Generalized unitarity and W+3 jet production at the Tevatron

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Outline

- Introduction
- Review of generalized D-dimensional unitarity
- Implementation
- W+3 jets at the Tevatron
- Conclusions

Introduction

- New Physics is naturally searched for in hard collisions
- Hard collisions are well described by perturbative QCD
- Leading order computations typically do a good job provided that sensible choices of factorization and renormalization scales are made
- Reasonably accurate predictions emerge with next-to-leading order computations since
  - ``correct'' scales are dynamically generated
  - final states are relatively realistic
- NLO QCD predictions are indeed employed where available, for comparison with data. Typically, for hard processes, they describe data excellently

Introduction

- NLO QCD predictions are unavailable for many processes of interest
- For example, production of electroweak gauge bosons (Z,W) and 3 and 4 jets is measured at the Tevatron but NLO QCD predictions to those processes are unknown (except, since recently, for W+3 jet)
- There is a sharp cut-off
  - processes with less than three particles in the final state are relatively easy
  - processes with more than three particles in the final state are very difficult
- However, it is very useful to have NLO QCD description of high-multiplicity final states because
  - multi-particle final states become more abundant at the LHC
  - such final states are backgrounds to generic BSM searches
  - NLO QCD corrections are more relevant for high multiplicities

Introduction

- A typical NLO computation consists of two parts
  - one-loop virtual corrections
  - real emission corrections
- At the level of 2 -> 4 and higher-multiplicity processes, both of these parts are difficult. Indeed, for 2 -> 4
  - one-loop corrections include high-rank one-loop six-point functions
  - real emission corrections require 2-> 5 tree-level computations. Such computations approach the limit of complexity that dedicated tree-level integrators (Alpgen, Amegic, Madgraph) can handle
- In recent years, a breakthrough occurred in understanding of how one-loop corrections to multi-particle processes can be computed. Two parallel developments
  - optimization of traditional Passarino-Veltman reduction techniques
  - development of new computational methods related to unitarity

Introduction

- Any one-loop amplitude can be written as a linear combination of (known) scalar integrals. We require reduction coefficients.

$$\mathcal{A}^{1-\text{loop}} = \sum c_j I_j$$

- Key observations
  - unitarity constrains reduction coefficients
  - tree amplitudes are involved in the constraint

$$\mathrm{Im}\left(\mathcal{A}^{1-\text{loop}}\right) \propto \sum \left|\mathcal{A}^{\text{tree}}\right|^2 \qquad \sum c_j \mathrm{Im}(I_j) \propto \sum \left|\mathcal{A}^{\text{tree}}\right|^2$$

- It turns out that such equations can be used very efficiently to find all reduction coefficients

Introduction

- Unitarity as a tool for generic one-loop computations was introduced about 15 years ago. Here is brief historical summary
  - Bern, Dixon and Kosower (BDK) have been advocating importance of unitarity for one-loop computations; they demonstrated its usefulness by computing one-loop matrix elements for Z(W)+2 jets and for 5-parton scattering in QCD
  - The computational method emerged in the past three years
    - quadrupole cuts freeze loop momentum and give box coefficients directly (Cachazo, Britto, Feng)
    - Ossola-Pittau-Papadopoulos (OPP) tensor integral reduction technique
    - The OPP procedure meshes well with unitarity (Ellis, Kunszt, Giele)
    - D-dimensional unitarity (Giele, Kunszt, K.M.)

