effort results in a decrease in quality adjusted price while the same merger where effort is not contractible results in an increase in quality adjusted price. Thus, to accurately assess the impact of any given merger on consumer welfare one must not ignore the contractibility of product improving effort.

4 Conclusion

This paper analyzes the behavior of a multiproduct firm when the assumption that the managers responsible for each product are perfect agents for the shareholders is relaxed. In particular, it shows that when the managers must exert unobservable effort to improve product quality, it is optimal for each manager to retain more than half of the profit her product generates, inducing incentives for intra-firm price competition. Thus, a multiproduct firm will not fully exercise its market power. Moreover, in a linear demand model, when product improving effort has positive spillovers (it improves the quality of the other product as well as the product of the manager that exerts the effort) the degree of profit sharing can be small enough (possibly even zero) that quality adjusted price is lower when both products are under common ownership. This is most likely to occur when the external benefit from quality improving effort is around half the private benefit. In such cases, moral hazard improves social welfare and can make a merger that would have otherwise reduced consumer welfare beneficial for consumers. If spillovers are very large, then there is very little gain in terms of improving effort incentives from limiting profit sharing. As a result, there will be close to complete profit sharing. In these cases, moral hazard reduces the social benefit of common ownership and can turn a merger that would have otherwise benefited consumers (absent moral hazard) into one that harms them.

While many, though not all, of the results in the paper were derived assuming linear demand, I suspect that qualitatively similar results would hold for many other demand specifications as well. Of course, establishing this conjecture would require further simulations using different demand curves.

While the model in the paper assumed that managerial effort improved product quality, similar conclusions might sometimes apply when managerial effort is useful for cost reduction. Because costs are more readily observable, it will often be possible to condition the profit sharing arrangement on costs and revenues separately. Forcing contracts, where one division gets all the profits unless costs are at or below the first best level, may be infeasible, however, due to the possibility of sabotage. As a result, the only feasible contracts may be contracts where each division gets a given share of its revenue and pays a (possible different) share of its costs. By giving the manager a large fraction of the benefits from cost reduction but a smaller fraction of her own revenue, the firm can decouple cost reduction incentives from price competition incentives. If, however, a manager can reduce costs not only by exerting effort but also by reducing quality, there will be an efficiency loss associated with this decoupling. Thus, moral hazard will still induce less revenue sharing than would be otherwise optimal, though probably to a lesser extent than when the unobservable effort affects quality rather than costs.

Appendix

Proof of Proposition 1 The argument in the text proves that so long as $e'(1/2) > 0$ the result holds. To show that $e'(1/2) > 0$, I need to show that the marginal benefit of effort is increasing in α at $\alpha = 1/2$. Differentiating the left hand side of (6) with respect to α, imposing symmetry, and evaluating it at $\alpha = 1/2$ gives the following:

$$(p-c)(q_3 - q_4 + 4q_2 p_3) + \frac{1}{2}p_1(q_3 + q_4 + (p-c)(q_{31} + q_{32} + q_{41} + q_{42})) \tag{16}$$

When demand is given by $q(p^i - (e^i + \gamma e^{-i}), p^{-i} - (e^{-i} + \gamma e^i))$, this can be written as:

$$(p-c)((1-\gamma)(q_2 - q_1) + 4q_2 p_3) - \frac{1}{2}(1+\gamma)p_1(q_1 + q_2 + (p-c)(q_{11} + 2q_{12} + q_{22})) \tag{17}$$

Now, I need to get formulas for p_1 and p_3 to substitute into (17). By differentiating (4) with respect to α and imposing symmetry, one can solve for p_1, getting the following:

$$p_1 = \frac{(p-c)q_2}{\alpha\{2\alpha q_1 + q_2 + (p-c)(\alpha q_{11} + (1-\alpha)q_{22} + q_{12})\}} \tag{18}$$

At $\alpha = 1/2$ this becomes:

$$p_1 = \frac{4(p-c)q_2}{2(q_1 + q_2) + (p-c)(q_{11} + q_{22} + 2q_{12})} \tag{19}$$

Now, I differentiate (4) with respect to q^{-1}, impose symmetry, and solve for p_3. At $\alpha = 1/2$ this gives:

$$p_3 = \frac{\begin{array}{c}(q_2 + (p-c)q_{12})(q_3 + (p-c)(q_{13} + q_{24})) \\ -(q_1 + \frac{1}{2}(p-c)(q_{11} + q_{22}))(q_4 + (p-c)(q_{14} + q_{23}))\end{array}}{2((q_1 + \frac{1}{2}(p-c)(q_{11} + q_{22}))^2 - (q_2 + (p-c)q_{12})^2)} \tag{20}$$

