W+3 Jet Production at Hadron Colliders

NLO QCD corrections with BlackHat+SHERPA

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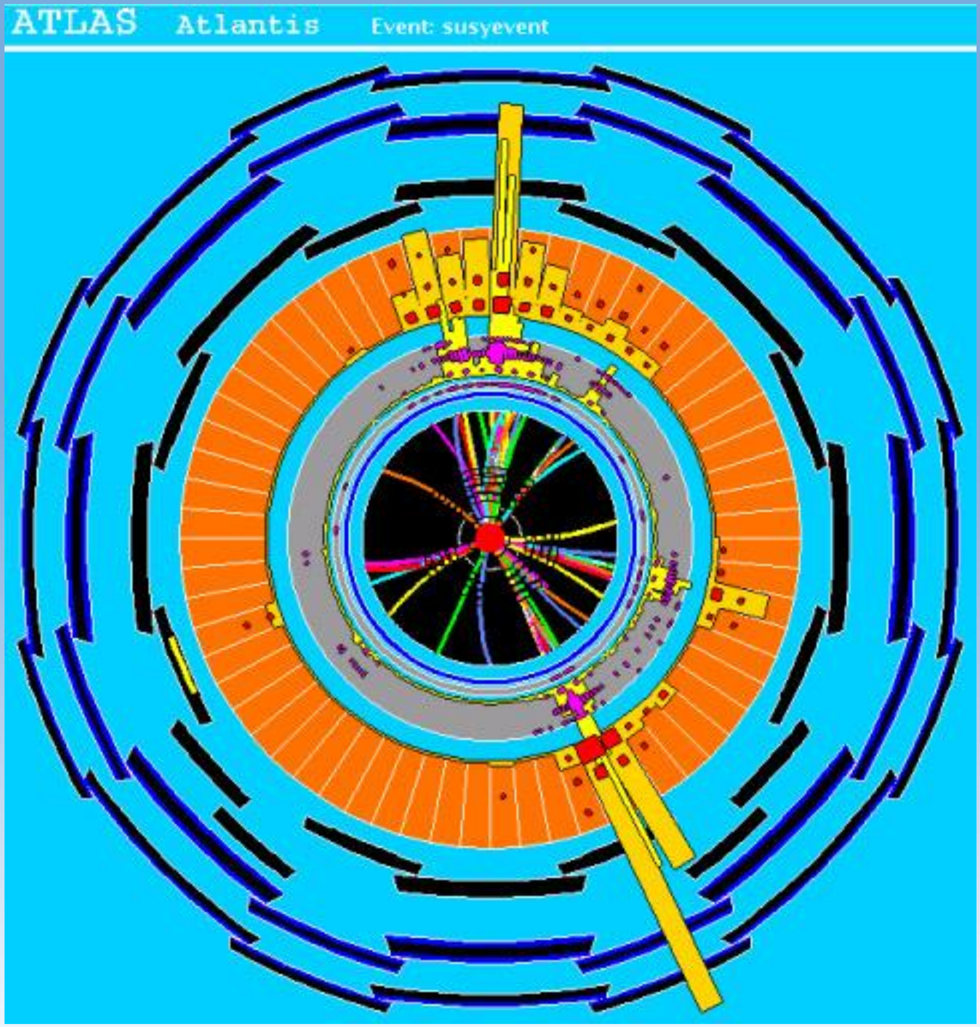
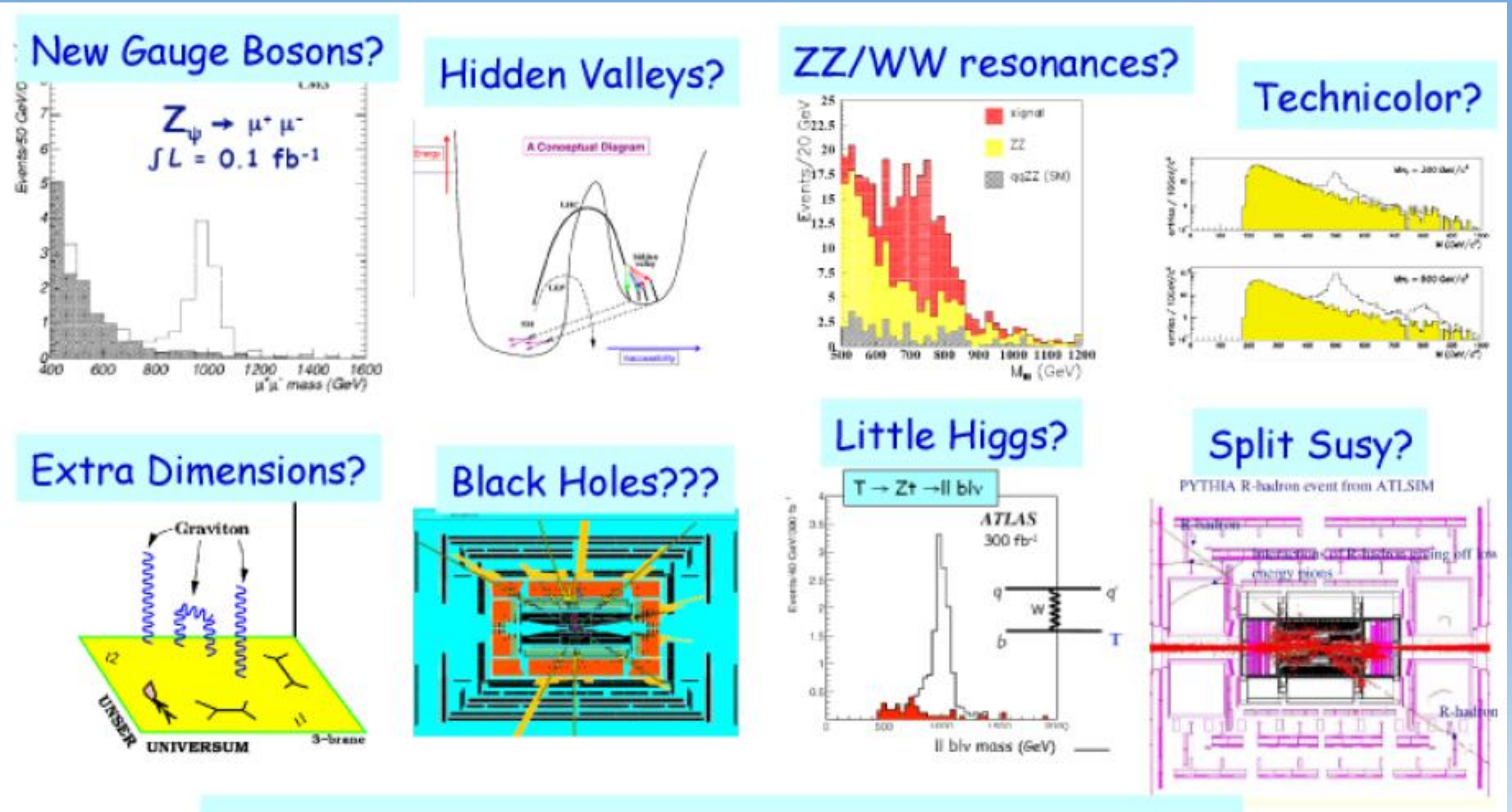
Fermilab Theory Seminar --- May 2009

Based on: **arXiv:0803.4180 ; arXiv:0808.0941 ; arXiv:0902.2760**

In collaboration with: *Carola Berger, Zvi Bern, Lance Dixon, Darren Forde, Tanju Gleisberg, Harald Ita, David Kosower and Daniel Maitre*

The questions ahead…





taken from J. Incandela

Summary:

