A tale of two right handed neutrinos

Gabriela Barenboim

Fermilab, August 6, 2009

$ Moneda nacional ($mn)	1 $mn
´70 peso Ley ($Ley)	1$Ley= 10^2 $mn
´83 peso argentino ($a)	1 $a =$10^2$ $Ley = 10^4 $mn
´85 Austral (A)	1 A = 10^4 $a =$10^6$ $Ley = 10^8 $mn
´92 peso ($)	1 $ = 10^4 A = 10^8 $a =$10^{10}$ $Ley = 10^{12} $mn

Unsolved issues in the standard model

- Horizon problem

  Why is the CMB so smooth ?

- The flatness problem

  Why is the Universe flat ? Why is $\Omega \sim 1$?

- The structure problem

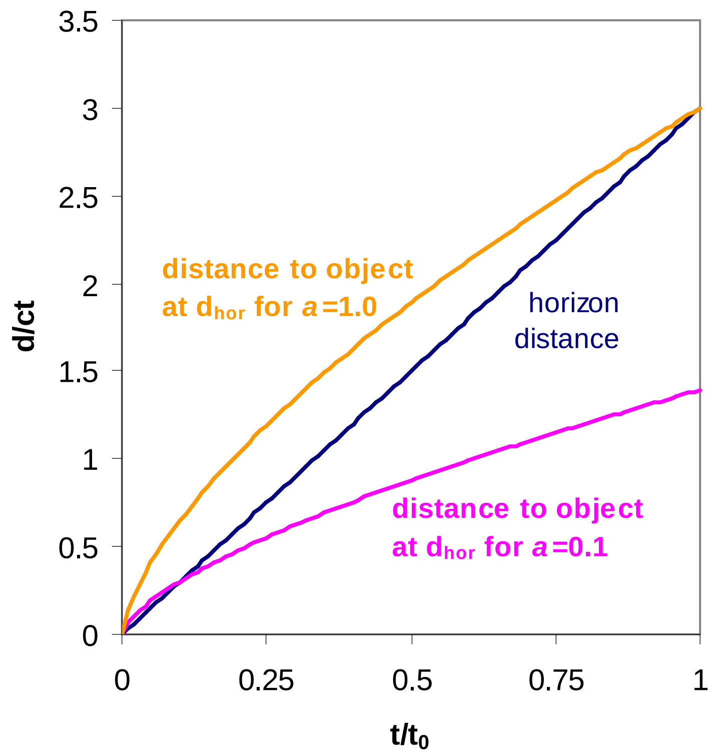
  Where do the fluctuations in the CMB come from ?

- The relic problem

  Why aren't there magnetic monopoles ?

Outstanding Problems

- Why is the CMB so isotropic?
  - consider matter-only universe:
    - horizon distance $d_H(t) = 3ct$
    - scale factor $a(t) = (t/t_0)^{2/3}$
    - therefore horizon expands faster than the universe
      - "new" objects constantly coming into view
  - CMB decouples at $1+z \sim 1000$
    - i.e. $t_{CMB} = t_0/10^{4.5}$
    - $d_H(t_{CMB}) = 3ct_0/10^{4.5}$
    - now this has expanded by a factor of 1000 to $3ct_0/10^{1.5}$
    - but horizon distance now is $3ct_0$
    - so angle subtended on sky by one CMB horizon distance is only $10^{-1.5}$ rad ~ 2°
  - patches of CMB sky >2° apart should not be causally connected



Outstanding Problems

- Why is universe so flat?
  - a multi-component universe satisfies

$$1-\Omega(t) = -\frac{kc^2}{H(t)^2 a(t)^2 R_0^2} = \frac{H_0^2(1-\Omega_0)}{H(t)^2 a(t)^2}$$

and, neglecting Λ,

$$\left(\frac{H(t)}{H_0}\right)^2 = \frac{\Omega_{\mathrm{r0}}}{a^4} + \frac{\Omega_{\mathrm{m0}}}{a^3}$$

  - therefore
    - during radiation dominated era $|1 - \Omega(t)| \propto a^2$
    - during matter dominated era $|1 - \Omega(t)| \propto a$
    - if $|1 - \Omega_0| < 0.06$ (WMAP) ... then at CMB emission $|1 - \Omega| < 0.00006$
  - we have a fine tuning problem!

Outstanding Problems

- Where is everything coming from ?

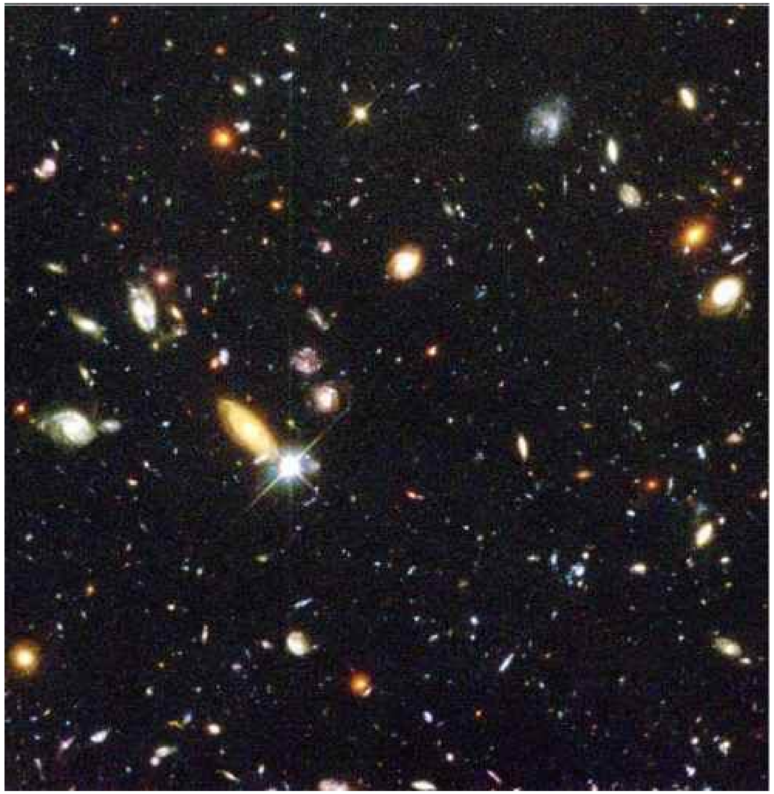
Models like CDM nicely explain how the fluctuations we can observe in the CMB grew to form galaxies.

They can also reproduce the observe large scale distribution of galaxies and clusters.

BUT .. why are there fluctuations in the first place ?

Outstanding Problems

- Where is everything coming from ?



