1 Physics Signals in a Muon-Induced Neutrino Beam

1.1 Wrong-Sign Muon Appearance

1.1.1 $\nu_e \to \nu_\mu$

A $\nu_e \to \nu_\mu$ oscillation can be detected by the appearance of a muon opposite in sign to that expected from the beam:

$$\mu^+ \quad \to \quad e^+ \bar{\nu}_e \nu_\mu$$

and we search for

$$\nu N \quad \to \quad \mu^+ X$$

1.1.2 $\nu_e \to \nu_\tau$

A $\nu_e \to \nu_\tau$ gives an identical signal. The only way to distinguish it from $\nu_e \to \nu_\mu$ is to note that the μ obtained from $\nu_\tau N \to \tau X$, $\tau \to \mu \bar{\nu}_\mu \nu_\tau$ will have a softer energy distribution than muons from a ν_μ charged-current interaction. With a good enough detector one can use the $p_\perp$ distribution to separate the two, since the muons from τ decay will generally have more transverse momentum.

1.2 Neutral-to-Charged Current Ratios

We first recall the Llewellyn-Smith relation. It gives the cross-section ratio of neutral Z^o exchange to W exchange in neutrino deep-inelastic scattering.

$$\begin{aligned} R_\nu &= \frac{\sigma(\nu, \mathrm{NC})}{\sigma(\nu, \mathrm{CC})} \\ &= \frac{1}{2} - \sin^2\theta_W + \frac{5}{9}(1+r)\sin^4\theta_W \end{aligned}$$

where r absorbs the QPM dependence into $r = \sigma(\bar{\nu}, \mathrm{CC})/\sigma(\nu, \mathrm{CC})$.

The neutral-to-charged current ratio is experimentally measured from the ratio between events without a visible muon (NC) and those with a detected muon (CC). For determinations of the weak mixing angle a Monte Carlo is used to obtain the physically interesting R_ν from the observed ratio. This measurement can be used for detection of oscillations, by *setting* the weak mixing angle to an externally measured value and determining the deviation of R_ν from the predicted value, preferably as a function of visible energy. An oscillation from $\nu_e \to \nu_\mu$ manifests itself in an apparent decrease in the number of events with no muon and a corresponding increase in the apparent charged current rate.

1.2.1 $\nu_e \to \nu_\mu$

Consider a beam in which a fraction α of the ν_e have oscillated into ν_μ and presume that all ν_e appear as neutral currents. Then the ratio in R_ν is given by

$$\begin{aligned} \Delta R_\nu &= R_\nu - R_\nu(oscillated) \\ &= \alpha \frac{\nu_e}{\nu_\mu} \end{aligned}$$

The ratio ν_e/ν_μ gives the ratio of interactions and must be computed from the properties of the muon beam and a detector simulation but is unity up to small corrections.

1.2.2 $\nu_e \to \nu_\tau$

In the case of $\nu_e \to \nu_\tau$ only 16% of the ν_τ interactions appear as charged currents, since $\tau \to e,$ hadrons have no outgoing muon. Therefore:

$$\begin{aligned} \Delta R_\nu &= R_\nu - R_\nu(oscillated) \\ &= \mathrm{BR}(\tau \to \mu) \times \alpha \frac{\nu_e}{\nu_\mu} \end{aligned}$$

Signals

Physics Channel	Disappearance Tests	Appearance Tests	$\tau \to \mu$	$\tau \to e$	$\tau \to$ hadrons
$\nu_\mu \to \nu_e$	CC Rate/Spectrum	NC/CC			
$\nu_\mu \to \nu_\tau$	CC Rate/Spectrum	NC/CC	NC/CC	NC/CC, η_3	NC/CC
$\bar{\nu}_e \to \bar{\nu}_\tau$	CC Rate/Spectrum	WSM	WSM	NC/CC, η_3	NC/CC
$\bar{\nu}_e \to \bar{\nu}_\mu$	CC Rate/Spectrum	WSM			

Backgrounds

Physics Channel	Disappearance Tests	Appearance Tests	$\tau \to \mu$	$\tau \to e$	$\tau \to$ hadrons
$\nu_\mu \to \nu_e$	Beam ν_e	Beam ν_e			
$\nu_\mu \to \nu_\tau$				Beam ν_e	NC/CC
$\bar{\nu}_e \to \bar{\nu}_\tau$		WSM	WSM	Beam ν_e	Electron Mis-ID
$\bar{\nu}_e \to \bar{\nu}_\mu$		WSM			

Detector Properties

Physics Channel	Disappearance Tests	Appearance Tests	$\tau \to \mu$	$\tau \to e$	$\tau \to$ hadrons
$\nu_\mu \to \nu_e$	Muon ID/$\vec{p}$				
$\nu_\mu \to \nu_\tau$	Muon ID/$\vec{p}$		Muon ID/$\vec{p}$	e ID	Muon ID
$\bar{\nu}_e \to \bar{\nu}_\tau$			Muon ID/$\vec{p}$	e ID	e ID
$\bar{\nu}_e \to \bar{\nu}_\mu$		Muon ID/$\vec{p}$			

Limiting Systematic

Physics Channel	Disappearance Tests	Appearance Tests	$\tau \to \mu$	$\tau \to e$	$\tau \to$ hadrons
$\nu_\mu \to \nu_e$	μ MisRec	e Mis-ID, π^o			
$\nu_\mu \to \nu_\tau$	μ MisRec, π/K		π/K	Beam ν_e	NC
$\bar{\nu}_e \to \bar{\nu}_\tau$			WSM	Beam ν_e	e Mis-ID
$\bar{\nu}_e \to \bar{\nu}_\mu$		μ MisRec			

§1. NC/CC means neutral-to-charged current rate *vs.* hadronic energy

§2. η_3 means electron ID from short EM shower *vs.* long hadronic shower or seeing it in a fine-grained calorimeter

§3. Muon ID/$\vec{p}$ means identifying the charge and momentum of a muon