1. Introduction:
   - Need for NLO and beyond
   - Calculation of Cross sections
   - Techniques and implementation:
     - BlackHat & SHERPA

2. W+ n jets (n=1,2,3) at NLO

3. Conclusions and Outlook

Why NLO?

$$\hat{\sigma}(\alpha_s, \mu_F, \mu_R) \;=\; [\alpha_s(\mu_R)]^{n_\alpha}\left[\hat{\sigma}^{(0)} + \frac{\alpha_s}{2\pi}\hat{\sigma}^{(1)}(\mu_F,\mu_R) + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}(\mu_F,\mu_R) + \cdots\right]$$

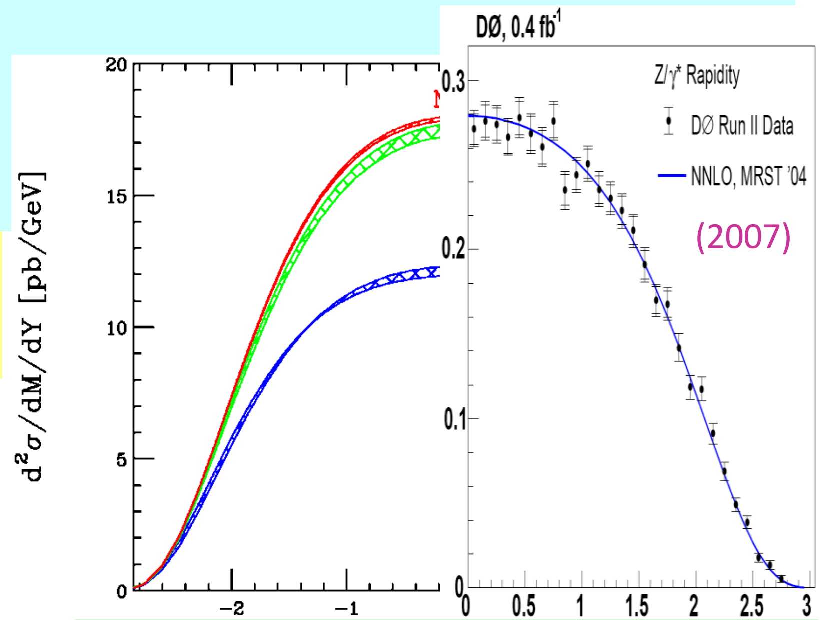
LO NLO NNLO

Because leading order (LO) predictions are only qualitative in normalization, due to **poor convergence** of expansion in $\alpha_s(\mu)$

Example: *Z* production at Tevatron
Distribution in rapidity *Y*

$$Y = \frac{1}{2}\ln\left(\frac{E+p_z}{E-p_z}\right)$$

$\frac{d\sigma}{dY}$ has $n_\alpha = 0$



still ~50% corrections, LO → NLO

by NNLO, a precision observable

Anastasiou, Dixon, Melnikov, Petriello (2004)

W+n jets: Comparing Rates

number of jets	CDF	LC NLO	NLO
1	53.5 ± 5.6	$58.3^{+4.6}_{-4.6}$	$57.8^{+4.4}_{-4.0}$
2	6.8 ± 1.1	$7.81^{+0.54}_{-0.91}$	$7.62^{+0.62}_{-0.86}$
3	0.84 ± 0.24	$0.908^{+0.044}_{-0.142}$	—

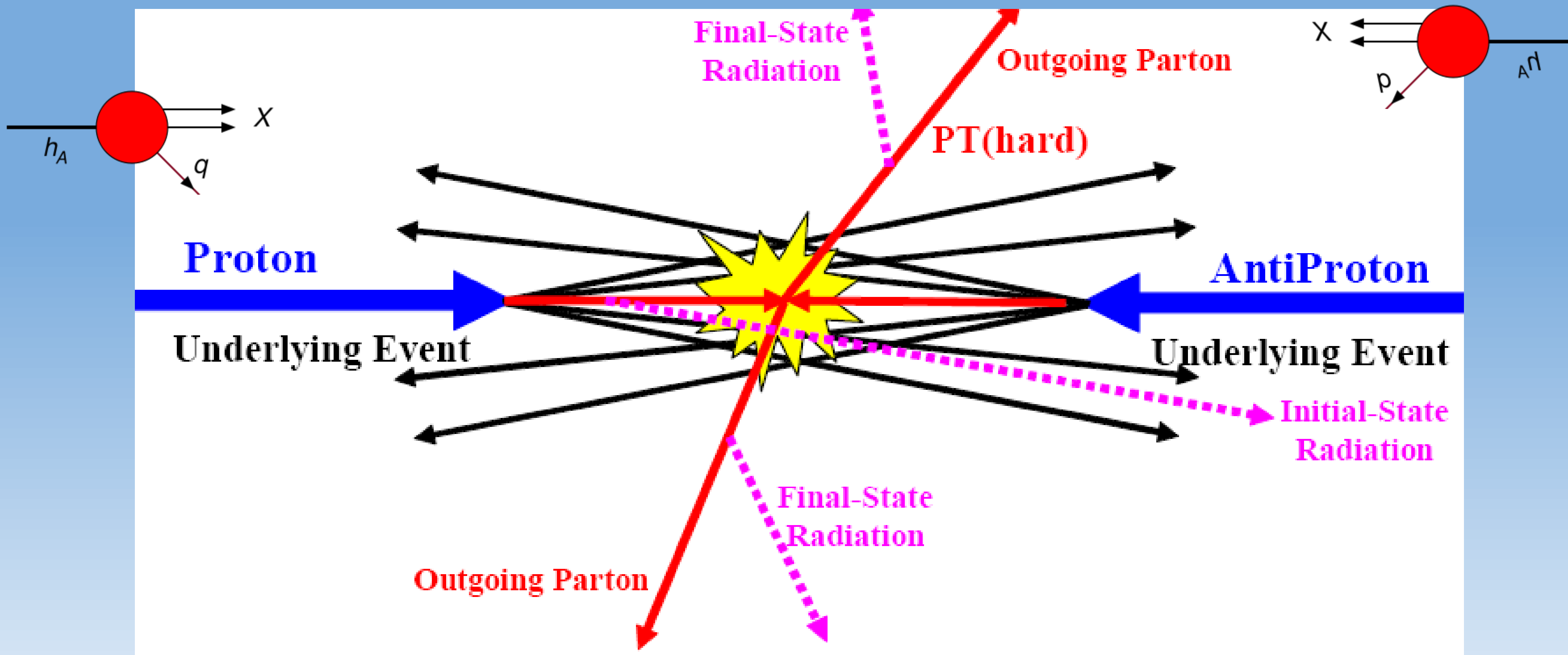
(LC approximation good to about 3%)

Reduction in Scale Dependence

Number of jets	LO	NLO
1	16%	7%
2	30%	10%
3	42%	11%

Scattering processes at hadron colliders:

