

Sterile neutrinos in cosmology and laboratories

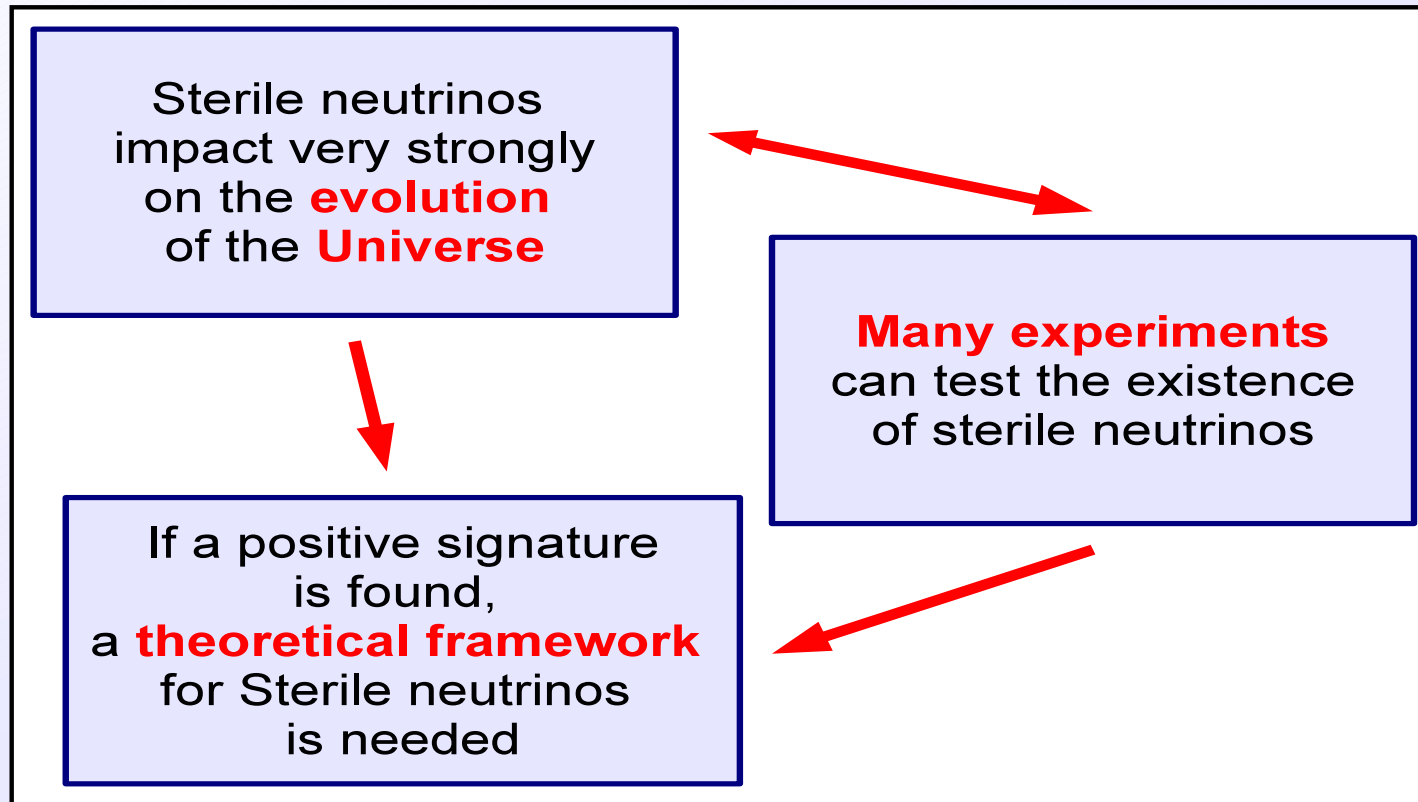
Theory Division - Fermilab

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1 – Outline



- Sterile neutrinos: formalism
- Production in the Early Universe
- Cosmological constraints
- Laboratory constraints

2 – Formalism

Sterile neutrinos do not have SM interactions but can mix with the active ones:

$$\begin{cases} |\nu_1\rangle = \cos\theta|\nu_e\rangle - \sin\theta|\nu_s\rangle \\ |N_2\rangle = \sin\theta|\nu_e\rangle + \cos\theta|\nu_s\rangle \end{cases}$$

More in general we have

$$\nu_{aL} = \sum_{m=1}^3 U_{am}\nu_{mL} + \sum_{m'=4}^n V_{am'}N_{m'L}^c$$

- This is the minimal extension of the Standard Model (phenomenological approach) which can account for DM (and in the ν MSM also for the baryon asymmetry).

The SM Lagrangian for massive neutrinos is:

$$\begin{aligned}
 -\mathcal{L} &= \left(\frac{g}{\sqrt{2}} W_{\mu}^{+} \sum_{a=1}^3 \bar{\nu}_{aL} \gamma^{\mu} l_L + h.c. \right) + \\
 &\quad \frac{g}{2 \cos W} Z_{\mu} \sum_{a=1}^3 \bar{\nu}_{aL} \gamma^{\mu} \nu_{aL}, \\
 &= \frac{g}{\sqrt{2}} W_{\mu}^{+} \left(U^{\dagger} \bar{\nu}_m \gamma^{\mu} P_L \ell + V^{\dagger} \bar{N}^c \gamma^{\mu} P_L \ell \right) + h.c. \\
 &\quad + \frac{g}{2 \cos W} Z_{\mu} \left(U^{\dagger} V \bar{\nu}_m \gamma^{\mu} P_L N^c + h.c. \right) \\
 &\quad + \frac{g}{2 \cos W} Z_{\mu} \left(U^{\dagger} U \bar{\nu}_m \gamma^{\mu} P_L \nu_n + V^{\dagger} V \bar{N}^c \gamma^{\mu} P_L N^c \right).
 \end{aligned}$$

Notation: N_4 is commonly called "sterile neutrino", although it is a massive state.

3 – Sterile neutrino production in the EU

The simplest mechanism of ν_s production in the EU is via oscillations.

[Dodelson and Widrow, 1992]

ν_a are kept in equilibrium due to the weak interaction with the other particles in the bath ($e^\pm \dots$) till

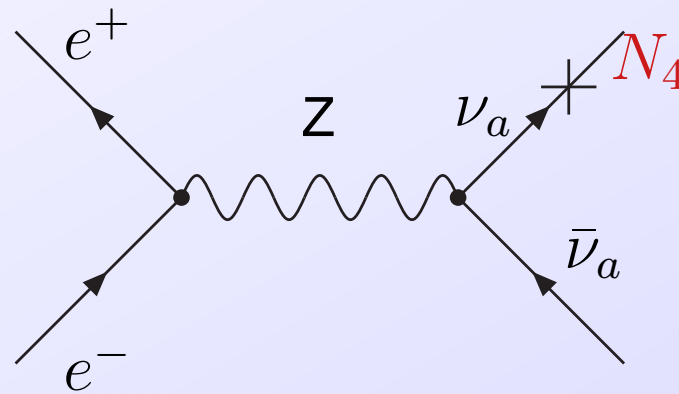
$$\Gamma_{\text{weak}} \sim H$$

- $\Gamma_{\text{weak}} = \langle \sigma \rangle n \sim G_F^2 T^2 T^3$

- $H = \sqrt{\frac{4\pi^3 g_*}{45}} \frac{T^2}{m_{\text{Pl}}}$

This corresponds to a temperature: $T_{\text{dec}} \sim \text{MeV}$

In an interaction involving active neutrinos, a N_4 can be produced **due to loss of coherence**



The "sterile" neutrino N_4 production

- depends on $|V_{a4}|^2 = \sin^2 \theta$
- is controlled by Γ_a and will stop at T_{dec}

The probability of $N_4 \sim \nu_s$ production can be computed by looking at the evolution of the states. In the flavour basis:

$$i \frac{d}{dt} \begin{pmatrix} |\nu_a\rangle \\ |\nu_s\rangle \end{pmatrix} = \left[U \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} U^\dagger + \begin{pmatrix} V_a & 0 \\ 0 & 0 \end{pmatrix} \right] \begin{pmatrix} |\nu_a\rangle \\ |\nu_s\rangle \end{pmatrix}$$

One needs to diagonalise H_m using a unitary matrix given in terms of $\sin \theta_m$

$$U_m = \begin{pmatrix} \cos \theta_m & \sin \theta_m \\ -\sin \theta_m & \cos \theta_m \end{pmatrix}$$

and finally the average probability of oscillation is given by

$$\langle P(\nu_a \rightarrow \nu_s; p, t) \rangle = \frac{1}{2} \langle \sin^2 2\theta_m \rangle$$

The mixing angle in the EU depends on

- **matter effects** due to an asymmetry in the weakly interacting particles

$$V_D \sim \frac{2\sqrt{2}\zeta(3)}{\pi^2} G_F T^3 (\mathcal{L} \pm \eta/4)$$

with $\mathcal{L} = (n_{\nu_\alpha} - n_{\bar{\nu}_\alpha})/\nu_\gamma$

- **finite temperature effects**

$$|V_T| = C_a G_F^2 T^4 E / \alpha$$

We have $(\Delta(p) = m_4^2/(2E))$

$$\sin^2 2\theta_m = \frac{\Delta^2(p) \sin^2 2\theta}{\Delta^2(p) \sin^2 2\theta + D^2 + (\Delta(p) \cos 2\theta - V_D + |V_T|)^2}$$

The production of sterile neutrinos can be computed solving the Boltzmann equation

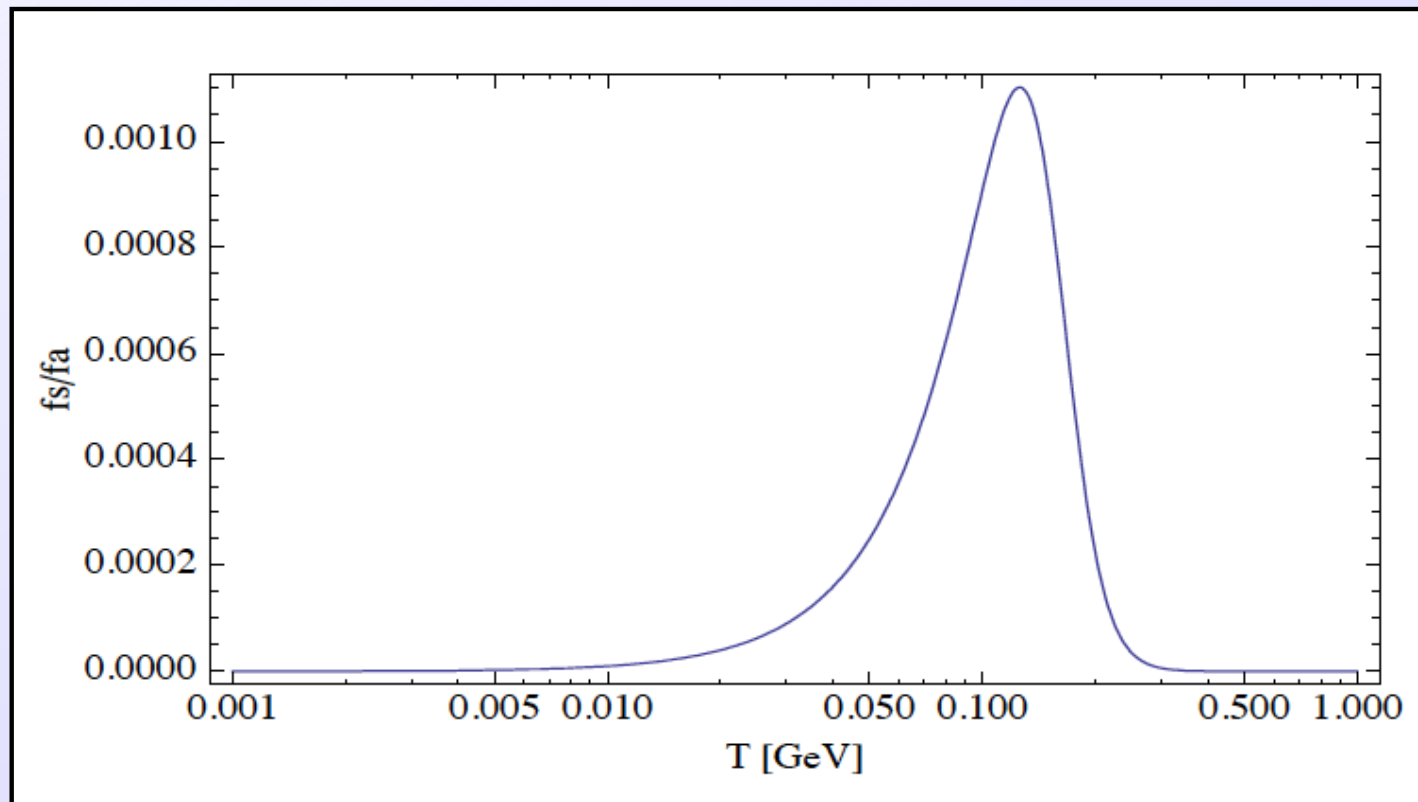
$$\frac{\partial}{\partial t} f_s(p, t) - Hp \frac{\partial}{\partial p} f_s(p, t) \simeq \frac{\Gamma_a}{2} \langle P(\nu_a \rightarrow \nu_s; p, t) \rangle [f_a(p, t) - f_s(p, t)]$$

with $\Gamma_a \sim G_F^2 p T^4$ and $f_a(p, T) = (1 + e^{E/T})^{-1}$ the equilibrium distribution for ν_a .

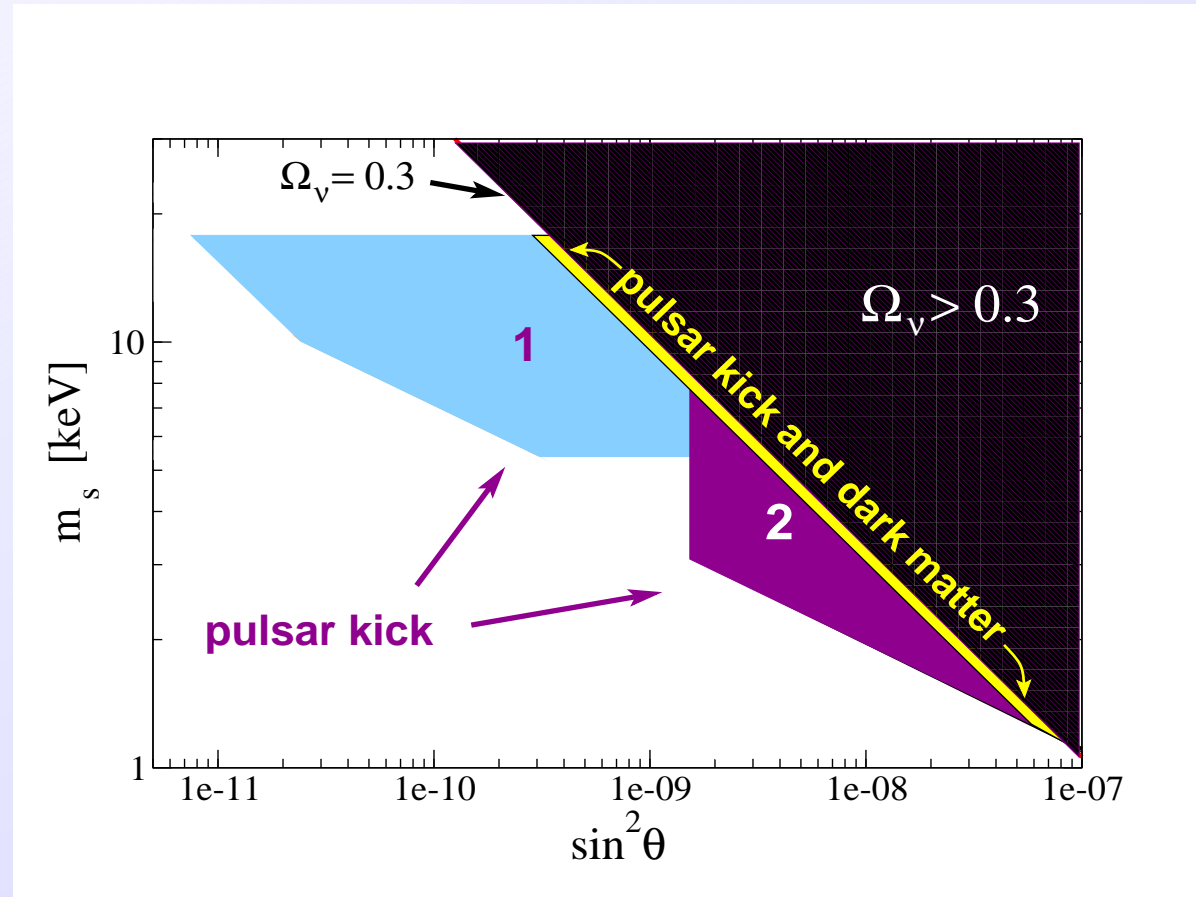
$$\left. \frac{\partial f_s}{\partial T} \right|_{E/T} = -\frac{\Gamma_a}{2HT} \sin^2 2\theta_m [f_a(p, T) - f_s(p, T)]$$

For the non-resonant production,

- at high T , the production is suppressed by $\sin^2 2\theta_m$
- at small T by $\Gamma_a \propto T^5$
- the maximum of production happens at $T_{\max} \simeq 133 \text{ MeV} \left(\frac{m_4}{1\text{keV}} \right)^{1/3}$



The final abundance is $\Omega_4 h^2 \simeq 0.3 \frac{\sin^2 2\theta}{10^{-8}} \left(\frac{m_4}{10\text{keV}} \right)^2$



[Fuller, Kusenko, Mocioiu, S.P., 2003; see also Dodelson, Widrow, 1992; Abazajian et al. 2001]

4 – A multiflavour approach

If more than one sterile neutrino is present, ν_h , it is convenient to use the density matrix formalism, $\hat{\rho}_j^i = |\nu_i\rangle\langle\nu_j|$. The Boltzmann equation can be rewritten as

$$\dot{\rho} = i [H_m + V_a, \rho] - \{\Gamma, (\rho - \rho_{\text{eq}})\}$$

- $H_m = U H_0 U^\dagger$
- $V_a = -C_a G_F^2 T^4 E / \alpha$
- $\{\Gamma, (\rho - \rho_{\text{eq}})\}$ describes the loss of coherence responsible for ν_h production

If the typical frequency of oscillation is faster than the expansion of the Universe, **the static approximation** is valid $\dot{\rho}_{ah} = 0$ and $\dot{\rho}_{sp} = 0$.