OPP reduction

- OPP suggested an interesting method to reduce tensor integrals to scalar integrals for a one-loop N-point function

$$\mathrm{Int}_N = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\mathrm{Num}(k)}{\prod_j^N D_i(k)} \qquad D_i = (p_i + k)^2 - m_i^2$$

$$\mathrm{Num(k)} = \sum_{i_1,i_2,i_3,i_4} d_{i_1,i_2,i_3,i_4}(k, p_{i_1}..) \prod_{j \neq i_1,i_2,i_3,i_4}^N D_j$$

$$+ \sum_{i_1,i_2,i_3} c_{i_1,i_2,i_3}(k, p_{i_1}..) \prod_{j \neq i_1,i_2,i_3}^N D_j + ..$$

OPP reduction

- Rules for writing down the reduction coefficients are simple. Consider c(k) which appears in the reduction to a three-point function
- A three-point function is described by two external momenta; those momenta define a two-dimensional subspace of a 4-dim space

$$R_4 = R_{||}(p_1, p_2) + R_{\perp}$$

- The loop momentum k is split accordingly
- Then c(k) = c0 + all possible traceless tensors, up to rank 3, defined on $R_{\perp}$
- Note that
  - highest rank of a contributing tensor is fixed by the fact that we deal with renormalizable theory
  - traceless-ness is required because otherwise numerator function can be reduced further

OPP reduction

- Consider reduction of a two-point function as an example

$$\mathrm{Int} = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{(ak)(bk)}{D_0 D_1}$$

$$D_0 = k^2 - m_0^2$$

$$D_1 = (p+k)^2 - m_1^2$$

OPP reduction

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$$\mathrm{Int} = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{(ak)(bk)}{D_0 D_1} \qquad \begin{aligned} D_0 &= k^2 - m_0^2 \\ D_1 &= (p+k)^2 - m_1^2 \end{aligned}$$

$$k = xp + k_\perp, \quad (pk_\perp) = 0$$

$$x = \frac{(m_1^2 - m_0^2 - p^2)}{2p^2} + \mathcal{O}(D_1, D_0) = x^* + \mathcal{O}(D_0, D_1)$$

OPP reduction

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$$\begin{aligned} (ak)(bk) = (x^*)^2 (ap)(bp) + x^* \left((ap)(bk_\perp) + (bp)(ak_\perp) \right) \\ + (ak_\perp)(bk_\perp) \end{aligned}$$

OPP reduction

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$$\begin{aligned}(ak)(bk) = (x^*)^2 (ap)(bp) + x^* \left((ap)(bk_\perp) + (bp)(ak_\perp)\right) \\ +(ak_\perp)(bk_\perp)\end{aligned}$$

$$(ak_\perp)(bk_\perp) = a_\mu b_\nu (k_\perp^\mu k_\perp^\nu - g_\perp^{\mu\nu} k_\perp^2) + a_\mu b_\nu g_\perp^{\mu\nu} k_\perp^2$$

$$k_\perp^2 = (x^*)^2 p^2 + m_0^2 + \mathcal{O}(D_0, D_1)$$

OPP reduction

- We conclude that

$$(ak)(bk) = b_0 + b_\mu^{(1)} k_\perp^\mu + b_{\mu\nu}^{(2)} (k_\perp^\mu k_\perp^\nu - g_\perp^{\mu\nu} k_\perp^2) + \mathcal{O}(D_0, D_1)$$

- We compute $\mathrm{Int} = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{(ak)(bk)}{(k^2 - m_0^2)(k+p)^2 - m_1^2}$

  by averaging over directions of $k_\perp$. Then both $b_\mu^{(1)}, b_{\mu\nu}^{(2)}$ do not contribute.

- Hence

$$\mathrm{Int} = b_0 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{D_0 D_1} + \mathrm{tadpoles}$$

- The easiest way to compute $b^{(0)}, b^{(1)}, b^{(2)}$ is to choose k such that both $D_{0,1}$ vanish since this automatically removes all the tadpole terms
- Such loop momentum can always be found if complex values of k are allowed

OPP reduction

- This procedure is very general
  - reduction coefficients d(k), c(k),b(k), a(k) can be decomposed into irreducible representations of rotation group in their particular (1-dim, 2-dim, 3-dim,4-dim) transverse spaces
  - since tensors are traceless, integrals over directions of the loop momenta vanish; hence ``spin zero'' , k-independent components of d(k), c(k), b(k), a(k) are the reduction coefficients
  - solutions are projected on a particular master integral by choosing momentum k in a way that a particular combination of inverse propagators vanishes. All possible combinations of inverse propagators should be considered to project on all possible master integrals.
  - construction is iterative: we first compute d(k), then c(k), etc.

