The neutral kaon mixing parameter from lattice QCD

Jack Laiho
Washington University
(work done in collaboration with
Christopher Aubin and Ruth Van de Water)

FNAL
Aug 13, 2009

Flavor physics

Extensions of the Standard Model generically predict new CP-violating phases and flavor changing interactions.

Without additional assumptions, absence of new physics places stringent constraints on energy scale of new physics.

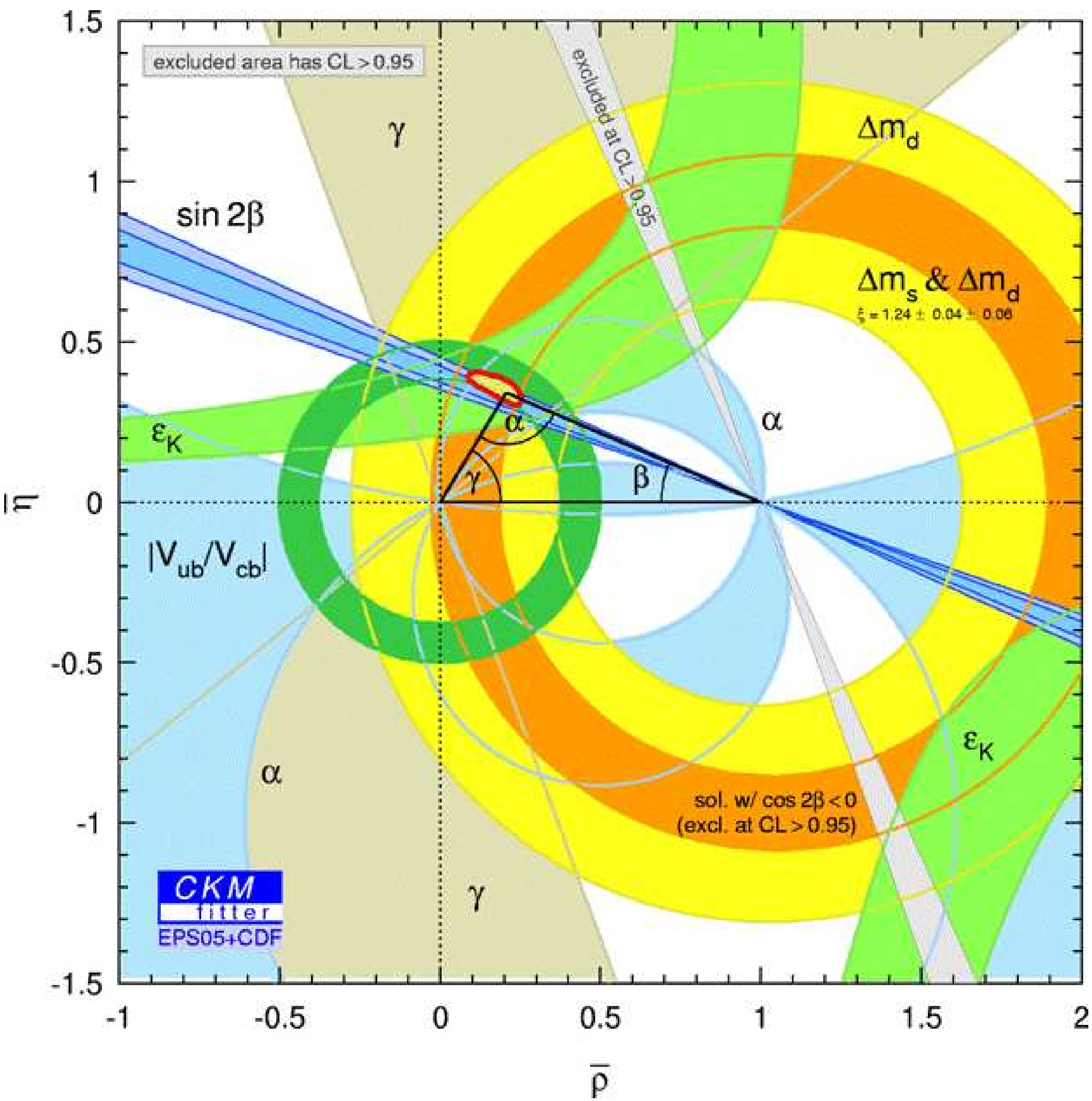
The flavor sector

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\text{flavor}} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\text{mass}}, \quad (1)$$

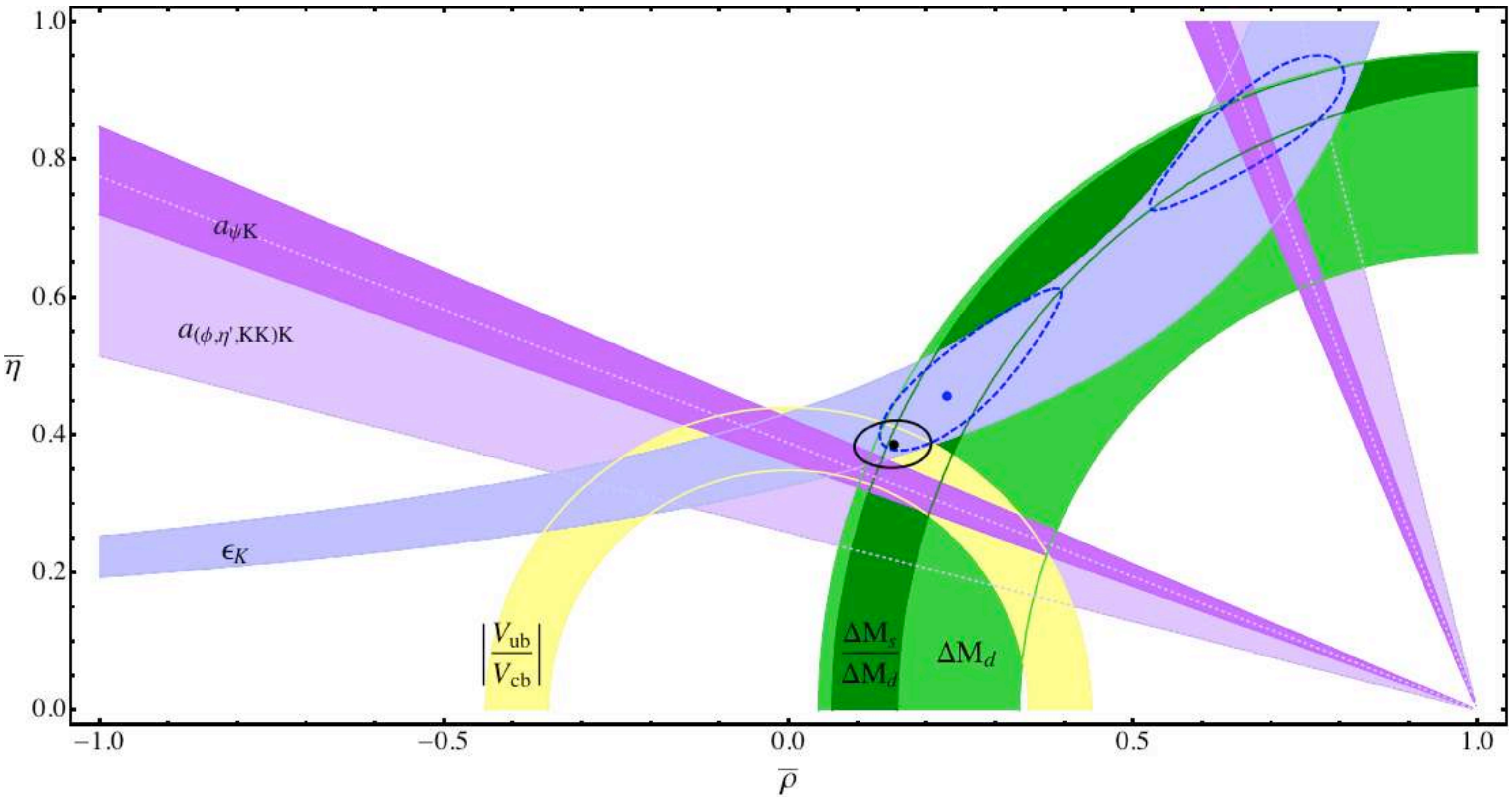
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \quad (2)$$