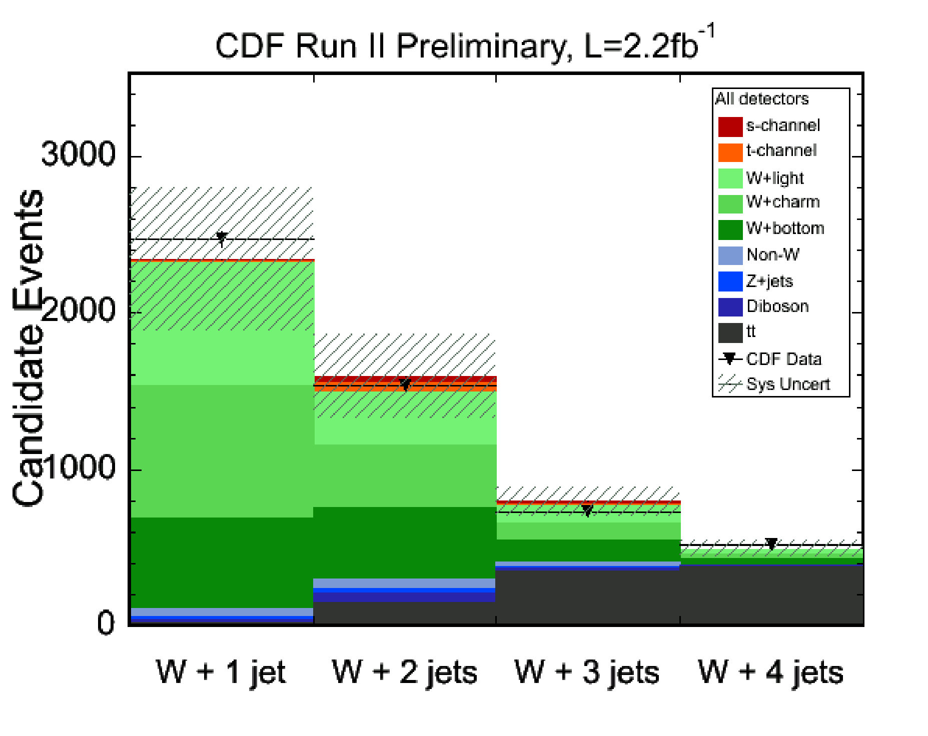
A multi-layered problem



taken from Rick Field

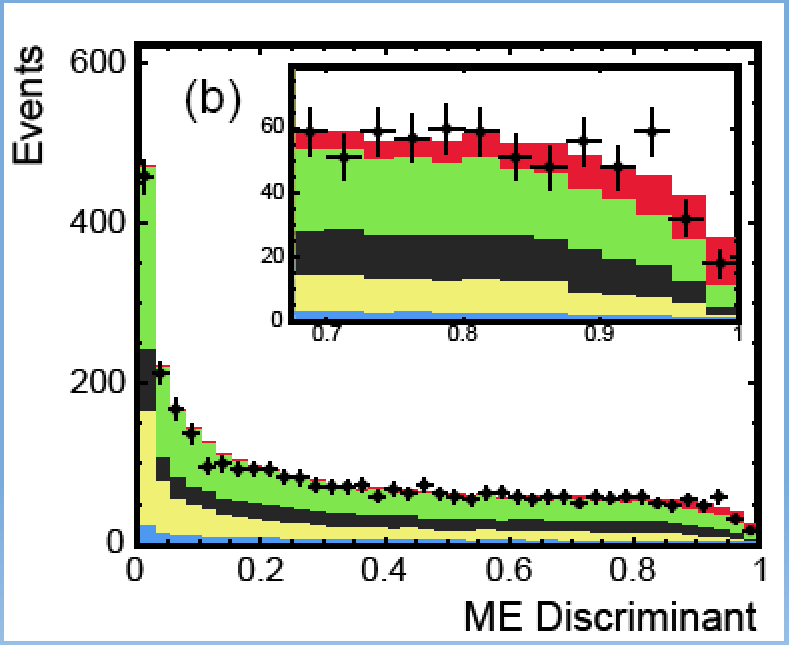
Tevatron: Single Top Production

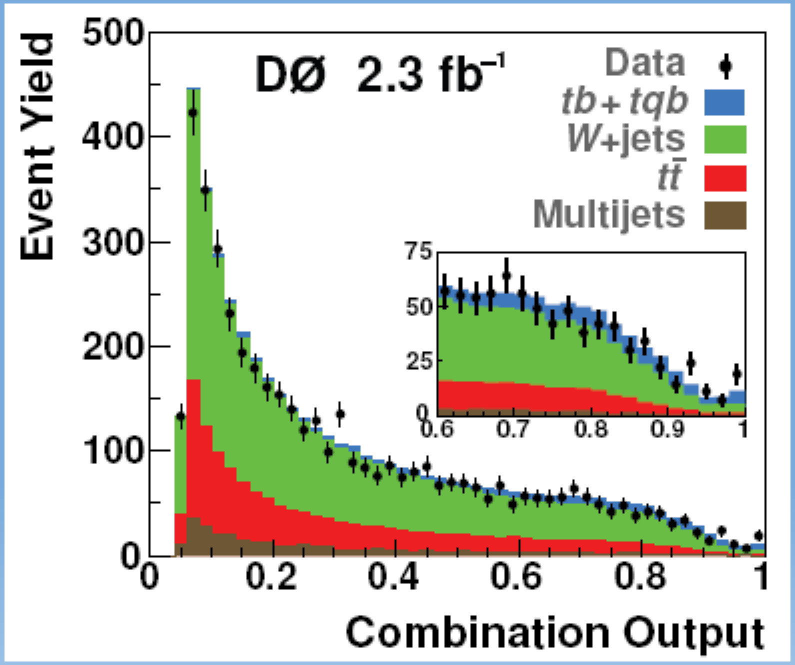
T. Aaltonen et al. [CDF Collaboration], arXiv:0809.2581



Matrix element method uses full information of LO matrix elements to pull the signal out of background.

Tevatron: Single Top Production





CDF 5 sigma discovery!

D0 5 sigma discovery!

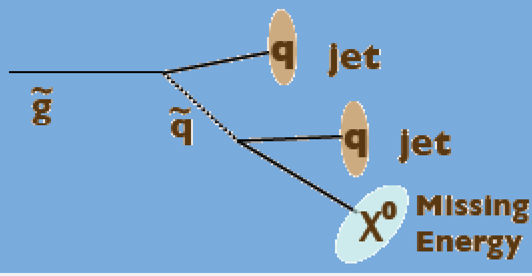
arXiv:0903.0885

arXiv:0903.0850

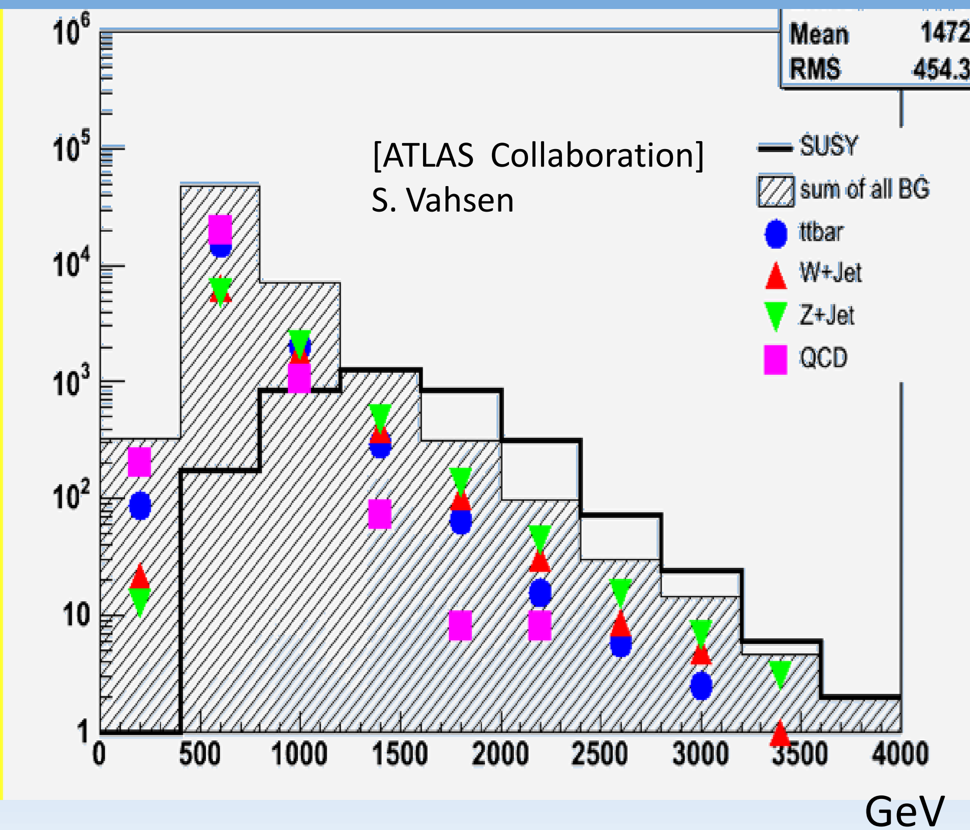
It should be possible to do better by using NLO matrix elements.
A goal is to provide experimenters with necessary theoretical tools for a wide variety of processes.