Generalized unitarity

- OPP procedure applied to full one-loop amplitudes rather than Feynman integrals leads to unitarity. Indeed, any one-loop amplitude can be written as

$$\mathcal{A}^{1\ \text{loop}} = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\mathrm{Num}_D(k,p)}{\prod\limits_i d_i}$$

- We express this amplitude as a linear combination of master integrals and find reduction coefficients following the OPP procedure

- The OPP procedure determines reduction coefficient from a set of loop momenta for which certain combinations of inverse Feynman propagators vanish. If this happens, certain (virtual) particles go on their mass shells and the one-loop amplitude factorizes into products of tree amplitudes

$$\frac{\mathrm{Num}(k,p)}{\prod d_i} \to \prod_i A_i^{\mathrm{tree}}(\{p\},\{k_i\})$$

- These amplitudes are entirely conventional but, as a rule, have to be evaluated at complex on-shell momenta

Generalized unitarity

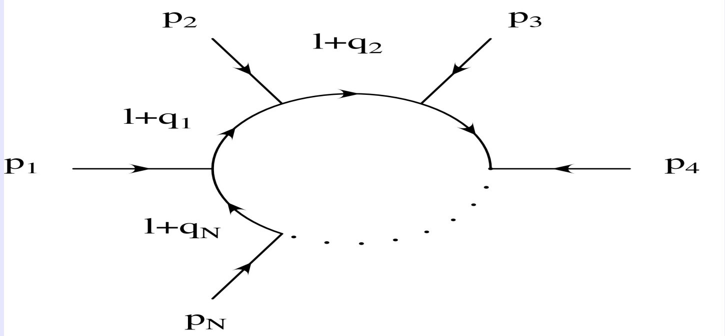
- If we use four-dimensional loop momentum to calculate cuts and on-shell conditions we obtain the *cut-constructible part of the amplitude*
- The neglected part is known as the *rational part*
- A general approach for calculating the rational part is based on numerical implementation of exact D-dimensional unitarity. It is independent of theoretical details such as particle's flavors, masses etc.
- The question is how to implement D-dimensional unitarity numerically. This is a valid question since conventional dimensional regularization requires working in D=4-2e space, and taking e->0 at the end of the calculation; it is hard to imagine how this can be implemented in a numerical program.

Generalized unitarity

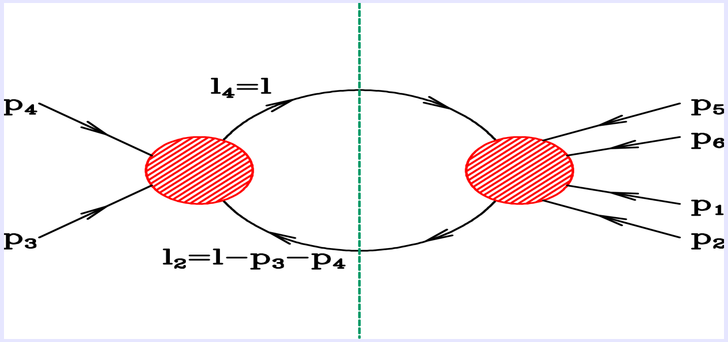
- We find that for one-loop computations conventional dimensional regularization is equivalent to the following construction
  - consider quantum field theory (QCD, EW, etc.) in integer Ds-dimensional space,   Ds > 4.
  - allow all external particles to propagate in a 4-dimensional space embedded into Ds-dimensional space
  - allow loop momentum to have support on the 5-dimensional space embedded in Ds-dimensional space. The 4-dimensional space, where external particles live, is a subspace of the 5-dimensional loop momentum space
- We find that knowledge of tree-level S-matrix for Ds=8 and Ds=6 and for complex on-shell momenta is sufficient to completely reconstruct any one-loop amplitude in any renormalizable four-dimensional quantum field theory
- Since calculations in integer-dimensional spaces are required, everryting is very similar to conventional 4-dimensional computations
- Important building blocks for efficient computations of on-shell scattering amplitudes such as Berends-Giele recurrence relations, are continued to D > 4 in a straightforward way

