Isospin Symmetry Breaking Effects in Hadron Masses

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- [C.Allton et al.] Pys.Rev.D76:014504[arXiv:0804.0473]
  "Physical Results from 2+1 Flavor Domain Wall QCD and SU(2) Chiral Perturbation Theory"
- [D. J. Antonio et al.] Phys. Rev. Lett. 100 (2008) 032001 [arXiv:hep-ph/0702042]
  "Neutral kaon mixing from 2+1 flavor domain wall QCD",
- [T. Blum, T. Doi, M. Hayakawa, TI, N. Yamada] ,
  Phys. Rev.D76 (2007) 114508,
  "Determination of light quark masses from the electromagnetic splitting of psedoscalar meson masses computed with two flavors of domain wall fermions"
- [R.Zhou, T.Blum, T.Doi, M.Hayakawa, TI, and N.Yamada] ,
  PoS(LATTICE 2008) 131.
  "Isospin symmetry breaking effects in the pion and nucleon masses"

Full QCD (including dynamical quarks)

- unquenched lattice QCD simulations

$$\text{Prob}[U_\mu] \propto \det \not{D} e^{-S_g},$$

- Quenched simulations

$$\text{quench: } \det \not{D} \to 1$$

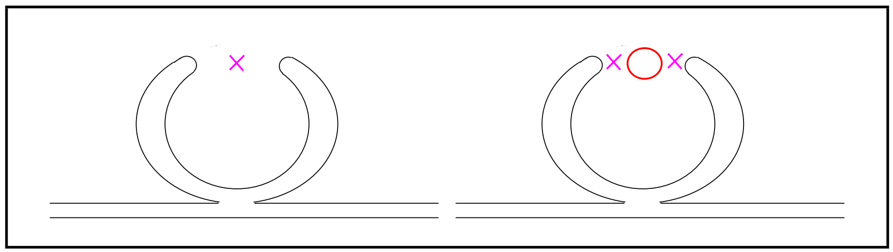
  ignores quark loops (sea quark loop) in QCD vacuum, and only using the external quarks (valence quarks) representing hadrons.

- This approximation causes the quenched pathologies.

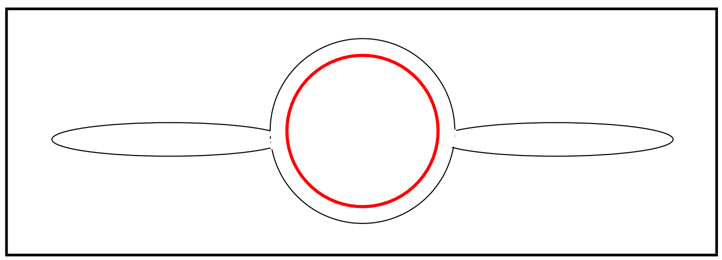
  - Lack of Unitarity.
  - quenched chiral divergences (η' loops):

    $$M_\pi^2 = 2B_0 m_q \left[1 - 2\delta \ln(m_f)\right]$$

    $\delta \propto m_f$ in Full QCD.



  - can't decays.
    $e.g.\ \rho \to \pi\pi$:



- quark mass with less than $\sim \Lambda_{QCD}$ should play a significant role :$N_F = 2 + 1$.

Unitarity violation in Non-singlet scalar meson (a_0)

- Point to point propagator of non-singlet scalar meson, $C_{a_0}(t)$, was found to be negative in quenched QCD, which is a clear signal of the unitarity violation in quenched QCD.

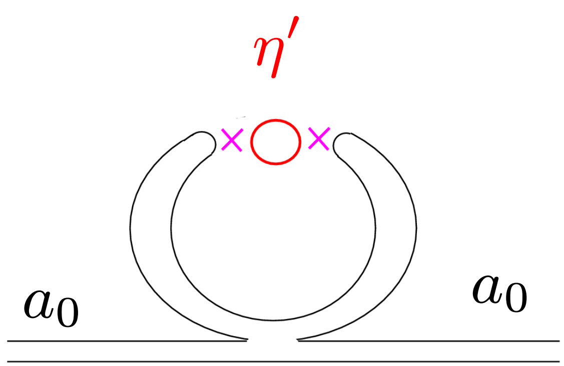
- In the language of mesons (ChPT),

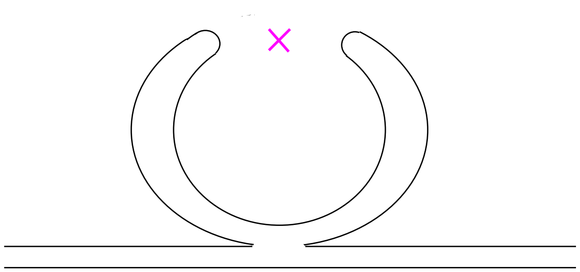
  $$a_0 \longrightarrow \eta' + \pi \longrightarrow a_0$$

  η' has double pole in (partially) quenched QCD.

  This contribution was argued to give a negative contribution (also finite size effect), and predicted using Quenched ChPT in finite volume.

  [Bardeen, Duncan, Eichten, Isgur, Thacker, PRD65 (02) 014509]





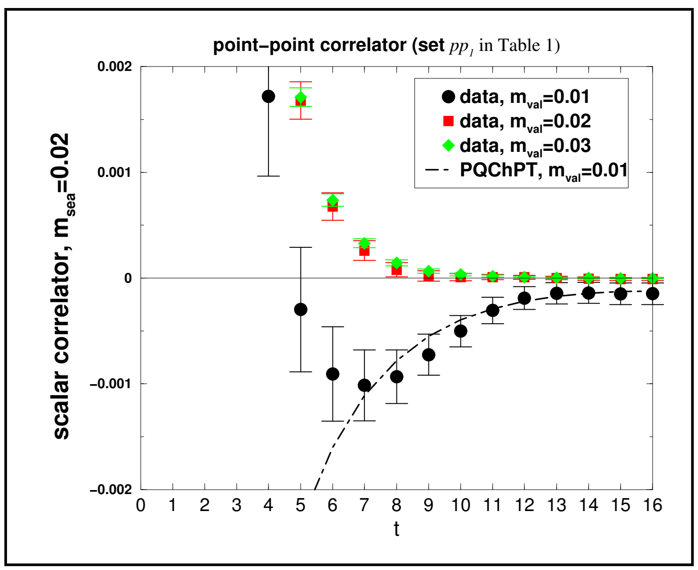
[S. Prelovsek, C. Dawson, T.I. K. Orginos, A. Soni, PRD70 (04) 094503]

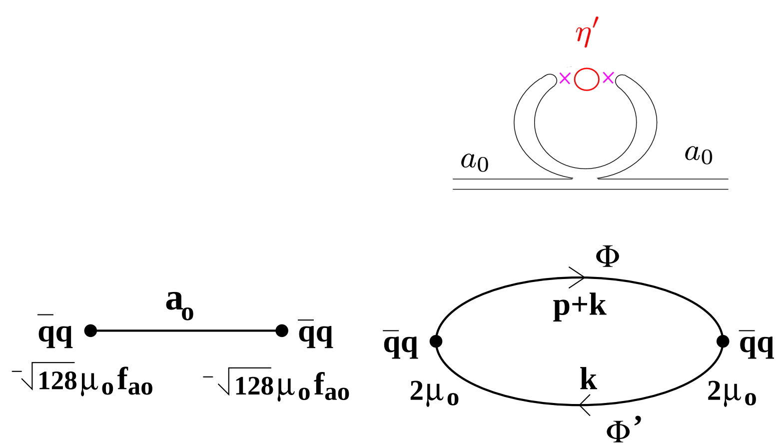
- By fixing m_{sea} and changing m_{val} we confirmed

$$
\begin{aligned}
C_{a_0}(t) = \left\langle a_0^\dagger(t) a_0(0) \right\rangle \quad &< \quad 0 \quad (m_v < m_s) \\
C_{a_0}(t) \quad &> \quad 0 \quad (m_v \geq m_s)
\end{aligned}
$$

- This behaviour could be understood by NLO Partially Quenched ChPT also.

$$
C_{a_0}(t) \to \frac{B_0^2}{2L^3} \frac{e^{-2M_{vs}t}}{M_{vv}^2} \frac{N_F}{2} - \frac{e^{-2M_{vv}t}}{M_{vv}^2} \frac{1}{M_{vv}^2} \left(\frac{M_{vv}^2 + M_{ss}^2}{N_F} + R M_{vv} t \right), \quad R = \frac{M_{ss}^2 - M_{vv}^2}{N_F}
$$





(a) (b)

Dynamical quark effects

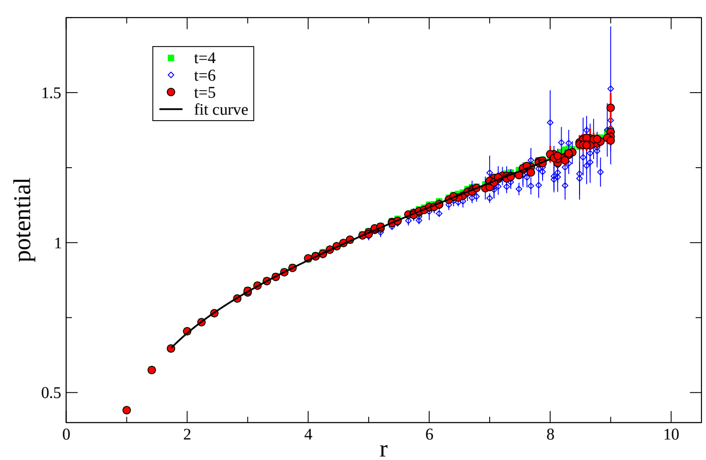
Quenching error (dynamical quark effect) is not a minor issue.

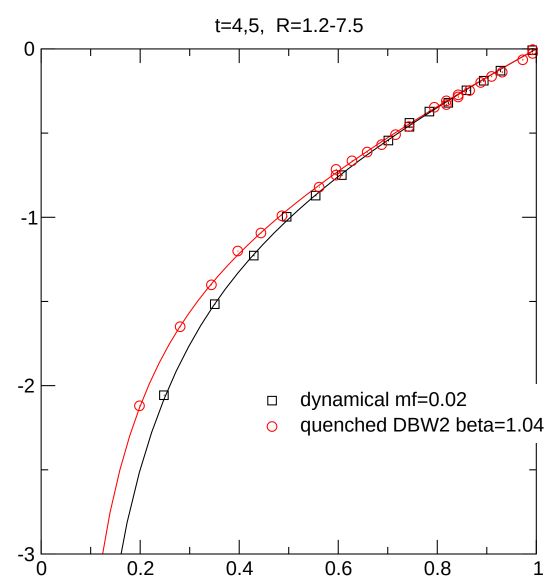
Other quantities very sensitive to dynamical quarks
[M. Golterman, TI, Y. Shamir, PRD74 (05) 114508]

- $I = 0$ $\pi\pi$ scattering length (Nucleon-Nucleon potential ?)

$$\frac{\Delta E_{I=0}}{2M} = -\frac{7\pi}{8f^2ML^3} + \frac{1}{2}B_0(ML)R, \quad R = \frac{1}{N_F}(M_{ss}^2 - M_{vv}^2) + \cdots$$

- Static quark potential $V_{Q\overline{Q}}(r)$ [K. Hashimoto, RBC (04) RBC-UKQCD(07) thesis (08)]
  In shorter distance, $r\Lambda_{QCD} \ll 1$, coupling is stronger for $N_F = 2$:
  $\alpha_S(r; N_F = 0) < \alpha_S(r; N_F = 2)$.
  asymptotic freedom $b_0 = (33 - 2N_F)/2$



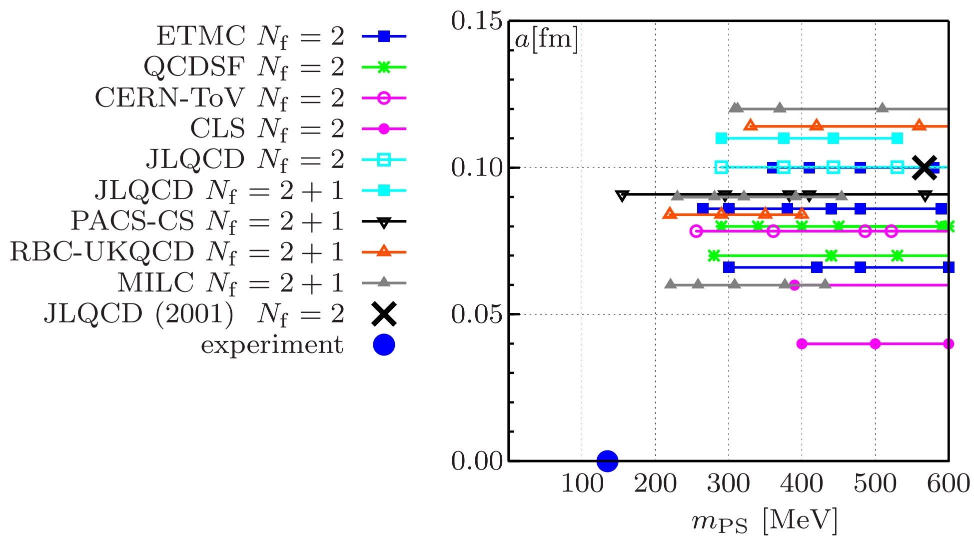


Various Lattice actions

- Improvements in algorithms & faster machine.
- Vacuum polarization effects from the sea up, down, strange quarks, whose masses are lighter than Hadronic scale, $m_i < \Lambda_{\mathrm{QCD}}$, are turned on.
- Have entered Era of Dynamical quark simulations. *Truly the first principle calculation*

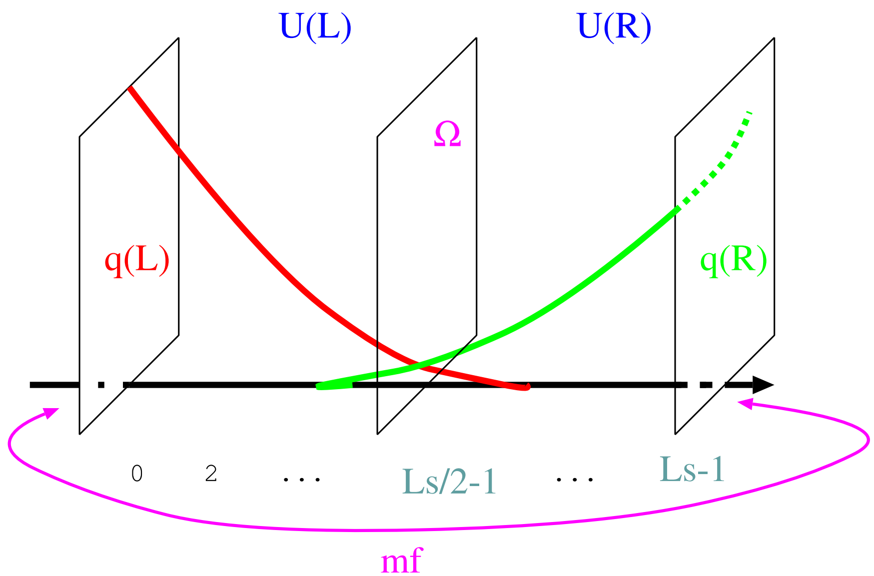
various Lattice quarks

- staggered
  [MILC, LHPC, J-Lab, FNAL...]
- 4D Wilson-types
  [CP-PACS, PACS-CS, BMW, ETM,...]
- DWF [RBC/UKQCD]
  [this talk]
- overlap [JLQCD]



[K.Jansen]

Dynamical Domain Wall Fermions (DWF)



- [Furman & Shamir NPB439 (95) 54]
- [Blum & Soni PRL79 (97) 3595]
- RBC (98-) CP-PACS (99-) quenched DWF spectrum, decay constant, B_K, ϵ'/ϵ

- DWF has exact flavor symmetry and a good chiral symmetry on $a > 0$.