When demand only depends on quality adjusted price, this becomes:

$$p_3 = \frac{\begin{array}{c}(q_1 + \frac{1}{2}(p-c)(q_{11} + q_{22}))(\gamma q_1 + q_2 + (p-c)(\gamma q_{11} + \gamma q_{22} + 2q_{12})) \\ -(q_2 + (p-c)q_{12})(q_1 + \gamma q_2 + (p-c)(q_{11} + q_{22} + 2\gamma q_{12}))\end{array}}{2((q_1 + \frac{1}{2}(p-c)(q_{11} + q_{22}))^2 - (q_2 + (p-c)q_{12})^2)} \tag{21}$$

Using these formulas for p_1 and p_3 in (17) and simplifying gives the following:

$$\frac{(p-c)(1-\gamma)(2q_1(q_1-q_2)+(p-c)(q_1+q_2)(q_{11}+q_{22}-q_{12}))}{-(2(q_1-q_2)+(p-c)(q_{11}+q_{22}-q_{12}))} \tag{22}$$

This is positive whenever $q_{11}+q_{22}-q_{12}<0$. Q.E.D.

Omitted Linear Demand Equations

Effort Equations Substituting in for prices using (14), differentiating (**??**) with respect to e^i when demand is given by (12) and then solving the resulting first order conditions for e^i gives the following:

$$e^i = \frac{\begin{array}{c} -\alpha(a-b_1(1-\tilde{b}_2)c)\{-4\alpha^2(\gamma+\alpha(1-\gamma))-\tilde{b}_2^3(1-\alpha)(1+\gamma) \\ +2\alpha\tilde{b}_2[2\alpha(1-\alpha)(1-\gamma)+\gamma]+\tilde{b}_2^2[1+4\alpha^2+\gamma-\alpha(3+\gamma)]\} \end{array}}{\begin{array}{c} b_1[2\tilde{k}(\tilde{b}_2-2\alpha)^2(\tilde{b}_2+2\alpha)+\alpha(1-\tilde{b}_2)(1+\gamma) \\ \{-4\alpha^2(\gamma+\alpha(1-\gamma))-\tilde{b}_2^3(1-\alpha)(1+\gamma) \\ +2\alpha\tilde{b}_2[2\alpha(1-\alpha)(1-\gamma)+\gamma]+\tilde{b}_2^2[1+4\alpha^2+\gamma-\alpha(3+\gamma)]\}] \end{array}} \tag{23}$$

Profit Sharing FOC Substituting in for prices using (14) and effort using (23), differentiating (**??**) with respect to α when demand is given by (12) gives the following first order condition for α:

$$\frac{\begin{array}{c} 8\tilde{k}^2(a-b_1(1-\tilde{b}_2)c)^2(\tilde{b}_2-2\alpha)^2\{\tilde{k}\tilde{b}_2^2(2\alpha-1)(\tilde{b}_2-2\alpha)(\tilde{b}_2+2\alpha) \\ +(1+\tilde{b}_2)\alpha^3[16\alpha^4(1-\gamma)(1-\alpha(1-\gamma))+8\tilde{b}_2\alpha^3(1-\gamma)(1-2\alpha(1-\alpha)(1-\gamma) \\ -\tilde{b}_2^5\alpha(1-\gamma^2)-\tilde{b}_2^4(1-\gamma)(2(2+\gamma)+4\alpha^2(4+\gamma)-\alpha(17+7\gamma)) \\ +4\tilde{b}_2^2\alpha(2-\gamma-4\alpha^3\gamma(1-\gamma)-\gamma^2+4\alpha^2(2-\gamma-\gamma^2)-\alpha(9-5\gamma-4\gamma^2)) \\ +2\tilde{b}_2^3(1+\gamma-2\gamma^2+\alpha^2(10+16\gamma-26\gamma^2) \\ -4\alpha^3(1+3\gamma-4\gamma^2)-\alpha(5+7\gamma-12\gamma^2))]\} \end{array}}{\begin{array}{c} b_1[2\tilde{k}(\tilde{b}_2-2\alpha)^2(\tilde{b}_2+2\alpha)+\alpha(1-\tilde{b}_2)(1+\gamma) \\ \{-4\alpha^2(\gamma+\alpha(1-\gamma))-\tilde{b}_2^3(1-\alpha)(1+\gamma)+2\alpha\tilde{b}_2[2\alpha(1-\alpha)(1-\gamma)+\gamma] \\ +\tilde{b}_2^2[1+4\alpha^2+\gamma-\alpha(3+\gamma)]\}]^3 \end{array}} \tag{24}$$