One can solve a system of linear equation and find ρ_{ah}, ρ_{sp} which will be given by (for real mixing matrix and no mixing in the sterile sector):

$$\rho_{ah} \simeq \rho_{eq} \frac{\sum_i m_i^2 / (2E) U_{ai} U_{hi}}{(H_{aa} - H_{hh} + i\Gamma_a)},$$

The density of sterile neutrino is then given by:

$$\dot{\rho}_{hh} = -2 \sum_a H_{ha} \text{Im}(\rho_{ah}) \propto \sum_a U_{ah}^2 \Gamma_a$$

- The integration can be performed as in the 2- ν case to find

$$\Omega_h h^2 = 7 \times 10^{-1} \frac{m_h^2}{10\text{keV}^2} \sum_a \frac{g_a}{\sqrt{C_a}} \frac{U_{aj}^2}{10^{-8}}$$

For mixing in the sterile sector, $\rho_{sp} \neq 0$ but an analytical approximation also holds. The growth of each ρ_{hh} can be followed separately.

- **The final abundance for each sterile neutrino** is the sum of the contribution of mixing with each active neutrino!!!

[Melchiorri, Mena, Palomares-Ruiz, SP, Slosar, Sorel, 2009]

5 – The low reheating case

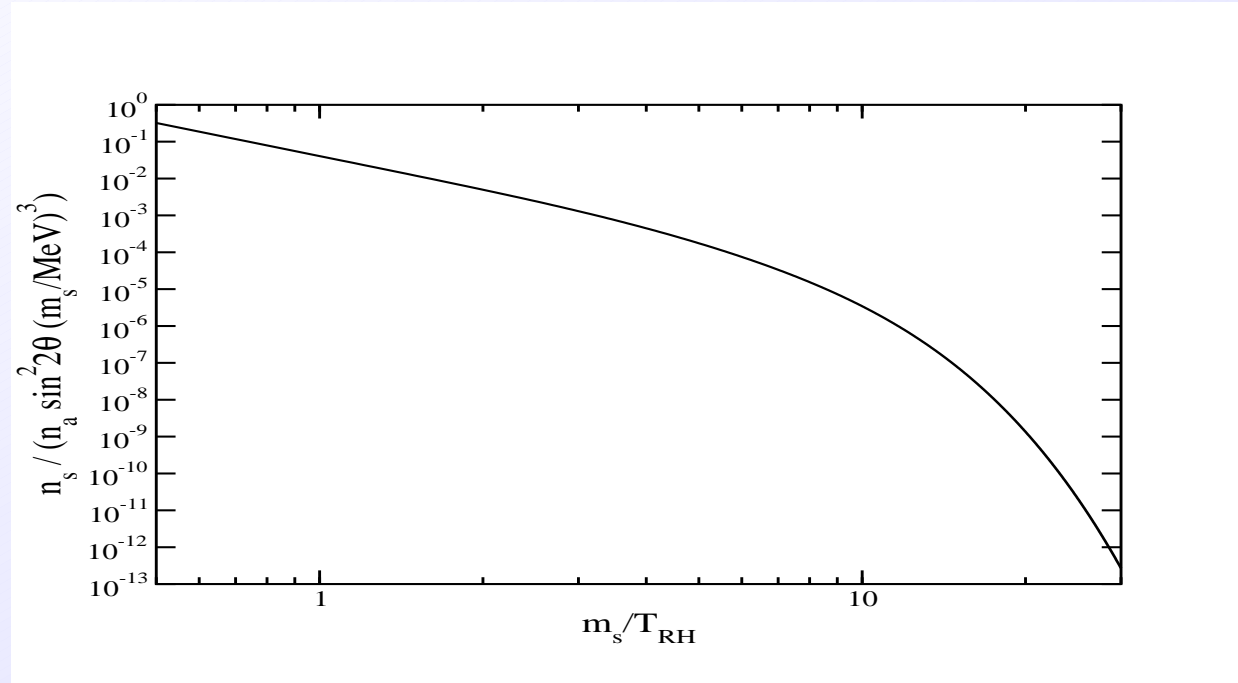
If the **reheating temperature is low** ($T_R \sim 5$ MeV), the production of sterile neutrinos in the Early Universe is **strongly suppressed**.

$$T_R \ll T_{\text{MAX}}$$

The allowed $\sin^2 \theta$ can be **much larger** than in the conventional case.

The sterile neutrinos can **give a signature** in future terrestrial experiments and astrophysical observations.

5 – The low reheating case



[Gelmini, Palomares-Ruiz, SP, 2005; Gelmini, Osoba, Palomares-Ruiz, SP, 2008]

For small masses, we have

$$\Omega_s h^2 \simeq 0.1 \left(\frac{\sin^2 2\theta}{10^{-3}} \right) \left(\frac{m_s}{1 \text{ keV}} \right)^3 \left(\frac{T_R}{5 \text{ MeV}} \right)^3$$

The production is significantly suppressed.

6 – Resonant sterile neutrino production

In presence of a large lepton asymmetry, $\mathcal{L} \equiv (n_\nu - n_{\bar{\nu}})/n_\gamma$, matter effects become important and the mixing angle can be resonantly enhanced. [Shi,

Fuller, 1998; Kishimoto, Fuller, 2008; Abazajian et al., 2001]

$$\sin^2 2\theta_m = \frac{\Delta^2(p) \sin^2 2\theta}{\Delta^2(p) \sin^2 2\theta + D^2 + (\Delta(p) \cos 2\theta - \frac{2\sqrt{2}\zeta(3)}{\pi^2} G_F T^3 \mathcal{L} + |V_T|)^2}$$

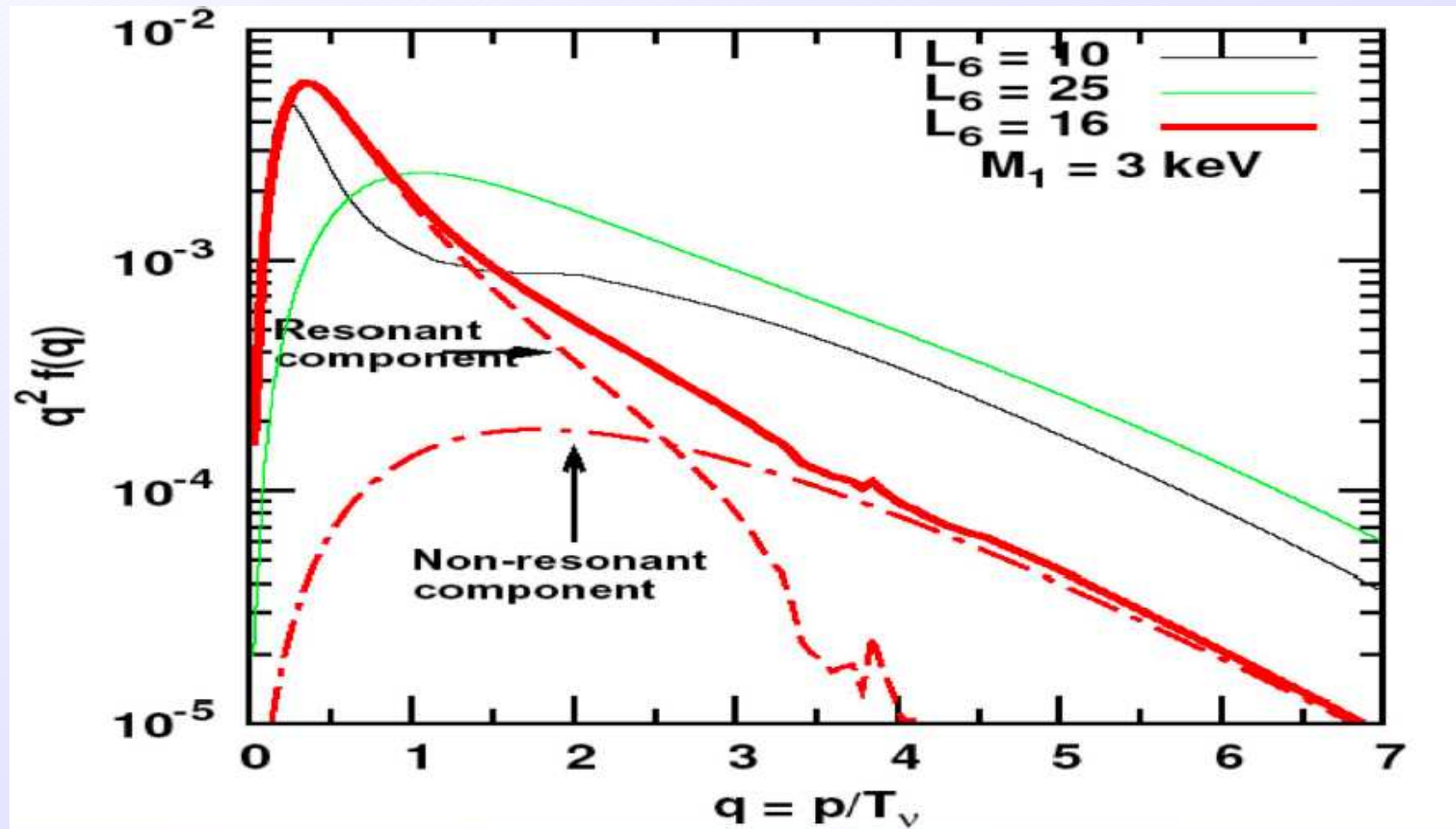
The mixing angle is maximal $\sin^2 2\theta_m = 1$ when the **resonant condition** is satisfied (with $\Delta(p) \equiv m_4^2/(2p)$)

$$\Delta(p) \cos 2\theta - \frac{2\sqrt{2}\zeta(3)}{\pi^2} G_F T^3 \mathcal{L} + |V_T| = 0$$

- The production (both resonant and incoherent) is enhanced with respect to the case of negligible lepton asymmetry and much smaller values of the vacuum mixing angles are required.

6 – Resonant sterile neutrino production

- The resonance enhancement of the conversion happens first for the lowest values of p/T and this results in a final distribution which peaks at much smaller values of p/T , with $\langle p/T \rangle \sim 0.6$.



This is a "cool" dark matter candidate.

7 – Sterile neutrino production summary

A population of keV sterile neutrinos, which constitute the DM, can be produced **out-of equilibrium** in the EU via various mechanisms:

- non-resonant production
- resonantly enhanced oscillations
- low reheating scenario
- boson decays

For a spectrum proportional to the equilibrium one, $p/T \sim 3.15$, while for the resonant production ($p/T \sim 2$) and for a subsequent injection of entropy in the thermal plasma ($p/T \sim 0.7$) it is much colder.

8 – Cosmological constraints

The impact of ν_s in the Early Universe depends critically on their life-time.

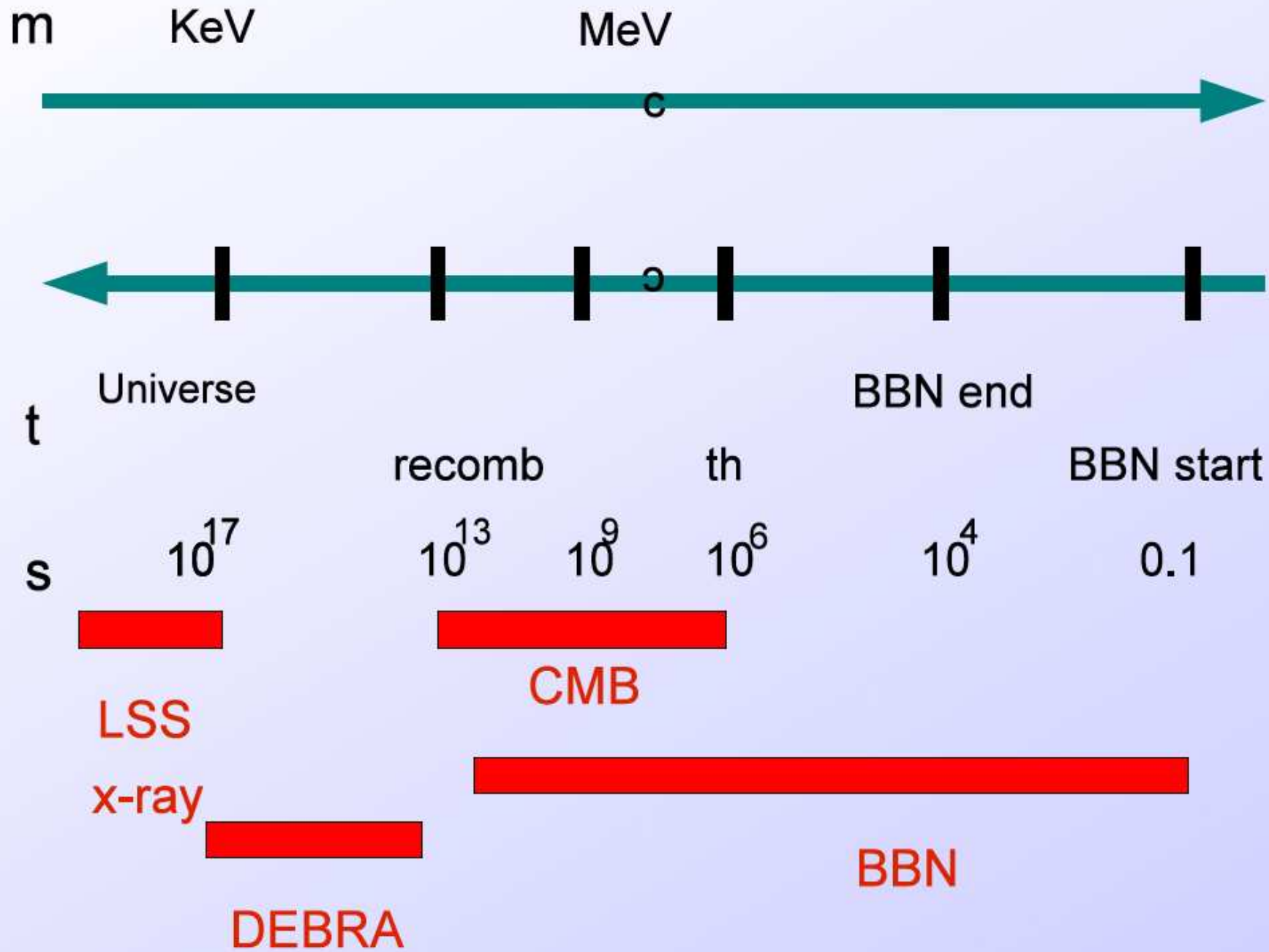
N decay due to their mixing with active neutrinos via CC and NC:

$$\begin{aligned}
 m_N < 1 \text{ MeV}: & \quad N \rightarrow \nu\nu\bar{\nu}; \\
 1 \text{ MeV} < m_N < m_\pi: & \quad N \rightarrow e^+e^-\nu; \\
 m_N > m_\pi^0: & \quad N \rightarrow \pi^0\nu\dots
 \end{aligned}$$