the LHC: an example of discovery

M L Mangano [arXiv:0809.1567]



Producing heavy colored particles



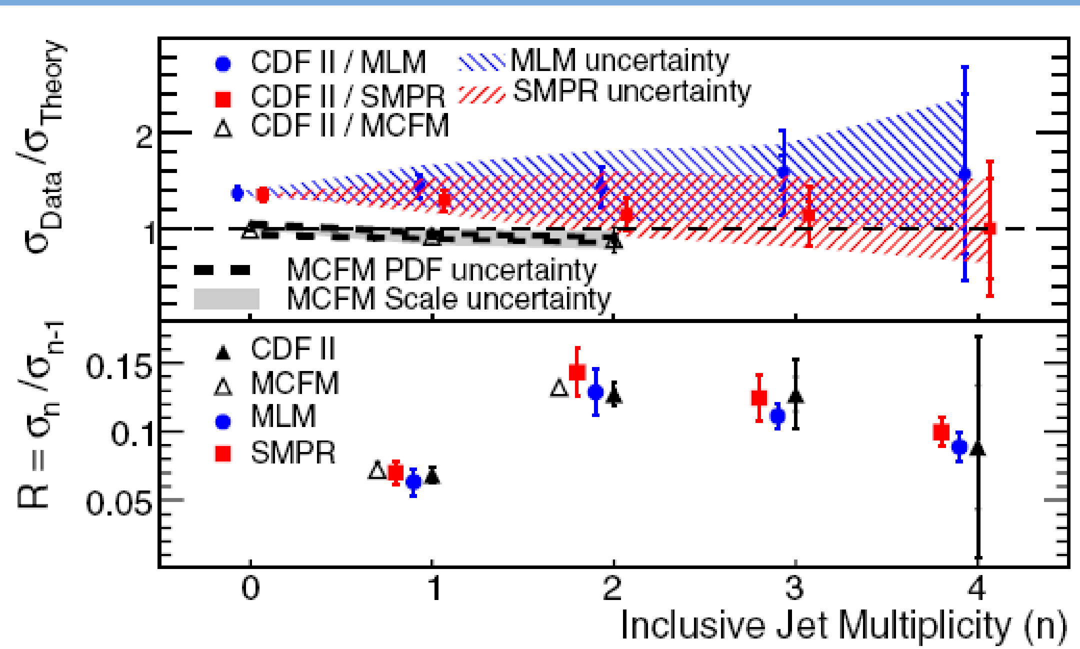
Multi-jet missing transverse energy final state.

Backgrounds:

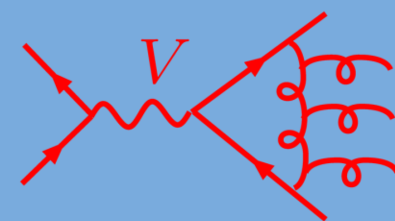
- Irreducible:
  - •Z(->neutrinos)+4 jets
- Reducible:
  - •W(->tau+neutrino)+3 jets
  - •W(->undetected leptons) +4 jets
  - • top pairs
- Instrumental:
  - • Multijets

How good are our tools?

T. Aaltonen et al. [CDF Collaboration], arXiv:0711.4044



Wanted: LHC studies with extra jets:



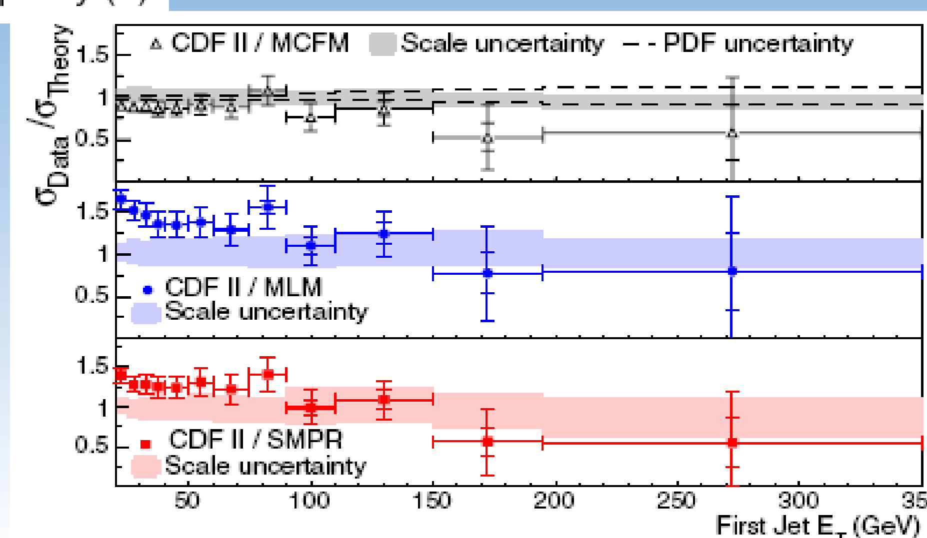
data from $320pb^{-1}$

SMPR-model: Madgraph+Pythia
MLM-model: Alpgen+Herwig

} LO,

MCFM; parton level; including Bern, Dixon, Kosower, Weinzierl 1-loop matrix elements; Full NLO by Campbell and Ellis

} NLO.



Wish-List

QCD: "Experimenters' Wish List"

Les Houches 2007

Process ($V \in \{Z, W, \gamma\}$)	Comments
4. $pp \to t\bar{t}\, b\bar{b}$	relevant for $t\bar{t}H$
5. $pp \to t\bar{t}$+2jets	relevant for $t\bar{t}H$
6. $pp \to VV\, b\bar{b}$,	relevant for VBF $\to H \to VV$, $t\bar{t}H$
7. $pp \to VV$+2jets	relevant for VBF $\to H \to VV$ VBF contributions calculated by (Bozzi/)Jäger/Oleari/Zeppenfeld.
8. $pp \to V$+3jets	various new physics signatures
NLO calculations added to list in 2007	
9. $pp \to b\bar{b}b\bar{b}$	Higgs and new physics signatures

- Five-particle processes under good control with Feynman diagram based approaches.

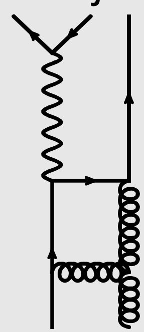
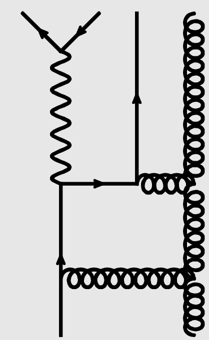
- Six-particle processes still difficult challenge.