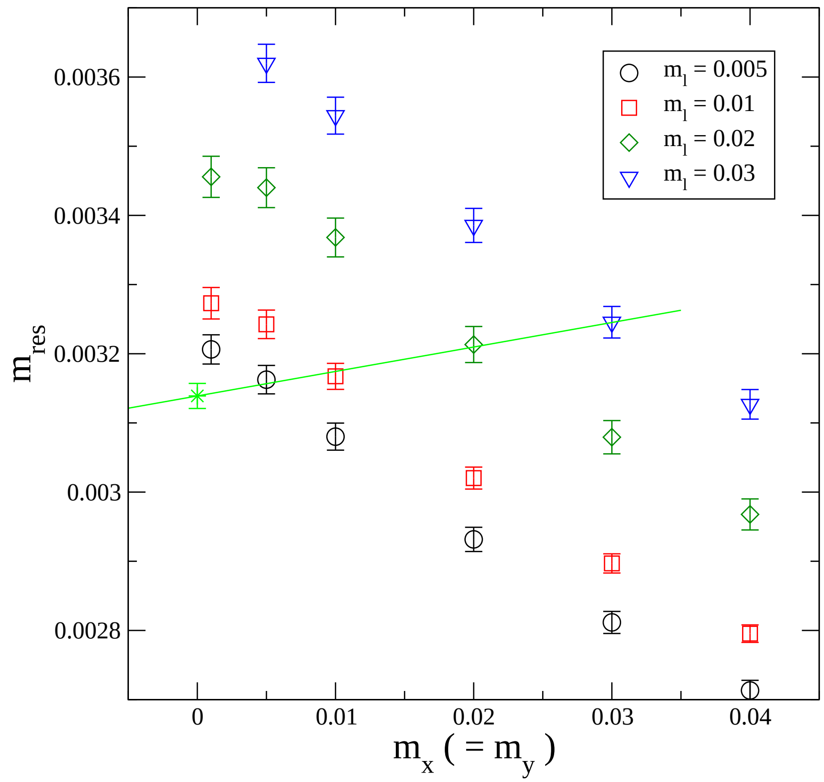
$S_f = \bar{q}\, \not{D} q = \bar{q}_L\, \not{D} q_L + \bar{q}_R\, \not{D} q_R$ chiral symmetry: $q_L \to e^{i\theta_L} q_L, q_R \to e^{i\theta_R} q_R$

- discretization error is small.(lattice spacing, $a > 0$)
  No local operator with dimension five preserving chiral symmetry.
  $\mathcal{O}_5 = F_{\mu\nu}\bar{q}\sigma_{\mu\nu}q, \bar{q}D^2q$ $\mathcal{L}_{lat} = Z\mathcal{L}_{cont.} + (a\Lambda_{\mathrm{QCD}})^2 O_6 + \cdots$
  O(a) error is suppressed. Results on relatively coarse lattice (large a ,smaller computational cost) is much closer to the continuum limit: $(a\Lambda_{QCD})^2 \sim 1\%$

- unphysical operator mixing is prohibited by χ-sym.

- Continuum-like ChPT and renormalization $\Longrightarrow$ Optimal for Hadron matrix elements comp.

m_{res}



- A measure of χ-sym breaking using PS density made of the mid-point quarks.

$$R(t) = \frac{\langle \sum_{\vec{x}} J^a_{5q}(\vec{x},t) P^a(\vec{0},0)\rangle}{\langle \sum_{\vec{x}} P^a(\vec{x},t) P^a(\vec{0},0)\rangle},$$

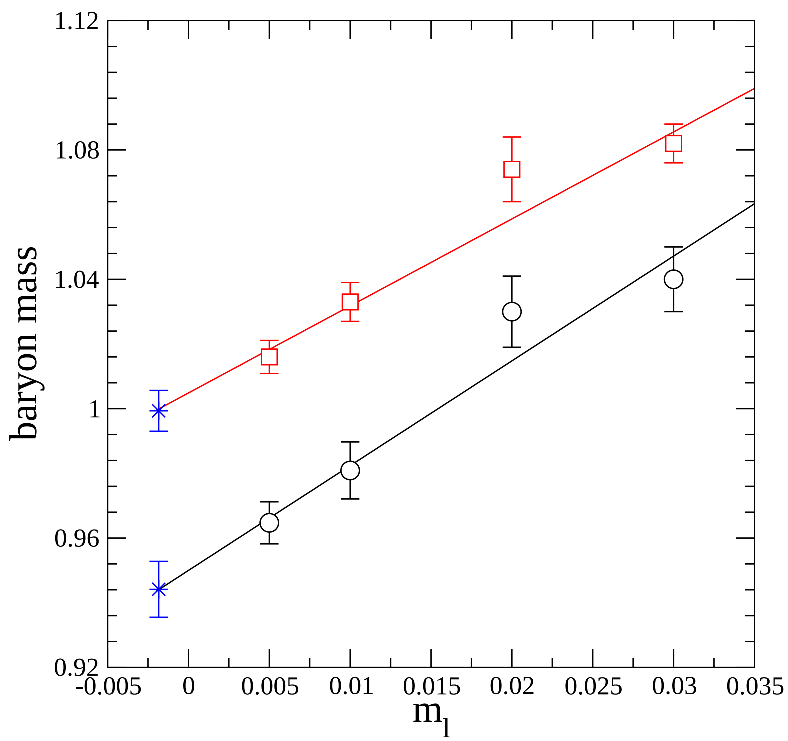
- Roughly two times physical u,d quark mass, $\sim$ 9 MeV.

$$m_{\mathrm{res}} = 0.00315(2).$$

- Universally correct shift of quark mass at NLO ChPT:

$$\tilde{m} = m + m_{\mathrm{res}}$$

Lattice spacing a from Ω mass



- Ω^- mass, 1672 MeV, is used to set the scale rather than m_ρ or r_0.

- At NLO ChPT, there is no log:

$$m_\Omega = m_\Omega^0 + c m_l + \cdots$$

- The single value of sea strange mass in our simulation, $m_h = 0.04$, turns out to be ~ 15 % heavier than the experimental.

- This systematic error is estimated by half of m_l dependence, which is $\sim$ - 1 % smaller than stat. error.

- We could measure the bounds from the reweighting method.

- After solving the coupled equations for Ω, π, K masses,

$$a^{-1} = 1.749(14)\text{GeV}$$

Pion sector

- f_{PS} is calculated from $\langle A_\mu J_5 \rangle$ using wall source. SU(2) PQChPT fit with cut $(m_x + m_y)/2 \le 0.01$ for $m_l = 0.005, 0.01$. χ^2 is degraded for larger cut. m_s dependence is estimated from conv'ed $SU(3)$ ChPT.

- PQChPT Low Energy Constants, $L_i^{(2)}$ and ChPT LEC $l^r_{3,4}$

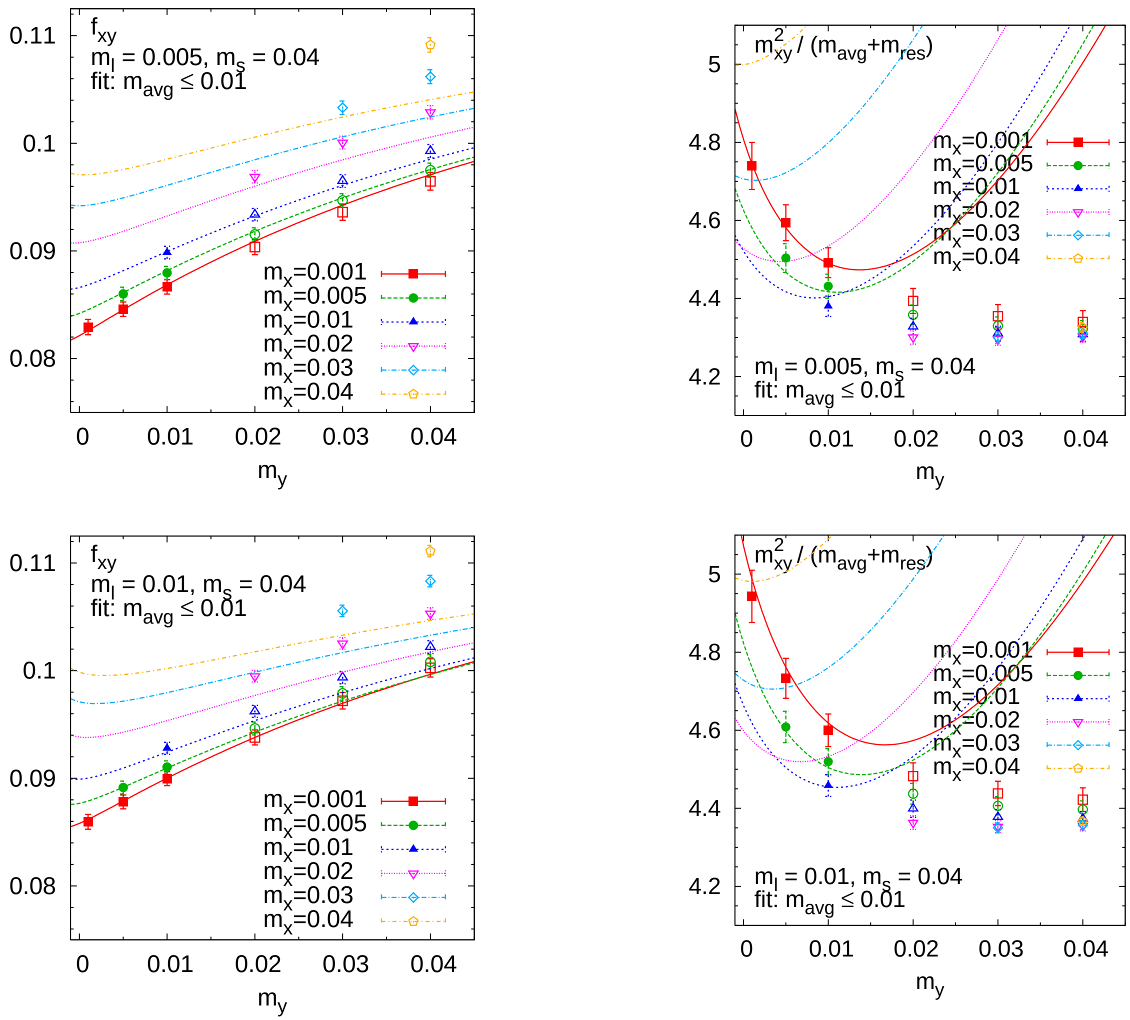
	B	f	$\bar{l}_3$	$\bar{l}_4$
	2.414(61)	0.0665(21)	3.13(.33)	4.43(.14)

Λ_χ	$L_4^{(2)}$	$L_5^{(2)}$	$(2L_6^{(2)} - L_4^{(2)})$	$(2L_8^{(2)} - L_5^{(2)})$
770 MeV	3.3(1.3)	9.30(.73)	0.32(.62)	0.50(.43)
1 GeV	1.3(1.3)	5.16(.73)	-0.71(.62)	4.64(.43)

	N_f	type	$\bar{l}_3$	$\bar{l}_4$
this work, direct $\mathrm{SU}(2)$ fit	2+1	DWF	3.13(.33)(.24)	4.43(.14)(.77)
this work, conv. from $\mathrm{SU}(3)$	2+1	DWF	2.87(.28)(--)	4.10(.05)(--)
MILC, direct $\mathrm{SU}(2)$ fit	2+1	stagg	2.85(.07)(--)	--
MILC, conv. from $\mathrm{SU}(3)$	2+1	stagg	0.6(1.2)	3.9(0.5)
ETMC	2	TM-Wilson	3.44(.08)(.35)	4.61(.04)(.11)
CERN	2	impr. Wilson	3.0(0.5)(0.1)	4.1(0.1)(--)
CERN NNLO				3.3(0.8)(--)
phenom.			2.9(2.4)	4.4(0.2)