$$V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = 0 \quad (3)$$

Constraining the Unitarity Triangle

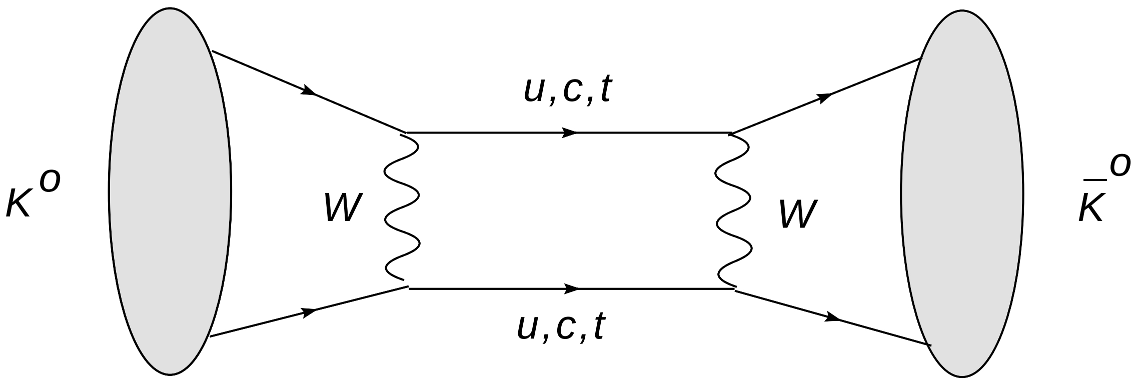


Constraining the Unitarity Triangle



Plot from Lunghi and Soni, Phys. Lett. B666:162-165, 2008 (arXiv:0803.4340)

Nonperturbative input needed



$$\epsilon_K = (\text{known factor})\,(\text{CKM factor})(\text{QCD factor}) \tag{4}$$

$$|\epsilon_K| = C_\epsilon \kappa_\epsilon B_K A^2 \overline{\eta} \{-\eta_1 S_0(x_c)(1-\lambda^2/2) + \eta_3 S_0(x_c, x_t) + \eta_2 S_0(x_t) A^2 \lambda^2 (1-\overline{\rho})\}$$

Lattice QCD

- Allows non-perturbative calculations from first principles
- Path integral can be evaluated on a computer using Monte Carlo methods
- Simulations require a finite-sized grid with lattice spacing a and size L
- Even with today's computers this is still a difficult task! However, unquenched calculations, including the fermion determinant, are now the norm.

Treatment of light quarks

QCD Light quarks are expensive. One must extrapolate to the physical u and d quark masses.

For this one uses chiral perturbation theory (ChPT).

Typically one uses partially quenched ChPT (not as bad as it sounds!) When the number of light quarks is the physical value the low energy constants of ChPT are the physical ones.

One also uses lattice ChPT where the discretization effects are incorporated by introducing additional operators consistent with the lattice symmetries. This is essential for doing chiral fits to lattice data!

Types of fermions on the lattice

- Wilson Fermions: Introduces an additional “irrelevant” term to the action. Hard breaking of chiral symmetry. Bad for light fermions but not a problem for heavy ones.
- Staggered Fermions: Identifies some of the extra fermions with the 4 different spin components of a single fermion. This brings us down to 4 extra fermions. This factor of 4 is eliminated by taking the 4th root of the fermion determinant. Some open theoretical issues with this. No one would bother except staggered is much cheaper than all the alternatives!
- Domain Wall Fermions: Solves chiral symmetry problem by using Wilson type quarks in five dimensions. More costly because of the extra dimension. There is a small chiral symmetry breaking due to the finiteness of the fifth dimension.
- Overlap Fermions: Exact chiral symmetry. Very expensive.

Staggered fermions

- Staggered fermions are the cheapest fermions on the market at the present time.

- The staggered action has extra unphysical species of fermions (called "tastes") due to lattice artifacts which vanish in the continuum limit. There is no rigorous proof that staggered quarks recover QCD in the continuum limit. There has been much recent theoretical progress, and the recovery of the correct continuum limit appears plausible. (Sharpe, hep-lat/0610094; Kronfeld, arXiv:0711.0699.)

- Extra "tastes" complicate the analysis with staggered fermions, as compared to "chiral" fermions such as domain-wall or overlap, which are many times more expensive.

- Staggered chiral perturbation theory gives good control over staggered discretization effects (MILC, arXiv:hep-lat/0407028).

Mixed action project

In collaboration with Christopher Aubin and Ruth Van de Water (hep-lat/0609009).
Mixed action: MILC lattices with 2+1 flavors of improved (asqtad) staggered quarks in the sea sector and domain wall quarks in the valence sector. This is the method adopted by the LHP Collaboration (hep-lat/0409130).

Advantages

- Has best of both worlds. Cheap configurations, and the good chiral properties of the valence sector make things nearly as simple as using chiral quarks throughout. Non-perturbative renormalization goes through the same way as for chiral quarks.
- A large number of ensembles with different volumes, sea quark masses and lattice spacings exist and are publicly available.

Mixed action calculations

Due to small chiral symmetry breaking, domain wall fermions get a small residual additive quark mass renormalization.

$$m_{dw}^2 = 2\mu_{\rm dw}(m_v + m_{\rm res}) \ , \tag{5}$$

In 1-loop Mixed Action χPT only two parameters beyond those of domain-wall:

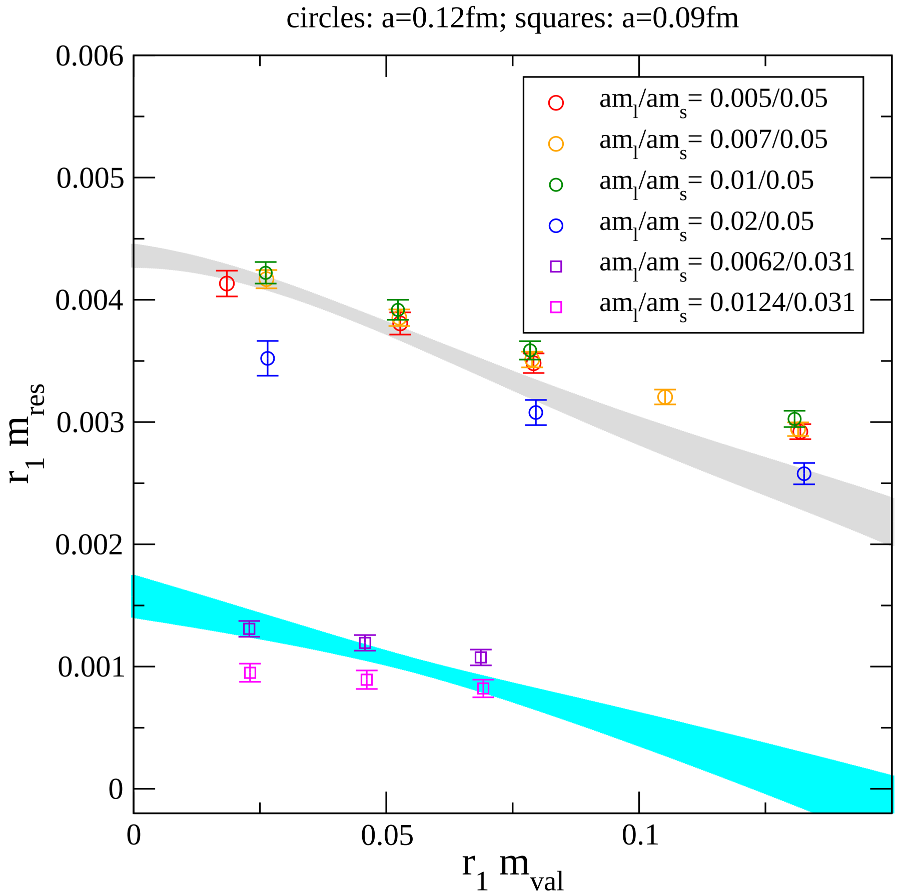
$$m_I^2 = 2\mu_{\rm stag} m_s + a^2\Delta_I \ , \tag{6}$$

$$m_{mix}^2 = \mu_{\rm dw}(m_v + m_{\rm res}) + \mu_{\rm stag} m_s + a^2\Delta_{\rm mix}, \tag{7}$$