What Has Been Done?: *2 to 3 field…*

- Most physics results done from Feynman diagram approach:
  - QCD corrections to vector boson pair production (W^+W^-, $W^{\pm}Z$ & ZZ) via vector boson fusion (VBF). (Jager, Oleari, Zeppenfeld)+(Bozzi)
  - QCD and EW corrections to Higgs production via VBF. (Ciccolini, Denner, Dittmaier)
  - $pp \longrightarrow$ Higgs$+2$ jets. (via gluon fusion Campbell, Ellis, Zanderighi), (via weak interactions Ciccolini, Denner, Dittmaier). $pp \longrightarrow$ Higgs$+3$ jets (leading contribution). (Figy, Hankele, Zeppenfeld)
  - $pp \longrightarrow t\bar{t}H$. (Beenakker, Dittmaier, Krämer, Plümper, Spira, Zerwas), (Dawson, Jackson, Reina, Wackeroth)
  - $pp \longrightarrow ZZZ$. (Lazopoulos, Petriello, Melnikov) $pp \longrightarrow t\bar{t}Z$ +(McElmurry)
  - $pp \longrightarrow WWZ,\ WWW$. (Hankele, Zeppenfeld, Campanario, Oleari, Prestel)
  - $pp \longrightarrow WW+j+X$. (Campbell, Ellis, Zanderighi), (Dittmaier, Kallweit, Uwer)
  - $pp \longrightarrow W/Z\, b\bar{b}$. (FFC, Reina, Wackeroth)
  - $pp \longrightarrow t\bar{t}+jet$. (Dittmaier,Uwer,Weinzierl)
  - …

What Has Been Done?: *2 to 3 field...*

- One case stands alone, using Unitarity Techniques:
  - QCD corrections to ***W/Z + 2 jets*** (Bern, Dixon, Kosower - 1997)+(Campbell, Ellis - 2002, included in MCFM)

	W+1jet  ~15 Years Required new techniques	W+2 jets 
Amplitudes :	Early 80's [Ellis, Martinelli, Petronzio]	1997 [Bern, Dixon, Kosower],
NLO Corrections :	Mid 80's [Arnold, Ellis, Reno],	MCFM 2002 [Campbell, Ellis]

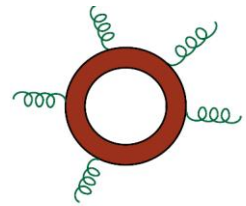
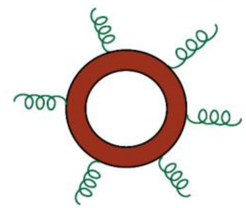
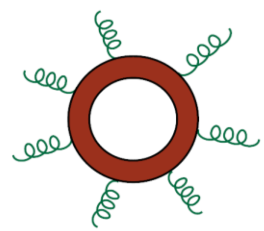
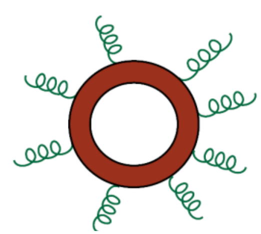
What Has Been Done?: *2 to 4 field…*

- Unitarity Based techniques:
  - Leading color QCD corrections to ***W + 3 jets***, some subprocesses with 3 gluons (Ellis, Melnikov, Zanderighi - 2009)
  - QCD corrections to ***W + 3 jets***, all subprocesses, leading color virtual matrix elements (Berger, Bern, Dixon, FFC, Forde, Gleisberg, Ita, Kosower, Maitre - 2009)


+


- Using Feynman Diagrams:
  - QCD corrections to $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}} + X$ (Bredenstein, Denner, Dittmaier, Pozzorini - 2009)

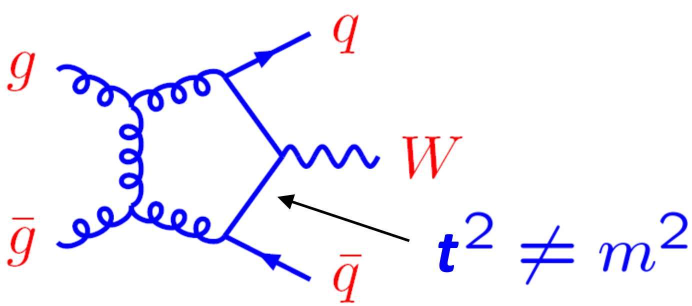
Strong growth in difficulty at one loop (NLO) with number of final-state objects

# of jets	# 1-loop Feynman diagrams (gluons only)	
3		810
4		10,860
5		168,925
6		3,017,490

Think off-shell, work on-shell!

Avoid the source of computational complexity in Feynman Diagrams:

$$\int \frac{d^d t}{(2\pi)^d} \frac{t^{\mu_1} \cdots t^{\mu_n}}{[t^2 - m_0^2][(t+q_1)^2 - m_1^2] \cdots [(t+q_1+\cdots+q_{m-1})^2 - m_{m-1}^2]}$$



And use the decomposition in terms of scalar integrals: $A_n = C_n + R_n$

with $$C_n = \sum_i d_i I_4^i + \sum_i c_i I_3^i + \sum_i b_i I_2^i + \sum_i a_i I_1^i$$

On-Shell Methods

Unitarity Method

- Unitarity Approach:
  - Bern, Dixon, Dunbar, Kosower, hep-ph/9403226, hep-th/9409265.
  - Recent Advances using spinorial integration techniques:
    - Cachazo, Svrcek, Witten; Britto, Cachazo, Feng; Britto, Feng, Mastrolia
- Generalized Unitarity:
  - Bern, Dixon, Kosower, hep-ph/9708239, hep-ph/0001001.
  - Britto, Cachazo, Feng, hep-th/0412103.
  - Recent Advances: classification of surface terms.
    - del Aguila and Pittau, hep-ph/0404120.
    - Ossola, Papadopoulos and Pittau, hep-ph/0609007.
    - Forde, 0704.1835; Badger, 0806.4600, 0807.1245 .
    - Ellis, Giele, Kunszt 0708.2398; Giele, Kunszt and Melnikov, 0801.2237; Ellis, Giele, Kunszt, Melnikov, 0806.3467; Ellis, Giele, Kunszt, Melnikov,Zanderighi 0810.2762

Amplitudes and more amplitudes

- Past years progress using unitarity and related techniques,
  - $gg \rightarrow gggg$ amplitude. (Bern,Dixon,Kosower), (Britto,Feng,Mastrolia), (Bern,Bjerrum-Bohr,Dunbar,Ita), (Berger,Bern,Dixon,Forde,Kosower), (Bedford,Brandhuber,Spence,Travaglini) (Xiao,Yang,Zhu) ,(Berger,Bern,Dixon,Forde,Kosower), (Giele,Kunszt,Melnikov)
  - Lots of gluons (Berger, Bern, Dixon, FFC, Forde, Ita, Kosower, Maître), (Giele, Zanderighi), (Lazopoulos), (Giele, Winter)
  - *W+3* (7 point) amplitudes (Berger, Bern, Dixon, FFC, Forde, Ita, Kosower, Maître), (Ellis, Giele, Kunszt, Melnikov, Zanderighi)
  - All *2->4* wish listed amplitudes (Hameren, Papadopoulos, Pittau)
  - ...
- Numerical packages under construction:
  - BlackHat Berger, Bern, Dixon, FFC, Forde, Ita, Kosower, Maître
  - CutTools Ossola, Papadopoulos, Pittau
  - Rocket Ellis, Giele, Kunszt, Melnikov, Zanderighi
  - Lazopoulos
  - Giele and Winter

Reminder: one-loop basis.

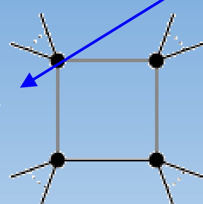
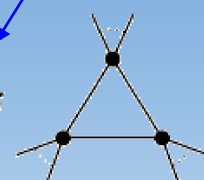
All external momenta in *D=4*, loop momenta in *D=4-2ε* (dimensional regularization).