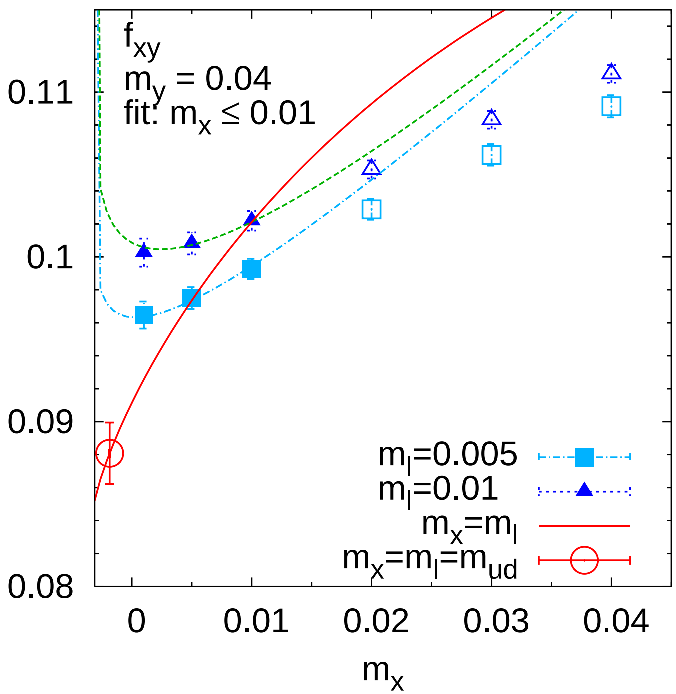
Pion sector results

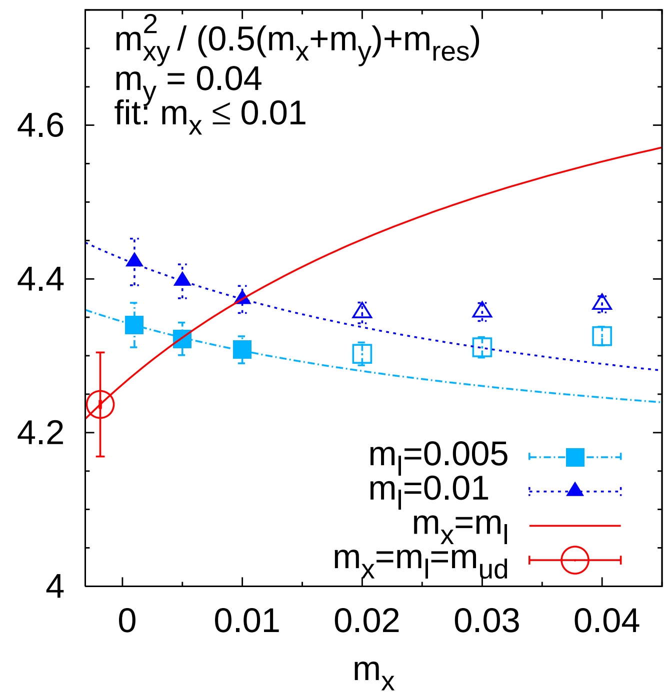
- The physical averaged u, d quark masses $m_{ud} = (m_u + m_d)/2$ is obtained by requiring extrapolated $m_{\mathrm{PS}} = 135.0$ MeV, corresponding to the neutral pion π^0 to avoid the leading QED effect.



Kaon sector

- m_{xy}^2 and f_{xy} are fit using heavy Kaon+$SU(2)$ PQChPT for $m_x \in [0.001, 0.01]$ and $m_y = 0.04$ to determine LECs $f^{(K)}, B^{(K)}, \lambda_{1,2,3,4}$ which are linear functions of m_h, m_y.

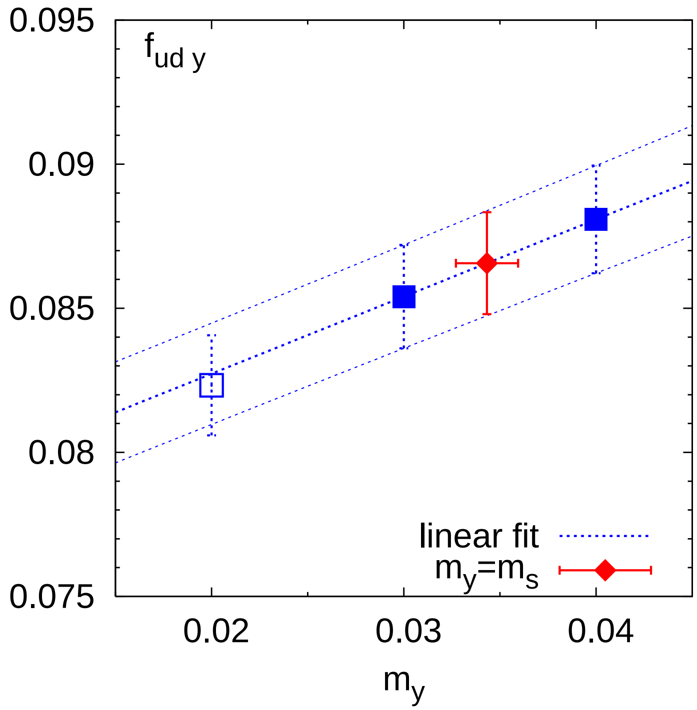


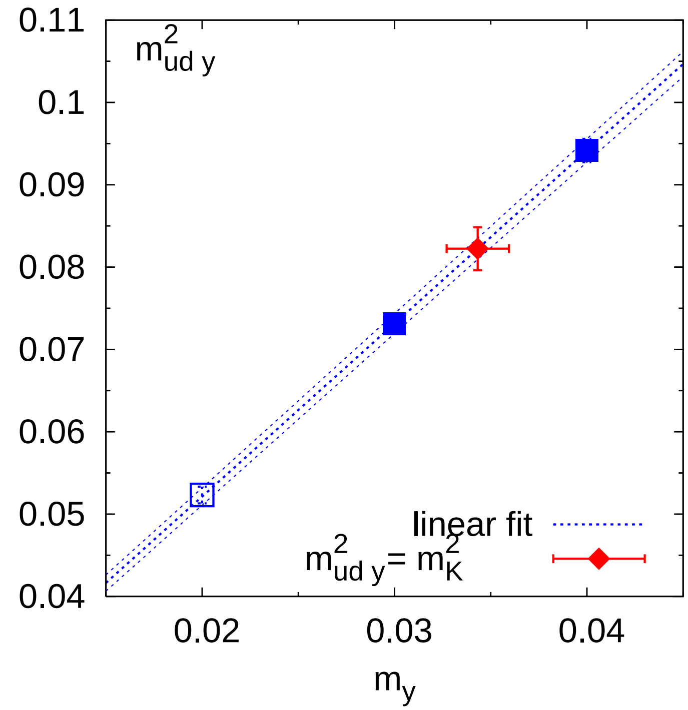


Kaon sector (contd.)

- By extrapolating $m_l \to m_{ud}$, obtained from pion sector, for each valence strange masses, $m_y = 0.04$ and 0.03, $m_K(m_y)$ and $f_K(m_y)$ is obtained.

- By requiring $m_K(m_y = m_s) = 495.7$ MeV, which is $\sqrt{M_{K^0}^2 + M_{K^\pm}^2}$ to avoid effect from $m_u - m_d$.





Determination of m_{ud}, m_s, a^{-1}

To determine m_{ud}, m_s, a^{-1}, the coupled three equations for the interpolated/extrapolated masses:

- $m_\Omega(m_y = m_s; m_h)a^{-1}$ =1672 MeV
- $m_{PS}(m_x = m_y = m_l = m_{ud}; m_h)a^{-1}$=135.0 MeV
- $m_{PS}(m_x = m_l = m_{ud}; m_y = m_s; m_h)a^{-1}$= 495.7 MeV

are solved iteratively until converged.

- Results of physical points ($\tilde{m}_X \equiv m_X + m_{\text{res}}$)

a^{-1} [GeV]	a [fm]	m_{ud}	$\widetilde{m}_{ud}$	m_s	$\widetilde{m}_s$	$\widetilde{m}_{ud} : \widetilde{m}_s$
1.729(28)	0.1141(18)	-0.001847(58)	0.001300(58)	0.0343(16)	0.0375(16)	1:28.8(4)

Quark Masses

- We calculate $Z_m = 1/Z_S$ in RI-MOM scheme non-perturbatively, then match to $\overline{MS}$ at 2GeV perturbatively

$$Z_m(2GeV) = 1.656(48)(150)$$

- Now we have RI-SMOM scheme, which would reduce the systematic error (IR effect, perturbative error, and the lack of $m_s \to 0$). A new statistical error reduction method using momentum volume source is also developed.

	N_f	type	$m_{ud}^{\overline{MS}}(2\,GeV)[MeV]$	$m_s^{\overline{MS}}(2\,GeV)[MeV]$	$m_s : m_{ud}$
using non-perturbative renormalization					
this work	2+1	DWF	$3.72(0.16)(0.33)_{ren}(0.18)_{syst}$	$107.3(4.4)(9.7)_{ren}(4.9)_{syst}$	$28.8(0.4)(1.6)_{syst}$
RBC	2	DWF	$4.25(0.23)(0.26)_{ren}$	$119.5(5.6)(7.4)_{ren}$	$28.10(0.38)$
ETMC	2	TM-Wilson	$3.85(0.12)(0.40)_{syst}$	$105(3)(9)_{syst}$	$27.3(0.3)(1.2)_{syst}$
QCDSF	2	impr. Wilson	$4.08(0.23)(0.19)_{syst}(0.23)_{scale}$	$111(6)(4)_{syst}(6)_{scale}$	$27.2(3.2)$
using perturbative renormalization					
MILC	2+1	stagg.	$3.2(0)(0.1)_{ren}(0.2)_{EM}(0)_{cont}$	$88(0)(3)_{ren}(4)_{EM}(0)_{cont}$	$27.2(0.1)(0.3)_{syst}$
PACS-CS	2+1	impr. Wilson	$2.3(1.1)$	$69.1(2.5)$	$30(?)$
JLQCD	2+1	impr. Wilson	$3.54(^{+.64}_{-.35})_{total}$	$91.1(^{+14.6}_{-6.2})_{total}$	$25.7(?)$

Electromagnetic Splittings

QED + QCD simulations

[R.Zhou, T.Blum, T.Doi, M.Hayakawa, T.Izubuchi, S.Uno and N.Yamada] , in preparation
[R.Zhou, T.Blum, T.Doi, M.Hayakawa, T.Izubuchi, and N.Yamada] ,
"Isospin symmetry breaking effects in the pion and nucleon masses" PoS(LATTICE 2008) 131.

[T. Blum, T. Doi, M. Hayakawa, TI, N. Yamada] ,
"Determination of light quark masses from the electromagnetic splitting of psedoscalar meson masses computed with two flavors of domain wall fermions"
Phys. Rev.D76 (2007) 114508 (38 pages)
"The isospin breaking effect on baryons with Nf=2 domain wall fermions"
PoS(LAT2006) 174 (7 pages)
"Electromagnetic properties of hadrons with two flavors of dynamical domain wall fermions"
PoS(LAT2005) 092 (6 pages)
"Hadronic light-by light scattering contribution to the muon g-2 from lattice QCD: Methodology"
PoS(LAT2005) 353(6 pages)

Isospin Breaking Effects

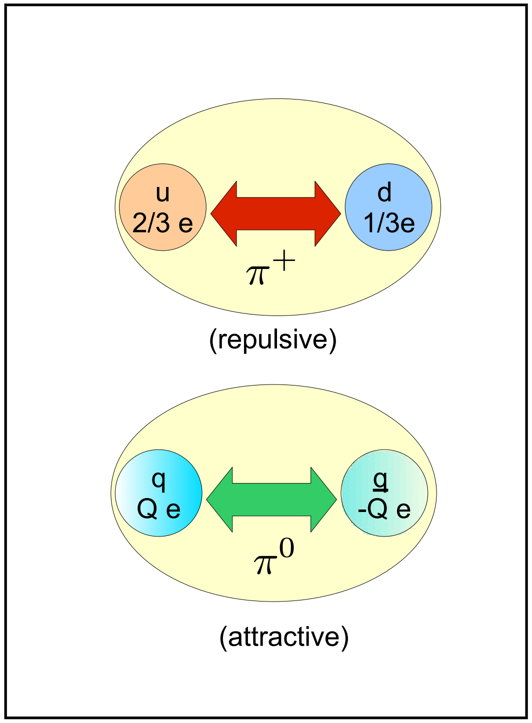
- The first principle calculations of isospin breaking effects due to electromagnetic (EM) and the up, down quark mass difference are necessary for accurate hadron spectrum, quark mass determination.

- Isospin breakings are measured very accurately :

  $m_{\pi^\pm} - m_{\pi^0} = 4.5936(5)\text{MeV},$

  $m_N - m_P = 1.2933317(5)\text{MeV}$

- From $\Gamma(\pi^+ \to \mu^+\nu_\mu, \mu^+\nu_\mu\gamma)$ + V_{ud}(exp)
  $f_{\pi^+} = 130.7 \pm 0.1 \pm 0.36$MeV   PDG 2004



- the last error is due to the uncertainty in the part of $\mathcal{O}(\alpha)$ radiative corrections that depends on the hadronic structure of the π meson.

$$\Gamma(PS^+ \to \mu^+\nu_\mu, \mu^+\nu_\mu\gamma) \propto [1 + C_{\text{PS}}\,\alpha]_{\text{had. struc.}}$$

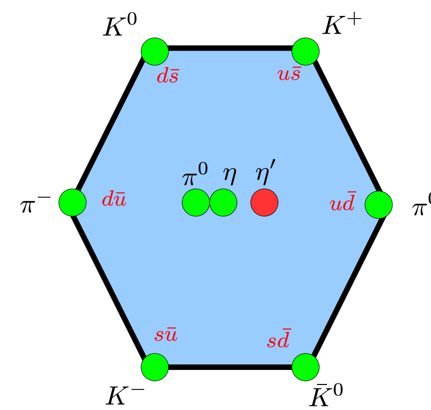
$$C_\pi \sim 0 \pm 0.24, \quad C_\pi - C_K = 3.0 \pm 1.5$$

$c.f.$ Marciano 2004, MILC, RBC/UKQCD : V_{us} from f_π/f_K (Lattice) + $\Gamma(\pi_{l2})/\Gamma(K_{l2})$.

Isospin Breaking Effects (contd.)

- PS meson spectrum and quark masses.

  - Asymmetry due to Quark mass differences : $m_u \neq m_d \neq m_s$
  - Asymmetry due to QED interactions : $Q_u = 2/3e, Q_d = Q_s = -1/3e$
  - QCD axial anomaly makes m_η' heavy.