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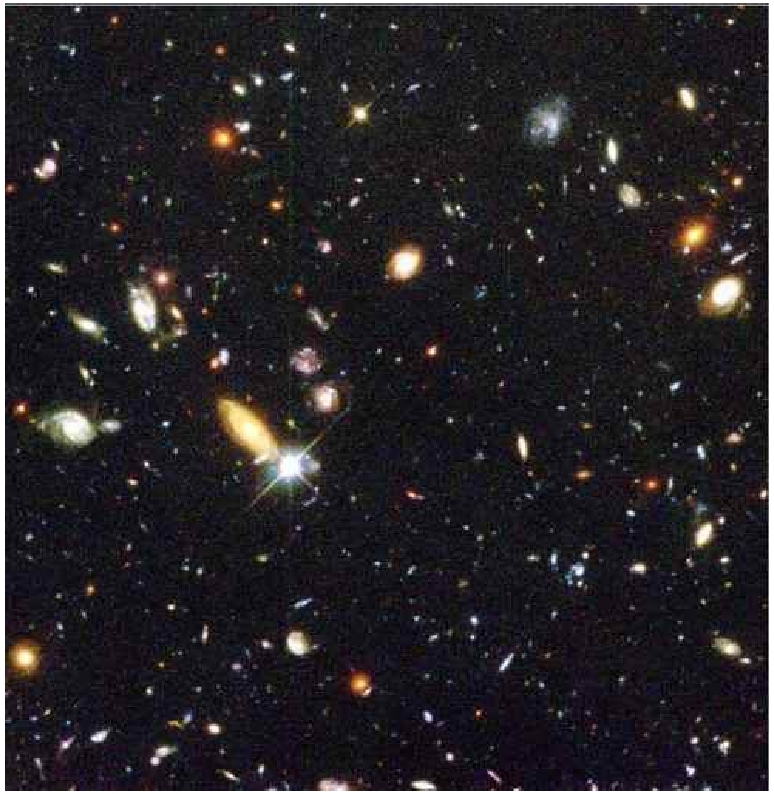
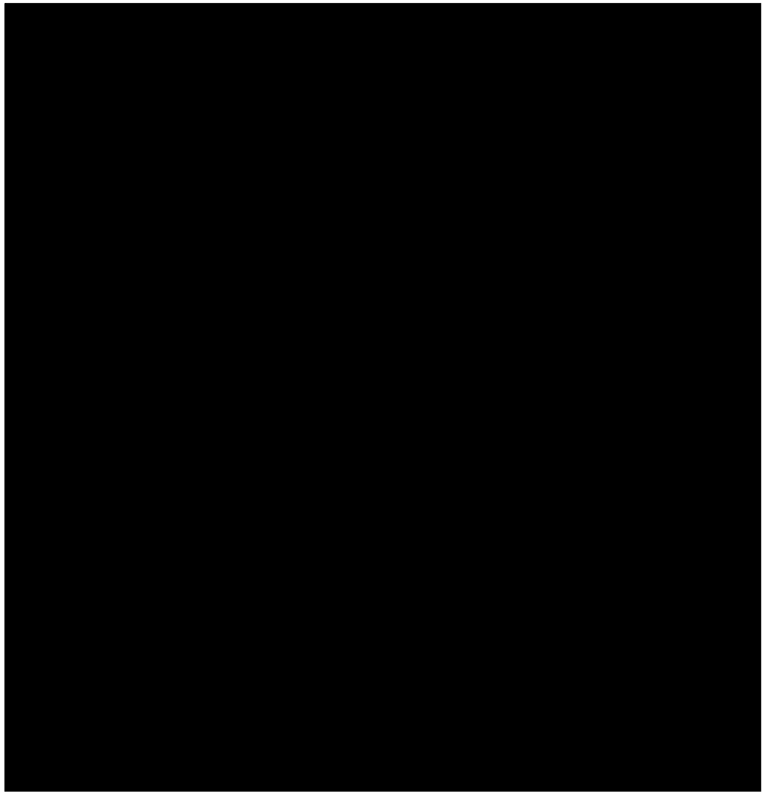




Outstanding Problems

- The monopole problem
  - big issue in early 1980s
    - Grand Unified Theories of particle physics → at high energies the strong, electromagnetic and weak forces are unified
    - the symmetry between strong and electroweak forces 'breaks' at an energy of ~10^{15} GeV ($T \sim 10^{28}$ K, $t \sim 10^{-36}$ s)
      - this is a phase transition similar to freezing
      - expect to form 'topological defects' (like defects in crystals)
      - point defects act as magnetic monopoles and have mass ~10^{15} GeV/c^2 (10^{-12} kg)
      - expect one per horizon volume at $t \sim 10^{-36}$ s, i.e. a number density of 10^{82} m^{-3} at 10^{-36} s
      - result: universe today completely dominated by monopoles (not!)

The concept of inflation

The idea (A. Guth and A. Linde, 1981): Shortly after the Big Bang, the Universe went through a phase of rapid (exponential) expansion. In this phase the energy and thus the dynamics of the Universe was determined by a term similar to the cosmological constant (vacuum energy).

Why would the Universe do that ?

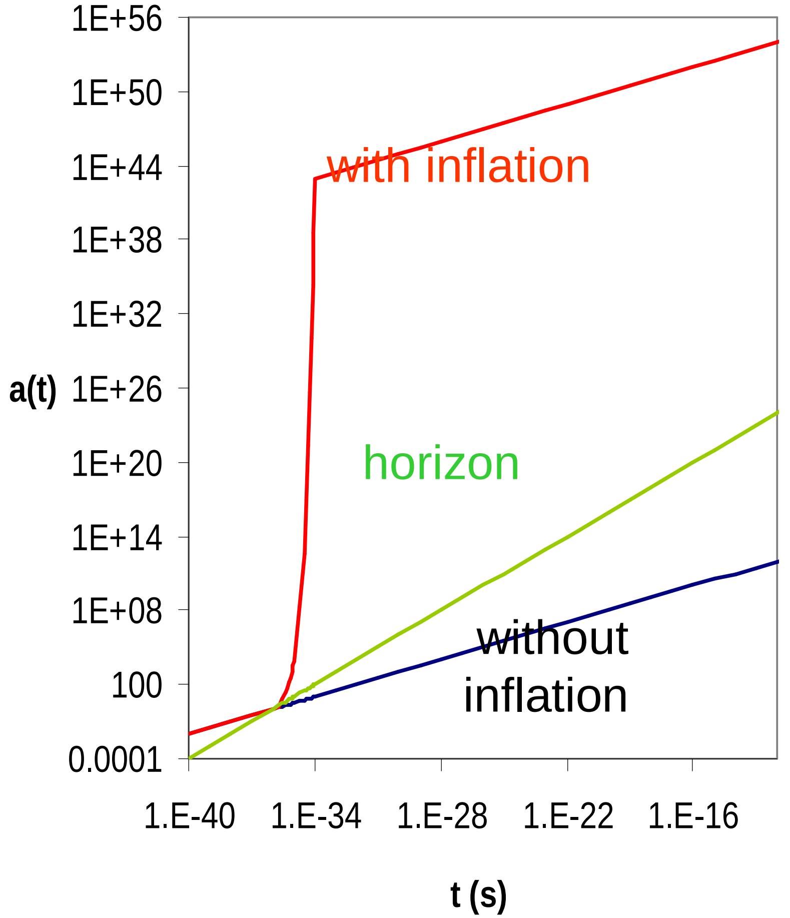
Why does it help ?