The decay rates is given by:

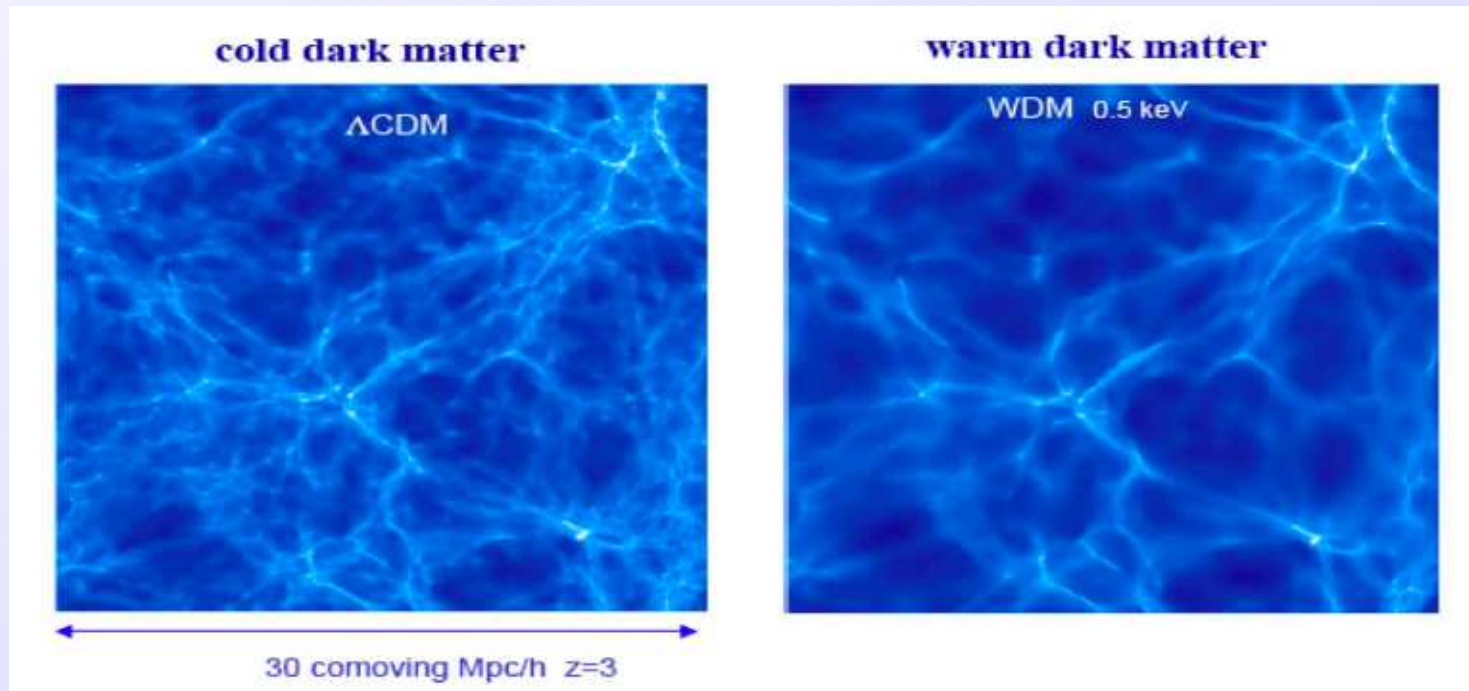
$$\begin{aligned}
 \Gamma^{3\text{-body}} & \propto \frac{1}{768\pi^3} G_F^2 m_N^5 |V_{l4}|^2 \\
 \Gamma^{2\text{-body}} & \propto \frac{1}{16\pi} G_F^2 m_N^3 |V_{l4}|^2
 \end{aligned}$$

8 – Cosmological constraints



9 – Large scale structure formation constraints

KeV sterile neutrinos behave as a **WDM component**: at large scales the formation of structure happens as for CDM but perturbations at small scales get erased due to the free-streaming.



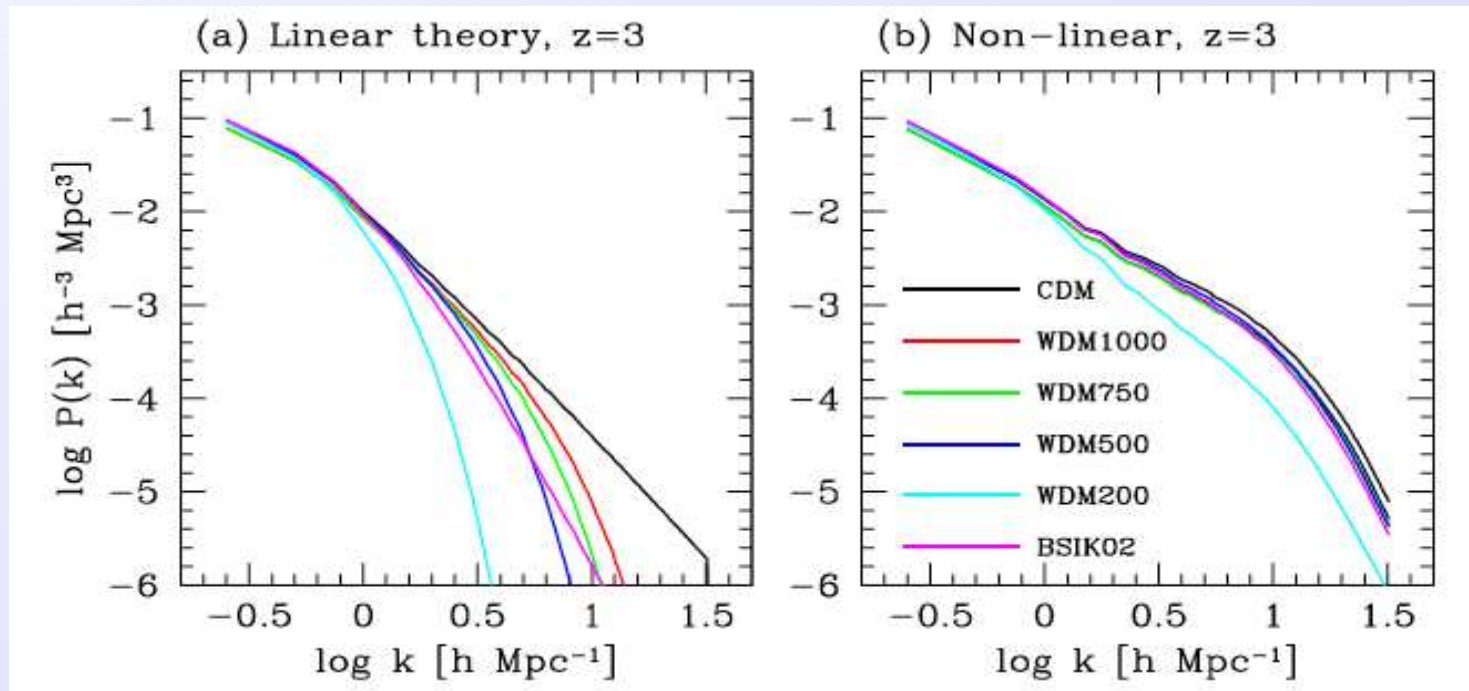
[see, e.g. Haehnelt]

9 – Large scale structure formation constraints

The amount of structures in the Universe at a certain scale is measured by the matter power spectrum $P(k)$:

$$P(k) = \left\langle \left| \frac{\delta\rho_m}{\bar{\rho}_m} \right|^2 \right\rangle$$

This can be probed by observations of the matter distribution in the Universe.



[see e.g. Narayanan et al., 2000]

10 – Indirect searches

- **KeV N_4** : they can decay into $3\nu_a$ and $\nu_a\gamma$ due to the mixing with active neutrinos. [Boehm and Vogel, 1987; Barger et al., 1995; Pal, Wolfenstein, 1982]

The decay rate is given by

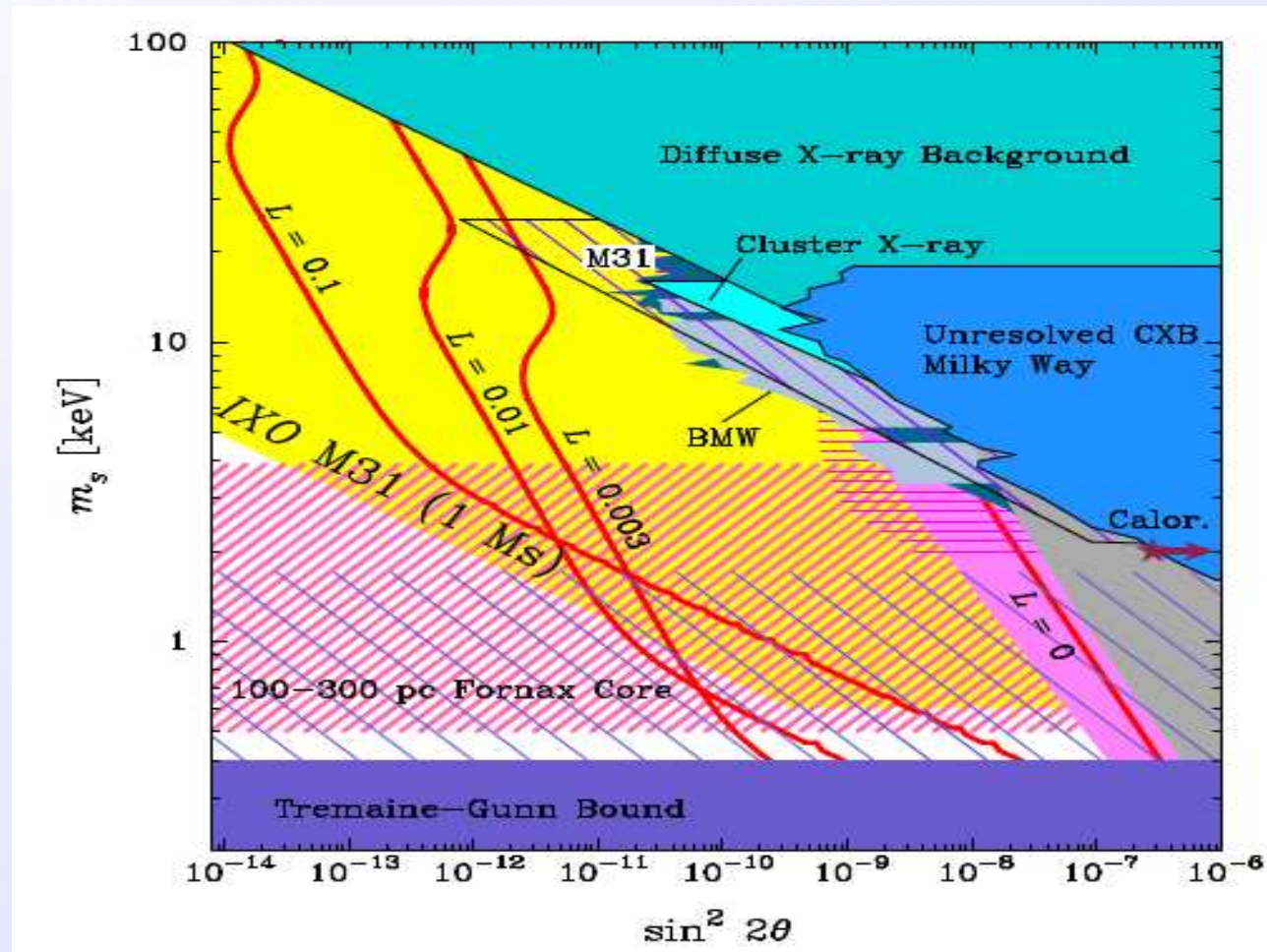
$$\Gamma_{3\nu} \simeq \sin^2 2\theta G_F^2 \frac{m_4^5}{768\pi^3} \sim 10^{-30} \text{s}^{-1} \frac{\sin^2 2\theta}{10^{-10}} \left(\frac{m_4}{\text{keV}} \right)^5$$

$$\Gamma_{\nu\gamma} \simeq \sin^2 2\theta \alpha G_F^2 \frac{9m_4^5}{2048\pi^4} \sim 10^{-32} \text{s}^{-1} \frac{\sin^2 2\theta}{10^{-10}} \left(\frac{m_4}{\text{keV}} \right)^5$$

The photon from the decay carries away an energy $E_\gamma = \frac{m_4}{2}$

Searches of a γ line in X-ray observatories which point towards objects with DM overdensities (dwarf galaxies, M31, the center of our galaxy) or DEBRA.