Implementation

- Here is how it works



  - employ color decomposition to have external particles ordered
  - specify all possible cuts (a cut is a collection of propagators that may vanish simultaneously) by examining the highest level integral that contributes to a given process/color ordering
  - each cut produces sums of products of tree amplitudes that can be computed for arbitrary D, Ds and complex momenta



  - from that, coefficients of master integrals are reconstructed

Implementation

- A number of attempts to employ unitarity and OPP ideas for one-loop computations (Blackhat, OPP, Lazopoulos, Giele & Winter)
- FORTRAN 90 program Rocket
- Currently, Rocket can compute the following one-loop amplitudes
  - N-gluon scattering amplitudes
  - two quark (massless and massive)+ N-gluon scattering amplitudes
  - W boson + two quarks + N-gluons
  - W boson + four quarks + 1 gluon
  - tt+Ngluons, ttqq+N gluons (Schulze)
- Rocket was interfaced with MCFM to compute W +3 jet production cross-section at the Tevatron

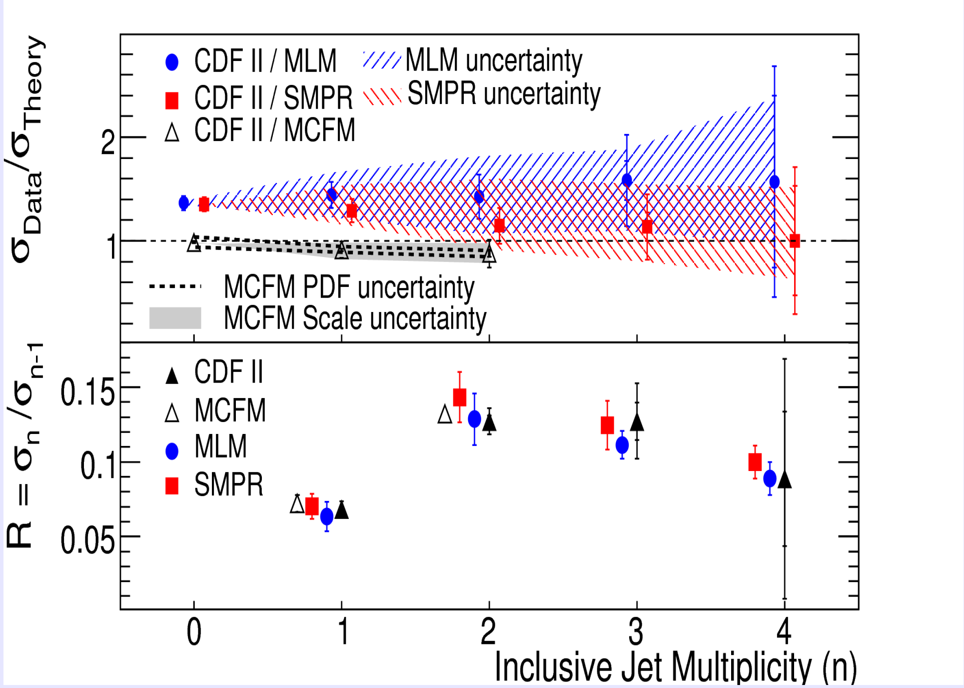
W+3 jet

- We would like to have a proof of concept that new methods for NLO QCD computations can compete with traditional methods
- We find NLO QCD corrections to W + 3 jets to be a case worth exploring because
  - measured at the Tevatron with reasonable accuracy
  - relevant for phenomenology (background to tt, single top, Higgs searches, SUSY searches)
  - large number (1480) of diagrams that contribute to virtual corrections
  - high-rank six point functions (in fact highest rank, studied so far)
  - all one-loop amplitudes required for this calculation are implemented in Rocket