Inflation and the horizon

- Assume large positive cosmological constant Λ acting from t_{inf} to t_{end}

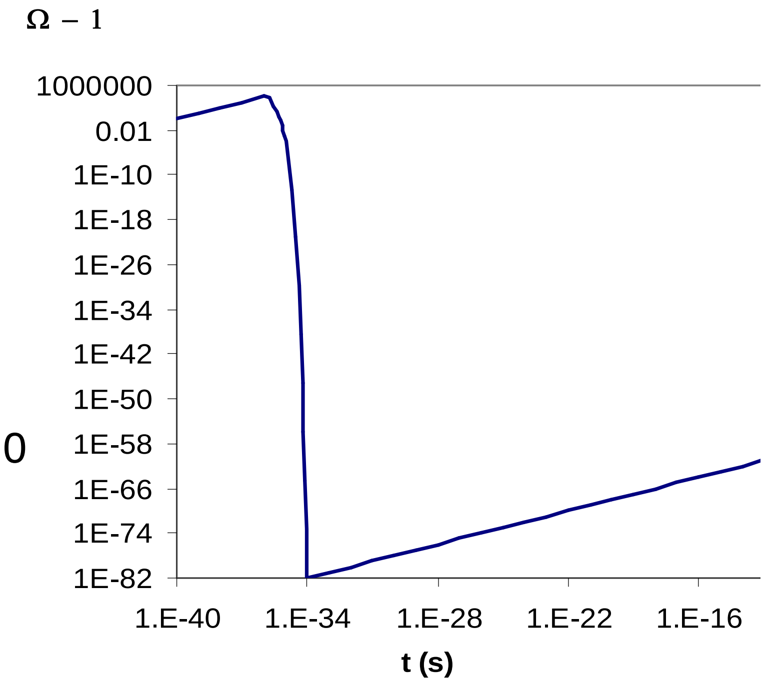
- then for $t_{inf} < t < t_{end}$
  $a(t) = a(t_{inf}) \exp[H_i(t - t_{inf})]$
  - $H_i = (\frac{1}{3}\Lambda)^{1/2}$
  - if Λ large *a* can increase by many orders of magnitude in a very short time

- Exponential inflation is the usual assumption but a power law $a = a_{inf}(t/t_{inf})^n$ works if $n > 1$



Inflation and flatness

- We had
$$1-\Omega(t) = -\frac{kc^2}{H(t)^2 a(t)^2 R_0^2} = \frac{H_0^2(1-\Omega_0)}{H(t)^2 a(t)^2}$$
  - for cosmological constant H is constant, so $1 - \Omega \propto a^{-2}$
  - for matter-dominated universe $1 - \Omega \propto a$
- Assume at start of inflation $|1 - \Omega| \sim 1$
- Now $|1 - \Omega| \sim 0.06$
  - at matter-radiation equality $|1 - \Omega| \sim 2 \times 10^{-5}$, $t \sim 50000$ yr
  - at end of inflation $|1 - \Omega| \sim 10^{-50}$
  - so need to inflate by $10^{25} = e^{58}$



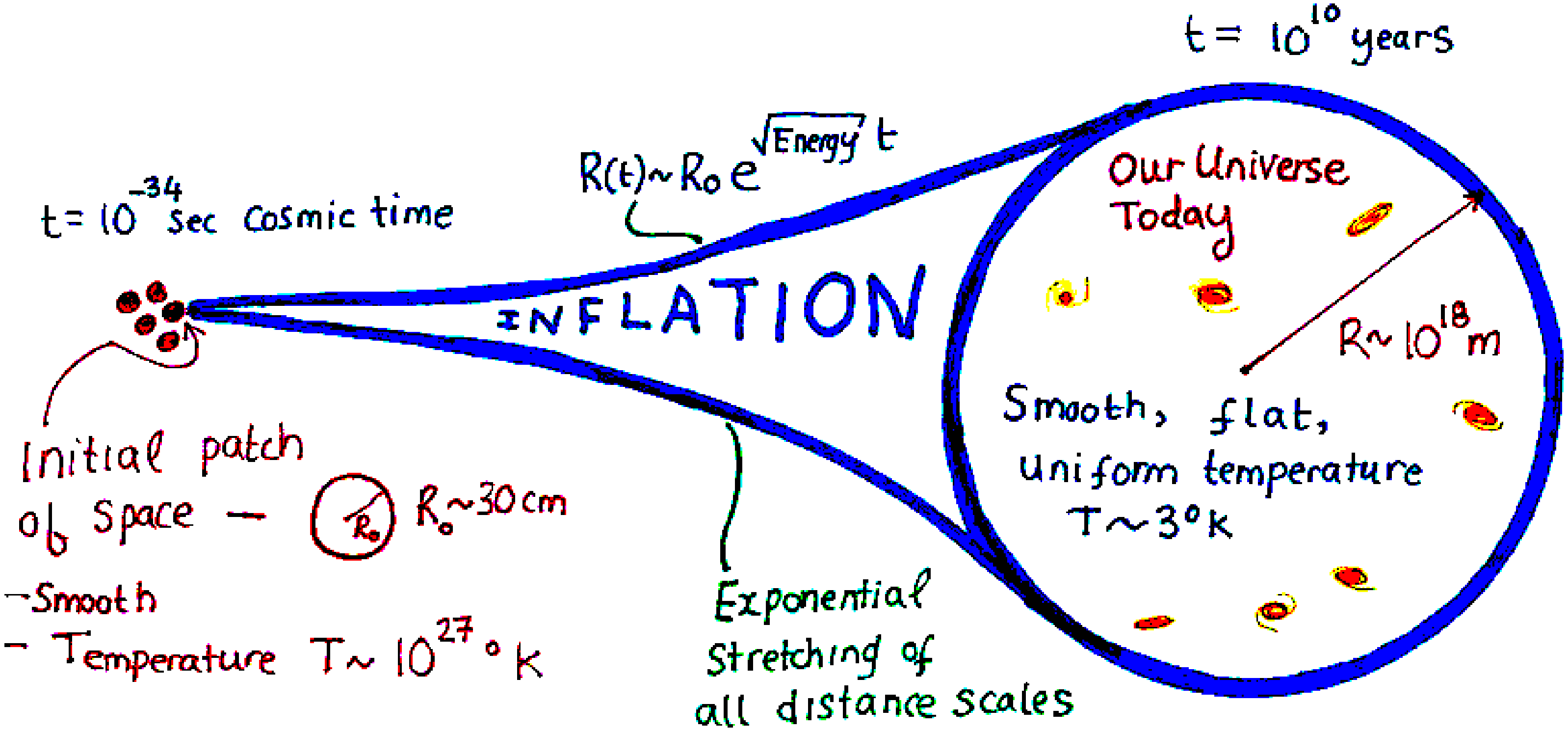
Inflation and the structure problem

- Before inflation: quantum fluctuations

- Inflation amplifies quantum fluctuations to macroscopic scales

- After inflation macroscopic fluctuations (as can be observed in the CMB radiation) provide the seeds from which galaxies form.

Inflation and the relic problem



What powers inflation?