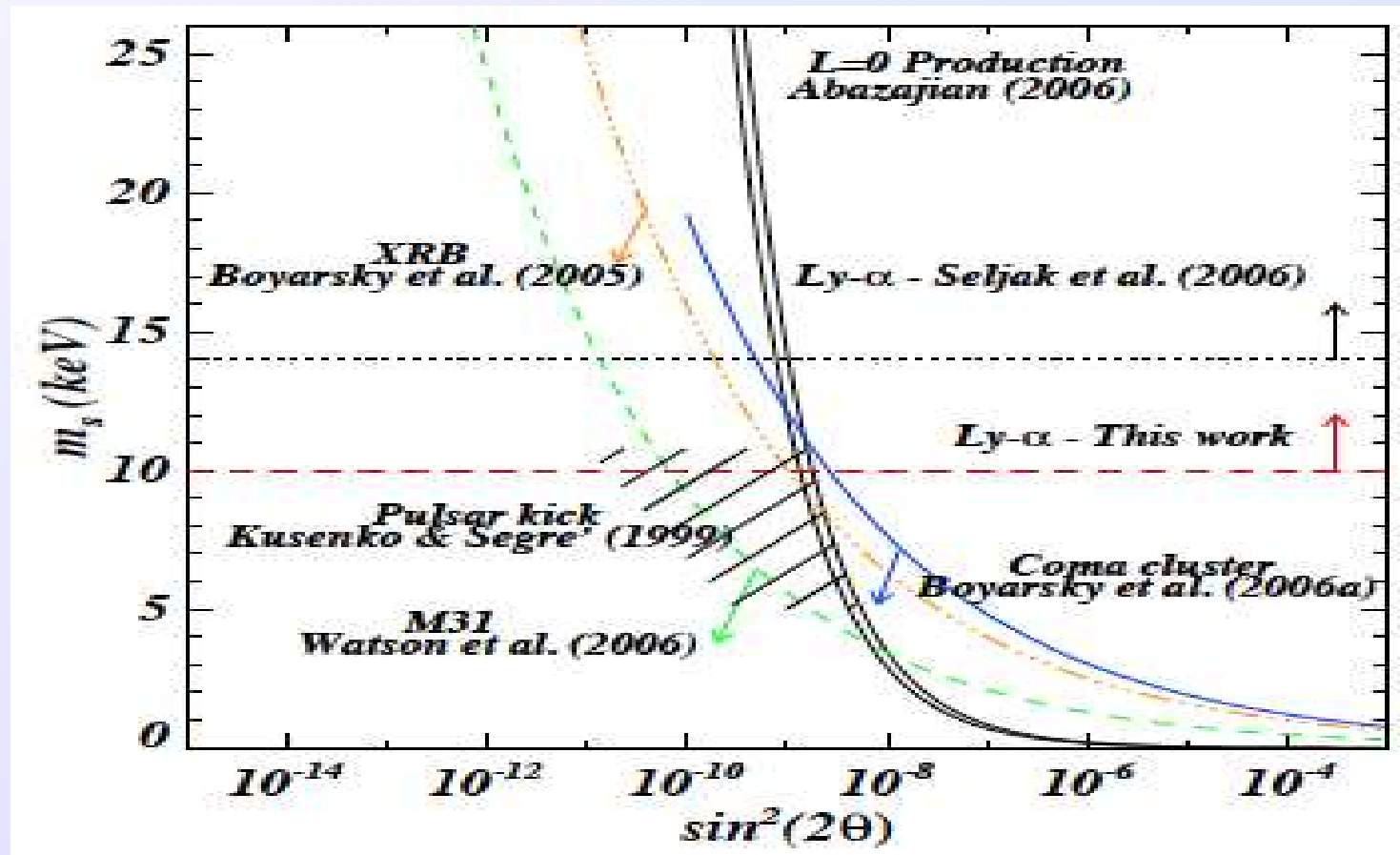
10 – Indirect searches



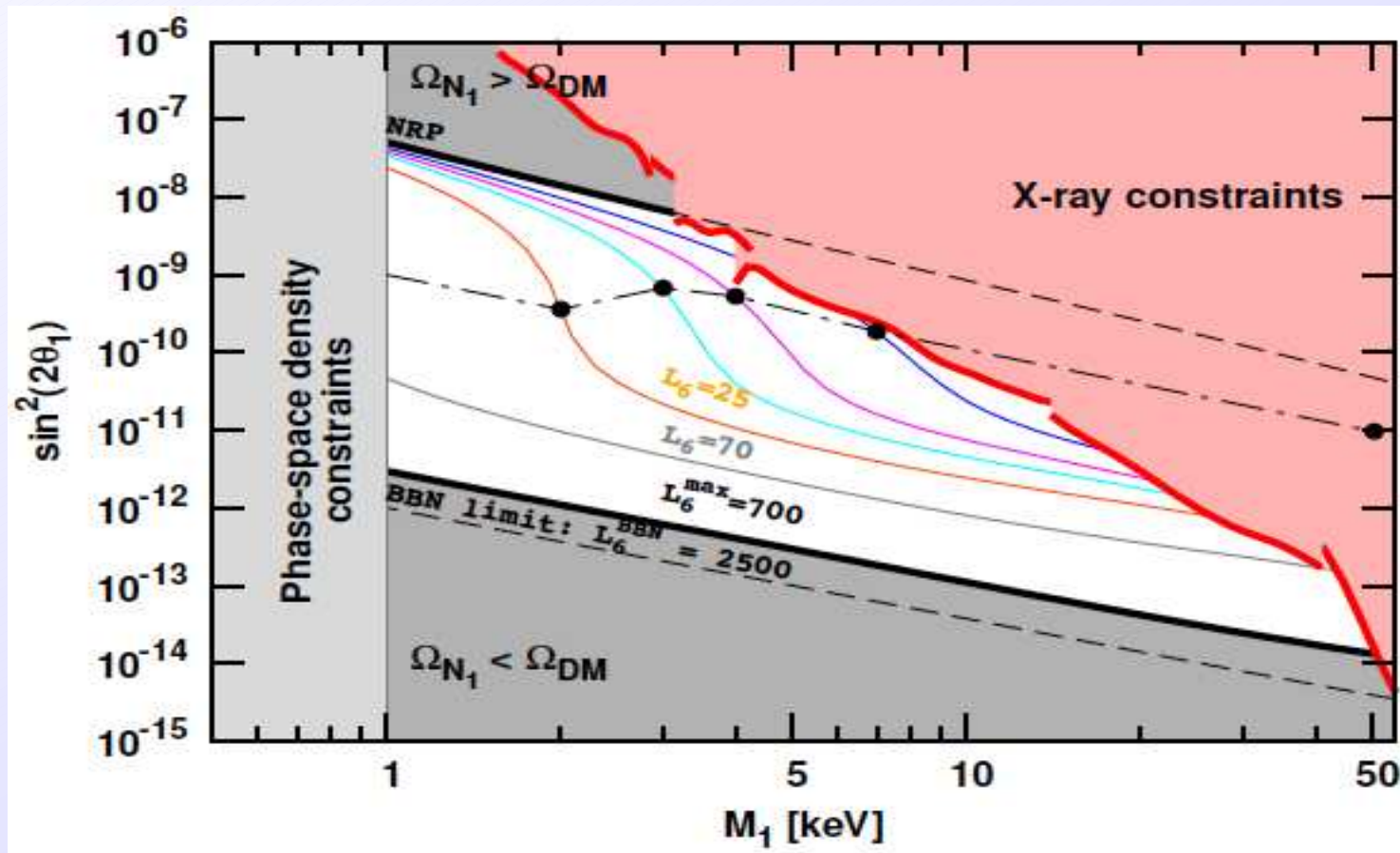
[Abazajian, 0903.2040; See also Abazajian, Fuller, Patel, 2001; Abazajian, Fuller, Tucker, 200; Dolgov, Hansen, 2002; Boyarsky et al., 2006, 2007; Loewenstein et al., 2008; Watson et al., 2006]

Tension between LSS and x-ray constraints

For a WDM sterile neutrino from DW non-resonant production, the lower bound on m_4 from LSS observations and the upper bound coming from x-ray searches exhibit a strong tension.

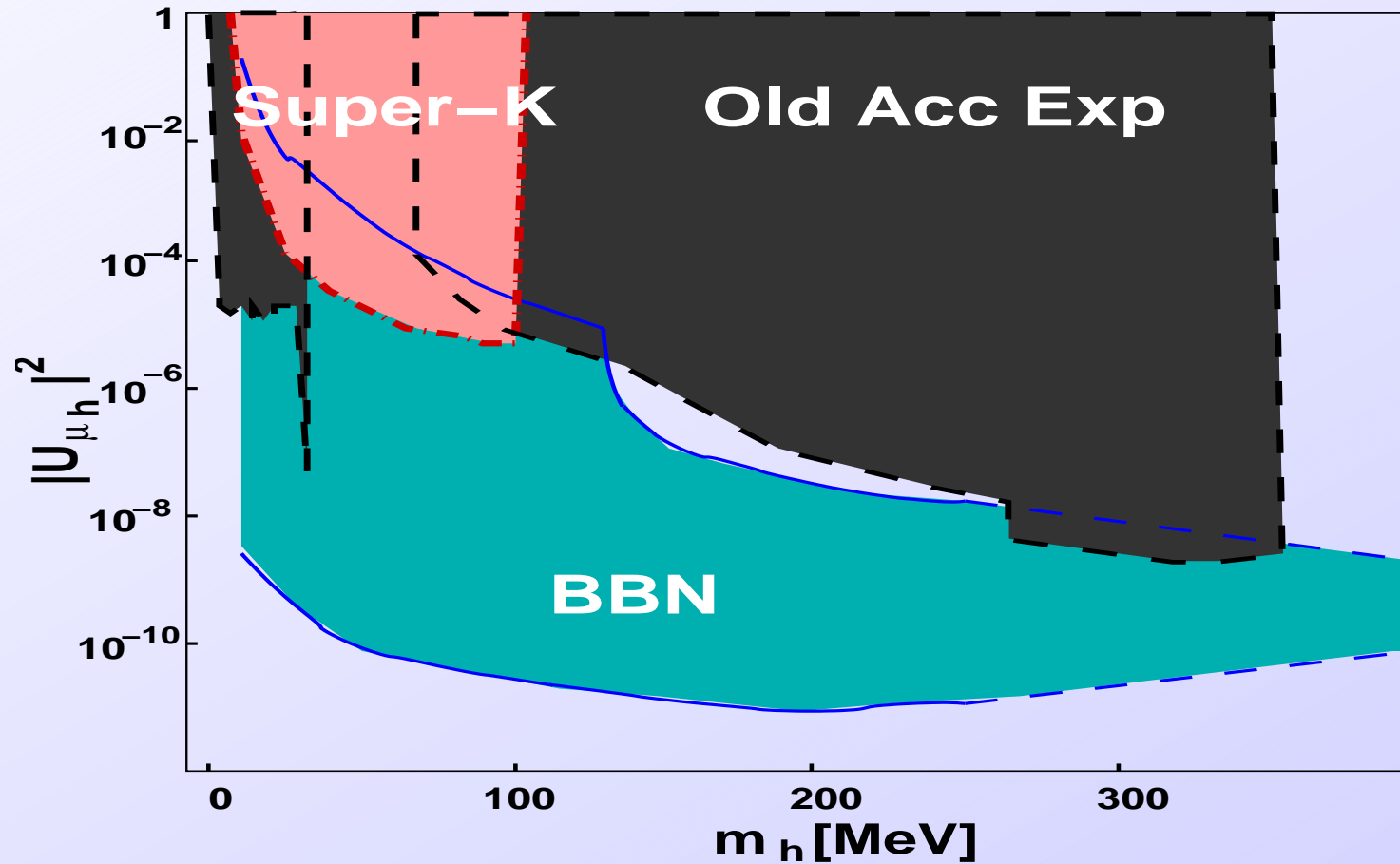


This tension can be avoided if a colder DM component is present or if the spectrum is peaked at smaller p (as in RP or boson decay).



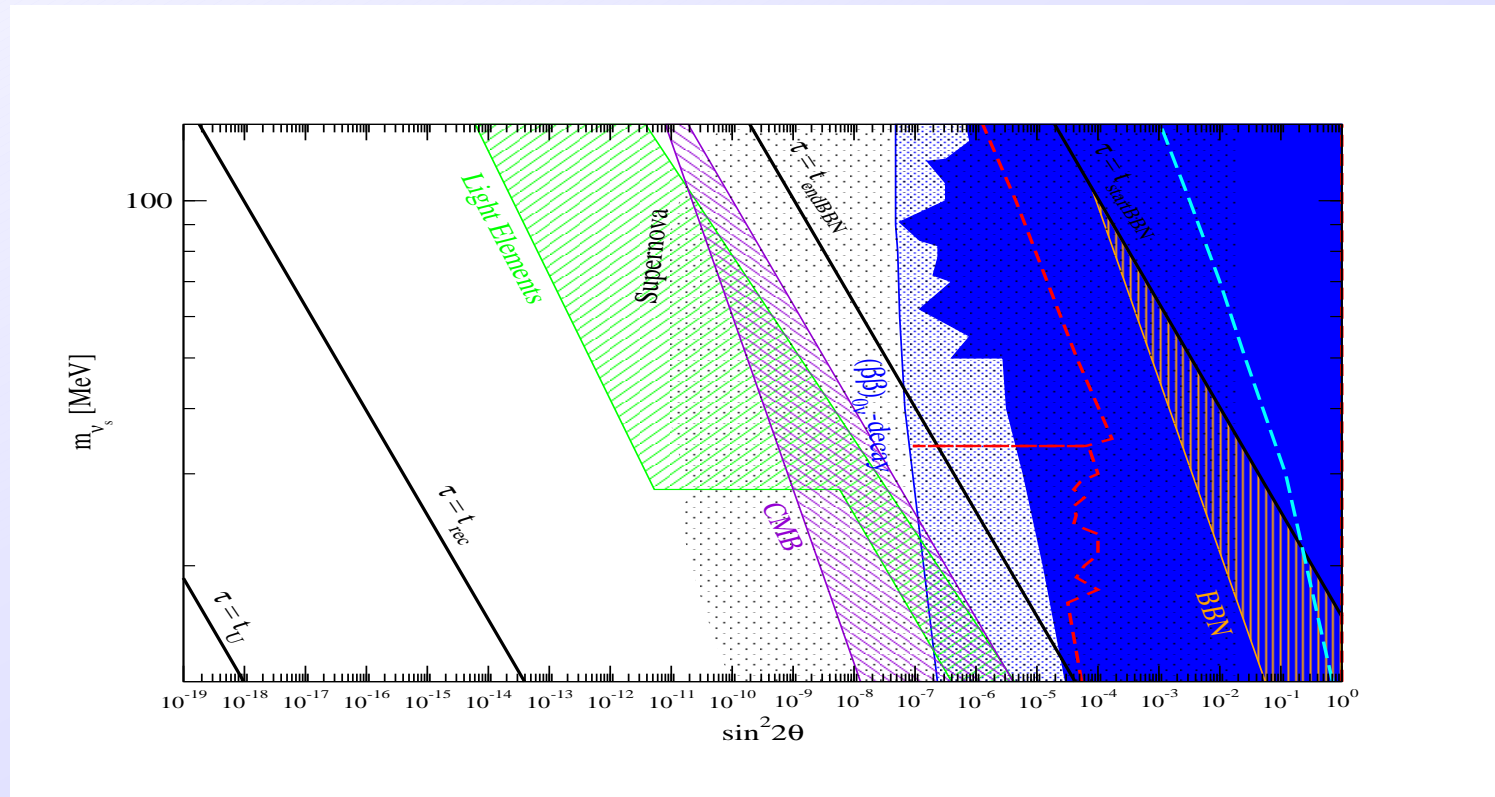
[Boyarsky et al., 2009]

- **MeV N_4** : they produce copious amounts of γ ($N_4 \rightarrow \pi^0 \nu \rightarrow \gamma \gamma \nu$) and affects BBN.



These bounds can be avoided if

- sterile neutrinos couple predominantly to other light non-standard model particles;
- the production in the EU was suppressed by a low reheating temperature.



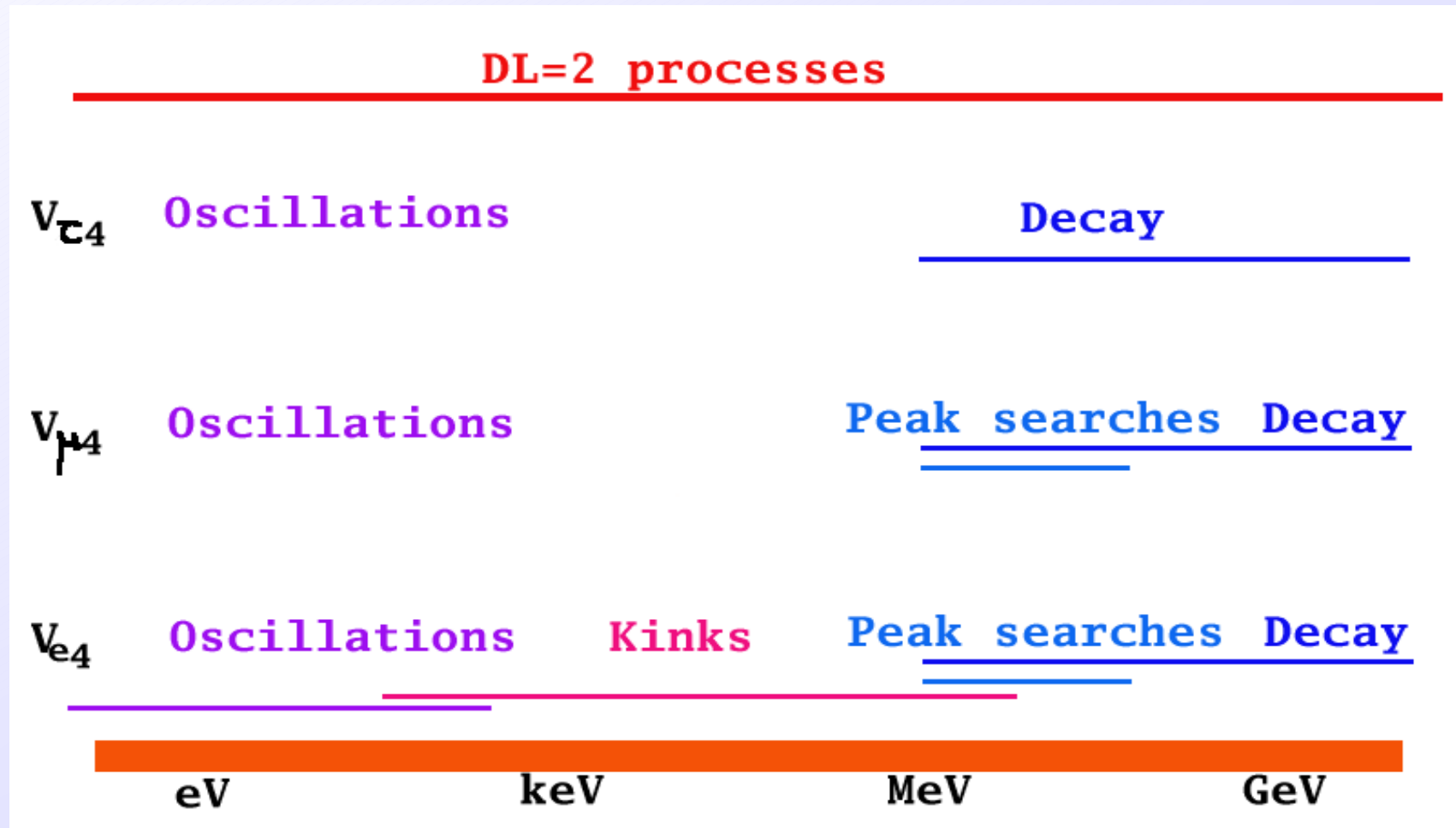
[Gelmini, Osoba, Palomares-Ruiz, SP, 2008]

11 – Summary of cosmological constraints

- Sterile neutrinos in the KeV range are a viable WDM (to CDM) candidate.
- If produced via non-resonant oscillations, tension between LSS constraints and x-ray searches.
- Other mechanisms of production (with smaller mixing angles and/or colder spectra) avoid this tension.