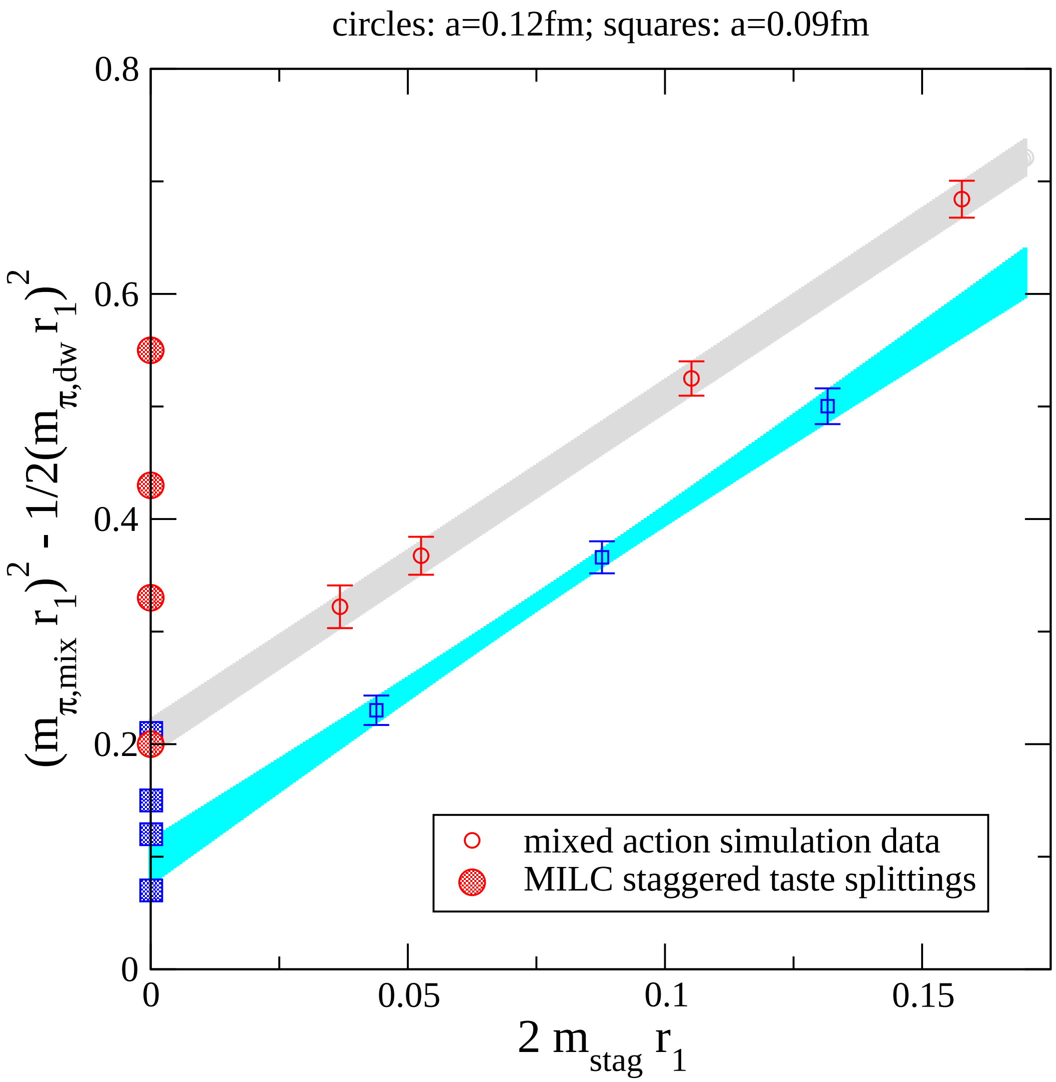
Testing the method

- Study lattice artifacts m_{res} and Δ_{mix}
- Worst-case scenario for quenching artifacts-the isovector-scalar correlator. Can mixed-action chiral perturbation theory describe it?
- How do we do for even simpler quantities like f_K and f_π, the pseudoscalar decay constants?