- We need $H_{inf}(t_{end} - t_{inf}) \geq 58$
  - if $t_{end} \sim 10^{-34}$ s and $t_{inf} \sim 10^{-36}$ s, $H_{inf} \sim 6 \times 10^{35}$ s^{-1}

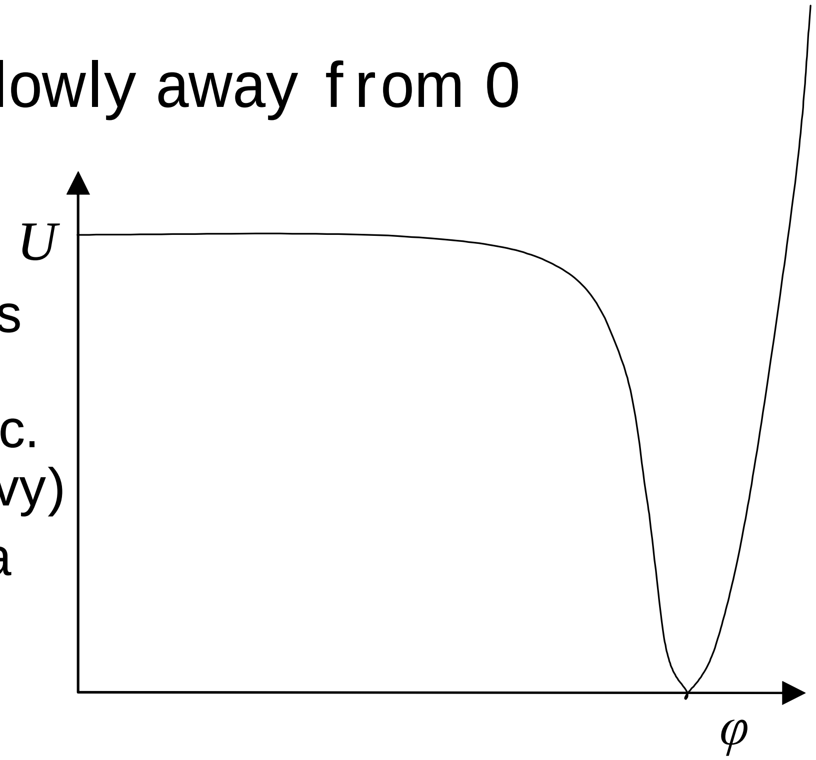
  - energy density $\rho_\Lambda \sim 6 \times 10^{97}$ J m^{-3} ~ 4×10^{104} TeV m^{-3}
    - cf. current value of $\Lambda \sim 10^{-35}$ s^{-2}, $\rho_\Lambda \sim 10^{-9}$ J m^{-3} ~ 0.004 TeV m^{-3}
- We also need an equation of state with negative pressure

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\rho + 3P)$$

accelerating expansion needs $P < 0$

Inflation with scalar field

- Need potential U with broad nearly flat plateau near $\varphi = 0$
  - metastable false vacuum
  - inflation as φ moves very slowly away from 0
  - stops at drop to minimum (true vacuum)
    - decay of inflaton field at this point reheats universe, producing photons, quarks etc. (but not monopoles – too heavy)
    - equivalent to latent heat of a phase transition

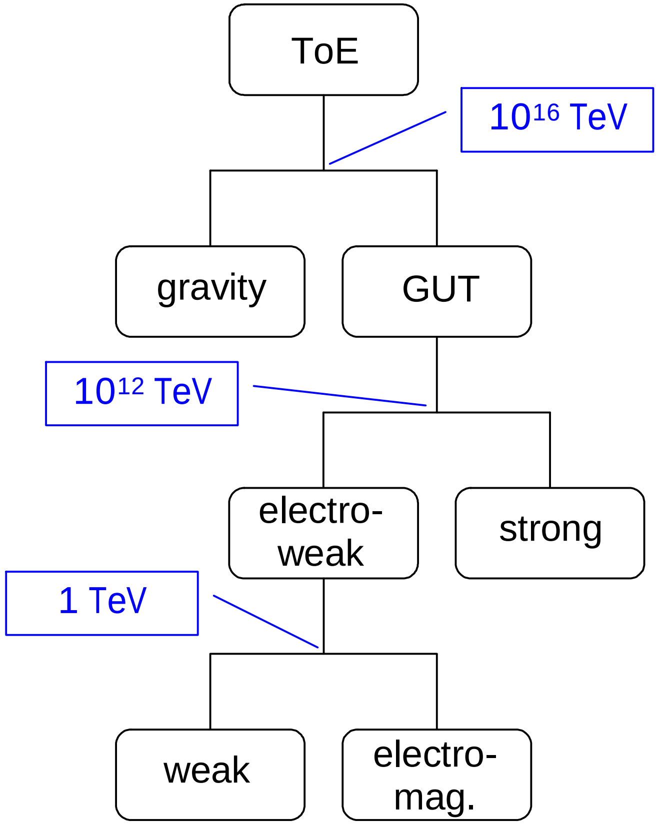


Inflation and particle physics

- At very high energies particle physicists expect that all forces will become unified
  - this introduces new particles
  - some take the form of scalar fields φ with equation of state

$$\rho_\varphi = \frac{1}{2\hbar c^3}\dot{\varphi}^2 + U(\varphi)$$

$$P_\varphi = \frac{1}{2\hbar c^3}\dot{\varphi}^2 - U(\varphi)$$



if $\dot{\varphi}^2 \ll 2\hbar c^3 U(\varphi)$ this looks like Λ

Life without a fundamental scalar

Good news : Bardeen, Hill and Lindner used a top quark condensate to replace the Higgs. The theory can predict both: the top mass and EWSB scale.

Bad news: a lot of fine tunning was needed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Constructing the scalar field

The four fermion effective interaction for the right handed neutrino below the scale Λ takes the form

$$G\,(\overline{\nu^c}_R\,\nu_R)\,(\overline{\nu}_R\,\nu^c_R)$$

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When the right handed neutrinos condense

$$-\,m_0^2\ \Phi^\dagger\Phi\ +\ g_0\ (\bar{\nu}^C_R\ \nu_R\ \Phi\ +\ \text{h.c.}\ )$$

with $G = g_0^2\ /\ m_0^2$

Let's keep the scalar field and integrate the short distance components of the right handed neutrino

$$g_0 \left(\nu^C_R \, \nu_R \, \Phi + \text{h.c.}\right) + Z_\Phi \left|D_\mu \Phi\right|^2 - m_\Phi^2 \, \Phi^\dagger \Phi - \lambda_0 \left(\Phi^\dagger \Phi\right)^2$$

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where

$$Z_\Phi = N_f\,g_0^2 / (4\pi)^2\,\ln(\Lambda^2/\mu^2)$$

$$m_\Phi^2 = m_0^2 - 2\,N_f\,g_0^2 / (4\pi)^2\,(\Lambda^2 - \mu^2)$$

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rescale the scalar field $\Phi \to \Phi / (Z_\Phi)^{1/2}$

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$$g = g_0 / (Z_\Phi)^{1/2}$$

$$m^2 = m_\Phi^2 / (Z_\Phi)$$

$$\lambda = / \lambda_0 (Z_\Phi)^2$$

where

$$Z_\Phi = N_f g_0^2 / (4\pi)^2 \ln (\Lambda^2 / \mu^2)$$

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Finally ...