- Could $m_u \simeq 0$, which would explain the very small Neutron EDM ? (Strong CP problem) [D.Nelson,G.Fleming, G.Kilcup,PRL90:021601,2003.]

- Positive mass difference between Neutron (udd) and Proton (uud) stabilizes proton thus make our world as it is. $m_N - m_P = 1.2933317(5)$MeV

- $m_\rho^+ - m_\rho^0$, $\Gamma_{\rho^+}, \Gamma_{\rho^0}$ are related to the conversion of $\Gamma(\tau \to$ Hadrons$)$ to $\Gamma(e^+e^- \to$ Hadrons$)$ to determine leading QCD correction to muon $g-2$.

EM splittings

- Axial WT identity with EM for massless quarks $(N_F = 3)$,

$$\mathcal{L}_{\text{em}} = e A_{\text{em}\,\mu}(x) \bar{q} Q_{\text{em}} \gamma_\mu q(x), \;\; Q_{\text{em}} = \text{diag}(2/3, -1/3, -1/3)$$

$$\partial^\mu \mathcal{A}^a_\mu = ie A_{\text{em}\,\mu}\, \overline{q}\, [T^a,\, Q_{\text{em}}]\, \gamma^\mu \gamma_5 q - \frac{\alpha}{2\pi} tr \left(Q^2_{\text{em}} T^a \right) F^{\mu\nu}_{\text{em}} \widetilde{F}_{\text{em}\,\mu\nu}\,,$$

neutral currents, four $\mathcal{A}^a_\mu(x)$, are conserved (ignoring $\mathcal{O}(\alpha^2)$ effects): π^0, K^0, $\overline{K^0}$, η_8 are still a NG bosons.

- ChPT with EM at $\mathcal{O}(p^4, p^2 e^2)$:

$$M^2_{\pi^\pm} = 2mB_0 + 2e^2 \frac{C}{f_0^2} \qquad +\mathcal{O}(m^2 \log m, m^2) + I_0 e^2 m \log m + K_0 e^2 m$$

$$M^2_{\pi^0} = 2mB_0 \qquad +\mathcal{O}(m^2 \log m, m^2) + I_\pm e^2 m \log m + K_\pm e^2 m$$

Dashen's theorem :
The difference of squared pion mass is independent of quark mass up to $\mathcal{O}(e^2 m)$,

$$\Delta M^2_\pi \equiv M^2_{\pi^\pm} - M^2_{\pi^0} = 2e^2 \frac{C}{f_0^2} + (I_\pm - I_0) e^2 m \log m + (K_\pm - K_0) e^2 m$$

C, $K_\pm$, K_0 is a new low energy constant. $I_\pm$, I_0 is known in terms of them.

QCD+QED lattice simulation

- In 1996, Duncan, Eichten, Thacker carried out SU(3)×U(1) simulation to do the EM splittings for the hadron spectroscopy using quenched Wilson fermion on $a^{-1} \sim 1.15$ GeV, $12^3 \times 24$ lattice. [Duncan, Eichten, Thacker PRL76(96) 3894, PLB409(97) 387]

- Using $N_F = 2$ Dynamical DWF ensemble (RBC) would have a benefits of chiral symmetry, such as better scaling and smaller quenching errors.

- Especially smaller systematic errors due to the the quark massless limits, $m_f \to -m_{res}(Q_i)$, has smaller Q_i dependence than that of Wilson fermion, $\kappa \to \kappa_c(Q_i)$.

- Generate Coulomb gauge fixed (quenched) non-compact U(1) gauge action with $\beta_{QED} = 1$. $U_\mu^{EM} = \exp[-iA_{\text{em}\,\mu}(x)]$.

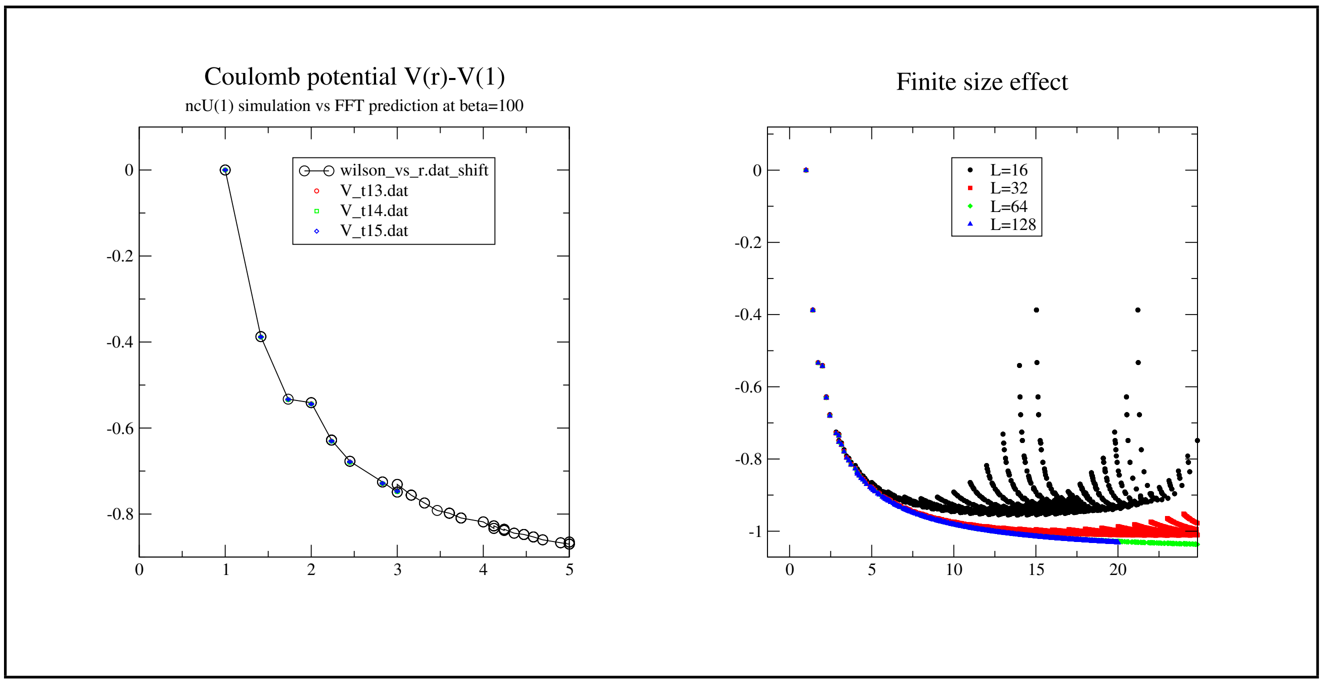
- Quark propagator, $S_{q_i}(x)$ with EM charge $Q_i = q_i e$ with Coulomb gauge fixed wall source

$$D\left[(U_\mu^{EM})^{Q_i} \times U_\mu^{SU(3)}\right] S_{q_i}(x) = b_{src}, \quad (i = \text{up,down})$$

$$q_{\text{up}} = 2/3, \quad q_{\text{down}} = -1/3$$

photon field on lattice

- non-compact $U(1)$ gauge is generated by using Fast Fourier Transformation (FFT). Coulomb gauge $\partial_j A_{\mathrm{em}\,j}(x) = 0$, $\tilde{A}_{\mathrm{em}\,\mu=0}(p_0, 0) = 0$ with eliminating zero modes. ($N_F = 2+1$: Feynman gauge)
- static lepton potential on $16^3 \times 32$ lattice ($\beta_{QED} = 100$, 4,000 confs) vs lattice Coulomb potential.
- L=16 has significant finite volume effect for $ra > 6 \sim 1.5 r_0 \sim 0.75$ fm. It would be worth considering for generation of U(1) on a larger lattice and cutting it off.



simulation parameters

- $N_F = 2$ Dynamical DWF configuration for QCD
- $a^{-1} = 1.691(53)$ GeV.
- degenerate quark mass at dynamical quark mass points, $m_{val} = m_{sea} = (0.02), 0.03, 0.04 \sim$ 50%, 75%, 100% of m_{strange}.
- $16^3 \times 32$ or (1.9 fm)^3.
- $Ls = 12$, $m_{res}a = 0.0013$ or a few MeV.
- EM charge: $e = 1.0, 0.6, 0.3028 = \sqrt{4\pi/137}$
- $\sim$ 94$\longrightarrow$ 190 configurations for each m
- one or two QED configuration per a QCD configuration.
- All 16 meson connected correlators + Neutron, Proton.

EM spectrum on lattice

- By neglecting $\mathcal{O}(\alpha^2)$ and $\mathcal{O}((m_u - m_d)^2)$, we approximate π^0 mass squared by that of π^3, which doesn't have the noisy disconnected diagram.

- We will not use π^0 mass to determine quark masses.

- The correlator for π^3, ρ^3 meson is calculated using the interpolation field of the $a = 3$ component of isospin:

$$C_{X^0}(t) = \frac{1}{2}\left[\left\langle J_X^{uu}(t) J_X^{uu\dagger}(0)\right\rangle_{conn} + \left\langle J_X^{dd}(t) J_X^{dd\dagger}(0)\right\rangle_{conn}\right], \quad X = \pi, \rho$$

- Chiral limit of DWF is defined through Axial Ward identity,
$m_f = -m_{res} = -\left\langle J_{5q}^a(t) P^a(0)\right\rangle / \langle P^a(t) P^a(0)\rangle$
$\mathcal{O}(\alpha)$ effect is parametrized in the generic form

$$m_{res}(\alpha) = m_{res}(0) + C_1(Q_1 - Q_2)^2 + C_2(Q_1 + Q_2)^2$$

for currents made of quarks of charges Q_1 and Q_2. $m_{res}(0), C_1, C_2 \to 0$ at $L_s \to \infty$.

Analysis methods

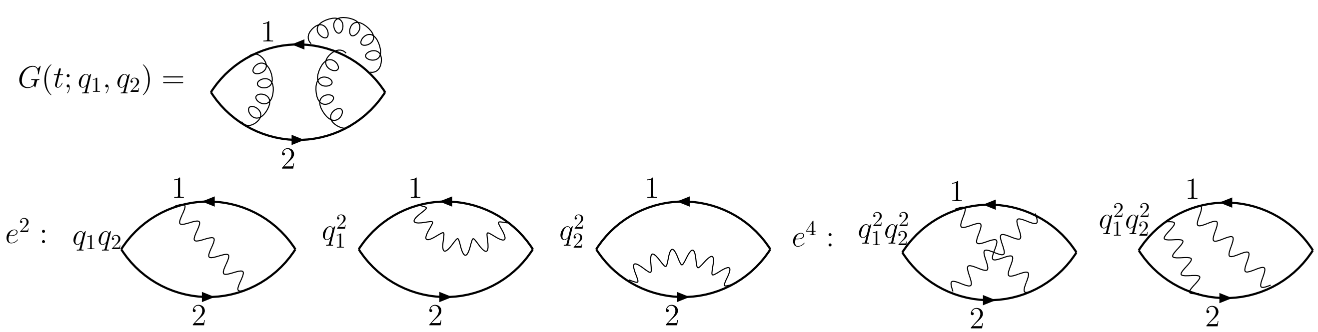
- Analysis method I :
  Fit correlator for each charge combination separately,
  then calculate the mass splittings under jackknife.

$$X = \pi, \rho, N : \Delta M_X = M_{X^\pm} - M_{X^0},$$

- Analysis method II :
  Subtract charged correlator by neutral correlator,
  and fit it by a linear function in t:

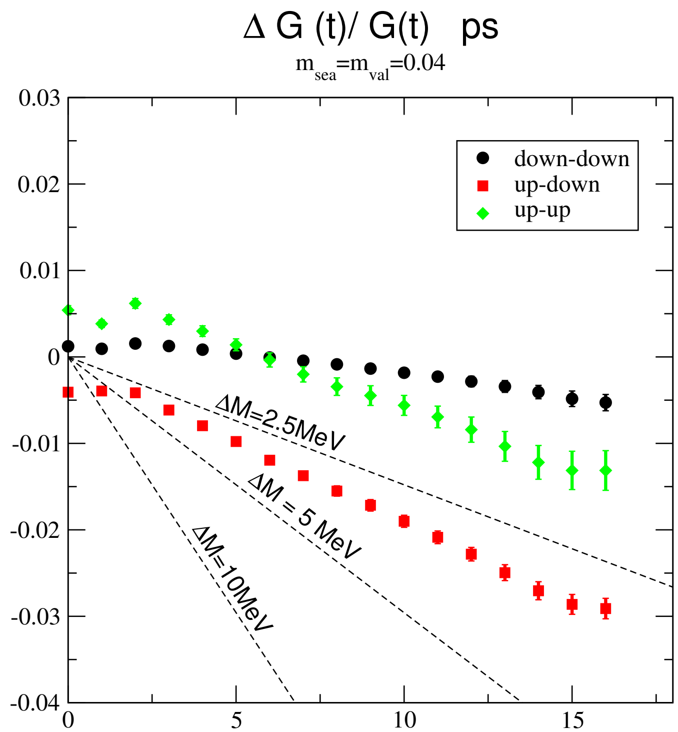
$$C_X(t) = A(e^2)e^{-M_X(e^2)t}$$

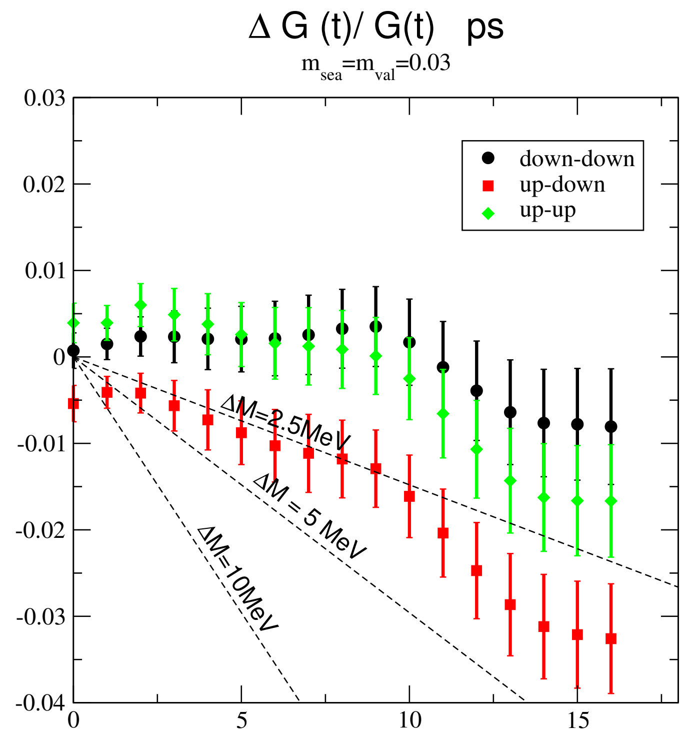
$$\frac{C_{X^\pm}(t) - C_{X^0}(t)}{C_{X^0}(t)} = \Delta M_X \times t + Const$$



propagator ratio

- $G(t) = \langle J_5(0) J_5(t) \rangle$ at $m = 0.04$ and 0.03.