Rational part

Cut part

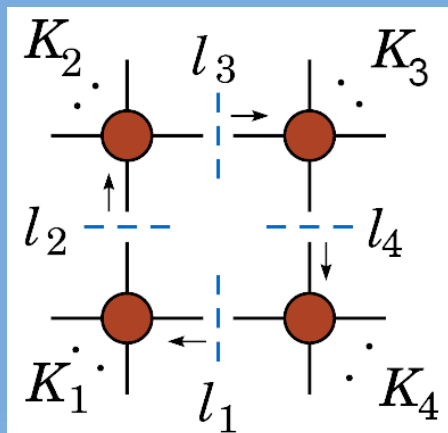
Process dependent D=4 rational integral coefficients

$$A = R + C$$

$$C = \sum_i b_i$$  $$+ \sum_i c_i$$  $$+ \sum_i d_i$$ 

- Cut Part from unitarity cuts in 4 dimensions.
- Rational part from on-shell recurrence relations.

Boxes: the simplest cuts



Britto, Cachazo, Feng hep-th/0412103;
Berger, Bern, Dixon, FFC, Forde, Ita, Kosower, Maitre 0803.4180; Risager 0804.3310.

$$d_i = \frac{1}{2}\sum_{\sigma=\pm} d_i^\sigma \,,$$
$$d_i^\sigma = A_{(1)}^{\rm tree} A_{(2)}^{\rm tree} A_{(3)}^{\rm tree} A_{(4)}^{\rm tree}\Big|_{l_i = l_i^{(\sigma)}}$$

$$(l_1^{(\pm)})^\mu = \frac{\langle 1^\mp|\, \not{K}_2 \not{K}_3 \not{K}_4 \gamma^\mu \,|1^\pm\rangle}{2\,\langle 1^\mp|\, \not{K}_2 \not{K}_4 \,|1^\pm\rangle}\,, \qquad (l_2^{(\pm)})^\mu = -\frac{\langle 1^\mp|\, \gamma^\mu \not{K}_2 \not{K}_3 \not{K}_4 \,|1^\pm\rangle}{2\,\langle 1^\mp|\, \not{K}_2 \not{K}_4 \,|1^\pm\rangle}\,,$$

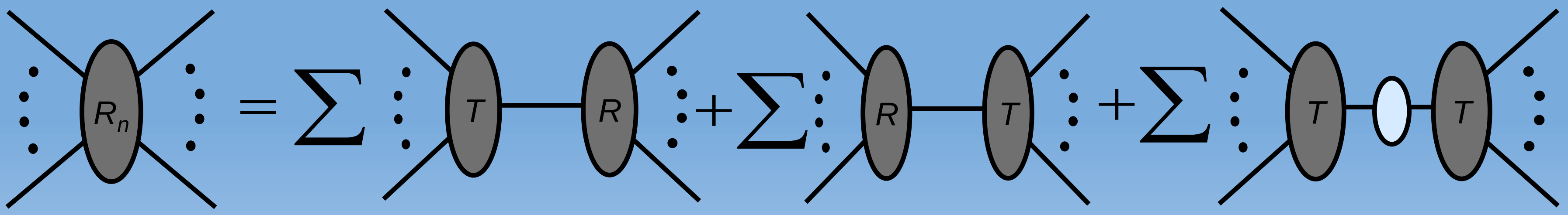
$$(l_3^{(\pm)})^\mu = \frac{\langle 1^\mp|\, \not{K}_2 \gamma^\mu \not{K}_3 \not{K}_4 \,|1^\pm\rangle}{2\,\langle 1^\mp|\, \not{K}_2 \not{K}_4 \,|1^\pm\rangle}\,, \qquad (l_4^{(\pm)})^\mu = -\frac{\langle 1^\mp|\, \not{K}_2 \not{K}_3 \gamma^\mu \not{K}_4 \,|1^\pm\rangle}{2\,\langle 1^\mp|\, \not{K}_2 \not{K}_4 \,|1^\pm\rangle}\,.$$

Un-physical (=spurious) singularities from parameterization.
Have to cancel eventually: role of rational term R.

Loop On-Shell Recursions.

Bern, Dixon, Kosower, Forde, Berger;
Bern, Bjerrum-Bohr, Dunbar, Ita

- At one-loop recursion using on-shell tree amplitudes, T, and rational pieces of one-loop amplitudes, R,



+ spurious residues

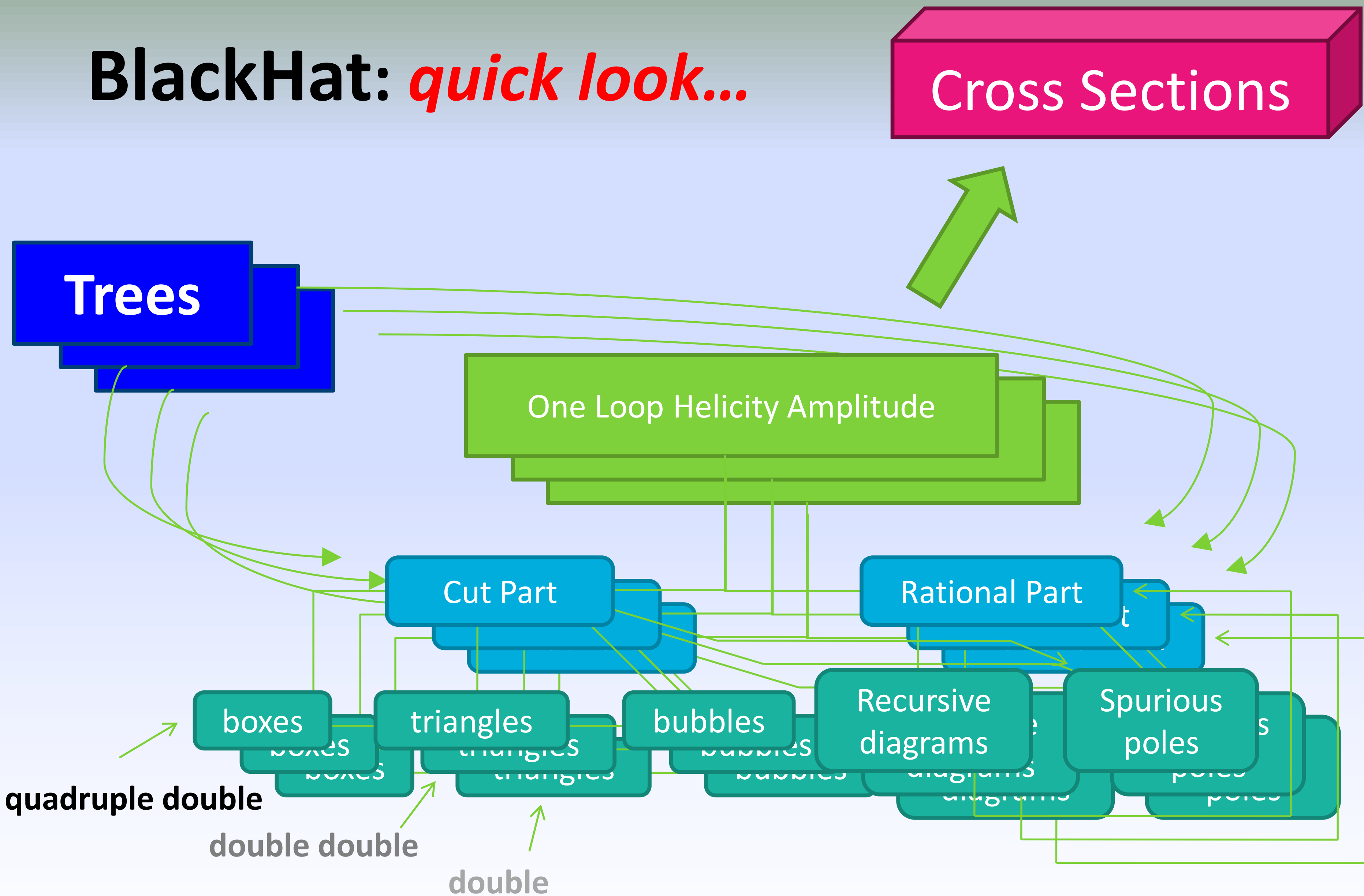
- Sum over all factorisations.
- In addition to tree recursion: sum over “spurious” residues.
- *Remark: Can be done for integral coefficients, auxiliary recursions…*