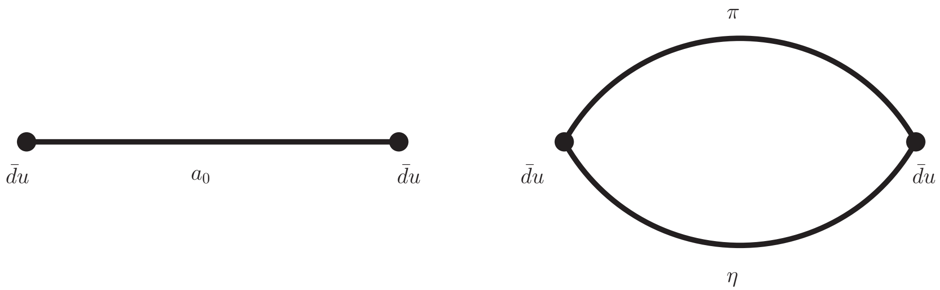
The residual mass



Determining the splitting Δ_{mix}



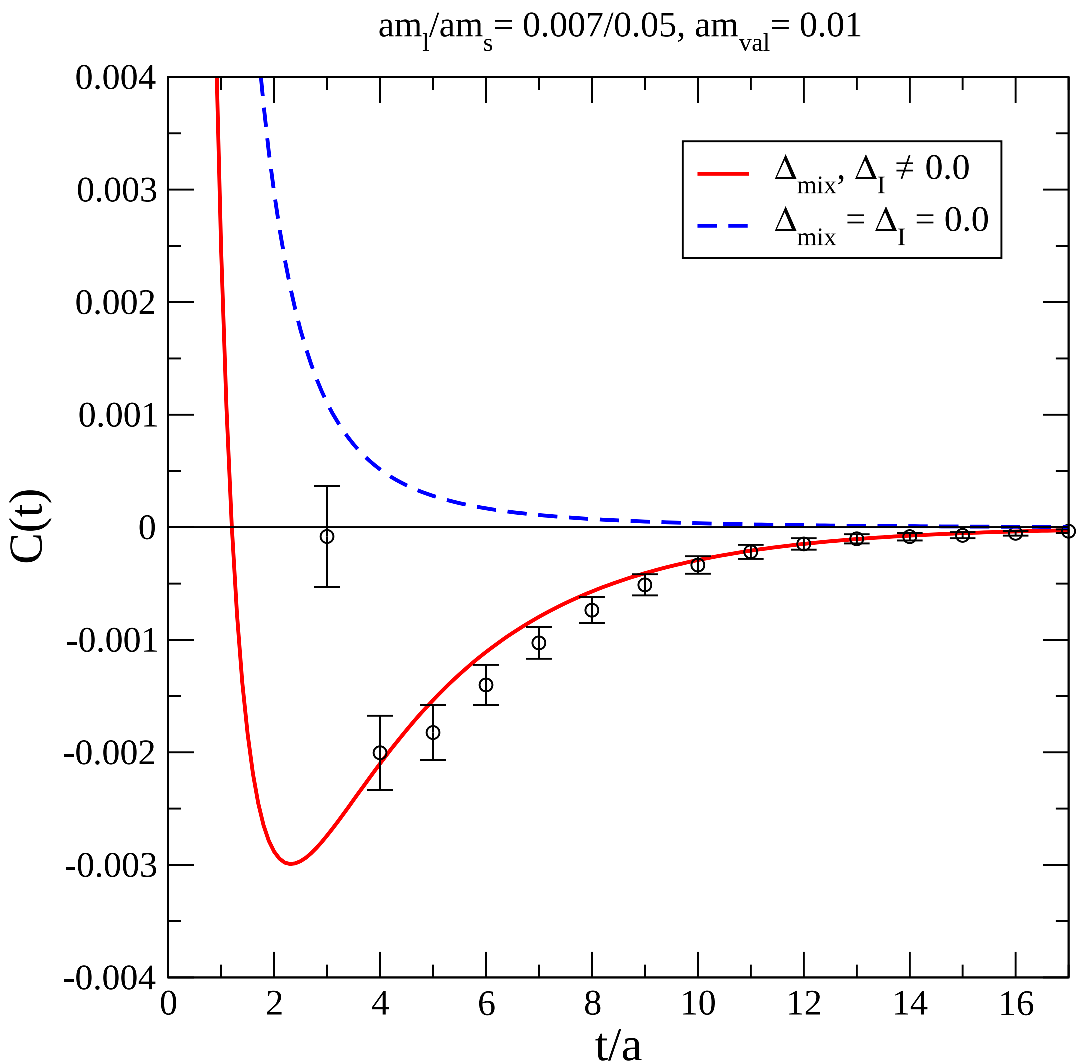
Scalar bubble prediction



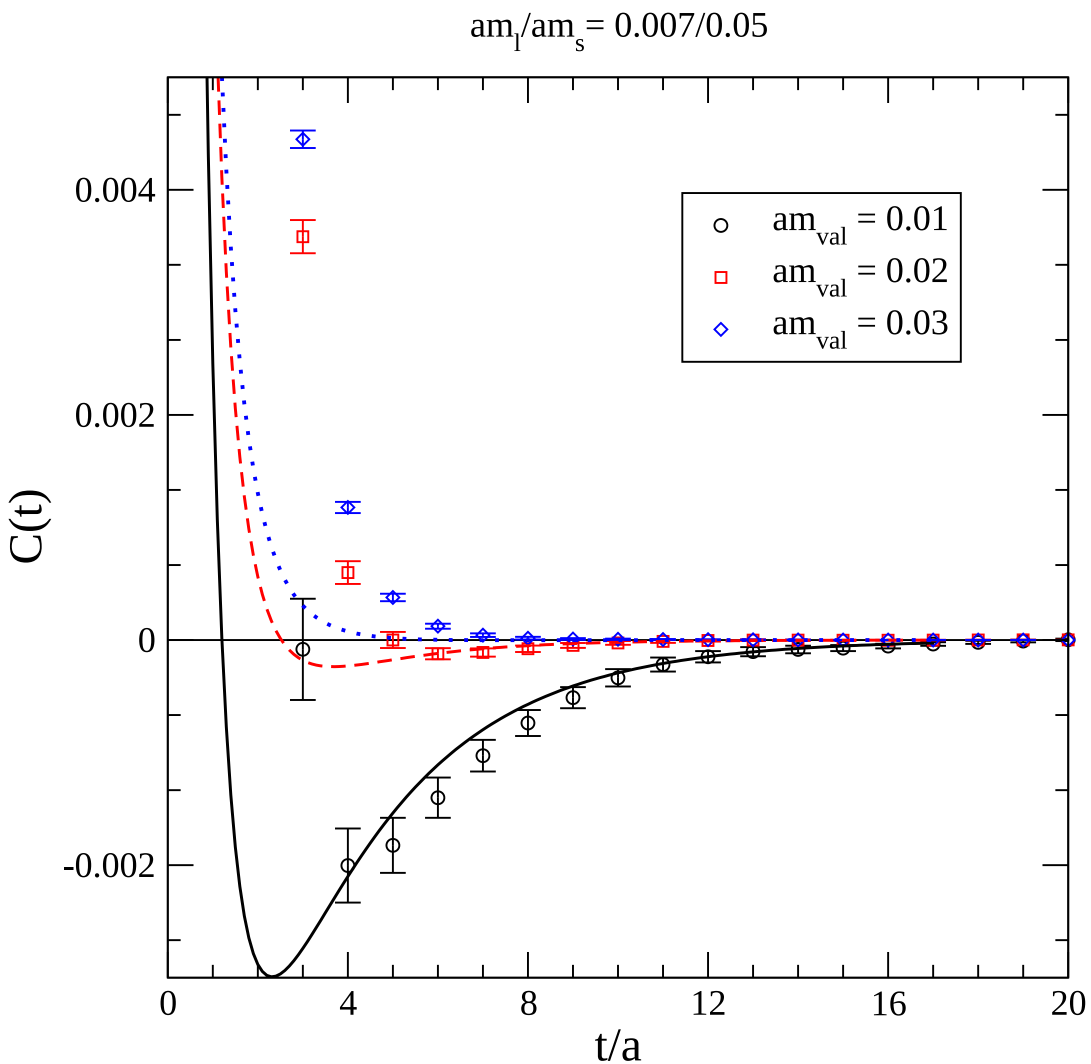
$$
\begin{aligned}
B(t) \quad = \quad & \frac{\mu^2}{3L^3}\sum_{\mathbf{k}}\Bigg[\frac{2}{9}\,\frac{e^{-(\omega_{vv}+\omega_{\eta_I})t}}{\omega_{vv}\omega_{\eta_I}}\,\frac{(m^2_{S_I}-m^2_{U_I})^2}{(m^2_{vv}-m^2_{\eta_I})^2} \\
& -\frac{e^{-2\omega_{vv}t}}{\omega^2_{vv}}\left[\frac{3m^2_{vv}(m^2_{vv}-2m^2_{\eta_I})+2m^4_{S_I}+m^4_{U_I}}{3(m^2_{\eta_I}-m^2_{vv})^2}\right] \\
& -\frac{e^{-2\omega_{vv}t}}{2\omega^4_{vv}}\,(\omega_{vv}t+1)\,\frac{(m^2_{U_I}-m^2_{vv})(m^2_{S_I}-m^2_{vv})}{m^2_{\eta_I}-m^2_{vv}}+\frac{3}{2}\,\frac{e^{-2\omega_{vu}t}}{\omega^2_{vu}}+\frac{3}{4}\,\frac{e^{-2\omega_{vs}t}}{\omega^2_{vs}}\Bigg]
\end{aligned}
$$