- Fluctuations due to SU(3) are comparable to that from U(1): by double the QED statistics: ΔM_π reduces by $\sim$ 4, 10, (30) % for $A_4, J_5, (N)$ resp. at $m = 0.04$.

$$\frac{\sigma_{QCD}^2 + 0.5\sigma_{QED}^2}{\sigma_{QCD}^2 + \sigma_{QED}^2} = (0.9)^2 \Longrightarrow \sigma_{QED}/\sigma_{QCD} \sim 0.85$$

$\mathcal{O}(e)$ error reduction

- On the infinitely large statistical ensemble, term proportional to odd powers of e vanishes. But for finite statistics,

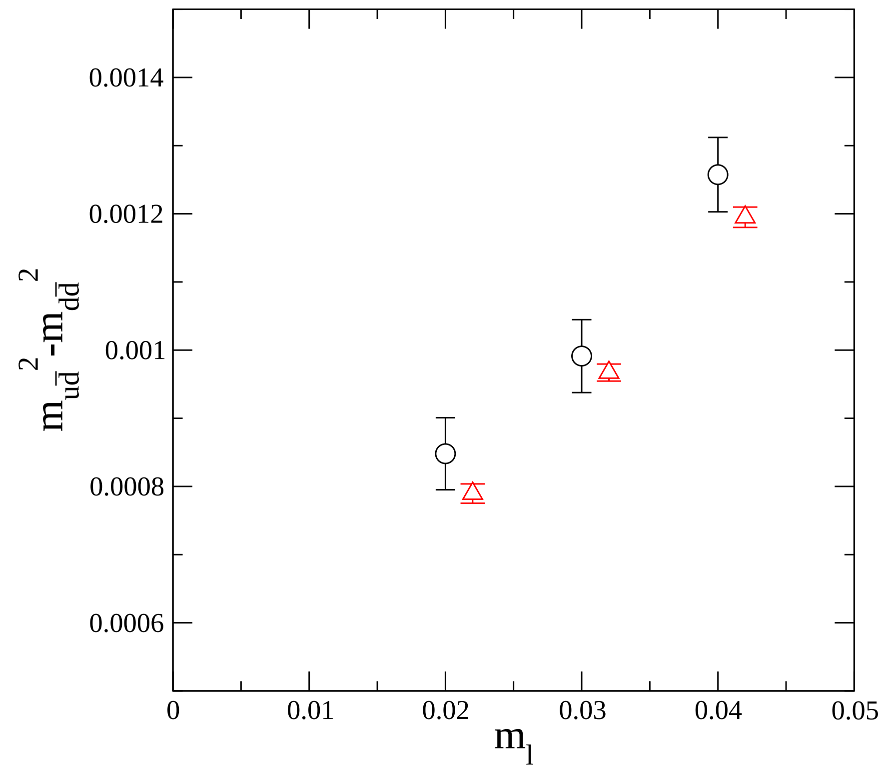
$$\langle O \rangle_e = \langle C_0 \rangle + \langle C_1 \rangle\, e + \langle C_2 \rangle\, e^2 + \cdots$$

$\langle C_{2n-1} \rangle$ could be finite and source of large statistical error as e^{2n-1} vs e^{2n}.

- By averaging $+e$ and $-e$ measurement on the same set of QCD+QED configuration,

$$\frac{1}{2}[\langle O \rangle_e + \langle O \rangle_{-e}] = \langle C_0 \rangle + \langle C_2 \rangle\, e^2 + \cdots$$

$\mathcal{O}(e)$ is exactly canceled.



Low energy constants

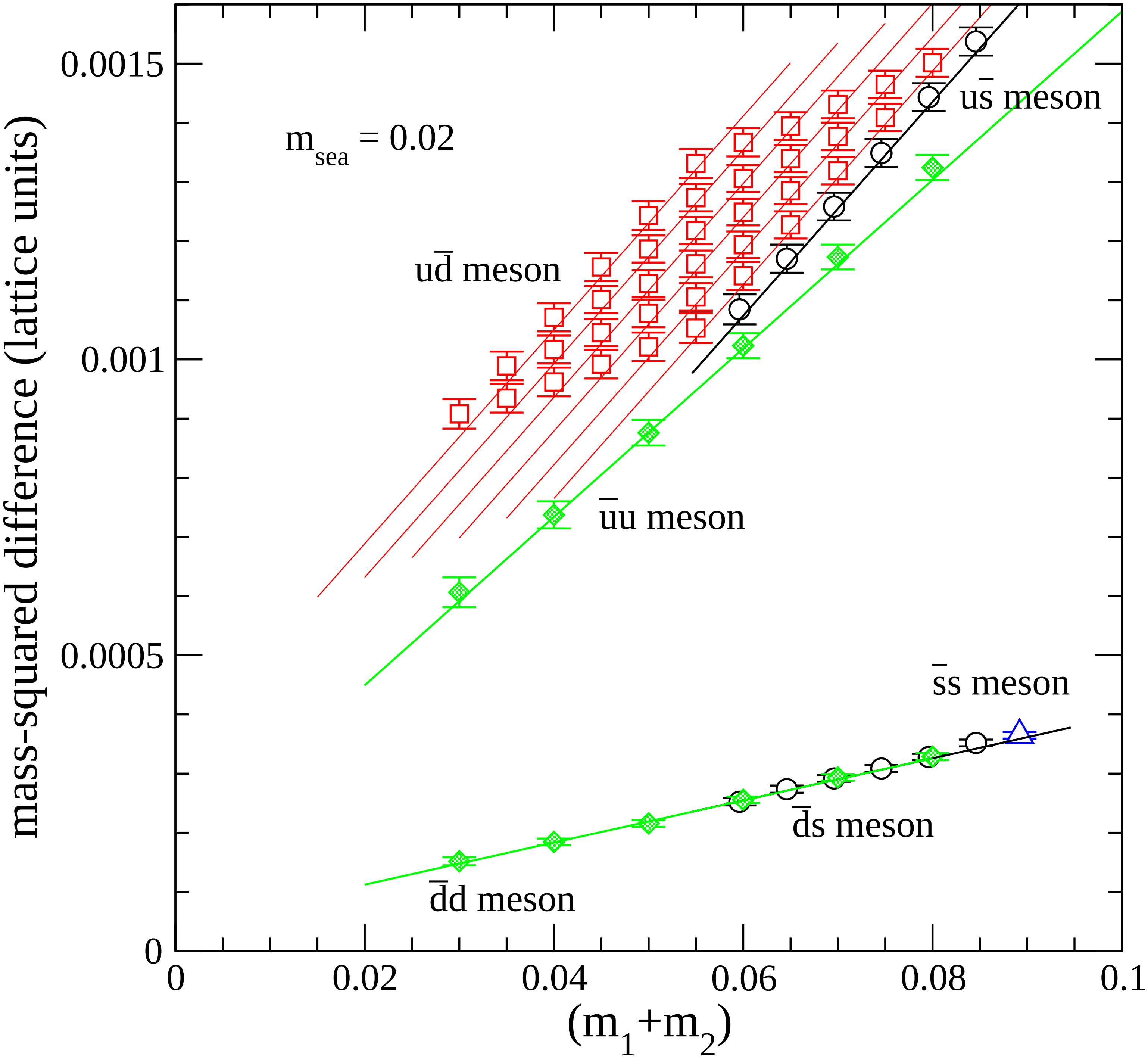
- Squared PS meson masses, made of valence quarks (m_i, Q_i) and (m_j, Q_j), are obtained at NLO

$$m_{ij}^2 = M_{ij}^2 + \Delta_{NLO}(M_{ij}^2) + \Delta_{em}(M_{ij}^2)$$

with $\mathcal{O}(\alpha)$ LEC δ's:

$$\begin{aligned}
\Delta_{em}(m_{ij}^2) &= \delta\,(Q_i - Q_j)^2 \\
&+ \delta_0\,(Q_i + Q_j)^2\,(m_i + m_j) \\
&+ \delta_+\,(Q_i - Q_j)^2\,(m_i + m_j) \\
&+ \delta_-\,(Q_i^2 - Q_j^2)\,(m_i - m_j) \\
&+ \delta_{sea}\,(Q_i - Q_j)^2\,(2\,m_{sea}) \\
&+ \delta_{mres}\,(Q_i + Q_j)^2
\end{aligned}$$

- While $\Delta_{NLO}(M_{ij}^2)$ included full NLO, we omit logarithmic term as there are no $N_F = 2$ PQChPT formula.
  $N_F = 2$ unitary case: [Urech, NPB433 (95) 234]
  $N_F = 2 + 1$ PQChPT: [Bijnens Danielsson, PRD75 (07) 014505]



mass-squared difference (lattice units)
m_{sea} = 0.02
$u\bar{d}$ meson
$u\bar{s}$ meson
$\bar{u}u$ meson
$\bar{s}s$ meson
$\bar{d}s$ meson
$\bar{d}d$ meson
0.0015
0.001
0.0005
0
0.02
0.04
0.06
0.08
0.1
(m_1+m_2)

Fit Results

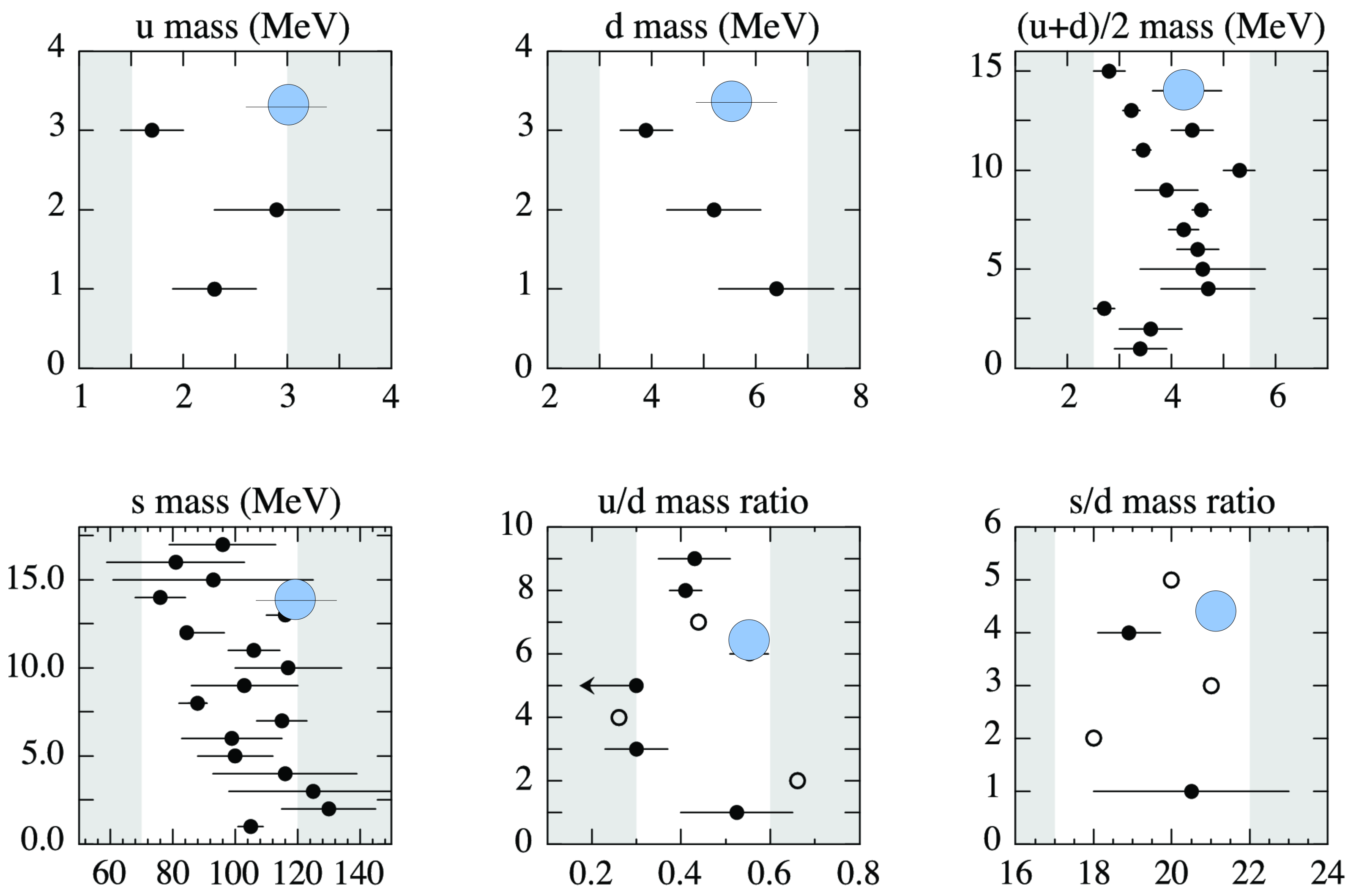
- Fit 61 masses for each sea quark point.
- δ_{sea} is poorly determined (uncorrelated). Fix $\delta_{sea} = 0$ for the main value,

fit range	$\delta \times 10^4$	δ_0	δ_+	δ_-	δ_{sea}
0.015-0.0446	4.62 (18)	0.0080 (12)	0.01129 (24)	0.01746(33)	-
	4.45 (56)	0.0080 (12)	0.01132 (23)	0.01741(29)	$2.5(8.4) \times 10^{-4}$
0.015-0.03	4.85 (21)	0.0077 (20)	0.01059 (32)	0.01696(40)	-
	6.46 (86)	0.0077 (20)	0.01048 (32)	0.01701(40)	-0.0028 (15)