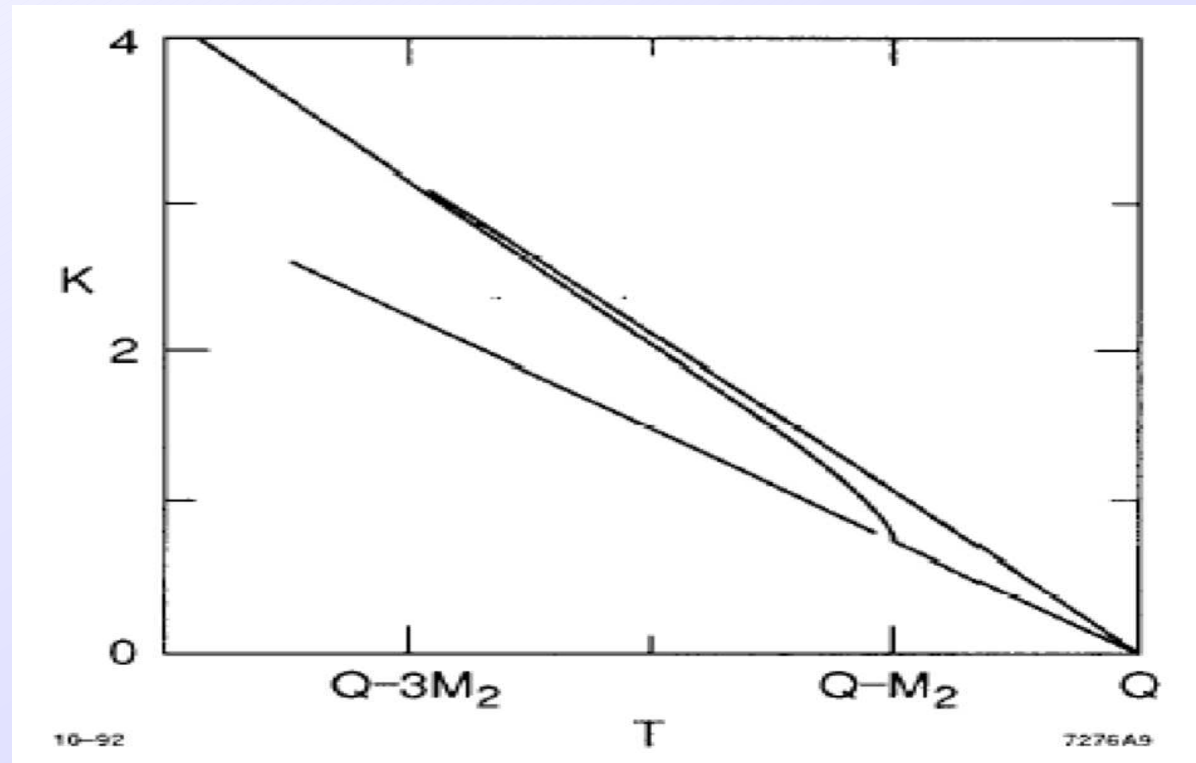
12 – Laboratory constraints

The signatures of N depend on its mass:

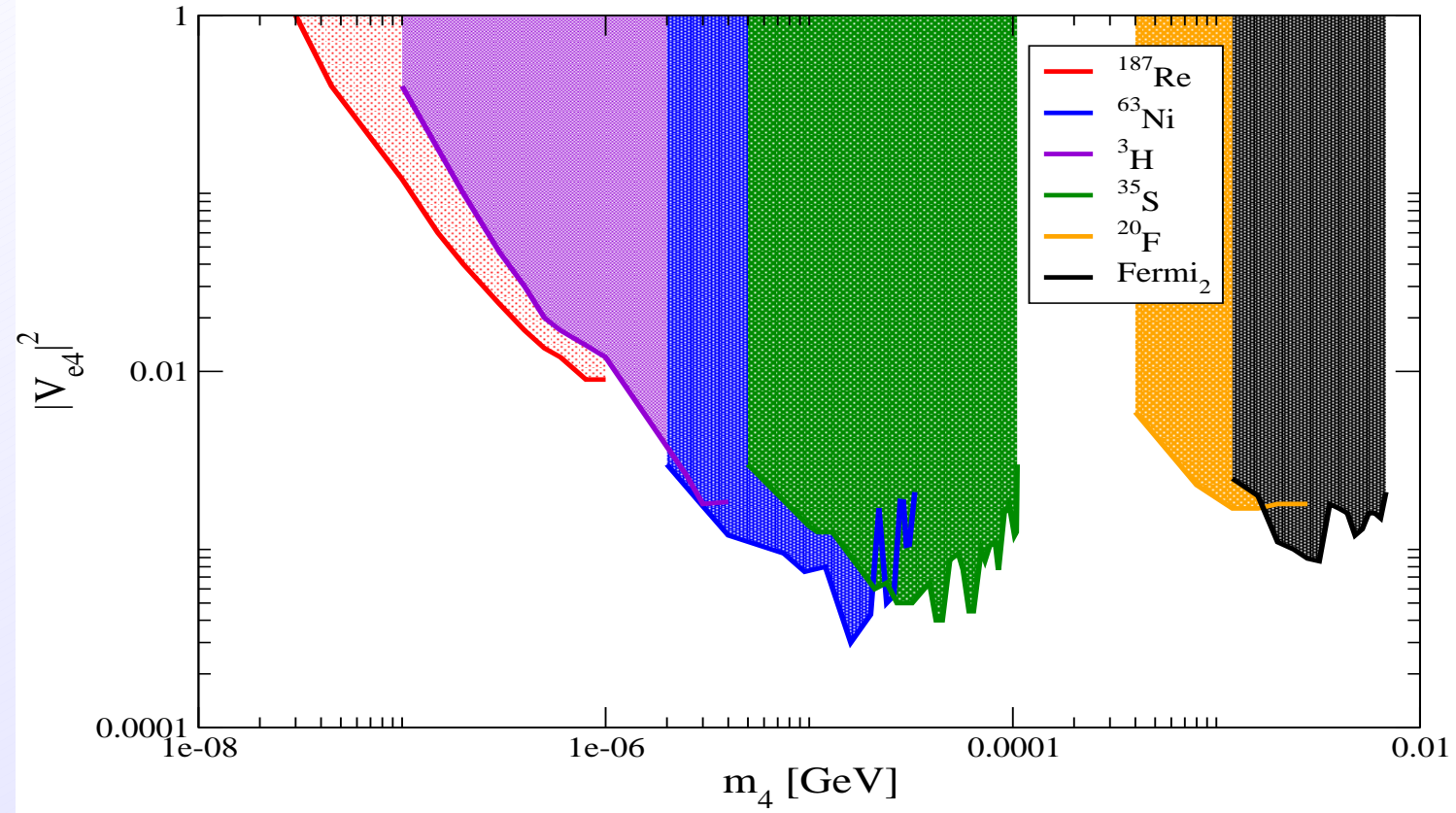


Kinks in the electron β -spectrum

For masses $m \sim 10 \text{ eV} - 1 \text{ MeV}$, the search of **kinks** in the β - spectrum is very sensitive to an admixture of heavy neutrinos in ν_e .



12 – Laboratory constraints

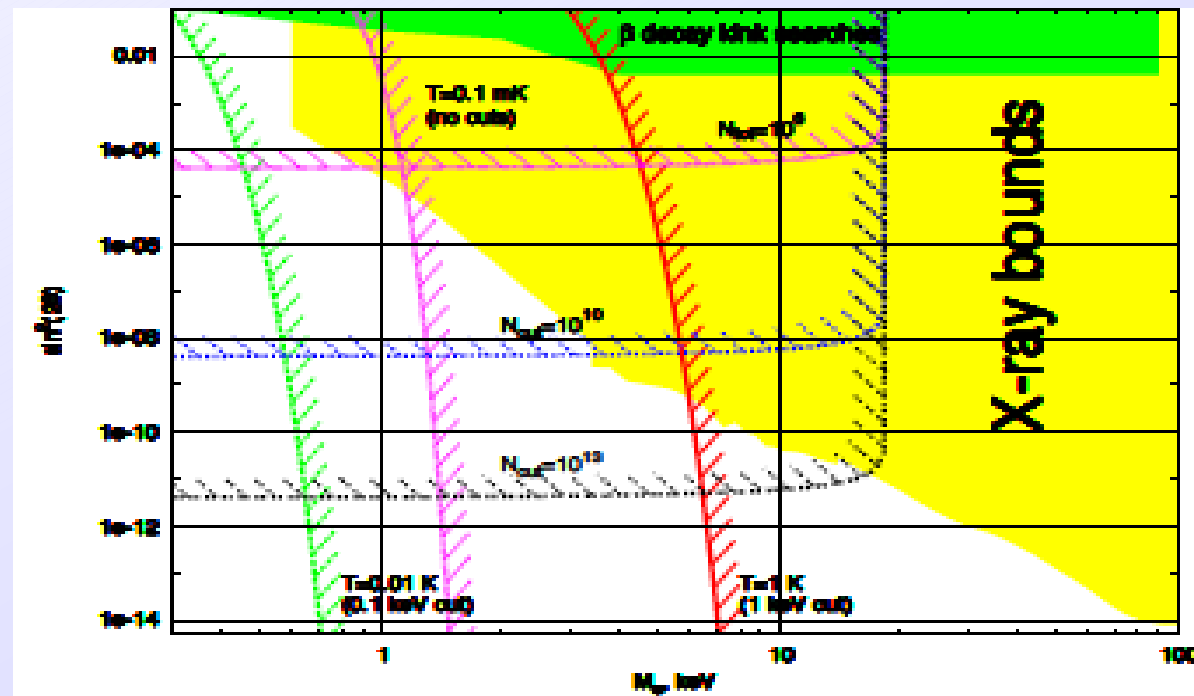


[Atre, Han, SP, Zhang, 2009]

Full kinematics in β -decay

A recent proposal is to study the full kinematics of the beta decay by reconstructing

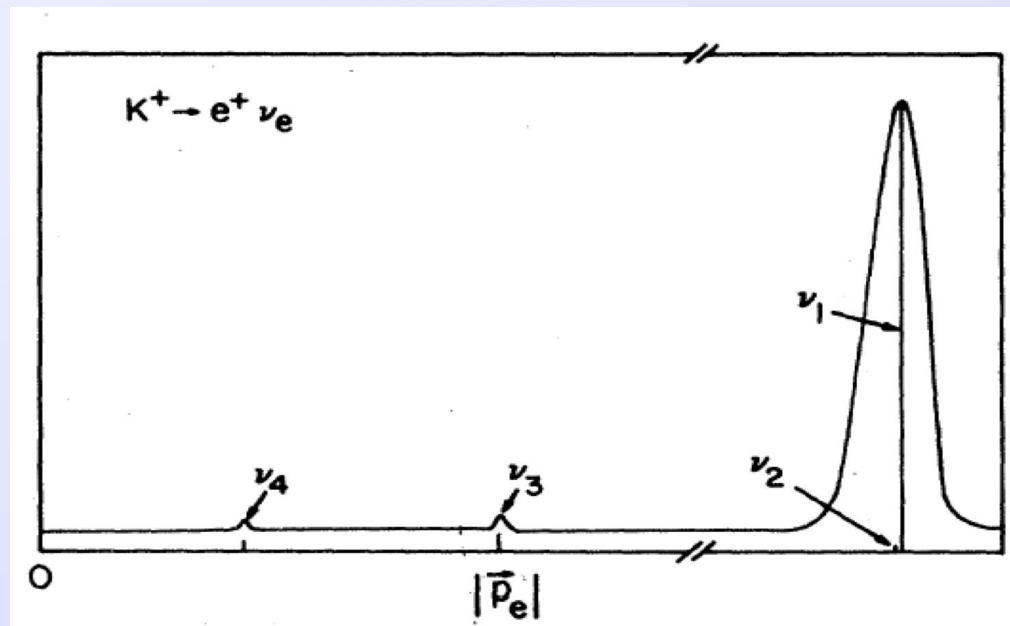
- the momentum of the initial and final ions
- the momentum of the emitted electron

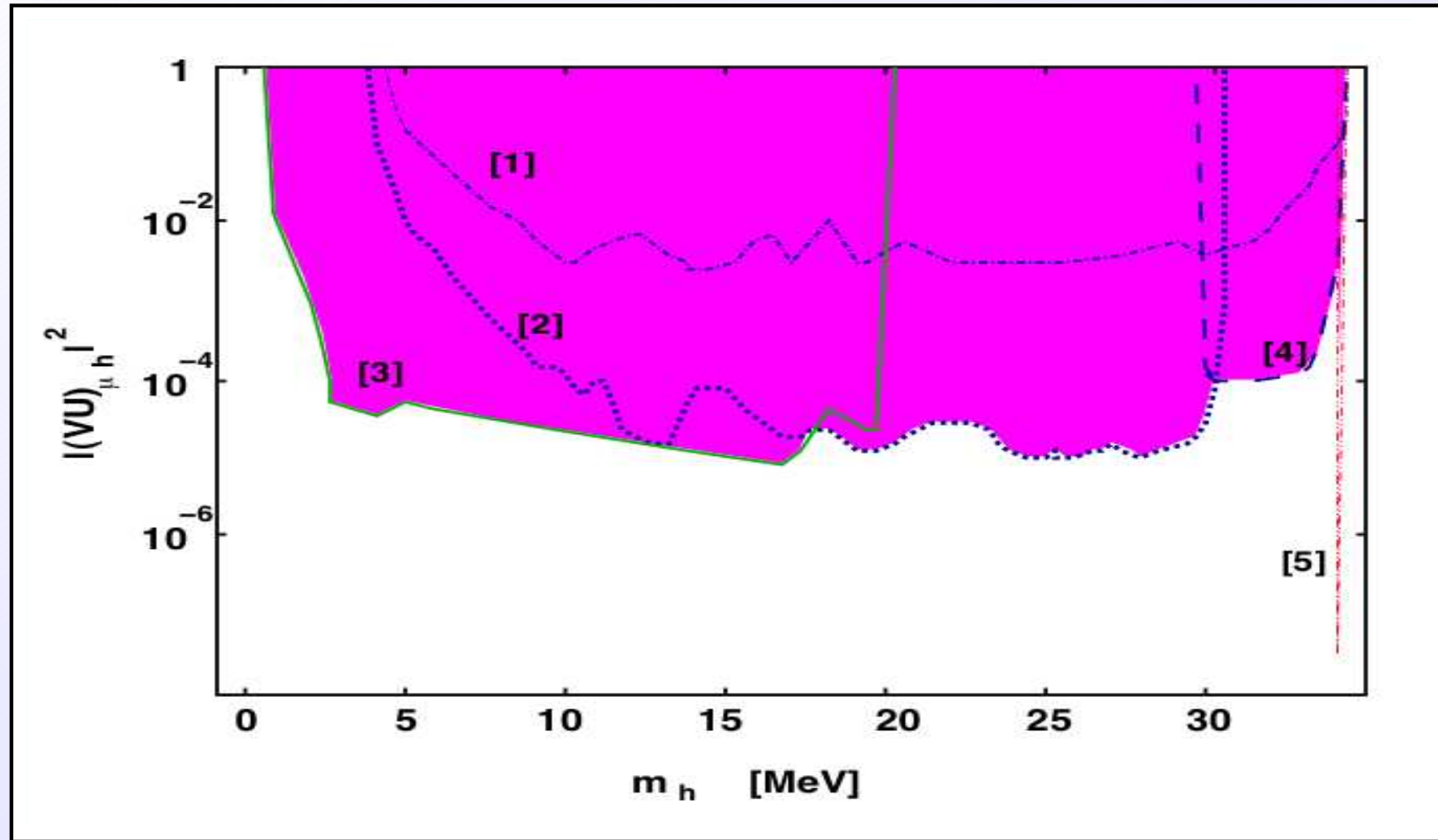


13 – Peak searches

If a heavy neutrino mixes with $\nu_{e,\mu}$, it would modify the **spectrum** of e and μ in **meson decays**. For ex., in $\pi \rightarrow \mu\nu_\mu$ a peak would appear at

$$T_i = (m_\pi^2 + m_\mu^2 - 2m_\pi m_\mu - m_{\nu_i}^2) / 2m_\pi,$$





[Kusenko, Pascoli and Semikoz, JHEP **0511** (2005) 028]

And similar bounds are available from K -decays.

14 – N -decays searches

N **decay** due to their mixing with active neutrinos via CC and NC
($N \rightarrow e^+ e^- \nu$, $N \rightarrow \pi^0 \nu \dots$).