$$g\,(\nu^C_R\,\nu_R\,\Phi + h.c.) + |D_\mu \Phi|^2 - V(\Phi)$$

with

$$V(\Phi) = m^2\,\Phi^\dagger\Phi - \lambda(\Phi^\dagger\Phi)^2$$

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Breaking the U(1)

The lowest dimension symmetry breaking operator constructed from the right handed neutrinos is given by

$$G'[\,(\overline{\nu}^{c}_{R}\,\nu_{R})^{2} + (\overline{\nu}_{R}\,\nu^{c}_{R})^{2}]$$

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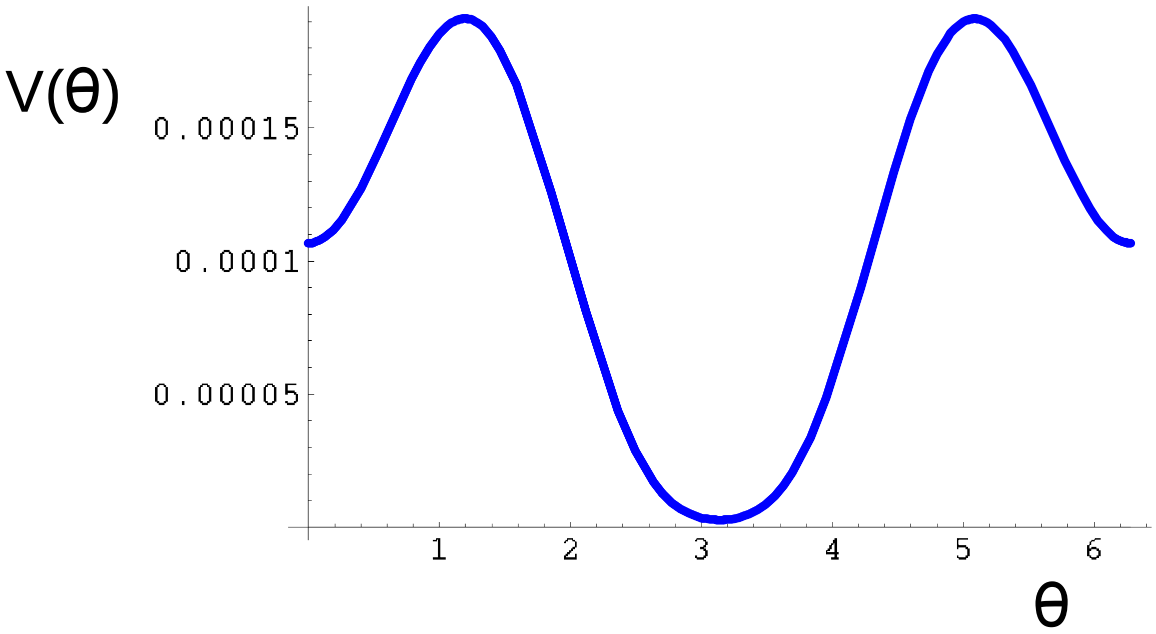
Resorting to the scalar field

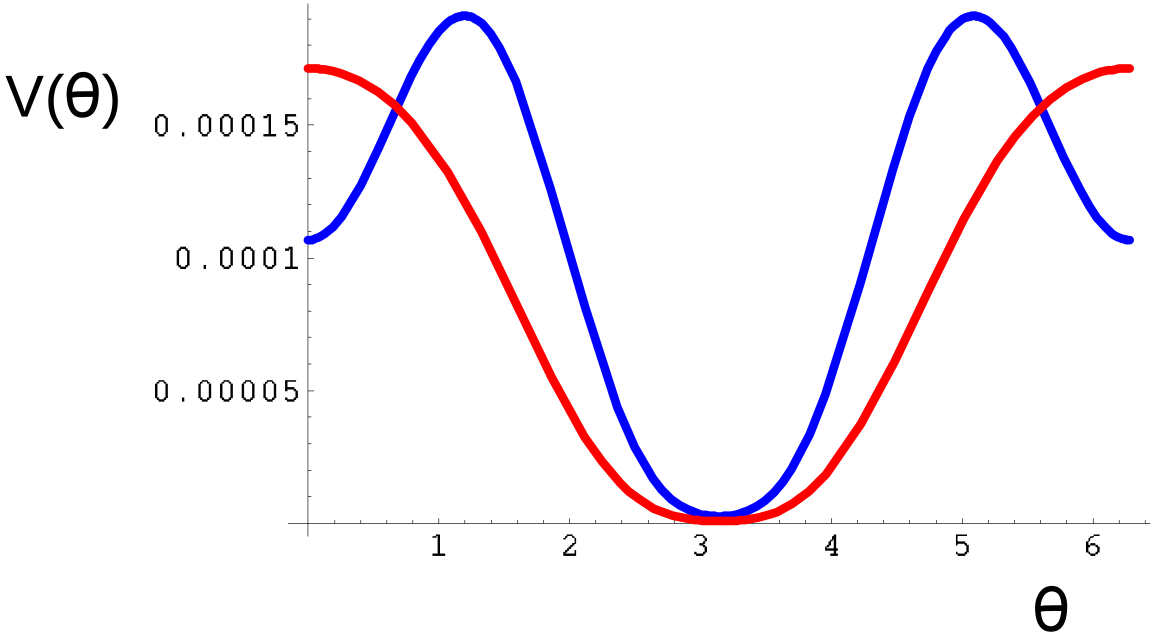
$$g'\,(\overline{\nu^c_R}\,\nu_R\,\Phi^\dagger + \overline{\nu_R}\,\nu^c_R\,\Phi)$$

at 1-loop

$$m_R^2(\theta) = (g^2 + g'^2 + 2\, g\, g' \,\mathrm{Cos}(\theta)\,)\, v^2$$

$$V(\theta) = -\frac{g^2\, g'^2\, v^4}{(16\, \pi^2)} \left[\frac{g^2 + g'^2}{2gg'} + \mathrm{Cos}(\theta)\right]^2 \ln \left[g^2 + g'^2 + 2gg'\, \mathrm{Cos}(\theta)\right]$$