Amplitudes From BlackHat

BlackHat: *A C++ framework of on-shell techniques for 1-loop amplitudes*

- Portability (standard libraries for unix systems)
- Modularity (object oriented)
- Malleability (to accept several routines – numerics and analytics)
- Numerical precision and efficiency
- Ready to use with existing Monte Carlo programs
  - Working already with automated real dipole subtraction from ***Sherpa*** (*with T. Gleisberg*)

BlackHat: *quick look…*



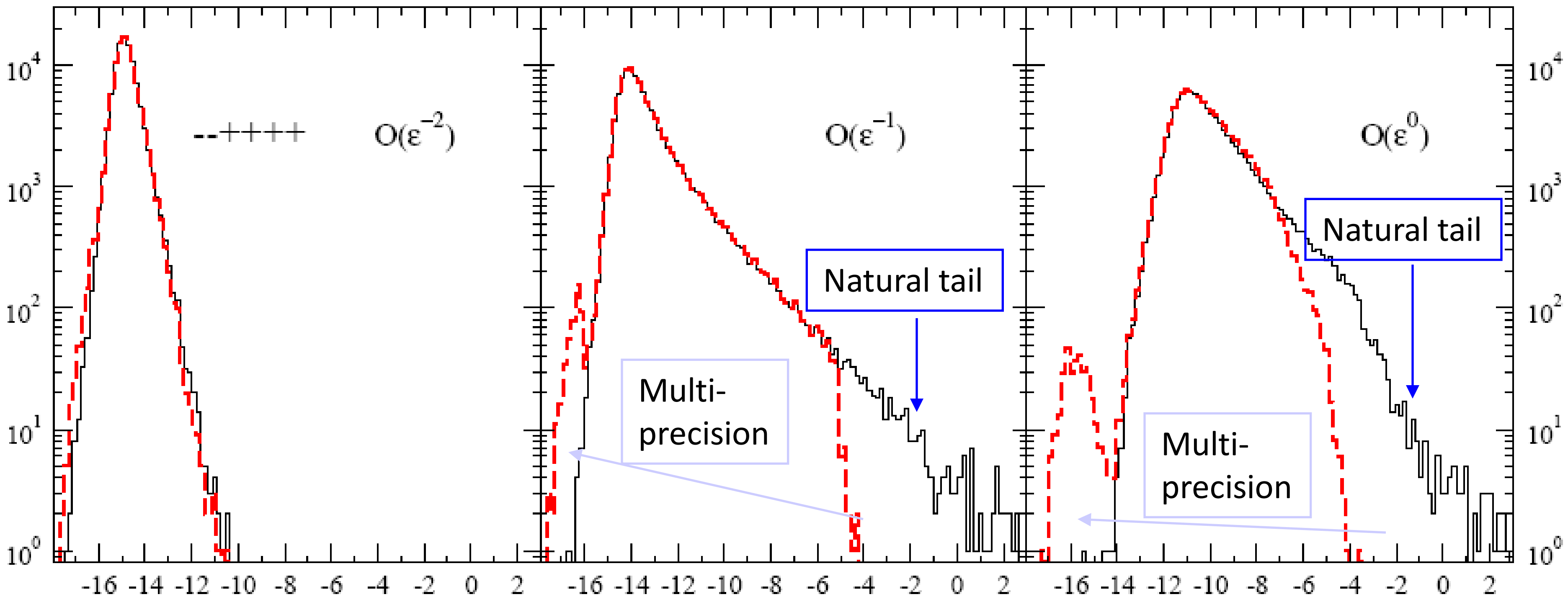
Multiprecision arithmetic gives excellent control over numerical stability…

Gluon amplitudes: The Tails.

Double-precision numerical computation.

Dynamic multi-precision computation.

Reference: analytic targets from Bern, Dixon, Dunbar, Kosower, hep-ph/9403226, hep-ph/9409265, hep-ph/0507005.



100 000 PS points, ET>0.01 s,
pseudo rapidity<3, separation cut >0.4

$$\log_{10}\left(\frac{|A_n^{\text{num}} - A_n^{\text{target}}|}{|A_n^{\text{target}}|}\right)$$

Watch Instabilities

- Monitor using known IR/UV pole structure of amplitudes

$$A_n^{loop} \sim \left[-\frac{n}{\epsilon^2} + \frac{1}{\epsilon}\left(-\frac{11}{3} + \sum_i \log\left(\frac{\mu}{s_{i,i+1}}\right)\right)\right] A_n^{tree}.$$

- Generalization for rational part (A consistency condition of spurious residues)

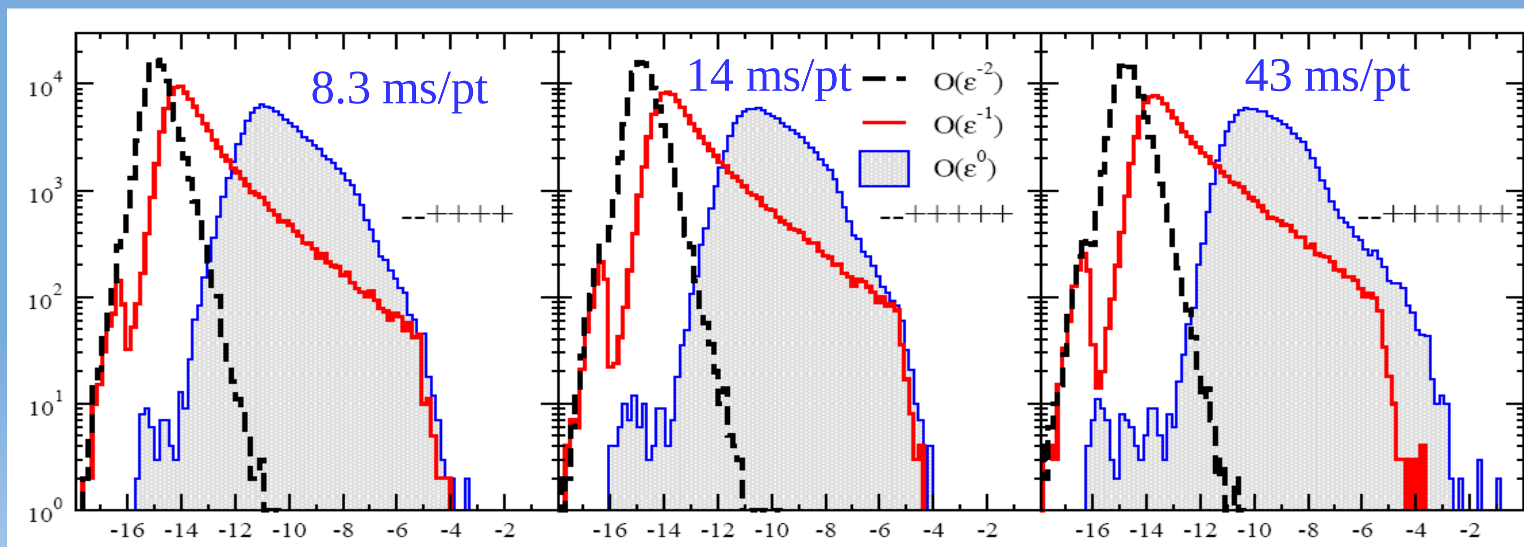
- Avoid instabilities with analytic tricks:
  - Use good loop momentum parametrizations & spinor variables