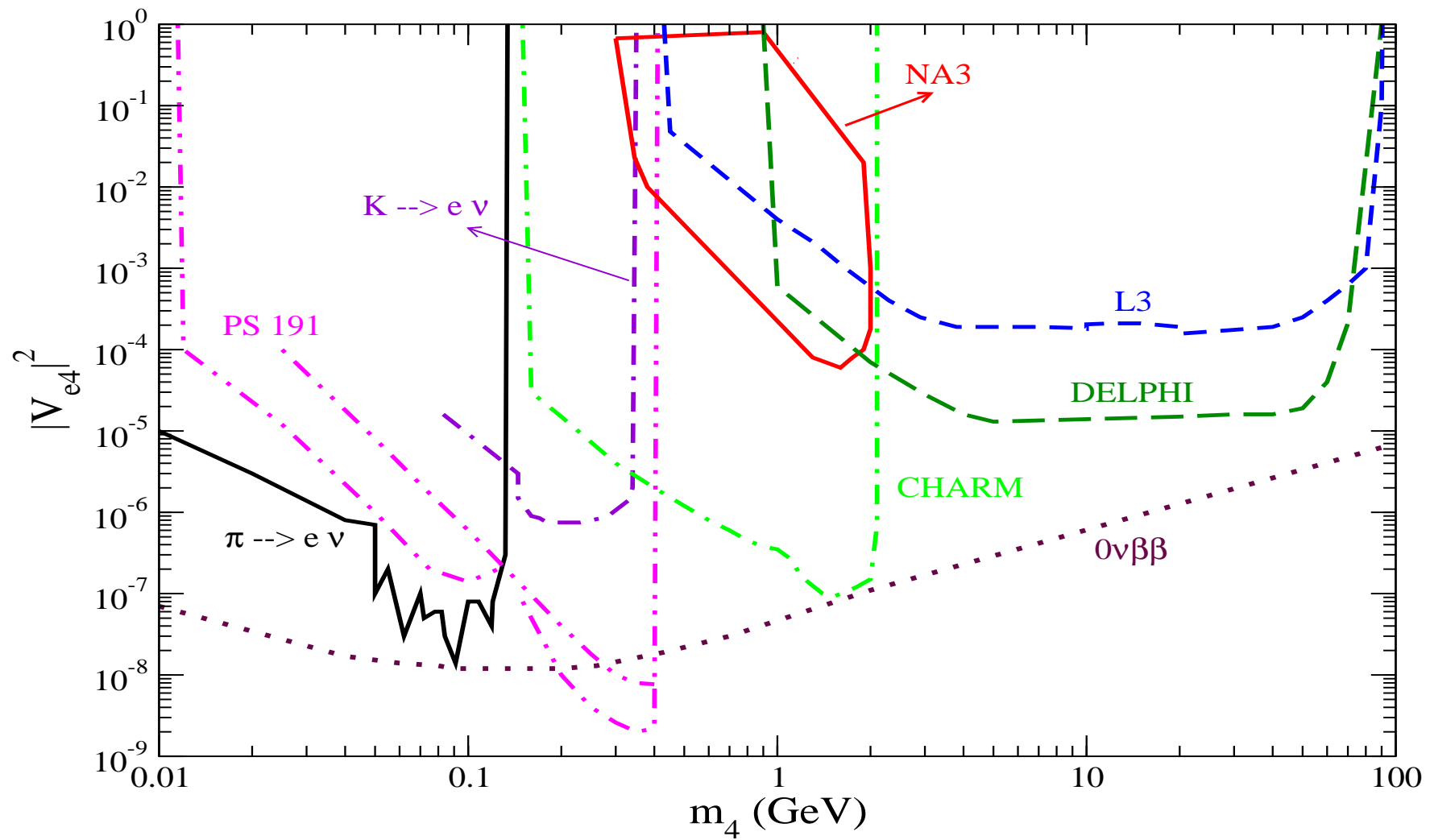
Experimentally: i) N_4 produced with $P = |V_{l4}|^2$ in ν -beams and colliders

ii) N_4 decays with $\Gamma_N(m_4, V_{l4})$

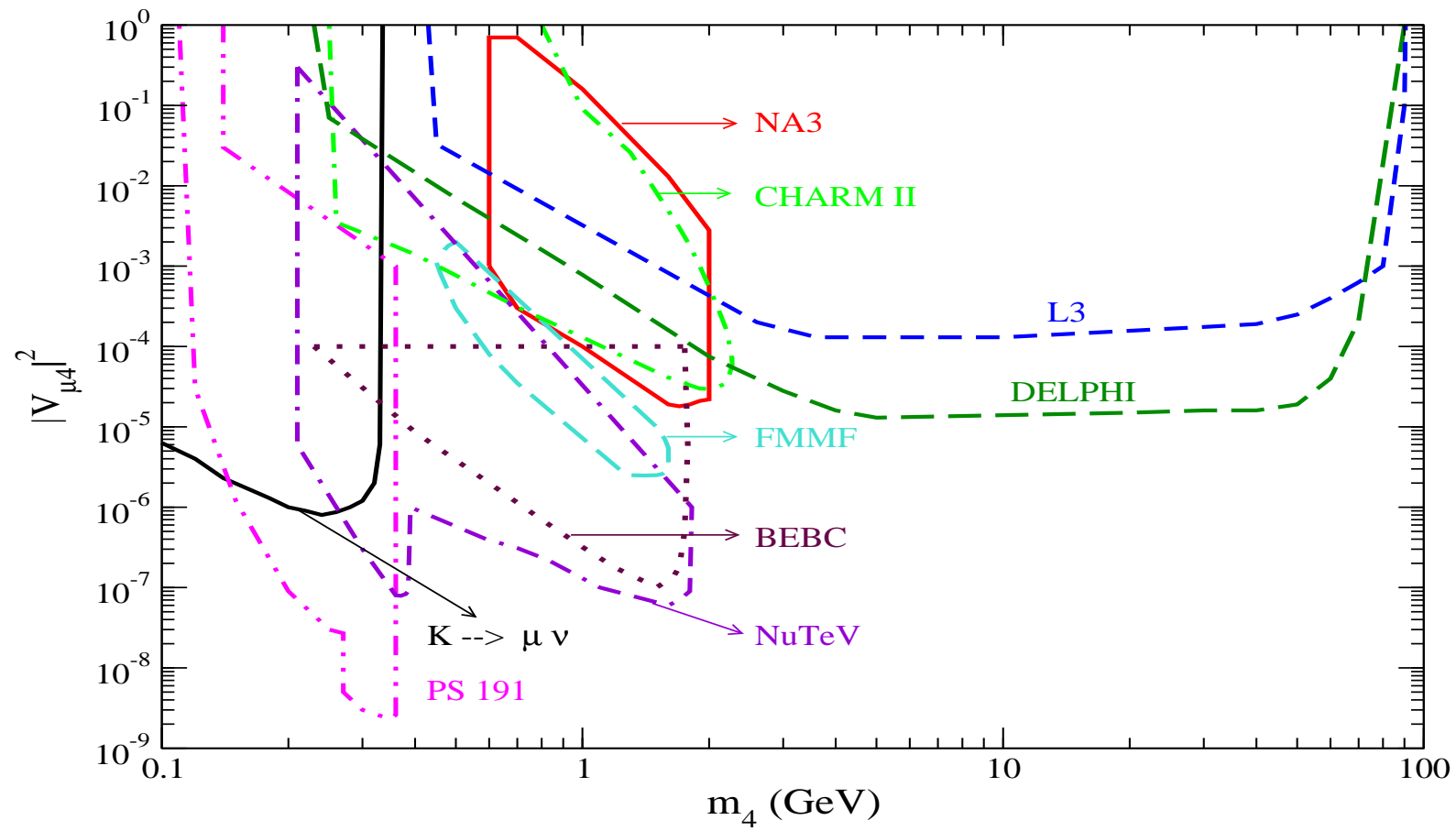
iii) search for SM decay products

- These bounds are less reliable than peak searches because in presence of non-SM decay channels they could be weakened and evaded.

- Present and future neutrino facilities could improve on these bounds, due to the large ν flux (MINOS, T2K and superbeams, neutrino factory).



[Atre et al., arXiv:0901.3589 [hep-ph]]



15 – Electroweak precision tests

- The presence of N affects processes below their mass threshold (due to breaking of unitarity in U). **Universality tests** allow to put constraints $|V_{\ell 4}|^2 < \text{few} \times 10^{-3}$.
- Indirect limits come from **lepton flavour violation processes** searches, as $\mu \rightarrow e\gamma$.

$$\text{Br}(\mu \rightarrow e\gamma) \simeq \frac{3\alpha}{8\pi} \left| \sum_{m'} V_{em'} V_{\mu m'}^* g(m_N) \right|^2 ,$$

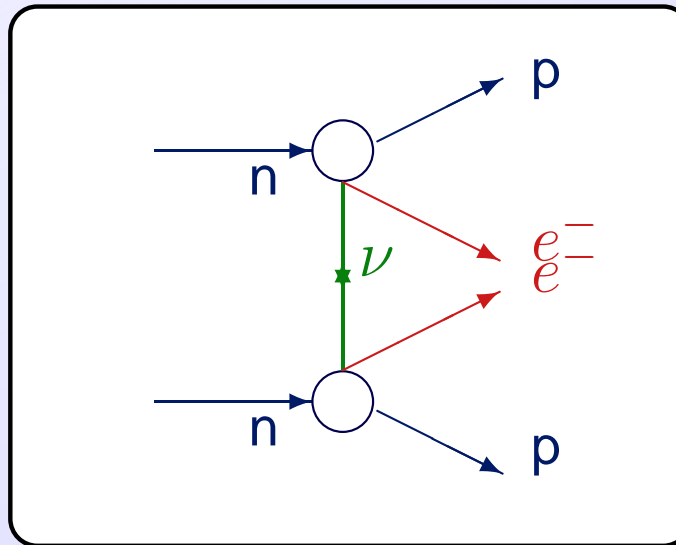
$\text{Br}(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11}$ implies

$$|V_{e4} V_{\mu 4}| < 0.015 [1 \times 10^{-5}] \text{ for } m_N = 10 \text{ GeV} [1000 \text{ GeV}] .$$

16 – $\Delta L = 2$ processes

$\Delta L = 2$ processes are sensitive to massive **Majorana** neutrinos. By far, the most sensitive is **neutrinoless double beta decay**:

$$(A, Z) \rightarrow (A, Z + 2) + 2e^-.$$



The **half-life time**, $T_{0\nu}^{1/2}$, of the $(\beta\beta)_{0\nu}$ -decay depends on

- small m_N : $\Gamma \propto (m_N \sin^2 \theta)^2$
- large m_N : $\Gamma \propto (m_N^{-1} \sin^2 \theta)^2$

For $m_N \ll 100$ MeV: $\left[T_{0\nu}^{1/2} \right]^{-1} \propto |M_F - g_A^2 M_{GT}|^2 |\langle m \rangle|^2$

- $|\langle m \rangle|$ **is the effective Majorana mass parameter:**

$$|\langle m \rangle| \equiv \left| \sum_{\text{light}} m_i U_{ei}^2 + m_4 |U_{e4}|^2 e^{i\alpha_{41}} \right|,$$

- The present best limit on $|\langle m \rangle|$ reads:

$$|\langle m \rangle| < (350 - 1050) \text{ meV} \quad \textbf{Heidelberg-Moscow}$$

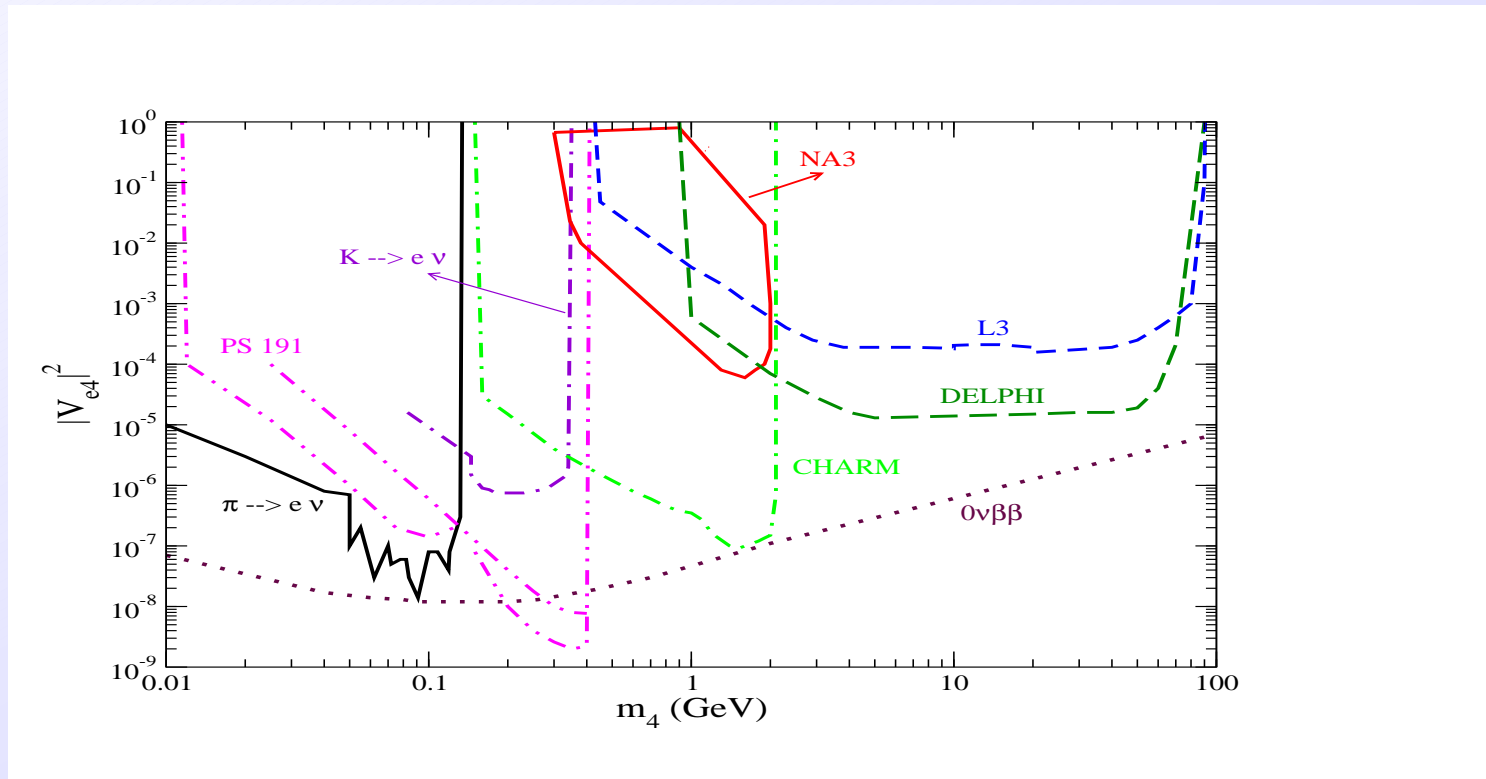
$$|\langle m \rangle| < (680 - 2800) \text{ meV} \quad \textbf{NEMO3}$$

$$|\langle m \rangle| < (200 - 1050) \text{ meV} \quad \textbf{CUORICINO}$$

- A claim of $(\beta\beta)_{0\nu}$ decay discovery has been published [Klapdor-Kleingrothaus et al. 2004, 2006]. It implies $|\langle m \rangle| \simeq 200 - 600$ meV .
- Future sensitivities to $|\langle m \rangle| \sim 10 - 30$ meV , (CUORE, Majorana, SuperNEMO, EXO, GERDA, COBRA, NEXT).