NLO QCD corrections to W+3 jet at the Tevatron were also recently computed by Blackhat/Sherpa collaboration (Berger, Bern, Febres Cordero, Dixon, Forde, Ita, Kosower, Maitre / Gleisberg)

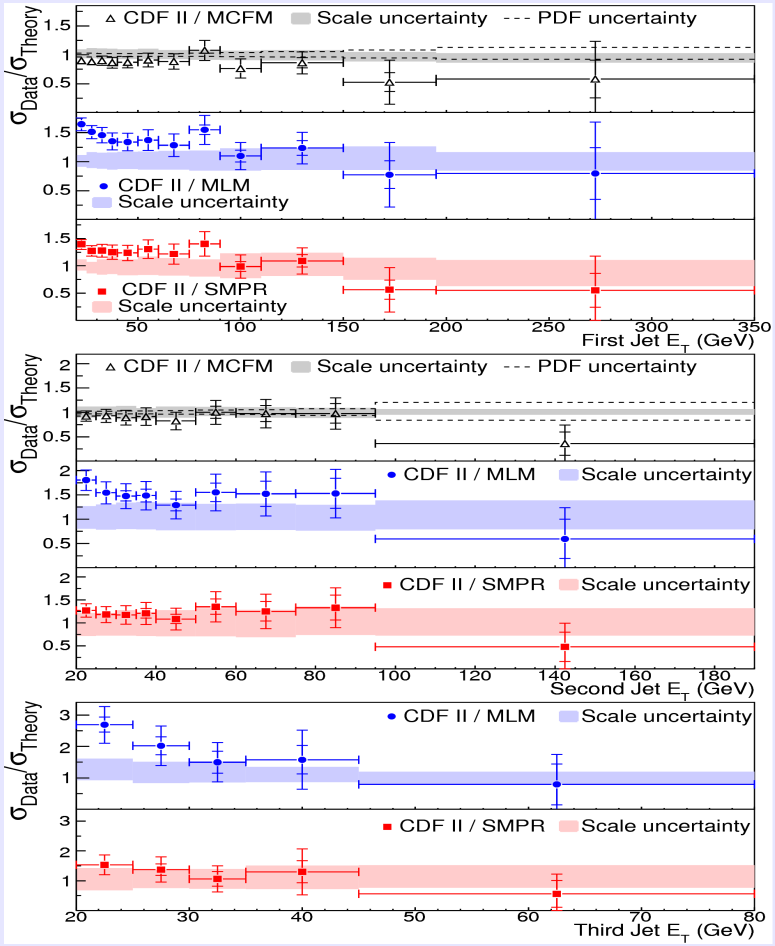
W+jets

- CDF performed careful studies of W+jet production
- Measurements are compared with theoretical predictions that include
  - ALPGEN interfaced with parton showers (MLM, etc.)
  - SHERPA
  - NLO QCD results for W+1 and W+2 jets (MCFM)
- General conclusion is that everything works, but NLO QCD predictions work best. No NLO QCD results for W+3 jets and W+4jets were/are available



W+jets

- CDF measures transverse energy distributions of the three hardest jets and compares this with theoretical predictions
- Such a comparison is helpful for understanding how well tree-level matrix elements matched to parton showers model hardest, next-to-hardest, etc. parton emissions
- An obvious conclusion from the comparison is that NLO QCD describes emissions of up to two partons very well, including region of relatively low transverse momentum
- Somewhat surprising is that matrix elements + parton showers underestimate data at low transverse momentum
- All this suggests that NLO QCD description of W+3 jets and W+4 jets is a way to go