Prelovsek, PR D73, 014506 (2006), hep-lat/0510080

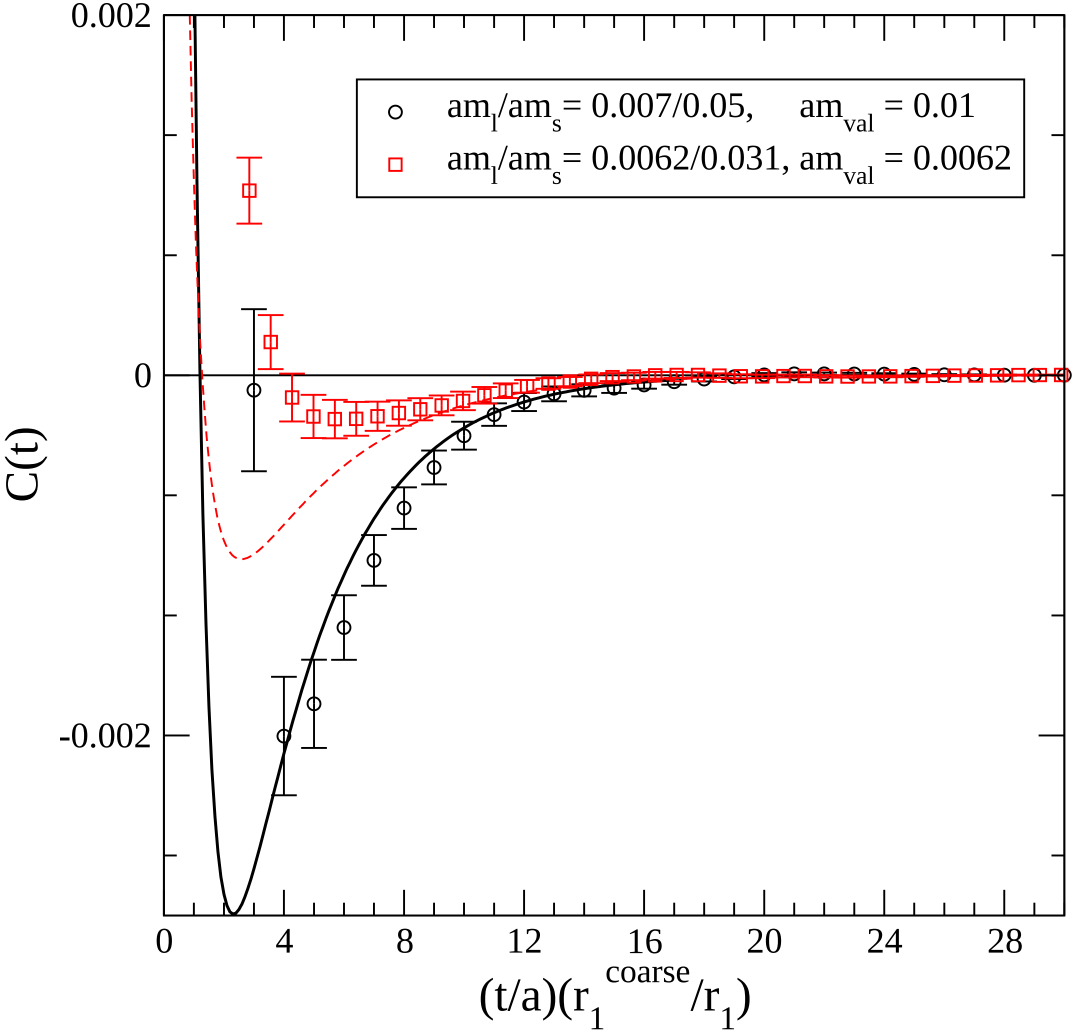
Scalar bubble prediction



Scalar bubble prediction vs. data



Scalar bubble prediction vs. data



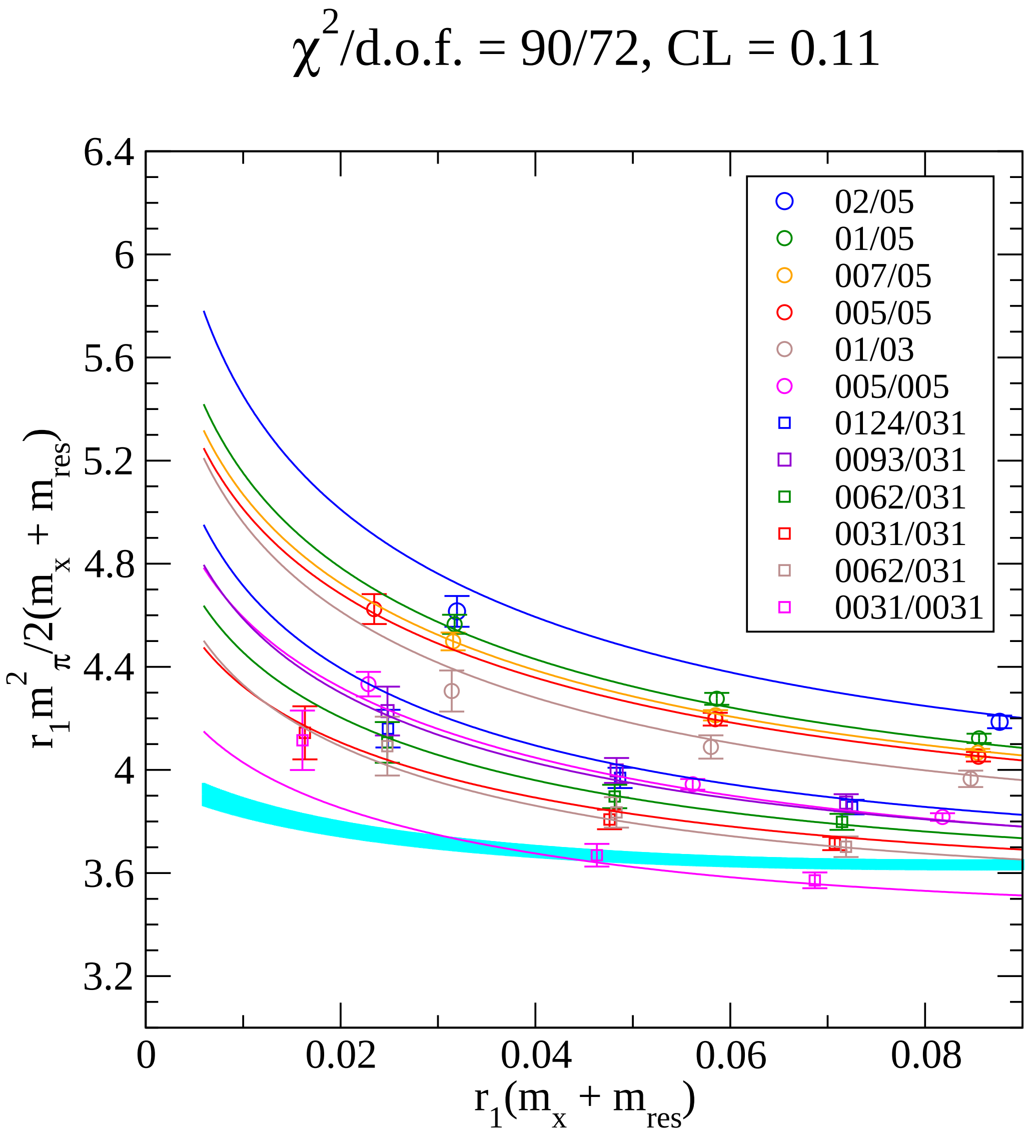
Approach to chiral fits

We have generated data with relatively high statistics so that we can resolve a correlation matrix and obtain reliable confidence levels in fits.

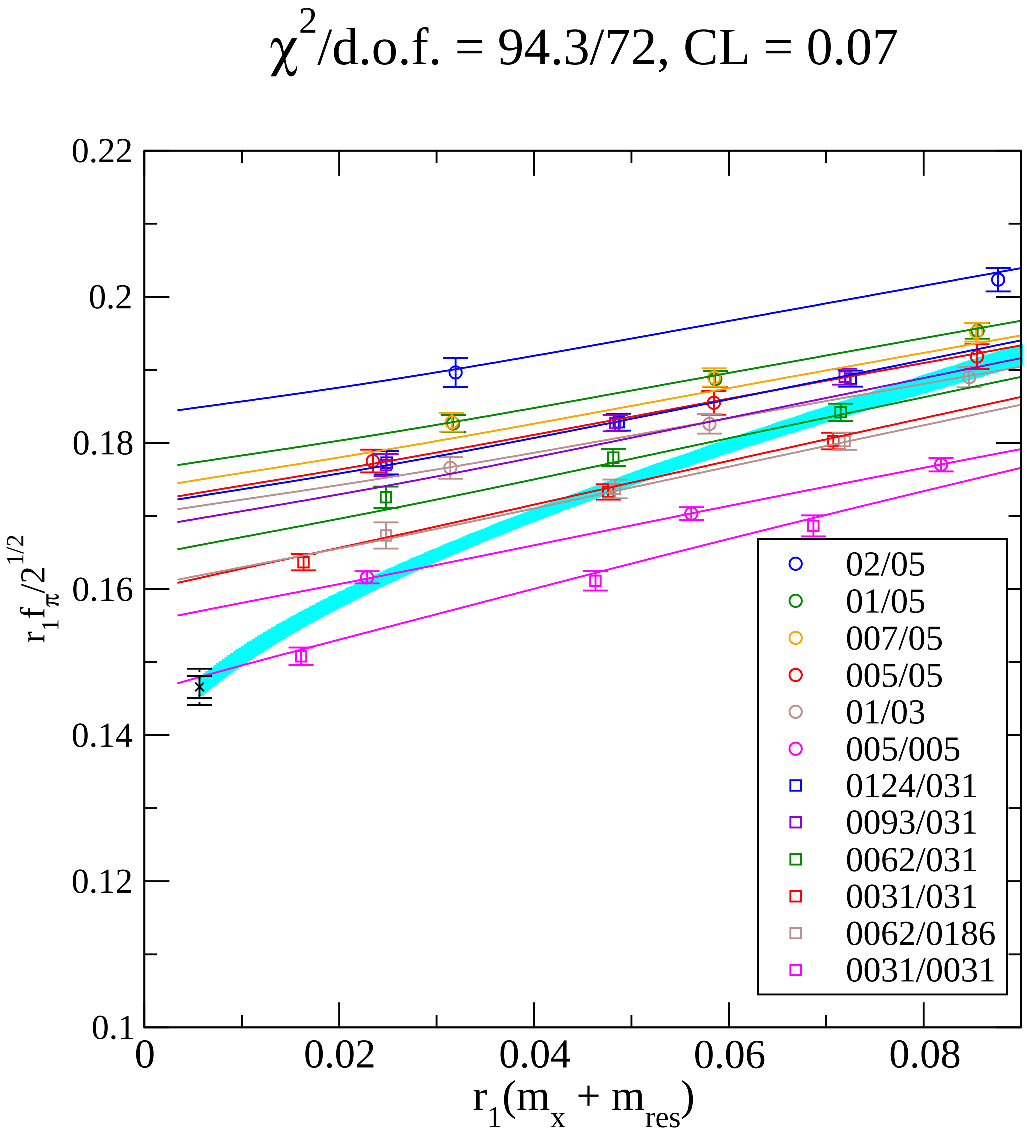
Using SU(3) chiral perturbation theory in order to interpolate about the strange quark mass and extrapolate in the light quark mass. We are using one-loop SU(3) mixed action χPT and higher order analytic terms.

Separate fits to m_π^2/m_q and f_π, where leading order μ is taken from linear fits to m_π^2 data, evaluated in region of data, rather than chiral limit. f_π evaluated at physical pion point.