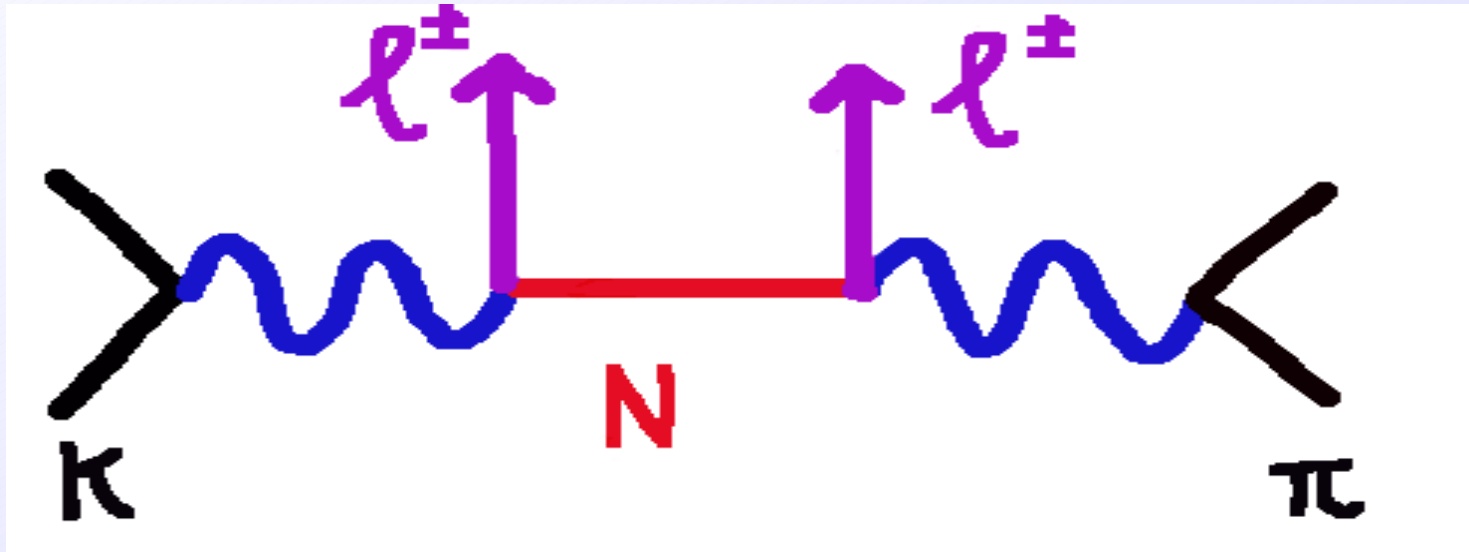
- These bounds translate into a limit on $\sin \theta$ as a function of m_4 :

$$\sin^2 \theta \lesssim \frac{|\langle m \rangle|}{m_4} \sim 10^{-4} \frac{3 \text{ keV}}{m_4}$$



- For more than one sterile ν , it is possible to have cancellations among different terms and the bounds can be strongly weakened.

For heavy masses, $m_4 \sim 100 \text{ MeV} - 1 \text{ GeV}$, $\Delta L = 2$ processes, as rare K and τ decays can be resonantly enhanced.



The intermediate N state is a real particle which propagates before decaying. It may exit the detector and therefore give no positive signal with probability

$$P = 1 - \exp(-L_{\text{exp}}\Gamma_N)$$

We give an estimated weaking factor for the mixing angle

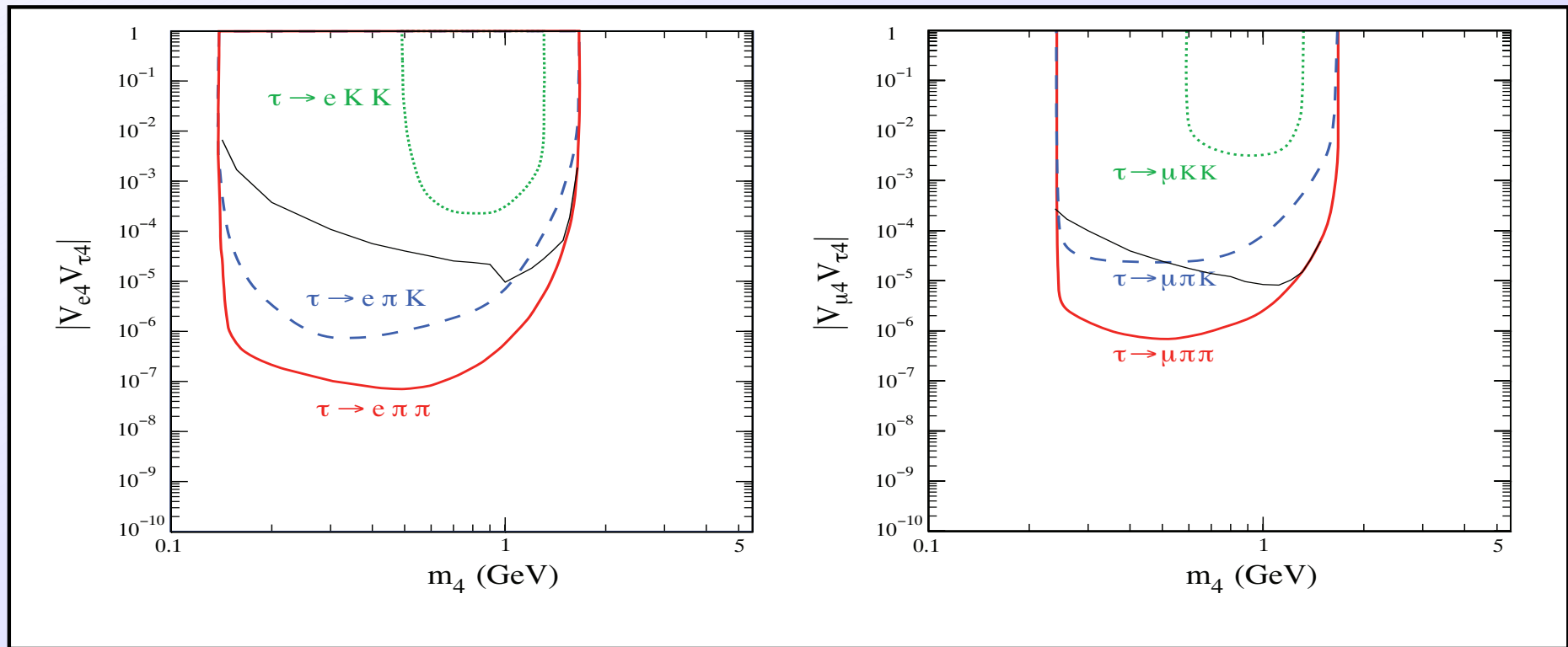
$$|V_{e4}V_{\tau4}| (= |V_{e4}|^2) = \sqrt{|V_{e4}|_\infty^2 / (L_{\text{exp}}\Gamma_{N0})}$$

LNV τ decays: $\tau^- \rightarrow \ell^+ M^- M^-$

The Branching ratio can be approximated as:

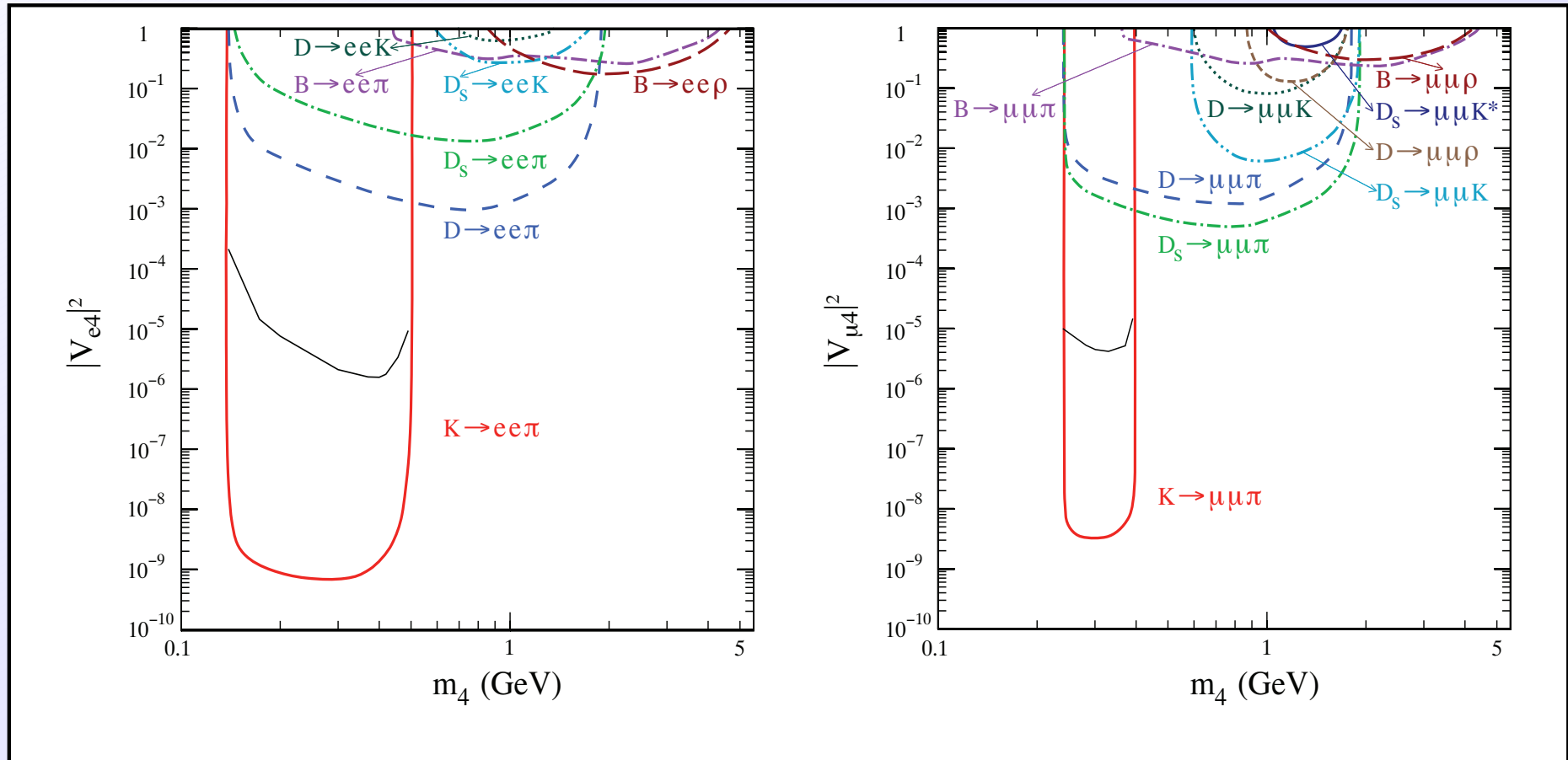
$$\text{Br} \sim G_F^2 f_{M_1}^2 f_{M_2}^2 |V_{M_1}^{CKM} V_{M_2}^{CKM}|^2 |V_{\tau 4} V_{\ell 4}|^2 \left(\frac{m_N}{\Gamma_{N_4}} \right),$$

$$\sim 10^{-3} \left(\frac{\text{GeV}}{m_N} \right)^4 |V_{\tau 4} V_{\ell 4}|$$



LNV meson decays

$$M^- \rightarrow \ell^+ \ell^+ M^-$$

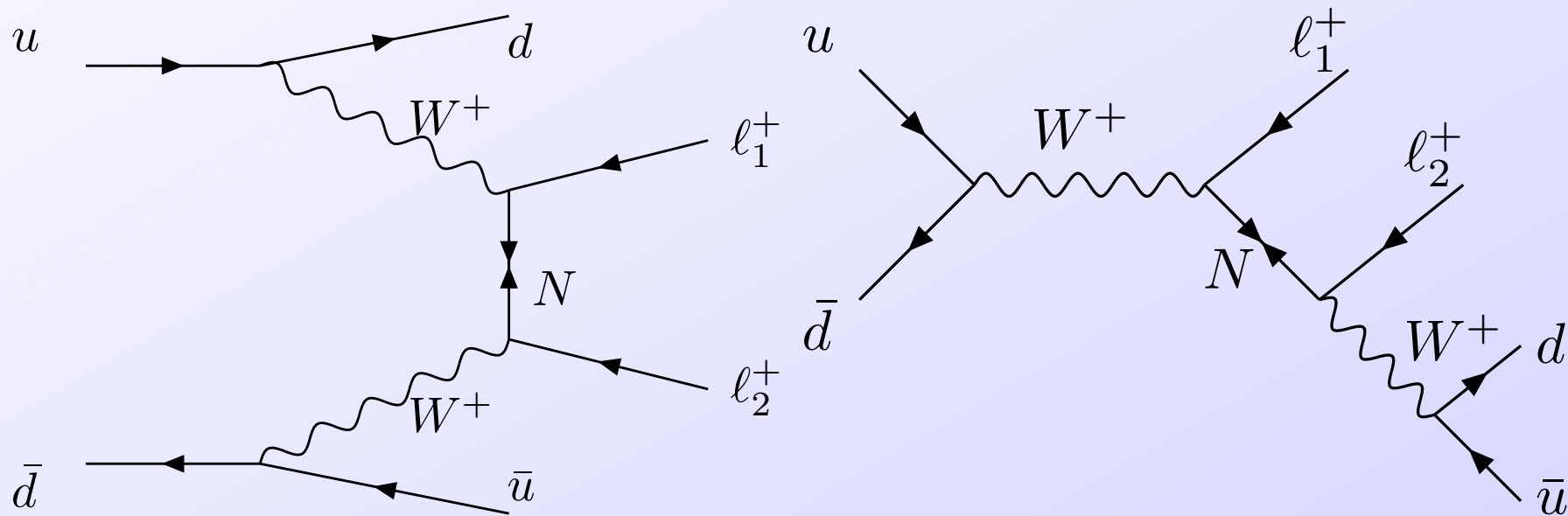


[Atre et al., arXiv:0901.3589 [hep-ph]]

17 – Collider signatures

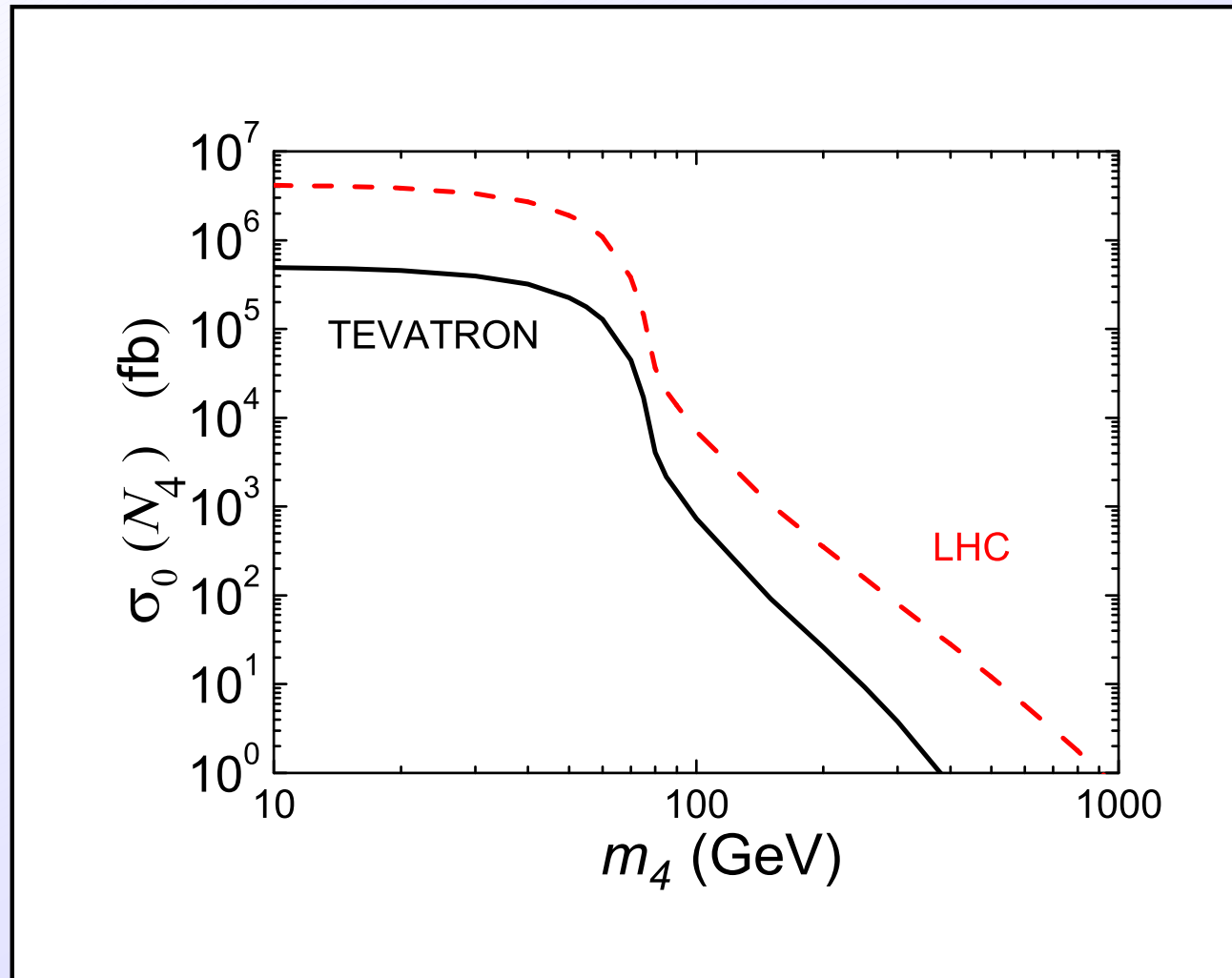
Search for **like-sign di-leptons** at Tevatron and LHC.

[Keung, Senjanovic PRL50; Dicus et al., PRD44; Datta et al., PRD50...]