W+3 jets

- I focus now on W+3 jets
- The total inclusive cross-section (W+ and W-, electron decay mode, pb)

$$\sigma^{W+3\mathrm{jet}}_{E_\perp^{3\mathrm{rd}}>25\ \mathrm{GeV}} = 0.84 \pm 0.10(\mathrm{syst}) + 0.21(\mathrm{stat}) \pm 0.05\ (\mathrm{lum})$$

- Jets are defined using JETCLU cone algorithm. This algorithm is not infra-red safe; we can use it for LO QCD computations but not for NLO computations
- It is not clear which IR safe jet algorithm would best correspond to JETCLU, and differences between different jet algorithms at the level of ten to fifteen percent are typical
- Given experimental uncertainty and jet algorithm mismatch, theoretical prediction at the level of 10 to 20 percent accuracy is a sensible goal but higher precision is harder to justify
- Of course, reaching 10-20 percent requires NLO QCD since theory uncertainty at LO is close to a factor 3

W+3 jets

- At 10-20 percent precision level, we can simplify the problem by working at leading color approximation $N_c \sim N_f \gg 1$
- Studies at leading order show that this approximation overestimates the full result by about ten percent. This ``ten percent overestimate'' seems to be very robust – it does not depend on choices of scales, choices of observables, etc.   Therefore, we can take our next-to-leading order result and re-weight it by the leading order full color to leading color ratio
- Other comments
  - virtual corrections are computed on a fixed grid
  - leading color approximation and symmetry of phase-space is used to reduce the number of independent structures that need to be computed
  - symmetrization requires a modification of Catani-Seymour dipole subtraction procedure
  - calculations are performed for three dynamical scales with the central value $\mu_0 = (p_{\perp,W}^2 + m_W^2)^{1/2}$
  - we use SIScone and anti-kt jet algorithm, on-shell W-bosons, CKM matrix is set to unity

W+3 jets

- Predictions for total cross-section become much more accurate at NLO

$$\sigma^{W+3j}_{\mathrm{LO},\mathrm{E}_\perp^{\mathrm{3rd}}>25\ \mathrm{GeV}} = 0.81^{+0.50}_{-0.28}\ \mathrm{pb}, \quad \mathrm{SIScone}$$

$$\sigma^{W+3j}_{\mathrm{NLO},E_\perp^{\mathrm{3rd}}>25\ \mathrm{GeV}} = 0.91^{+0.05}_{0.15}\ \mathrm{pb}, \quad \mathrm{SIScone}$$

$$\sigma^{W+3j}_{\mathrm{LO},\mathrm{E}_\perp^{\mathrm{3rd}}>25\ \mathrm{GeV}} = 1.01^{+0.62}_{-0.35}\ \mathrm{pb}, \quad \mathrm{anti}-k_\perp$$