V(θ)
0.00015
0.0001
0.00005
1
2
3
4
5
6
θ



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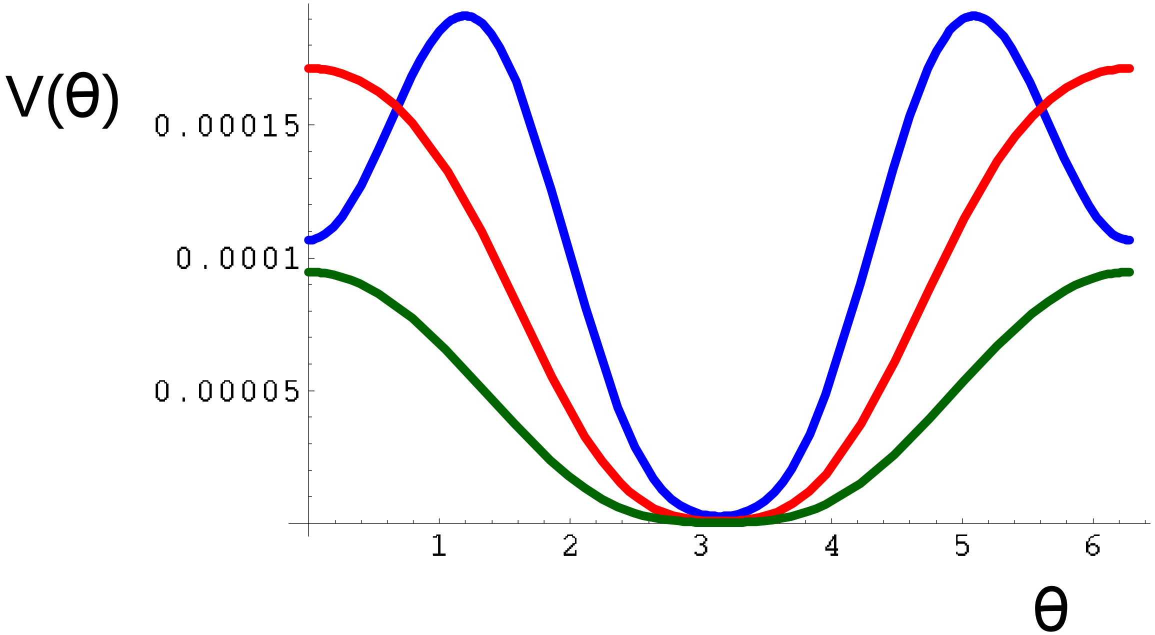

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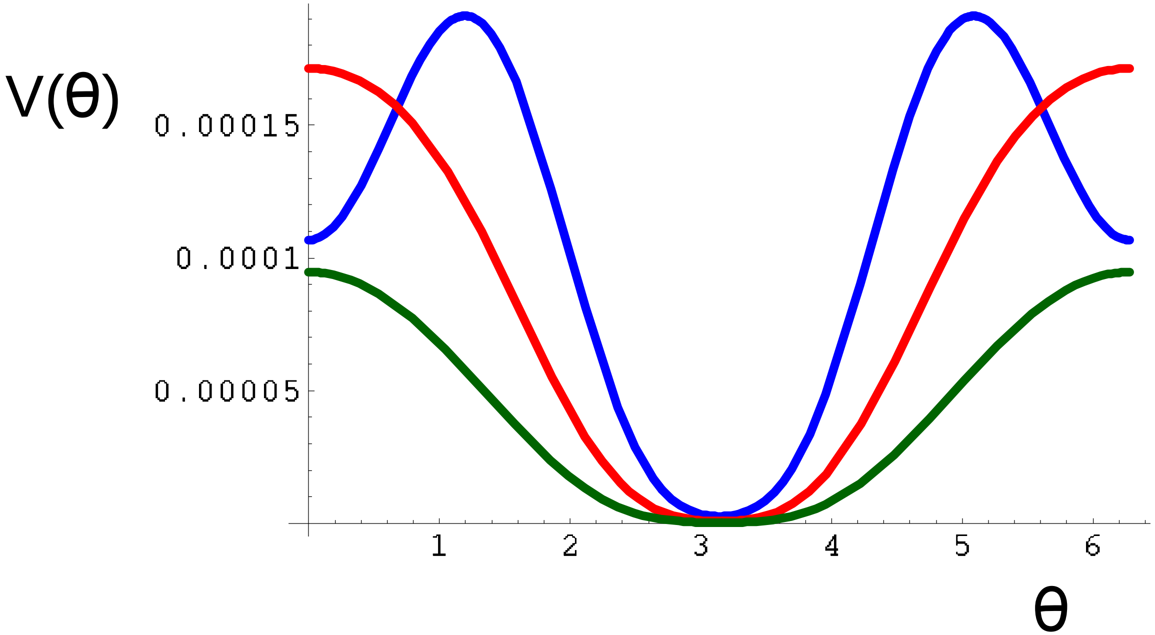

The potential has its minimum at $\theta = \pi$

$$m_\theta^2 = - \frac{g\, g'\, v^2}{32\, \pi^2} (g - g')^2 \left(1 + 2 \ln (g - g')^2 \right)$$

$$m_R^2(\theta) = (g^2 + g'^2 + 2\, g\, g' \, \mathrm{Cos}(\theta)\,)\, v^2$$

$$V(\theta) = -\frac{g^2\, g'^2\, v^4}{(16\,\pi^2)} \left[\frac{g^2 + g'^2}{2gg'} + \mathrm{Cos}(\theta)\right]^2 \ln\left[g^2 + g'^2 + 2gg' \, \mathrm{Cos}(\theta)\right]$$

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The scalar field and its circumstances

slow roll approximation

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The beginning of the end...

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}\left(\rho + 3P\right)$$

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$$N = \frac{8\pi}{3 M_{Pl}^2}\int \frac{V(\theta)}{V'(\theta)}\,d\theta$$

Inflation phenomenology

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Recall: quantum fluctuations become density perturbations

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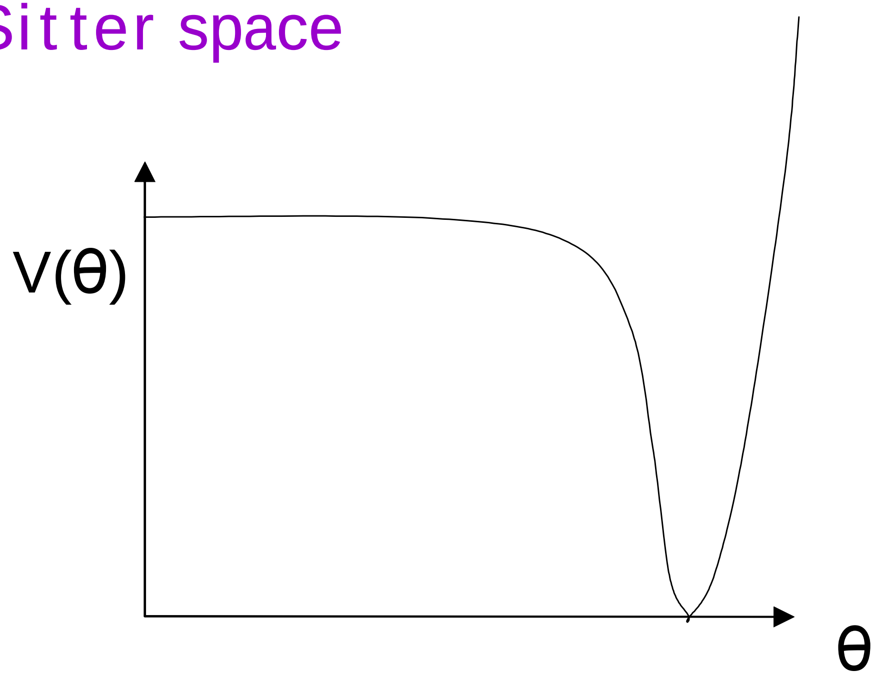
Quantum fluctuations in de Sitter space