No Moral Hazard Case: Merger Quality Adjusted Price Change

$$\Delta P^{i,NMH} = \frac{\tilde{k}(a-(1-\tilde{b}_2)b_1c)}{(1-\tilde{b}_2)b_1} \frac{\{2\tilde{k}\tilde{b}_2(\tilde{b}_2^2-4)+(1-\tilde{b}_2)[4\gamma(2+\gamma)+\tilde{b}_2^2(1-2\gamma-\gamma^2)]\}}{[2-2\tilde{b}_2-\tilde{b}_2^2+\tilde{b}_2^3-\tilde{k}(2-\tilde{b}_2)^2(2+\tilde{b}_2)](4\tilde{k}-(1-\tilde{b}_2)(1+\gamma)^2)} \tag{25}$$

$$\frac{\partial \Delta P^{i,NMH}}{\partial \gamma} = \frac{-4\tilde{k}(a-(1-\tilde{b}_2)b_1c)(1+\gamma)}{b_1(4\tilde{k}-(1-\tilde{b}_2)(1+\gamma)^2)^2} < 0 \tag{26}$$

Proof of Proposition 2 Evaluating (24) at $\alpha = 1$ and $\gamma = 0$, it becomes:

$$\frac{[\tilde{k}^2\tilde{b}_2(a-b_1(1-\tilde{b}_2)c)^2(2-\tilde{b}_2)^2] \{-8-4\tilde{b}_2(1+4\tilde{k})+8\tilde{b}_2^2(1-2\tilde{k})+\tilde{b}_2^3-\tilde{b}_2^4(4+\tilde{b}_2)(1-\tilde{k})\}}{b_1[-2+2\tilde{b}_2+\tilde{b}_2^2-\tilde{b}_2^3+\tilde{k}(2-\tilde{b}_2)^2(2+\tilde{b}_2)]^3} \tag{27}$$

The denominator is increasing in $\tilde{k}$. At $\tilde{k} = 1/4$ it is: $b_1\tilde{b}_2^3[\frac{4+2\tilde{b}_2-3\tilde{b}_2^3}{4}]^3 > 0$, so it is positive for any $\tilde{k} \geq 1/4$. The numerator has the sign of: $-8-4\tilde{b}_2(1+4\tilde{k})+8\tilde{b}_2^2(1-2\tilde{k})+\tilde{b}_2^3-\tilde{b}_2^4(4+\tilde{b}_2)(1-\tilde{k})$. This is decreasing in $\tilde{k}$, so it is largest at $\tilde{k} = 1/4$, where it is:

$$-8-8\tilde{b}_2+4\tilde{b}_2^2+\tilde{b}_2^3-\frac{3\tilde{b}_2^4}{4}(4+\tilde{b}_2) < 0 \tag{28}$$

The left hand side of the first order condition for α is negative whenever $\tilde{k} \geq 1/4$ if $\alpha = 1$ and $\gamma = 0$, so some profit sharing is always optimal when there are no spillovers.

Proof of Lemma 1 From (14) I can write the optimal percentage change in quality adjusted price from a merger, holding quality constant at e, as follows:

$$\frac{p^i(\alpha=1/2)-p^i(\alpha=1)}{p^i(\alpha=1)-e} = \frac{-\tilde{b}_2(a-(1-\tilde{b}_2)b_1(c-e))}{2(1-\tilde{b}_2)(a-b_1(c-e))} \tag{29}$$

Let $\widetilde{e} = e - c$, then the right hand side is:

$$\frac{-\tilde{b}_2(a+(1-\tilde{b}_2)b_1\widetilde{e})}{2(1-\tilde{b}_2)(a+b_1\widetilde{e})} \tag{30}$$

This is decreasing in $\widetilde{e}$. So the largest possible percentage increase in quality adjusted price for any given $\tilde{b}_2$ will occur at $\widetilde{e} = 0$, where the percentage price increase is: $\frac{\tilde{b}_2}{2-2\tilde{b}_2}$. This percentage price increase will be exactly ten percent when $\tilde{b}_2 = 1/6$. Q.E.D.

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