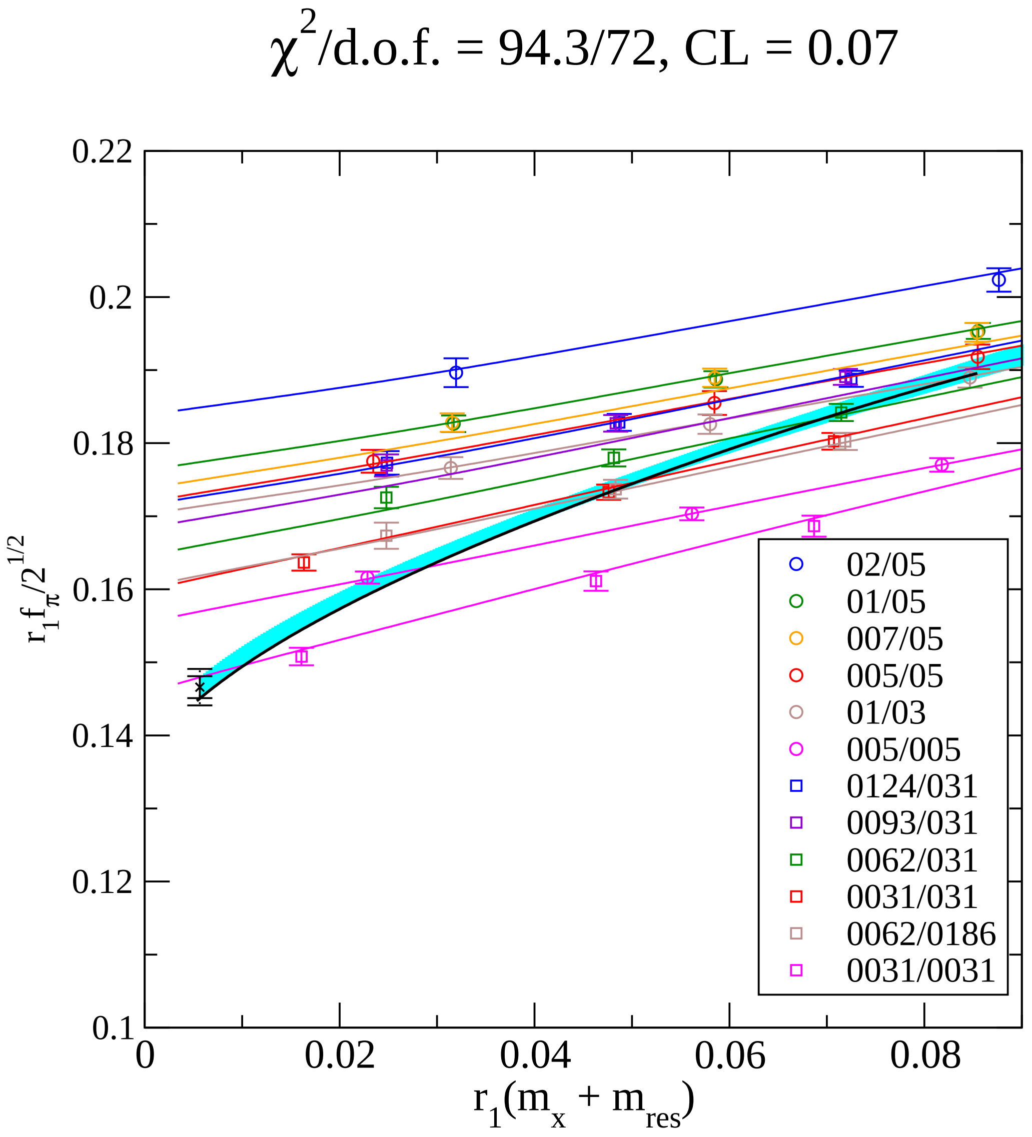
m_π^2/m_q chiral fit



f_π chiral fit



f_π chiral fit (compared w/ MILC)



Fit results to subset of data

Various "low-mass" fits to NLO χPT + NNLO analytic terms, leaving out different improvements.

type of f_π fit	χ^2/d.o.f.	C.L.
NNLO analytic	0.99	0.54
No NNLO	6.15	9×10^{-41}
No NLO logs	1.22	0.17
No FV	1.34	0.08
No taste-breaking	1.08	0.37

type of m_π^2/m_q fit	χ^2/d.o.f.	C.L.
NNLO analytic	1.12	0.31
No NNLO	6.30	4×10^{-42}
No NLO logs	2.43	4×10^{-7}
No FV	2.34	1×10^{-6}
No taste-breaking	1.50	0.02

Preliminary error budget

Table 1: Uncertainties are shown as percentages.

source	f_K	f_π	f_K/f_π
statistics	1.0	1.1	1.1
input r_1	0.7	0.9	0.3
chiral-continuum extrapolation	1.0	1.2	1.1
finite volume	0.6	0.9	0.9
total error	1.7	2.1	1.8

$f_\pi = 131.1(15)(23)$ MeV, $\quad f_K = 156.3(15)(20)$ MeV, $\quad f_K/f_\pi = 1.192(13)(17)$.

Preliminary quark masses

Using "partially non-perturbative" method to renormalize quark masses (inspired by Fermilab approach to renormalizing heavy-light currents). The ratio of currents Z_A/Z_S is close to one. The difference from 1 is computed using 1-loop lattice perturbation theory. Z_A is computed non-perturbatively. $Z_m = 1/Z_S$. Quark masses, in $\overline{MS}$ at 2 GeV are:

$$\begin{aligned} \widehat{m} &= 3.1(0)(2)(4)(0)\text{MeV}, \\ m_s &= 88(0)(5)(8)(0)\text{MeV}, \\ m_u &= 1.7(0)(2)(2)(1)\text{MeV}, \\ m_d &= 4.4(0)(2)(4)(1)\text{MeV}. \end{aligned} \tag{8}$$

$$\begin{aligned} \frac{m_s}{\widehat{m}} &= 28.9(3)(14)(0)(0), \\ \frac{m_u}{m_d} &= 0.39(1)(3)(0)(4). \end{aligned} \tag{9}$$

Errors are: statistical, lattice systematic, perturbative, electromagnetic.

Matching B_K to the continuum

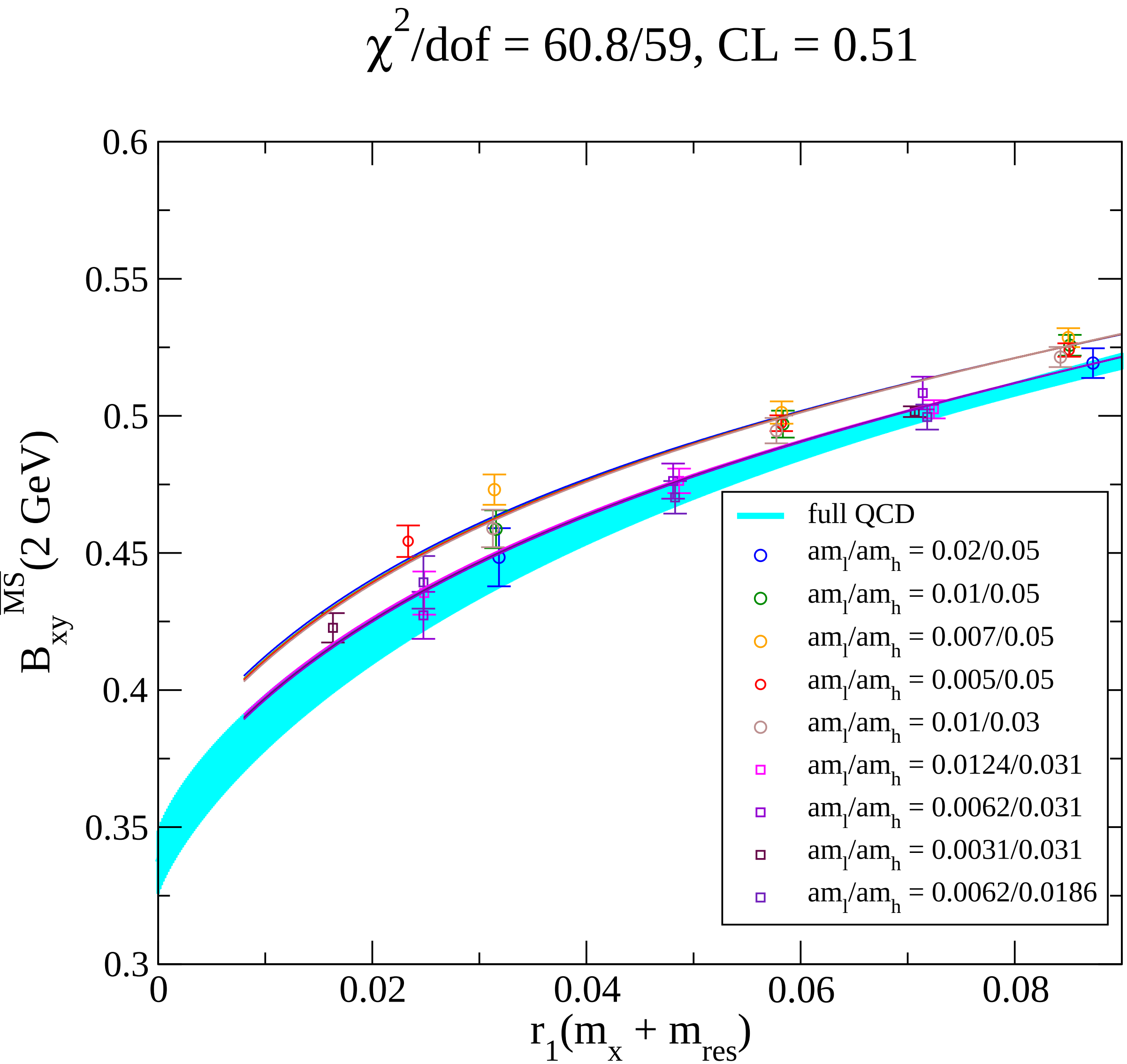
We must match the lattice renormalization scheme to the $\overline{MS}$ scheme.