- Experimental inputs: $m^2_{\pi^\pm}, m^2_{K^\pm}, m^2_{K^0}$ (exclude m_{π^0})
- Using non-perturbatively determined Z factor $1/Z_m = Z_S = 0.62(4)$

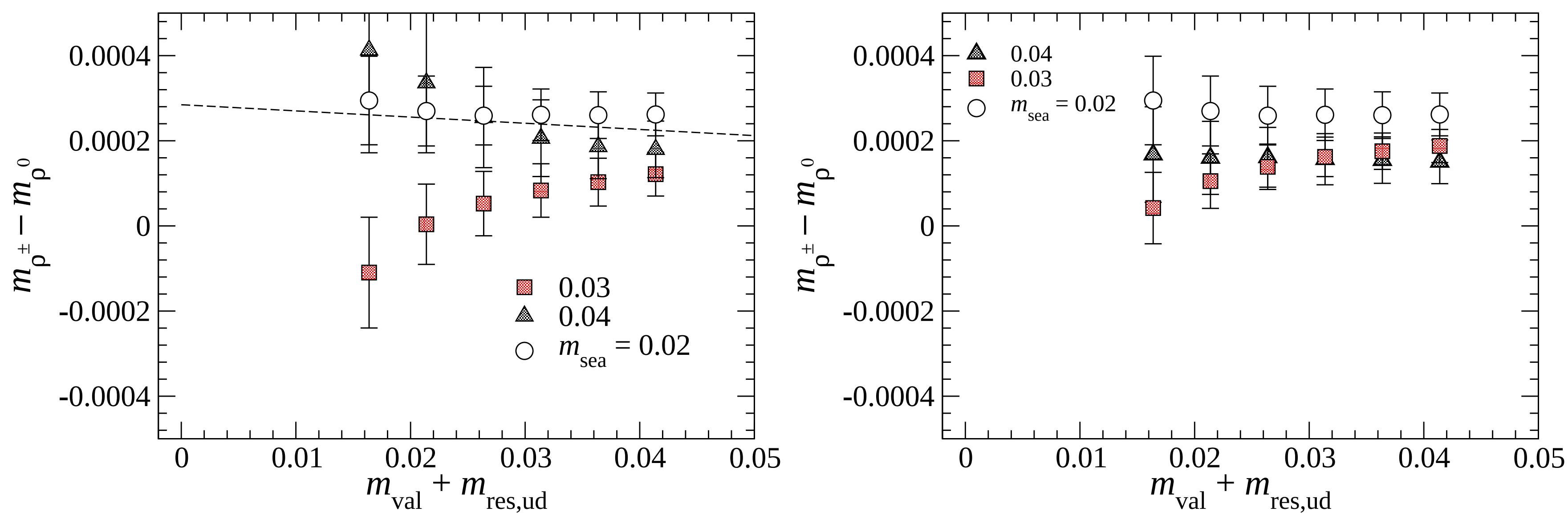
$$
\begin{aligned}
m_u^{\overline{MS}}(2\,GeV) &= 3.02(27)(19)\text{ MeV},\\
m_d^{\overline{MS}}(2\,GeV) &= 5.49(20)(34)\text{ MeV},\\
m_{ud}^{\overline{MS}}(2\,GeV) &= 4.25(23)(26)\text{ MeV},\\
m_s^{\overline{MS}}(2GeV) &= 119.5(56)(74)\text{ MeV},\\
m_u/m_d &= 0.550(31),\\
m_s/m_{ud} &= 28.10(38).
\end{aligned}
$$

Quark masses



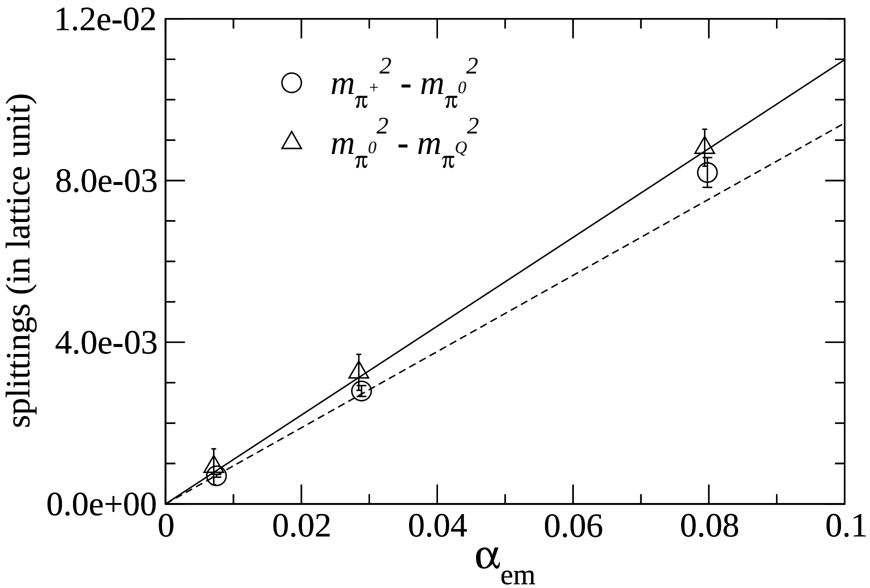
$\rho^{\pm} - \rho^0$ splittings

- ignore disconnected quark loops (ω).
- Non-monotonic in mass
- Depends on fit range
- Linear extrapolation for unitary points, $m_v = m_s$, yield a small but positive value ~ 0.5 MeV.
- Experimentally consistent with zero.



$\mathcal{O}(\alpha^4)$ signal

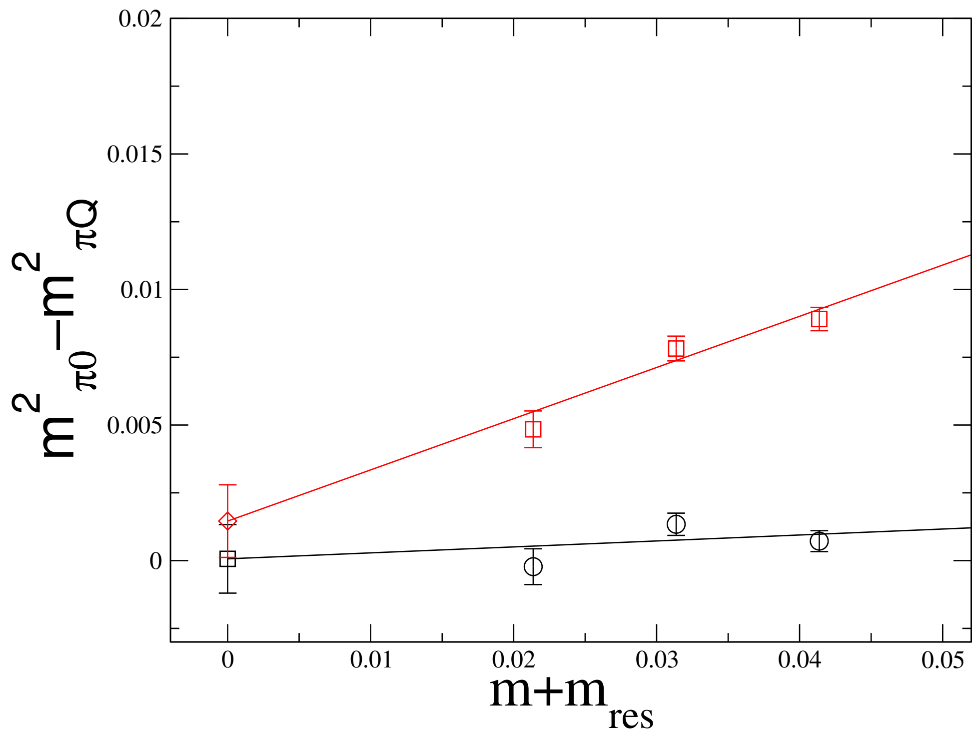
- $m^2_{\pi+} - m^2_{\pi 0}$ and $m^2_{\pi 0} - m^2_{\pi Q}$
- EM charge: $e = 1.0, 0.6, 0.3028 = \sqrt{4\pi/137}$
- $m^2_{\pi 0} - m^2_{\pi Q}$ is seen to have $\mathcal{O}(\alpha^4)$



Dashen's theorem

- $M_{\pi,Q}$: pure QCD pion ($e^2 = 0$)

$$M_{\pi Q}^2 = 2B_0 m_q M_{\pi 0}^2 - M_{\pi Q}^2 = I e^2 m_q \log m_q + K e^2 m_q$$



- O($e^2 m_q$) correction is measured, which was one of major uncertainties in quark mass determination.

Conclusion of $N_F = 2$ EM Spectrum

- QED+QCD calculation for $N_F = 2$ DWF is carried out.
- Using $m^2_{\pi^\pm}, m^2_{K^\pm}, m^2_{K^0}$ value from experiment, quark masses are determined.
- Using the LEC and quark mass,

$$m_{\pi^\pm} - m_{\pi^3} = \boxed{\underline{4.12(21)\ \text{MeV}}} \quad (\text{exp.}: 4.5936(5)\text{MeV})$$

  is derived. Difference from the quark mass difference is estimated as
  . 0.17(3) MeV [Gasser Leutwyler NPB250 (85) 465]
  . 0.30(20) MeV [Bijnens Prades, NPB490 (97) 239] .

- Various systematic errors are remaining, and will be addressed on $N_F = 2 + 1$ DWF ensemble on larger volume lattice [Feb. 08 Zhou, Blum, Doi, Hayakawa, TI, Yamada] .
- The deviation from the Dashen's theorem due to $\mathcal{O}(m\alpha)$,

$$\Delta_{EM} = \left(\frac{{m_{K^+}}^2 - {m_{K^0}}^2}{{m_{\pi^+}}^2 - {m_{\pi^0}}^2} \right)_{EM\ part} - 1 = \boxed{\underline{0.337(40)}} \quad \text{[full mass] or} \quad 0.264(43),$$

  Large N_C extended NJL model: $\Delta_{EM} = 0.85(24)$ [Bijnens Prades, NPB490 (97) 239] .

Systematic errors

- Quark mass used in the fit: 0.02$\sim$0.03 or 0.02$\sim$0.0467
- QCD's Z_m : $\Lambda_{QCD} = 250 - 300$ MeV, $\mathcal{O}(\alpha) \sim$ 1%.
- Disconnected loops: η' from DWF
  [K. Hashimoto TI PTP (08) *"η' meson from two flavor dynamical domain wall fermions"* (80 pages)]
- Quenched QED $\mathcal{O}(\alpha\alpha_S)$:
  ChPT and a clever combinations of masses [Bijnens Danielsson, PRD75 (07) 014505]
- One lattice spacing results, $\mathcal{O}(a^2)$. Uncertainty in a.
- Lack of strange sea quark.
- Finite Size Effect from vector-saturation model: $\Delta_{\pi,EM} = m^2_{\pi^+} - m^2_{\pi^0}$, to be

$$\Delta_{\pi,EM}(L) = \frac{3\,\alpha}{4\pi}\,\frac{1}{a^2}\,\frac{2^4 \cdot \pi^2}{N} \sum_{q \in \widetilde{\Gamma}'} \frac{(am_\rho)^2 (am_A)^2}{\widehat{q}^2 \left(\widehat{q}^2 + (am_\rho)^2\right) \left(\widehat{q}^2 + (am_A)^2\right)}\,,$$

$$\frac{\Delta_{\pi,EM}(\infty)}{\Delta_{\pi,EM}(L \approx 1.9\ fm)} = 1.10\,.$$

By varying δ by 10 %, quark masses are shifted by less than 1 %.

$N_F = 2+1$ QCD+QED simulation

- Including gluon's vacuum polarization effects from up, down, and strange quarks.
- Photon is still quenched (Feynman gauge).
  Reweighting idea: generate ensemble without QED and later measure

  $$\frac{\det D(U^{\text{QCD}}, U^{\text{QED}})}{\det D(U^{\text{QCD}}, 1)}$$

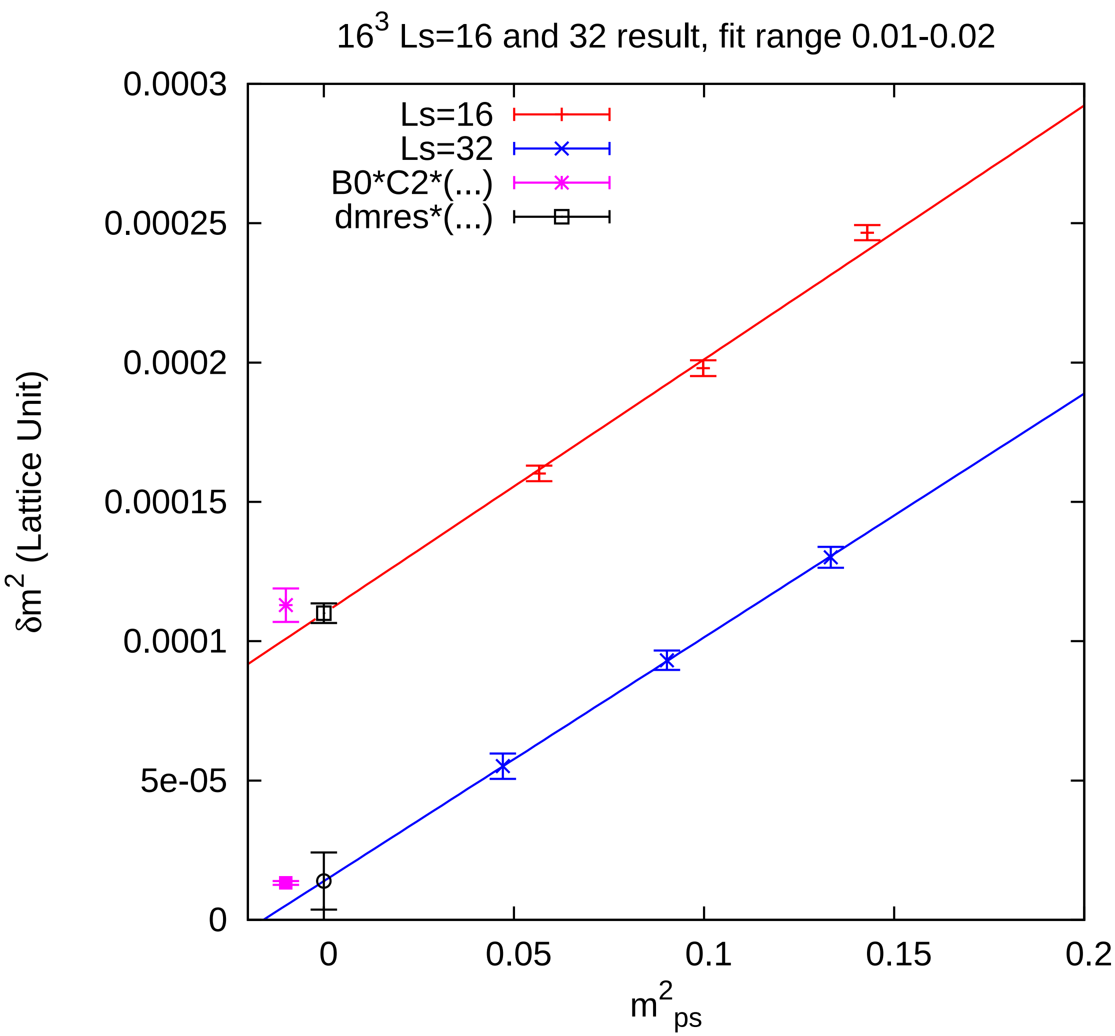
  as an observable, may work. [A.Duncan, E.Eichten, R.Sedgewick, PRD71:094509,2005]
- Both SU(3) and heavy Kaon+SU(2) PQChPTs with virtual photon and finite volume corrections are being examined.