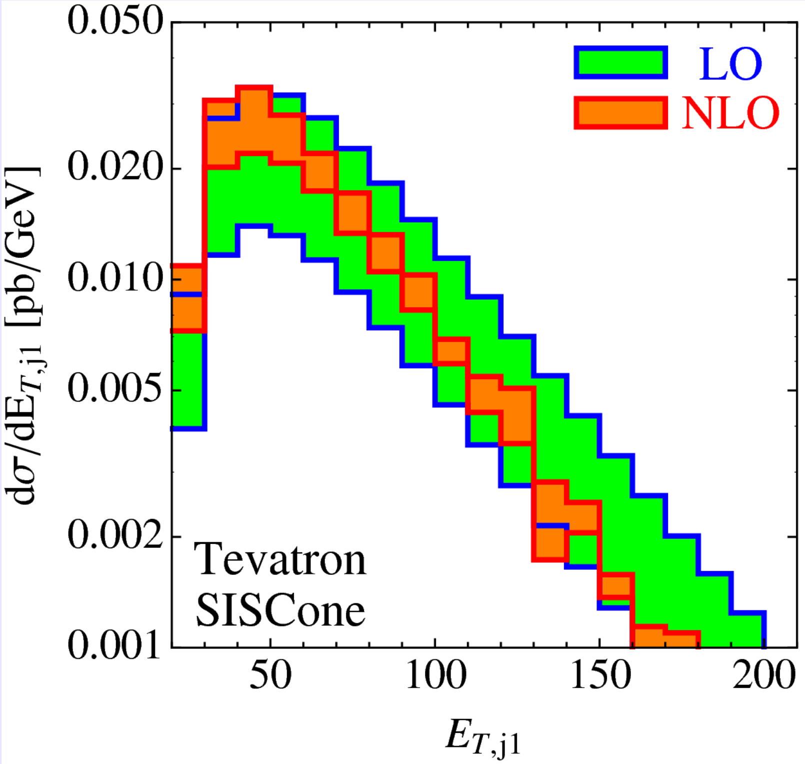
$$\sigma^{W+3j}_{\mathrm{NLO},\mathrm{E}_\perp^{\mathrm{3rd}}>25\ \mathrm{GeV}} = 1.00^{+0.01}_{-0.12}\ \mathrm{pb}, \quad \mathrm{anti}-k_\perp$$

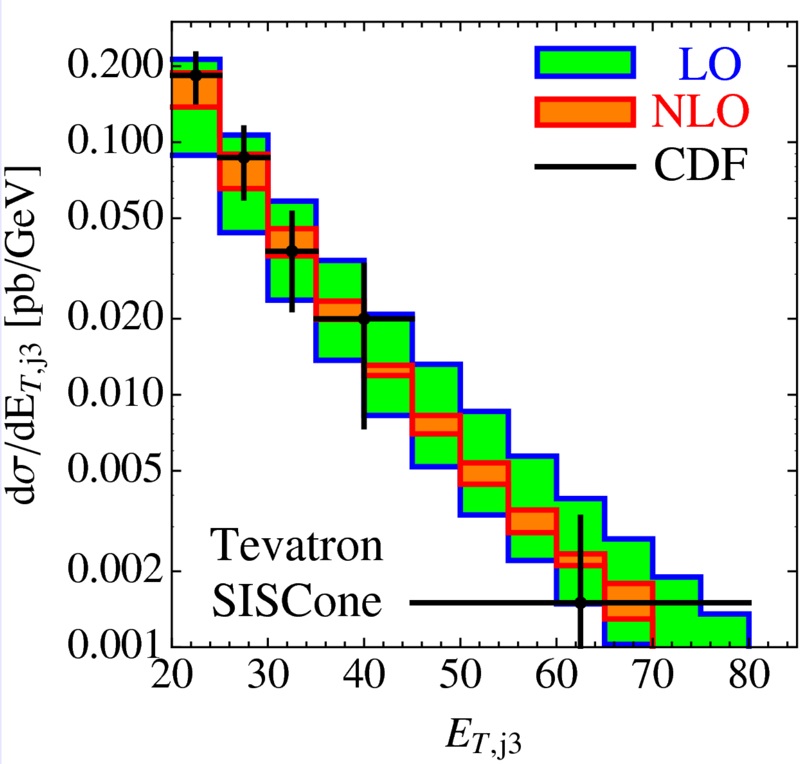
LO uncertainty is a factor 2.5, NLO uncertainty is about 20 percent