We use the Rome-Southampton non-perturbative renormalization method: First match to the regularization-independent (RI-mom) scheme non-perturbatively. Then match the RI-mom scheme to $\overline{MS}$ using known continuum perturbation theory to 1 loop order in α_s.

As a cross-check, we also match to the $\overline{MS}$ scheme directly using 1-loop lattice perturbation theory.

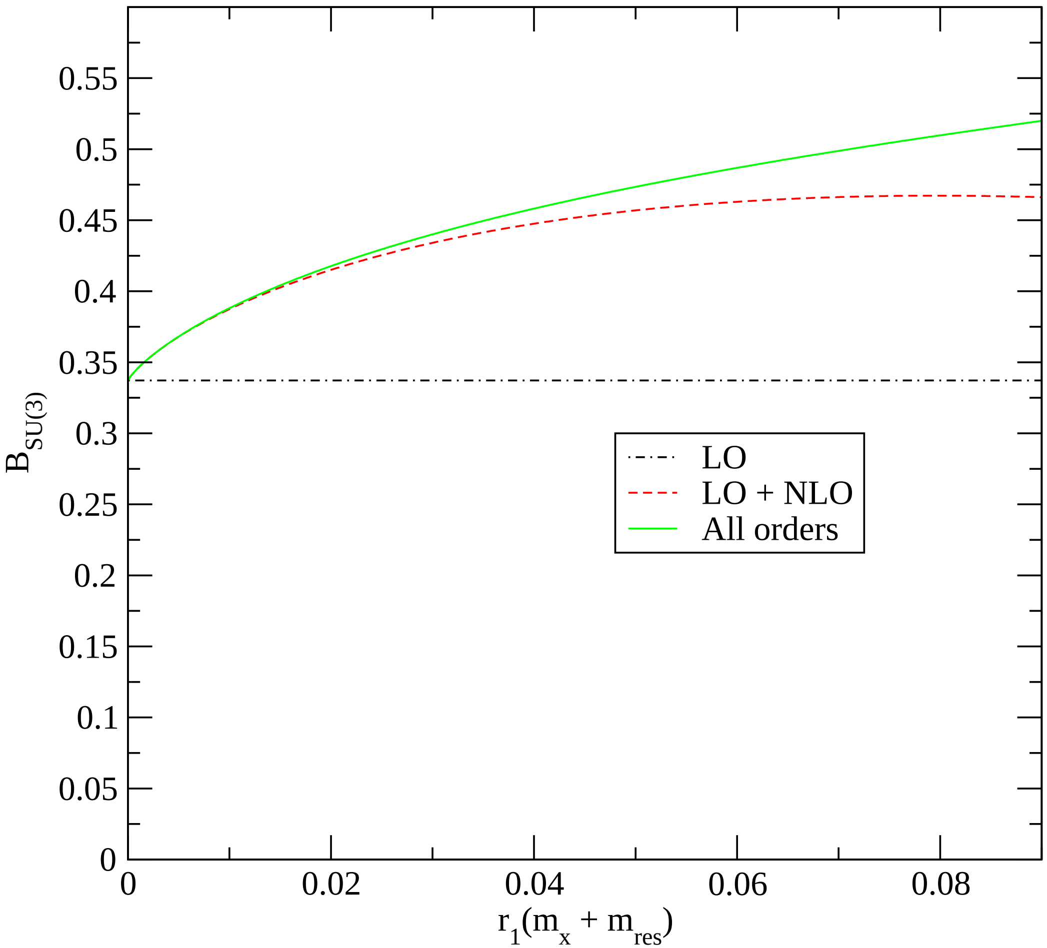
B_K chiral fit

Band is for degenerate valence masses in SU(3) limit



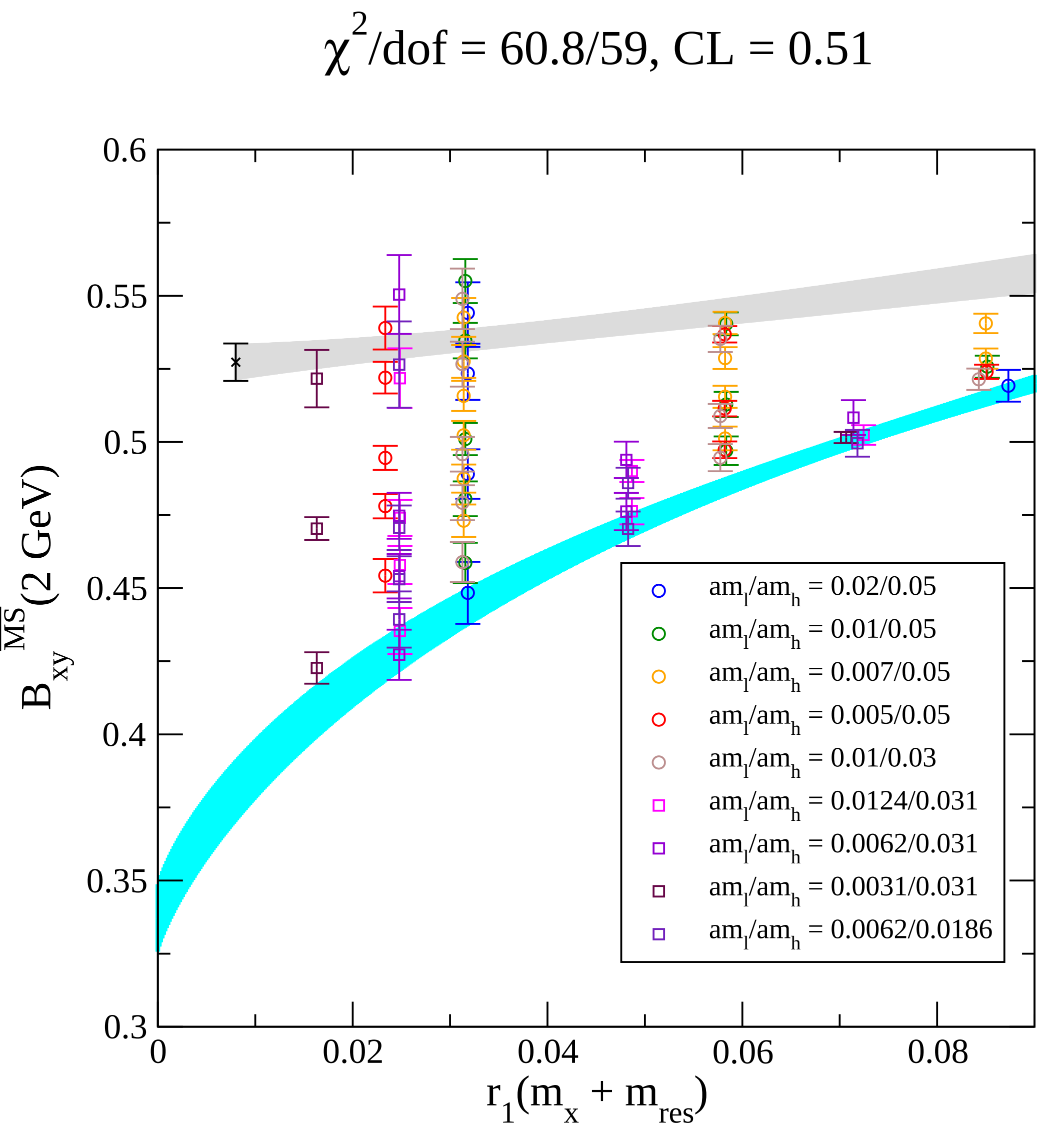
Convergence of ChPT for B_K

Curves are continuum QCD in SU(3) limit

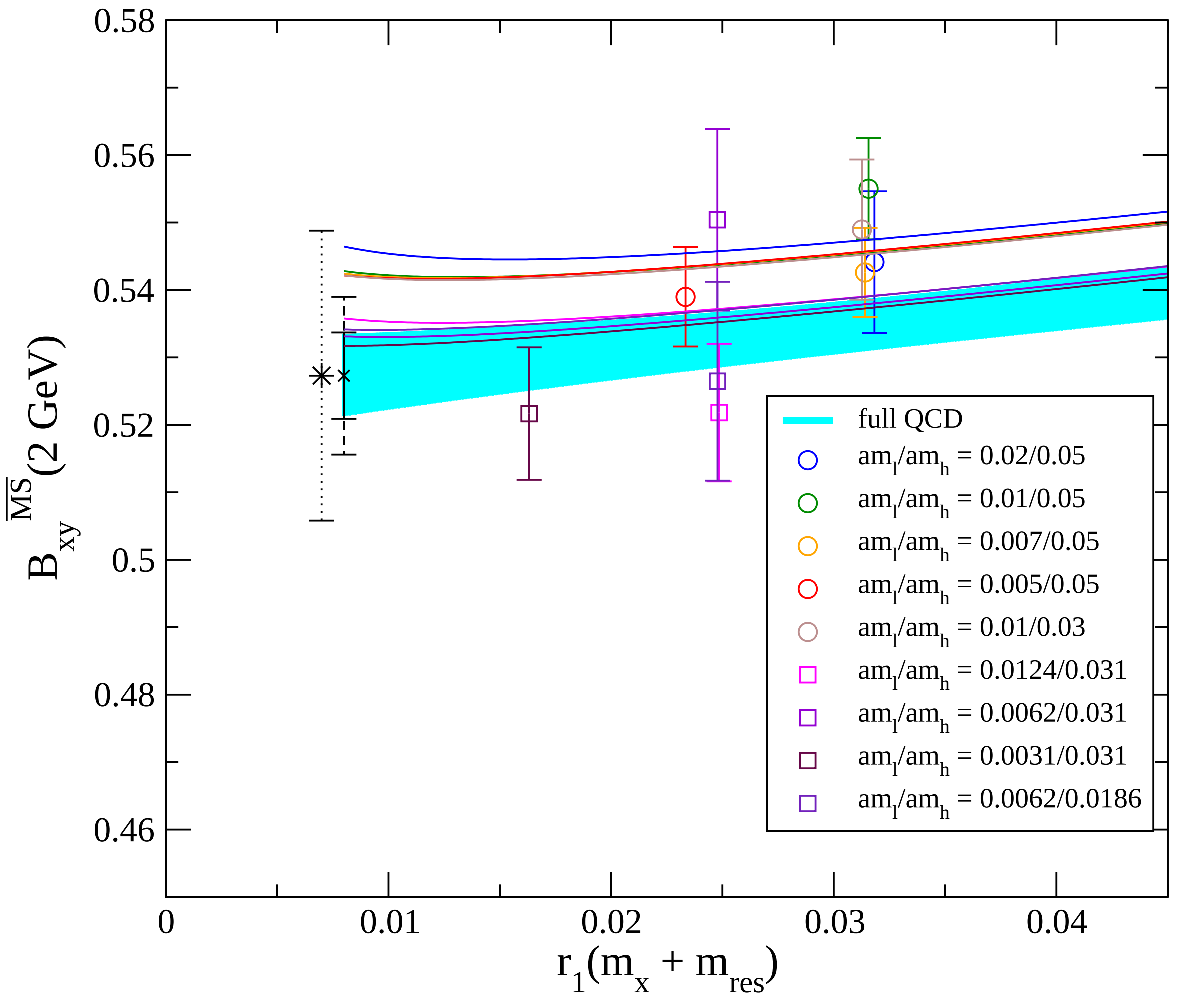


B_K extrapolation to physical point

Grey curve is full QCD at tuned sea and strange masses



B_K extrapolation to physical point



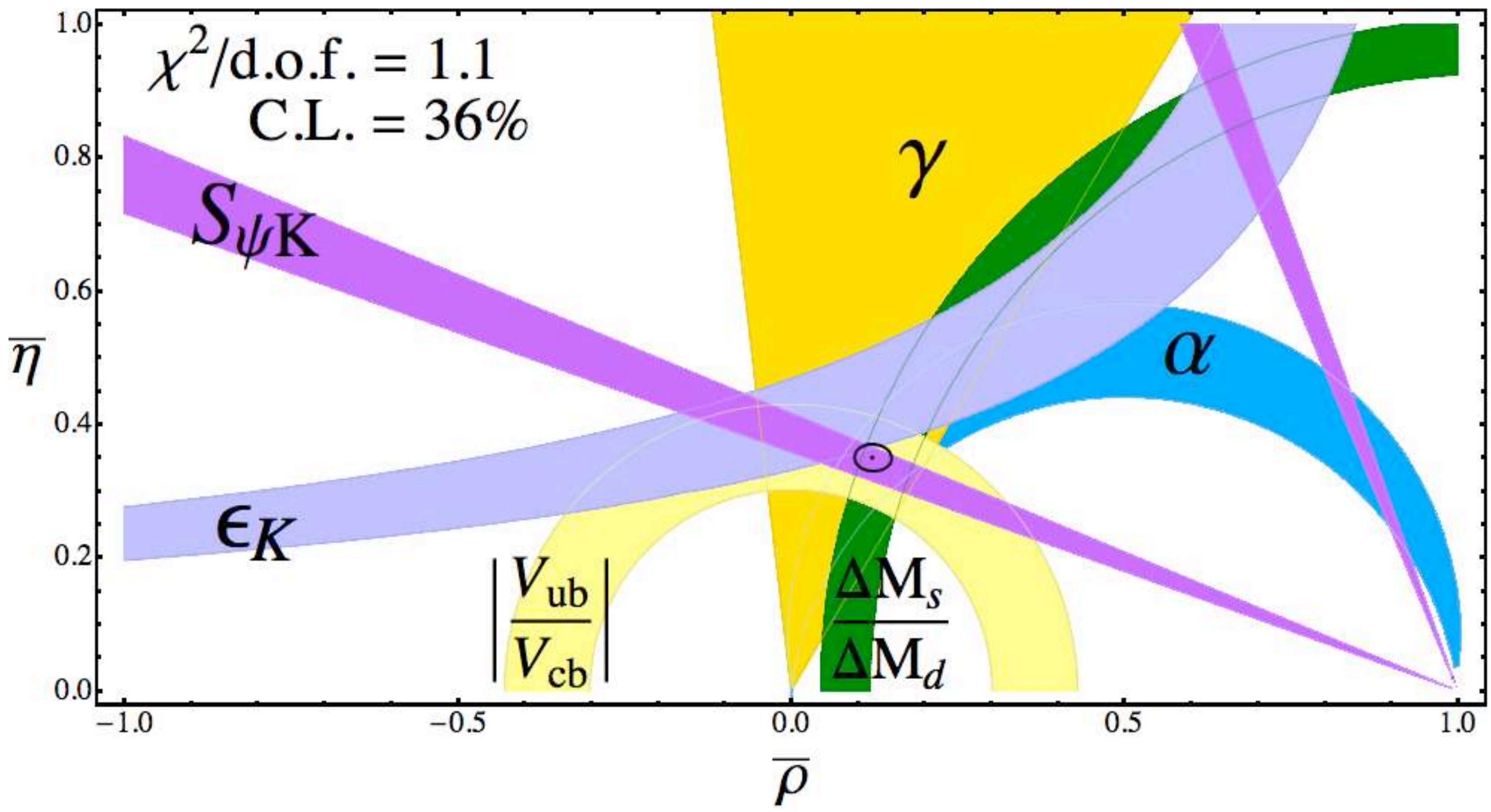
Result for B_K

uncertainty	B_K
statistics	1.2%
chiral & continuum extrapolation	1.9%