- Two physical volumes: (1.8 fm)^3, (2.7 fm)^3 to see the finite volume effects.
- Two $L_s = 16, 32$ to see the effect of the residual chiral symmetry breaking.
- Still one lattice spacings.

[R.Zhou, T.Blum, T.Doi, M.Hayakawa, T.Izubuchi, S.Uno and N.Yamada]

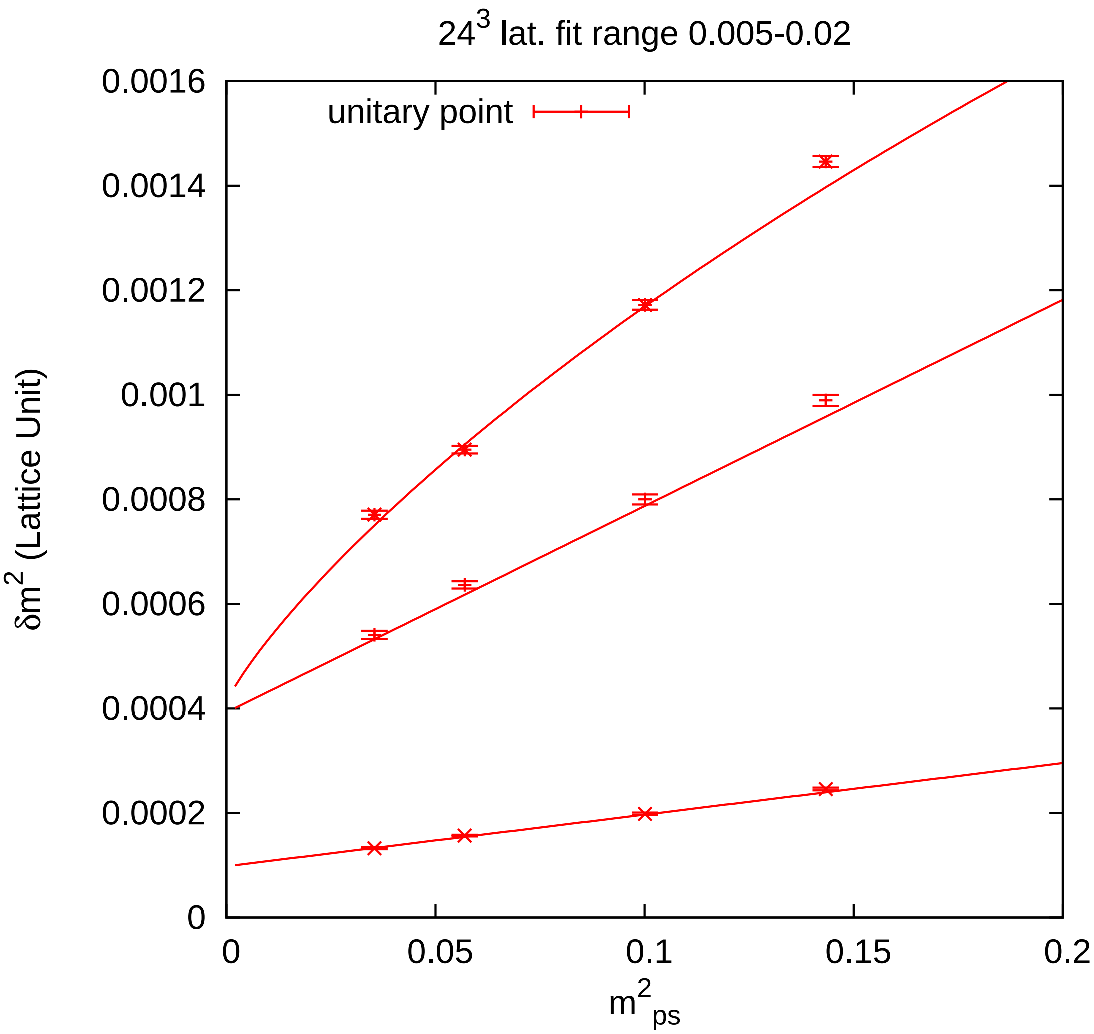
Effect of the residual chiral symmetry breakings in $N_F = 2 + 1$ QCD+QED simulations

- $\delta m^2 = M^2_{\mathsf{PS}}(e \neq 0) - M^2_{\mathsf{PS}}(e = 0)$



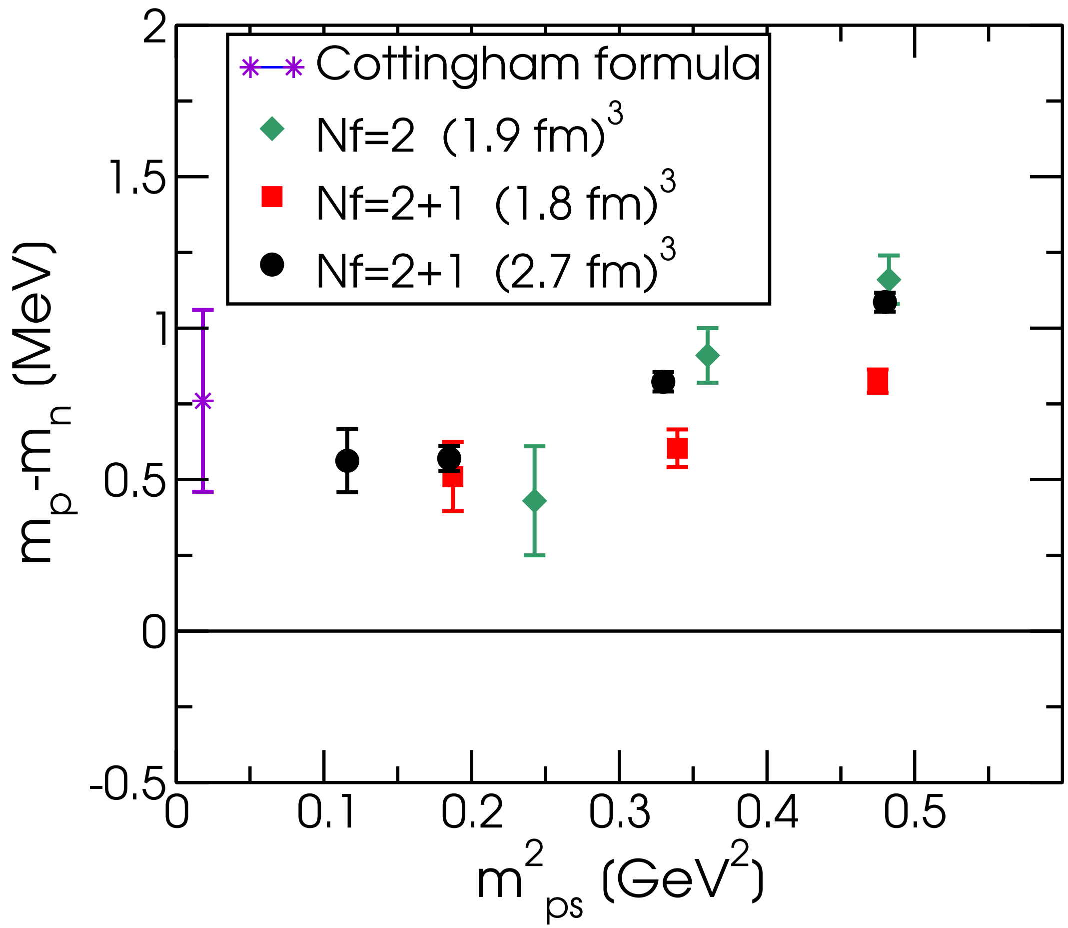
SU(3) PQChPT fit in $N_F = 2 + 1$ QCD+QED simulations

- SU(3) PQChPT fit.



Nucleon mass splitting in $N_F = 2, 2+1$ (Preliminary)

[R.Zhou, T.Blum, T.Doi, M.Hayakawa, TI, N.Yamada, (Preliminary)]



- Only EM effect, $m_u = m_d$ case, are shown. *c.f.* [Gasser Leutwyler, PR87(82)77]

$$M_N - M_p|_{\text{EM}} = -0.76(30) \text{ MeV} \tag{1}$$

$$M_N - M_p|_{\text{quark mass}} = 2.05(30) \text{ MeV} \tag{2}$$

Summary and Future perspective

- DWF QCD is now a practical tool for the accurate determination of important quantities for QCD and SM.
- Continuum-like chiral behavior and renormalization allow us to compute indispensable SM parameters, Hadronic matrix elements and form factor computation etc. that relate experiments and theory

$$m_u m_d, m_s, f_\pi, f_K, B_K$$

- Isospin breaking effects are interesting and important, which could now be addressed by QCD+QED simulations from the first principle.

Future plans

- Analysis on the finer lattice, $a \sim 0.08$ fm. $\mathcal{O}(a^2)$ is currently one of dominant errors.
- Weak matrix elements of Heavy-light meson, B, D
- EM spectrum, $(g_\mu - 2)$.
- New QCD ensemble on the large lattice with light pion:

Backup Slides

Measurement

m_l	Dataset	Range	Δ	N_{meas}	$t_{src\ locations}$
0.005	FPQ	900-4460	40	90	5, 59
	DEG	900-4460	40	90	0, 32
	UNI	900-4480	20	180	0, 32, 16
0.01	FPQ	1460-5020	40	90	5, 59
	DEG	1460-5020	40	90	0, 32
	UNI	800-3940	10	315	0, 32
0.02	DEG	1800-3560	40	45	0, 32
	UNI	1800-3580	20	90	0, 32
0.03	DEG	1260-3020	40	45	0, 32
	UNI	1260-3040	20	90	0, 32

- **FPQ** valence masses $m_x, m_y \in \{0.001, 0.005, 0.01, 0.02, 0.03, 0.04\}$
  source: Wall, sink: Wall or Local
- **DEG** $m_x = m_y \in \{0.001, 0.005, 0.01, 0.02, 0.03, 0.04\}$
  source: 16^3 Box , sink: Box or Local
- **UNI** $m_x, m_y \in \{m_l, m_h\}$, sea u,d (strange) quark mass $m_l(m_h)$
  source: Hydrogen S-wavefunction $r = 3.5a$, sink: H or Local
- PBC+ABC, PBC-ABC

f_π, f_K and $|V_{us}|$

- The ratio of the decay widths of K and π

$$\frac{\Gamma(K^+ \to \overline{\mu}\nu(\gamma))}{\Gamma(\pi^+ \to \overline{\mu}\nu(\gamma))} = \left|\frac{V_{us}}{V_{ud}}\right|^2 \frac{f_K^2}{f_\pi^2}\frac{m_K}{m_\pi}\left[\frac{1-m_\mu^2/m_K^2}{1-m_\mu^2/m_\pi^2}\right]^2 \times (1+\delta_{em})$$

- Our results,

$$f_\pi = 124.1(3.6)(6.9)\text{MeV}, \quad f_K = 149.6(3.6)(6.3)\text{MeV}$$

$$\frac{f_K}{f_\pi} = 1.205(18)_{\text{stat}}(62)_{\text{syst}}[= (14)_{\text{FV}}(48)_{a^2}(34)_\chi(12)_{m_s}]$$

- Systematic uncertainties

  FV evaluated from difference between PQChPT with descrete loop momentum and continuous one.

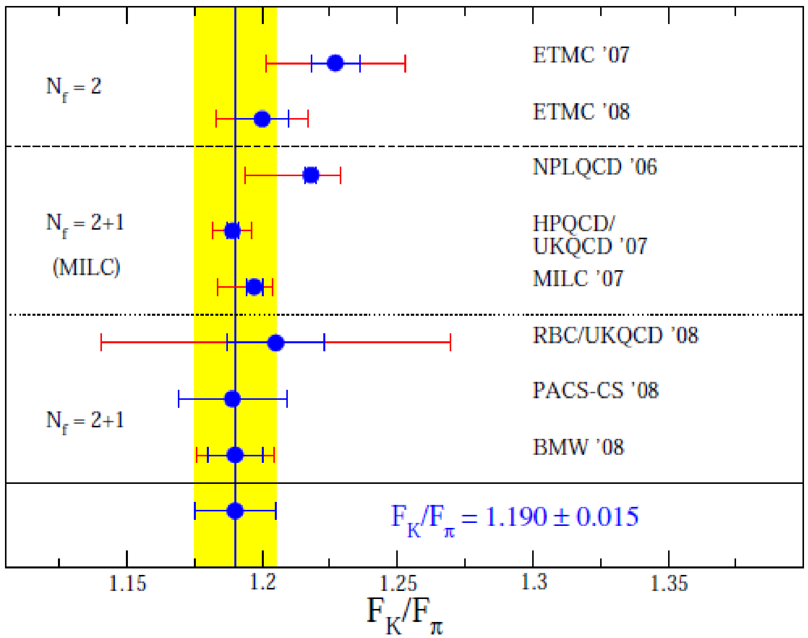
  a^2 $(a\Lambda_{QCD}) \sim 4\%$.