Difference between SIScone and anti-kt is 20 percent at LO and 10 percent at NLO

W+3 jets

- Kinematic distributions





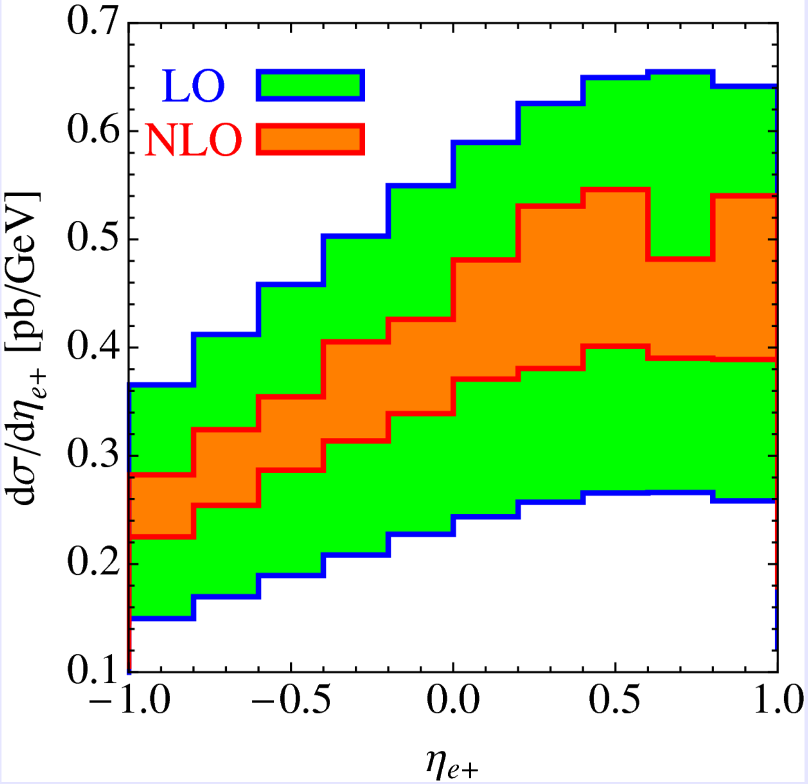
Characteristic change in shape of transverse energy distributions – NLO results are smaller than LO at high transverse energy and (may be) larger than LO results for low transverse energy

W+3 jets

- To understand this recall that the leading order predictions are computed with the renormalization/factorization scale $\mu_0 = (p_{\perp,W}^2 + m_W^2)^{1/2}$
- Typical transverse momentum of the W-boson is 40-80 GeV; it is rarely produced with a much higher transverse momentum
- Parton emissions are governed by the strong coupling constant at the scale of the relative transverse momentum of parton branchings
  - when jets are hard, transverse momentum of the W is smaller than jet momenta $\rightarrow$ leading order prediction is systematically higher;
  - when jets are soft, it is the other way around and LO predictions are smaller
- If the computation is performed at fixed, non-dynamical scale, the same effect should and does occur
- Good choice of scale for leading order computations should increase like transverse momentum of jets in the region of high $p_\perp$
- Bauer and Lange analyzed W +n jets final state using SCET focusing on transverse energy and transverse mass distributions.
  They suggest to use $\mu_0 = (m_{\mathrm{hadr}}^2/2 + m_W^2)^{1/2}$

W+3 jets

- However, we should keep in mind that ``good scale'' depends on an observable we look at; for example
  - as already mentioned, NLO transverse momentum distribution of the softest jet at low $p_\perp$ may exceed the LO result computed with fixed or dynamical scale
  - lepton rapidity distribution does not exhibit significant shape changes variations



Conclusions

- Generalized D-dimensional unitarity is a robust method for one-loop computations; it builds up complete one-loop amplitudes from on-shell tree-level amplitudes
- Very general method; applicable to massless and massive scattering amplitudes
- I described how D-dim unitarity can be used to compute NLO QCD corrections to W+3 jet production at the Tevatron. We use leading color approximation. We observe
  - substantial reduction in scale dependence at NLO QCD
  - instructive shape changes in NLO distributions compared to LO distributions; seem to imply negative corrections at high transverse momenta relative to LO predictions with fixed scale
  - these shape changes suggest better scale choices in leading order computations
- Ten percent difference between different IR-safe jet algorithms at NLO; for better theory/experiment comparison, identical jet algorithms should be used