scale and quark mass uncertainties	0.8%
finite volume errors	0.6%
renormalization factor	3.3%
total	4.1%

$$\widehat{B}_K = 0.724(8)(28)$$

Compare to $\widehat{B}_K = 0.720(13)(37)$ [5.6% error] RBC/UKQCD (PRL 100:032001, 2008) and $\widehat{B}_K = 0.83(18)$ [22% error] HPQCD (PRD 73, 114502, 2006).

Unitarity triangle fit (Lunghi Fitter)



How does new B_K compare with UT fits?

Work with E. Lunghi, and R. Van de Water, in preparation.

$$(\widehat{B}_K)_{\rm fit} = \begin{cases} 1.08 \pm 0.13 & |V_{cb}|_{\rm excl} \\ 0.875 \pm 0.085 & |V_{cb}|_{\rm incl} \\ 0.95 \pm 0.11 & |V_{cb}|_{\rm excl+incl} \end{cases} \tag{10}$$

Differs from new world average $\widehat{B}_K = 0.725 \pm 0.026$ by 2.6, 1.7, and 2.0 σ, respectively.

Conclusions

Good consistency with MILC calculations on decay constants and quark masses.

Agreement with RBC/UKQCD on B_K.

Some tension in the CKM fit. Improved determination of $|V_{cb}|$ is necessary.