  χ twice of difference between NLO $SU(2)$ ChPT and analytic NNLO for pion sector. Kaon sector's error is estimated as twice of $m_{cut} = 0.01$ and $m_{cut} = 0.02$.

  m_s estimated from shifts in $SU(2)$ LECs, which are converted from $SU(3)$ fits.

- $|V_{us}|^2 + |V_{ud}|^2 = 0.9980(54)$ using super-allowed nuclear β decay $|V_{ud}| = 0.97377(27)$.

Comparison with other results



[Lellouche Lattice 2008]

- World average 2008

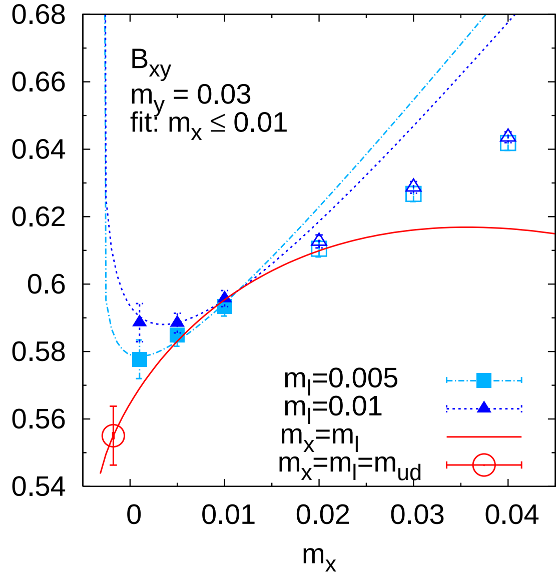
$$\frac{f_K}{f_\pi} = 1.190(15)[1.3\%]$$

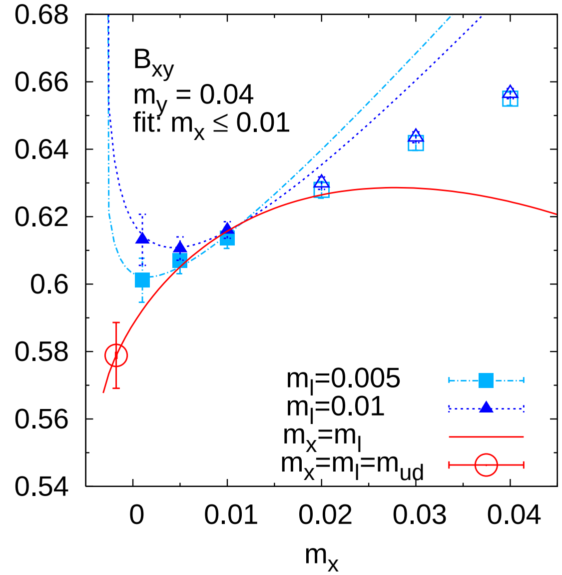
- Systematic error, $\mathcal{O}(a^2)$, for RBC-UKQCD's value may be overestimated, since

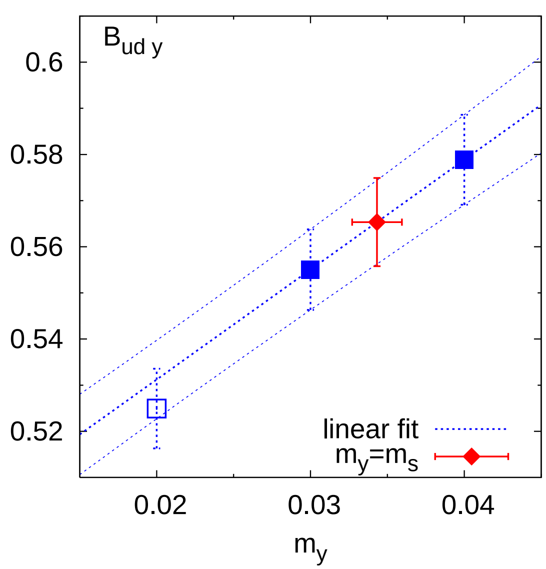
$$\left.\frac{f_K}{f_\pi}\right|_{m_s=m_{ud}} = 1$$

- Goal would be 0.5% to be comparable to K_{l3} determination.

B_K

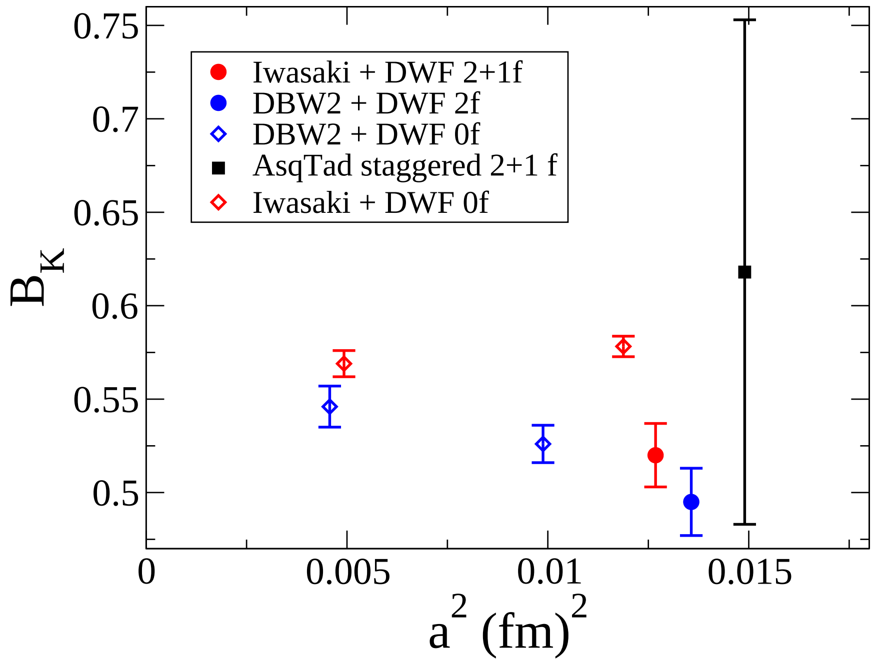






- Extrapolating to $m_x \longrightarrow m_{ud}$ using $SU(2)$ PQChPT
- Then interpolating to $m_y \longrightarrow m_s$
- NPR RI-MOM $Z_{B_K}^{\overline{\mathrm{MS}}}(2\mathrm{GeV})$ = 0.910(05)(13)$_{sys}$ (continuumm perturbative error dominates)

B_K (contd.)



- $$B_K^{\overline{\mathsf{MS}}}(2\mathsf{GeV}) = 0.514(10)(07)_{\mathsf{ren}}(24)_{\mathsf{sys}}$$
- 4% from $\mathcal{O}(a^2)$, 1% from $m_h \neq m_s$, 2% from ChPT.
- Now we are checking Z_{B_K} in the RI-SMOM scheme.

$SU(2)$ ChPT with Kaon

pion matrix, quark-mass matrix, and kaon field

$$\phi = \begin{pmatrix} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & -\phi^0/\sqrt{2} \end{pmatrix}, \quad M = \begin{pmatrix} m_l & 0 \\ 0 & m_l \end{pmatrix}, \quad K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix}$$

$$\xi = \exp(i\phi/f), \Sigma = \xi^2$$

$L \in SU(2)_L, R \in SU(2)_R$ and $U = U(L, R, \phi, K)$

$$\Sigma \longrightarrow L\Sigma R^\dagger, \quad \xi \longrightarrow L\xi U^\dagger = U\xi R^\dagger, \quad K \longrightarrow UK$$

LO chiral Lagrangian of kaons:

$$L^{(1)}_{\pi K} = D_\mu K^\dagger D^\mu K - M^2 K^\dagger K,$$

covariant derivative and the vector field:

$$D_\mu K = \partial_\mu K + V_\mu K, \quad V_\mu = (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger)/2 = [\phi, \partial_\mu \phi]/2f^2 + \cdots$$

f_K and m_K in $SU(2)$ ChPT

The axial current $\bar{q}\gamma_\mu\gamma_5 s$ is identified in LO as

$$j^5_\mu = \frac{f_K^{(K)}}{2}(D_\mu K)^\dagger(\xi + \xi^\dagger) + iL_{A2}K^\dagger A_\mu(\xi - \xi^\dagger).$$

where $f_K^{(K)}(m_h; m_y), L_{A2}(m_h; m_y)$ is LEC of $SU(2)$ ChPT, with sea (valence) strange quark mass $m_h(m_y)$.
At NLO, only $f_K^{(K)}$ term contributes to form tad pole loop at the current,

$$f_K = f_K^{(K)}(m_h)\left\{1 + c(m_h)\frac{\xi_l m_\pi^2}{f^2} - \frac{m_\pi^2}{(4\pi f)^2}\frac{3}{4}\log\frac{m_\pi^2}{\Lambda_\chi^2}\right\}$$

The tadpole from $KK\pi\pi$ coupling in the LO Lagrangian vanishes,

$$m_{xh}^2 = B^{(K)}(m_h)m_h\left\{1 + \frac{\lambda_1(m_h)}{f^2}\xi_l + \frac{\lambda_2(m_h)}{f^2}\xi_x\right\}$$

B_K in $SU(2)$ ChPT

$K^0 - \overline{K^0}$ mixing contains the QCD four quarks operator

$$\bar{s}_L \gamma_\mu d_L \bar{s}_L \gamma^\mu d_L$$

which transform under $SU(2)_L$ and symmetric under interchange $d_L \leftrightarrow d_L$: $SU(2)_L$ **triplet** and $SU(2)_R$ **singlet**. The Kaon field in the ChPT transforms as $K \longrightarrow UK, \xi \longrightarrow L\xi U \Longrightarrow \xi K \longrightarrow L\xi K$ so the four quark operator could be written as (a, b is $SU(2)$ flavor indices)

$$\mathcal{O}_{ab} = \beta \left[(\xi K)_a (\xi K)_b + (\xi K)_b (\xi K)_a \right]$$

By expanding $\mathcal{O}_{ab}$ in terms of ϕ at NLO,

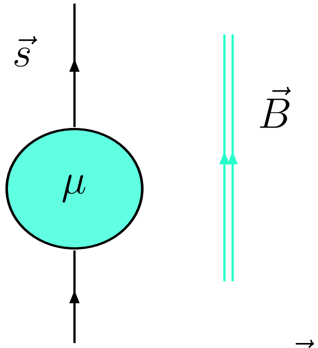
$$\mathcal{O}_{ab} = 2\beta K_a K_b - \frac{2\beta}{f^2} \left\{ (\phi K)_a (\phi K)_b + \frac{1}{2} \left[(\phi^2 K)_a K_b + K_a (\phi^2 K)_b \right] \right\} + \cdots$$

Only the second term contribute to B_K (the third term is the NLO of $(f_K)^2$), and

$$B_{xh} = B_{\mathsf{PS}}^{(K)}(m_h) \left\{ 1 + \frac{b_1(m_h)}{f^2} \chi_l + \frac{b_2(m_h)}{f^2} \chi_x - \frac{\chi_l}{32\pi^2 f^2} \log \frac{\chi_x}{\Lambda_\chi^2} \right\}$$

QCD + QED simulations

- muon anomalous magnetic moment $g_\mu - 2$ [BNL-E821] .
  g_μ gyromagnetic ratio: muon (spin 1/2)'s coupling to magnetic field



$E = -g_\mu \frac{e}{2m_\mu} \vec{s} \cdot \vec{B}$

$$a_\mu^{\text{exp}} = \frac{g_\mu - 2}{2} = 116,592,080(60) \times 10^{-11}$$

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{Had}} + a_\mu^{\text{EW}},$$

$$a_\mu^{\text{exp}} - a_\mu^{\text{th}} = (220 \pm 100) \times 10^{-11}$$

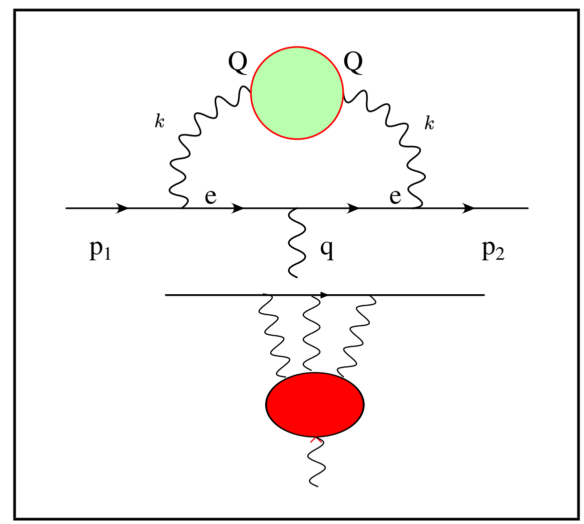
- Hadronic contributions dominates theory error.

$$a_\mu^{\text{Had}} = a_\mu^{\text{Had,LO}} + a_\mu^{\text{Had,HO}} + a_\mu^{\text{Had,LBL}}$$

$$a_\mu^{\text{had,LBL}} = 134(25) \times 10^{-11}$$

$$(\text{before}: 86(35) \times 10^{-11})$$

$$a_\mu^{\text{new}} \sim \mathcal{O}((m_\mu / M_{new})^2)$$



$a_\mu^{\text{Had,HO}}$ was explored by T. Blum in PRL 91, 2003,
C. Aubin & T. Blum new analysis using SChPT.