$$< (\delta\theta)^2 > \approx H^2$$



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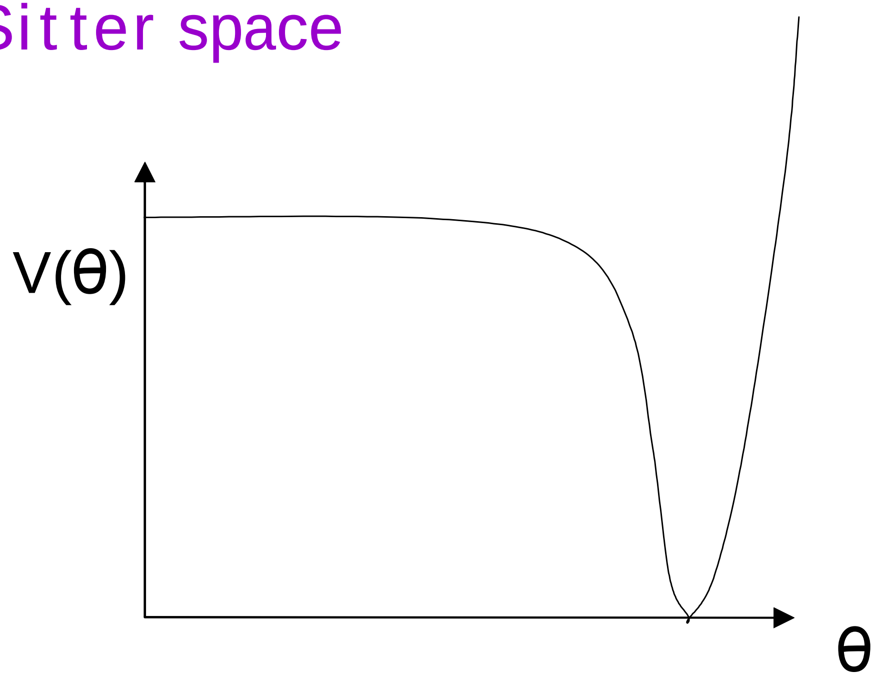

$$\frac{\delta\rho}{\rho} \approx \frac{H^2}{\dot{\theta}}$$

Scalar density fluctuations are given by different regions ending inflation at different times

Inflation phenomenology

Again: quantum fluctuations in de Sitter space

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The two helicity states of the gravitational waves follow independent equations of a scalar field. Hence gravity waves fluctuations

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Scalar density perturbations depend on the shape of the potential. Tensor perturbations just depend on the scale.