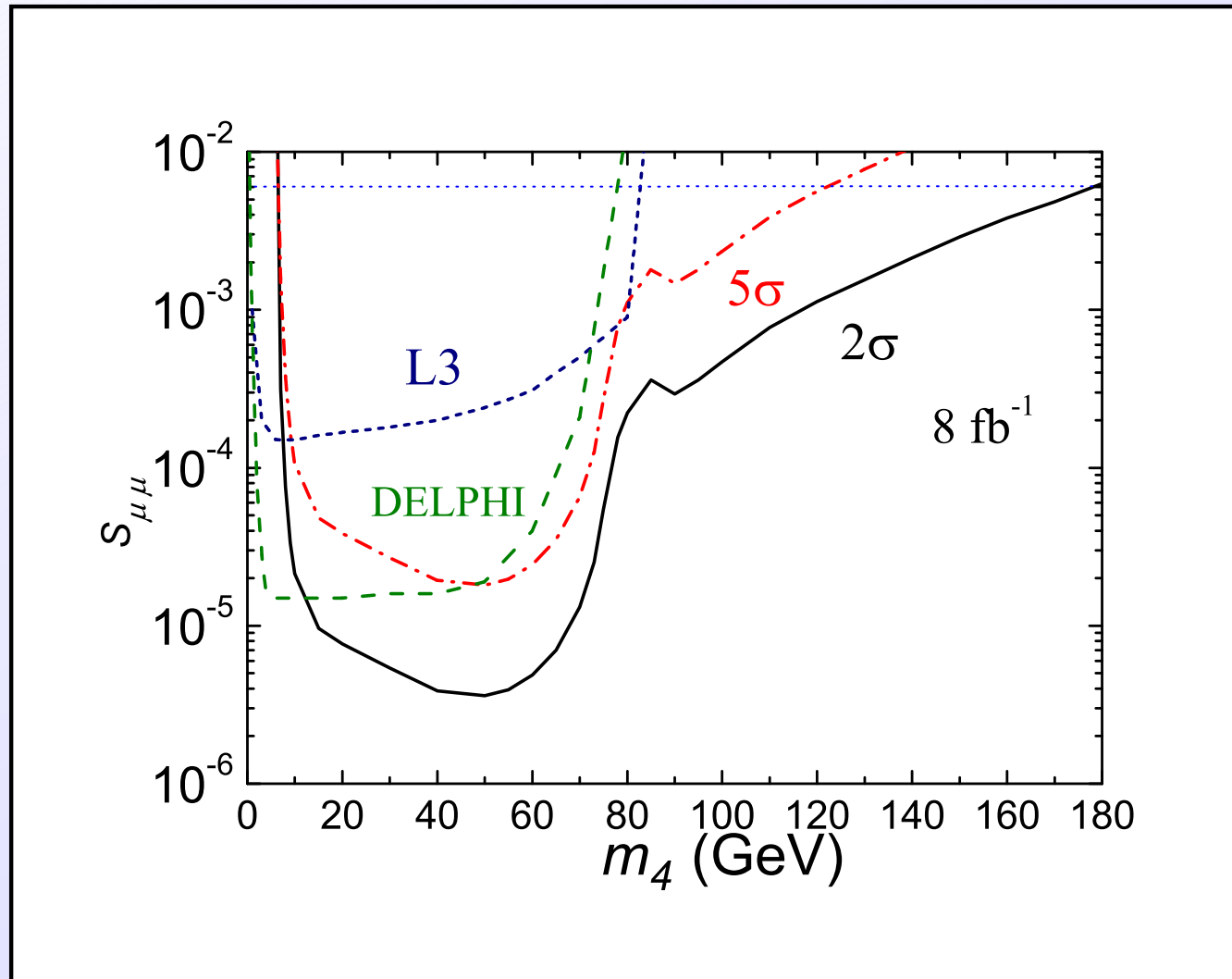


An excellent approximation for the cross section is

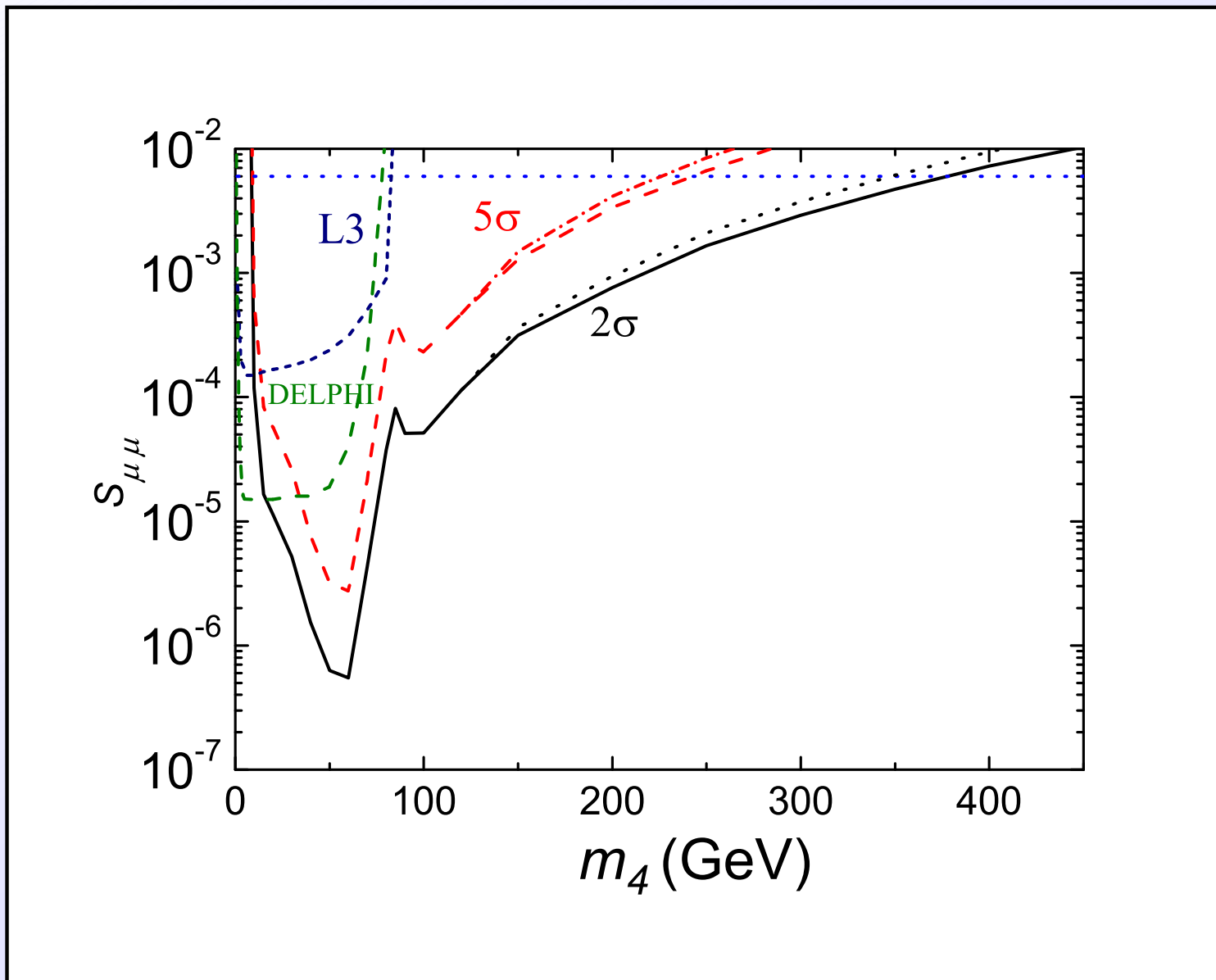
$$\sigma(pp \rightarrow \ell\ell W) \approx \sigma(pp \rightarrow \ell N_4) Br(N_4 \rightarrow \ell W) \simeq \frac{|V_{\ell_1 4} V_{\ell_2 4}|^2}{\sum_{\ell=e}^{\tau} |V_{\ell 4}|^2} \sigma_0$$



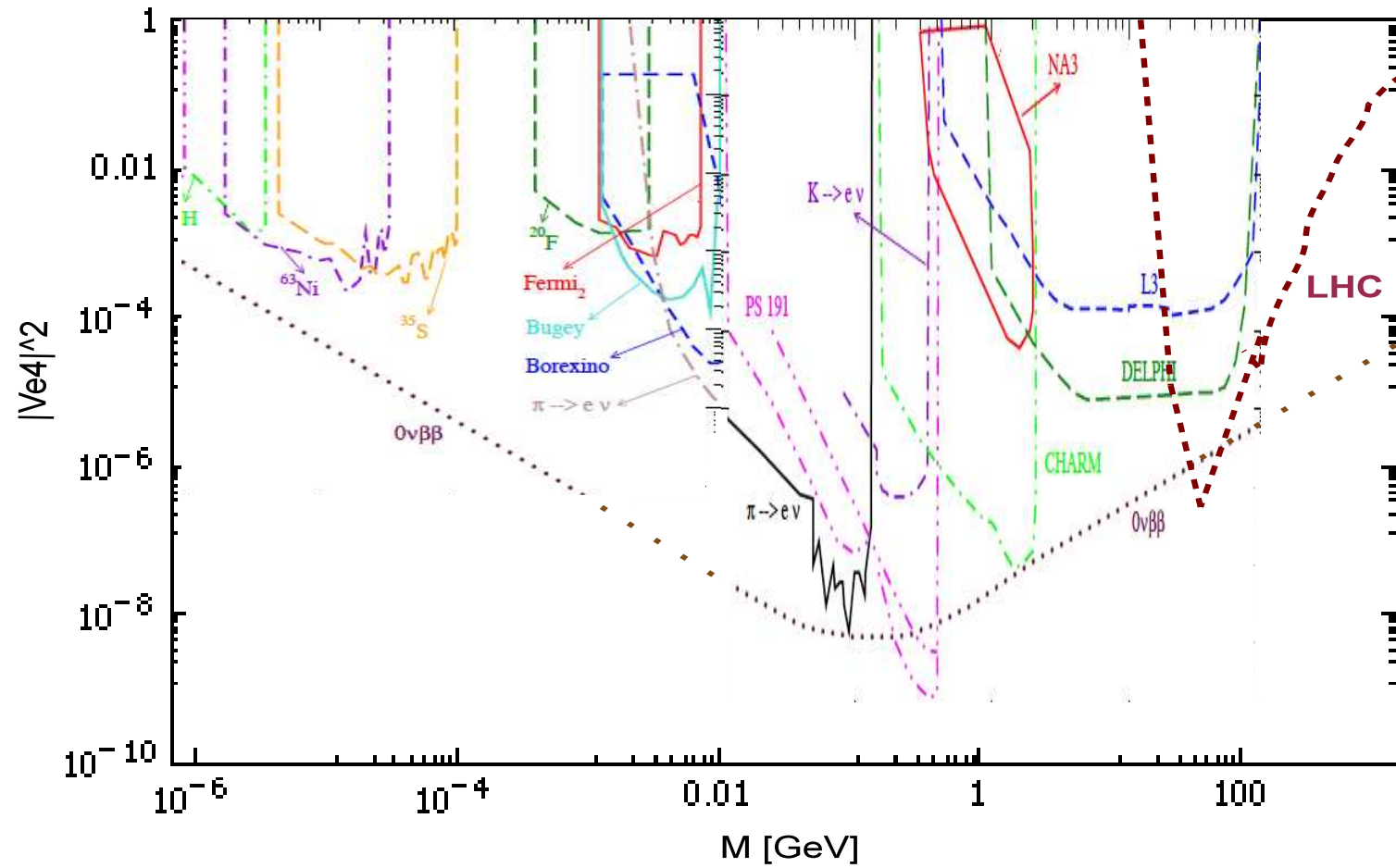
At Tevatron, the most promising channels are $p\bar{p} \rightarrow \mu^\pm \mu^\pm jj X$
 ($p\bar{p} \rightarrow \mu^\pm e^\pm jj X$):



At the LHC a similar analysis gives

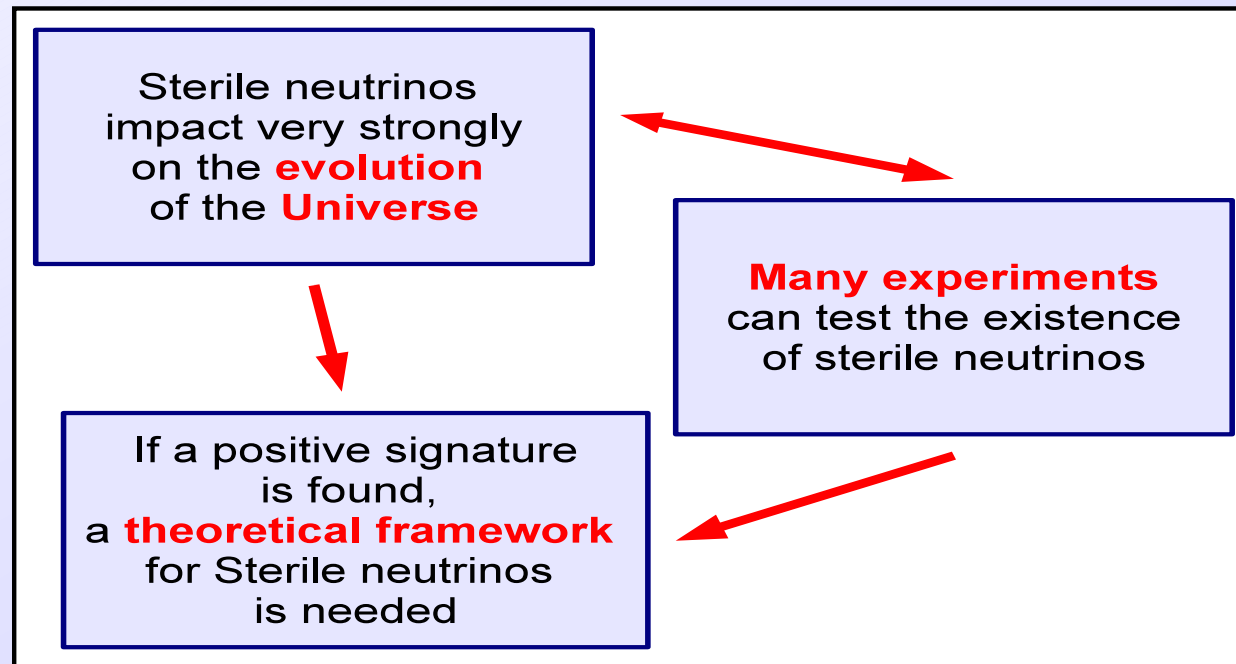


18 – Summary of laboratory constraints



19 – Conclusions

- Sterile neutrinos have a rich phenomenology and can severely impact the evolution of the Universe.
- keV sterile neutrinos can constitute the DM of the Universe and can be searched for in x-ray observatories.



20 – Sterile neutrino production summary

A population of keV sterile neutrinos, which constitute the DM, can be produced **out-of equilibrium** in the EU via various mechanisms:

- non-resonant production
- resonantly enhanced oscillations
- low reheating scenario
- boson decays

For a spectrum proportional to the equilibrium one, $p/T \sim 3.15$, while for the resonant production ($p/T \sim 2$) and for a subsequent injection of entropy in the thermal plasma ($p/T \sim 0.7$) it is much colder.

- Production via decay of new bosons S , with $m_S \sim 100$ GeV, which couple to N_4

$$\mathcal{L}_{S\nu} = f_a S N_4 N_4$$

When $\langle S \rangle = v_S \sim 100$ GeV,

- N_4 acquire a mass
- S can decay into N_4 at $T \sim m_S/\text{few}$

Because of the **entropy released** as the EU cools down, the density of N_4 gets diluted by a factor ~ 33 and the average momentum ~ 3 .

$$\frac{\langle p \rangle}{T} \sim 0.76 \left(\frac{110}{g_*(100\text{GeV})} \right)$$

This DM is much **more cold** than the standard DW scenario.

The ν MSM: this model is the minimal extension of the SM which incorporates DM, the baryon asymmetry and neutrino masses. It requires 3 sterile neutrinos:

- N_4 is the DM candidate with $m_4 \sim \text{few keV}$
- N_5 and N_6 are required for baryogenesis,
 $m_5 \simeq m_6 \sim 100 \text{ MeV} - 100 \text{ GeV}$

If N_4 are produced before N_5 and N_6 decays, the entropy release after N_4 production due to out-of-equilibrium decays of heavy sterile neutrinos cools down the DM.

[Asaka et al., 2006]

If N_4 are produced after, N_5 and N_6 decays can generate a large lepton asymmetry and the N_4 production is resonantly enhanced.

[Asaka et al., 2005; 2005; Shaposhnikov, 2008; Laine, Shaposhnikov, 2008; Asaka et al., 2007]