Inflation phenomenology

Scalar and tensor perturbations are characterized by their amplitude and spectra

scalar perturbations : $\delta\rho/\rho$, n_s

tensor perturbations: r, n_T

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$r = -8\, n_T$

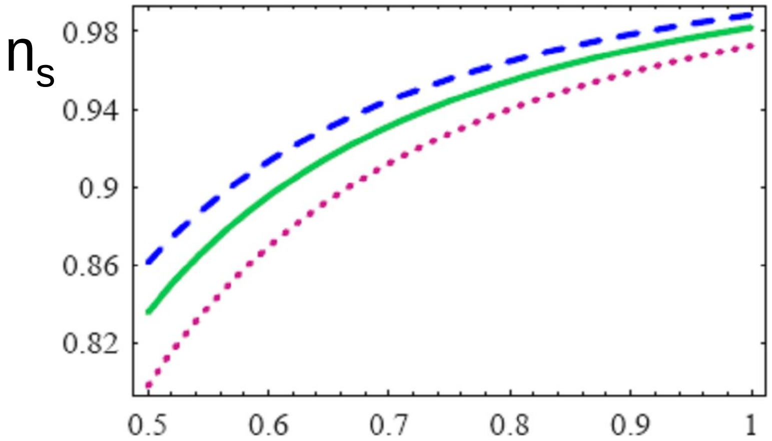
Inflation phenomenology

Scalar and tensor perturbations are characterized by their amplitude and spectra

scalar perturbations : $\delta\rho/\rho$, n_s $dn_s/ d \ln(k)$

tensor perturbations: r, n_T $r = -8\, n_T$

Inflation phenomenology



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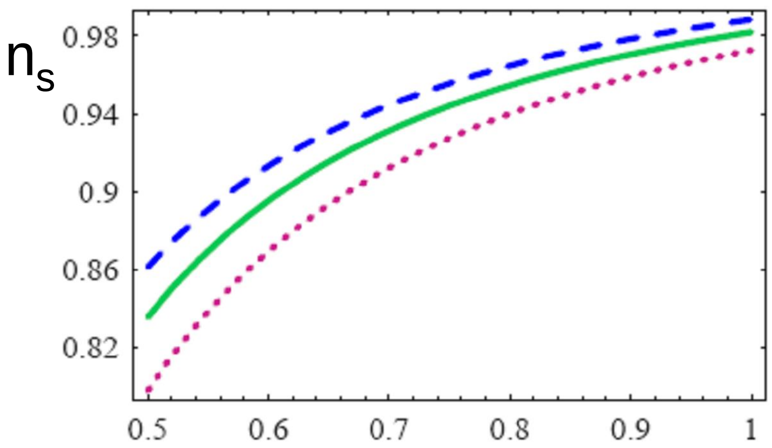
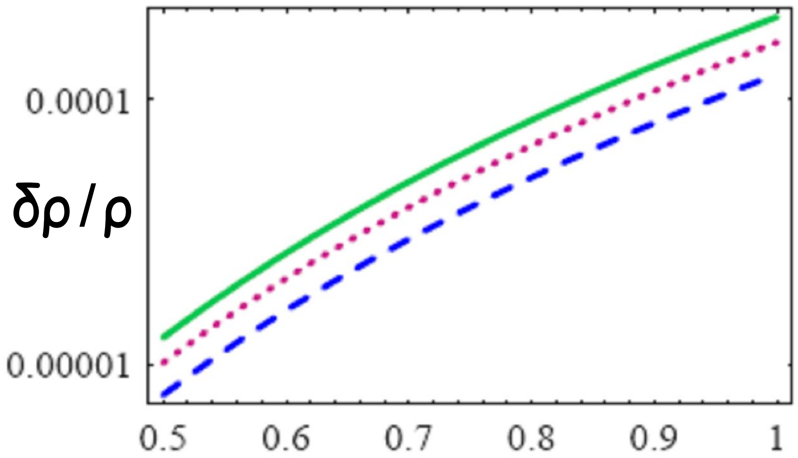
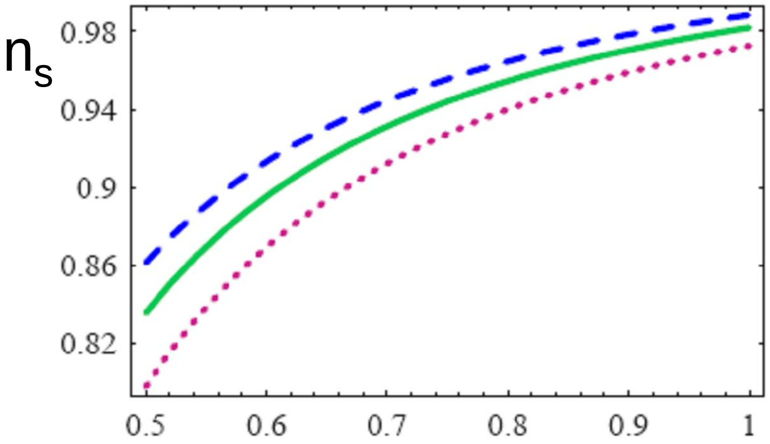
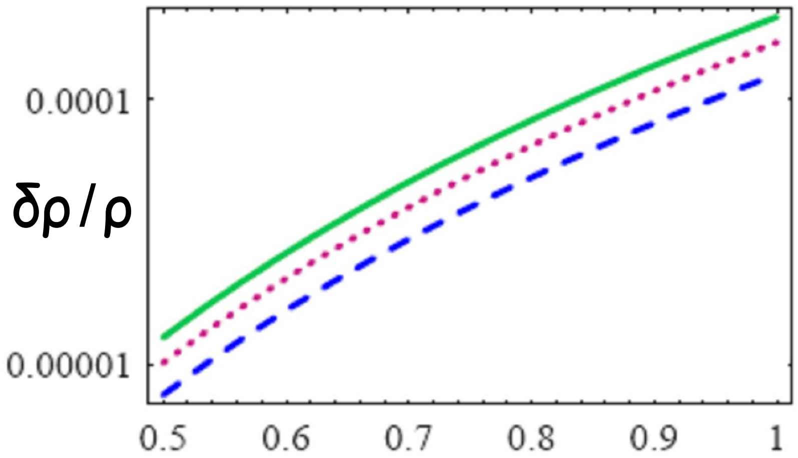
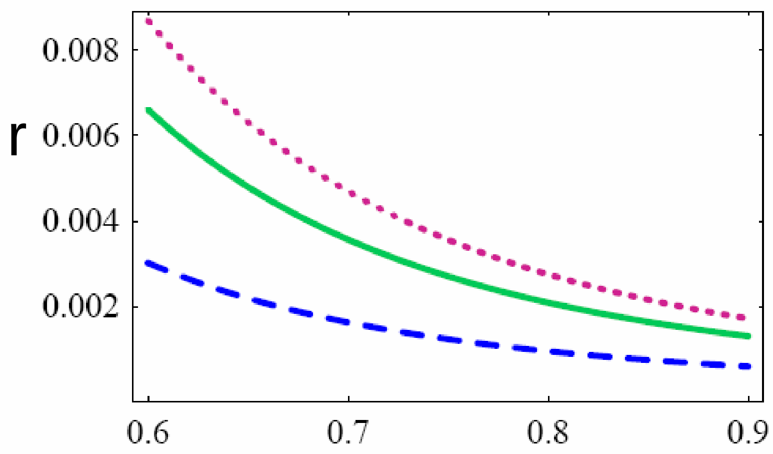




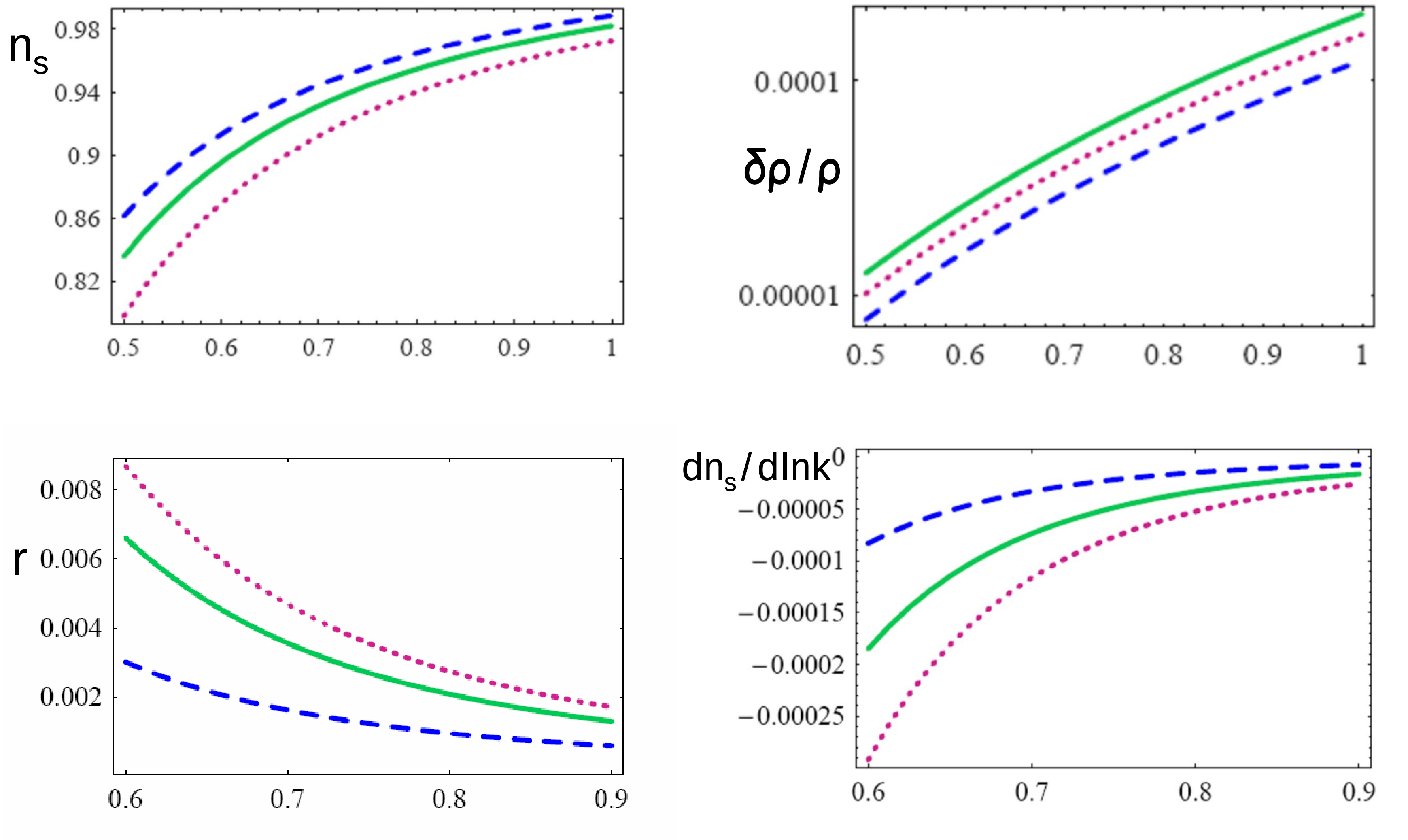
Inflation phenomenology







Inflation phenomenology



Conclusions

A theory ENTIRELY written in terms of neutrino degrees of freedom is equivalent to a theory containing Φ.

The resulting model is phenomenogically tighly constrained and can be (dis) probed in the near future.

The model with more neutrinos is EVEN more beautiful

(if such a thing is possible).