Scaling with number of legs

Berger, Bern, Dixon, FFC, Forde, Ita, Kosower, Maitre

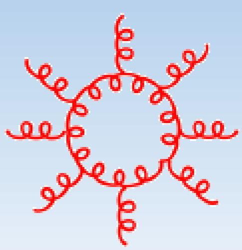
2.33 GHz Xeon

$--++++$ $\quad$ $--+++++$ $\quad$ $--++++++$



6 gluons $\quad$ 7 gluons $\quad$ 8 gluons

amusing count for 8 gluons

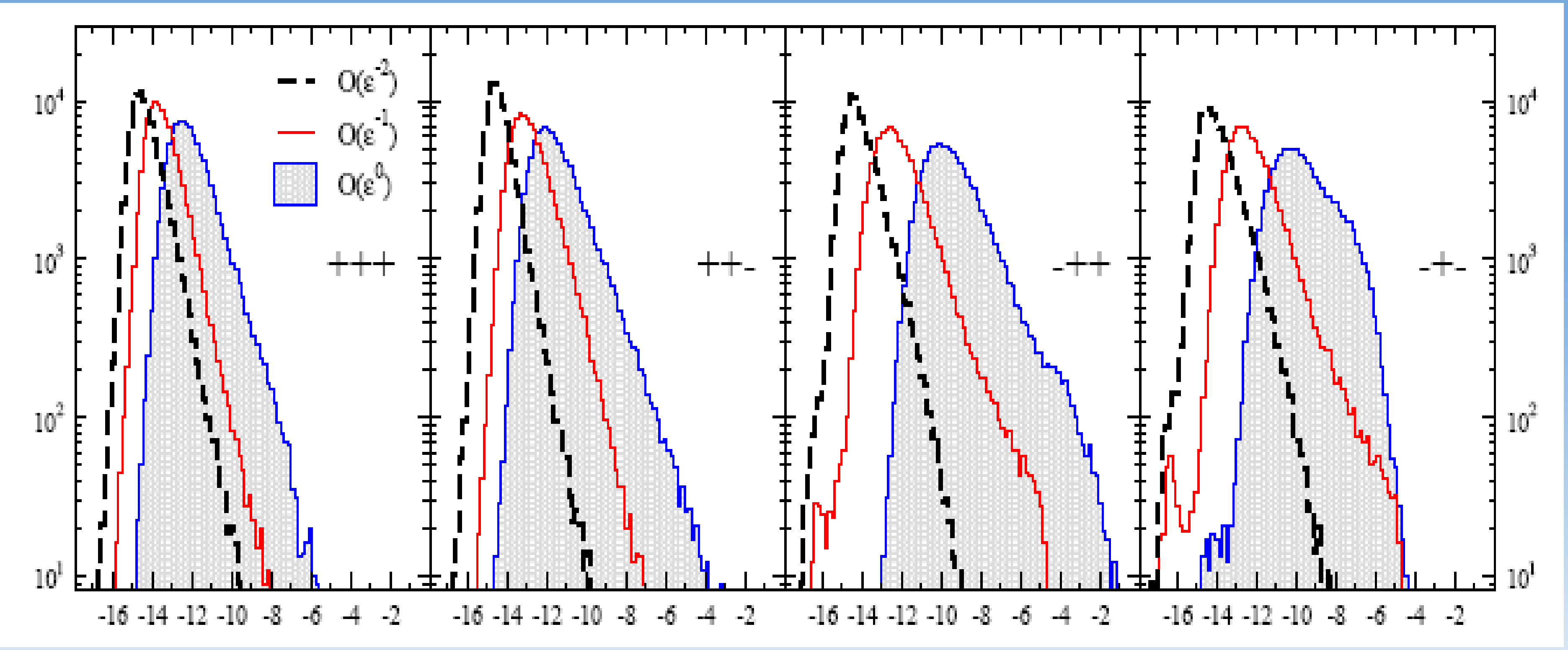


+ 3,017,489 Feynman diagrams

Z+3jets: Stability Study

Berger, Bern, Dixon, FFC, Forde, Ita, Kosower, Maître, arXiv:0808.0941[hep-ph]

100 000 PS points, ET>0.01 s,
pseudo rapidity<3, separation cut >0.4



$$\log_{10}\left(\frac{|A_n^{\rm num} - A_n^{\rm target}|}{|A_n^{\rm target}|}\right)$$

Ellis, Giele, Kunszt, Melnikov, Zanderighi:
confirmed leading color and completed subleading color.

Physics!!!

(Gleisberg, Hoeche, Krauss, Schoenherr, Schumann, Siegert, Winter)

NLO with **BlackHat+Sherpa**

$$\sigma^{\mathrm{NLO}} = \int_{m+1}\left[\mathrm{d}^{(4)}\sigma^{\mathrm{R}} - \mathrm{d}^{(4)}\sigma^{\mathrm{A}}\right] + \int_{m}\left[\int_{\mathrm{loop}}\mathrm{d}^{(d)}\sigma^{\mathrm{V}} + \int_{1}\mathrm{d}^{(d)}\sigma^{\mathrm{A}}\right]_{\epsilon=0}$$

(S. Catani, M.H. Seymour,1997)

(T. Gleisberg, F. Krauss, 2007)

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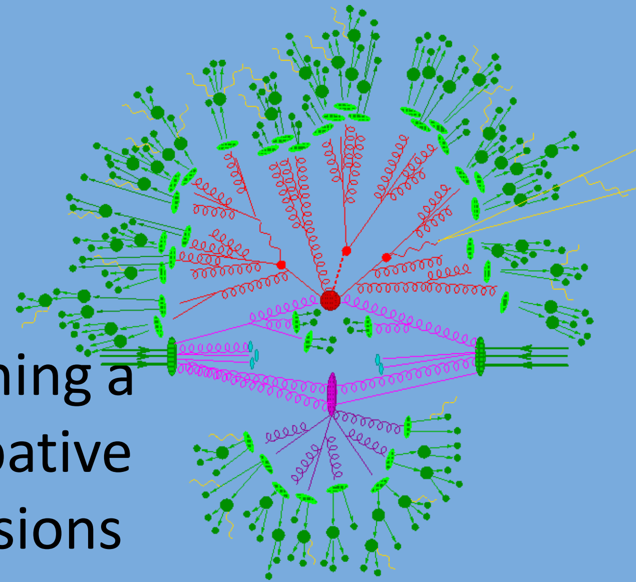
(a glance to NLO automation!)

SHERPA



- SHERPA is a full event generator, combining a number of perturbative and non-perturbative approaches to simulate high energy collisions (Gleisberg, Hoeche, Krauss, Schoenherr, Schumann, Siegert, Winter)
- Here just parts of the framework are used:
  - The automated tree-level matrix element generator AMEGIC++, includes automated dipole subtraction (Gleisberg, Krauss)
  - Phase space integration techniques
  - The event generation framework and the ANALYSIS package to evaluate generated events

W+Jets at the Tevatron: CDF Analysis

T. Aaltonen et al. [CDF Collaboration], arXiv:0711.4044, 320 pb^-1

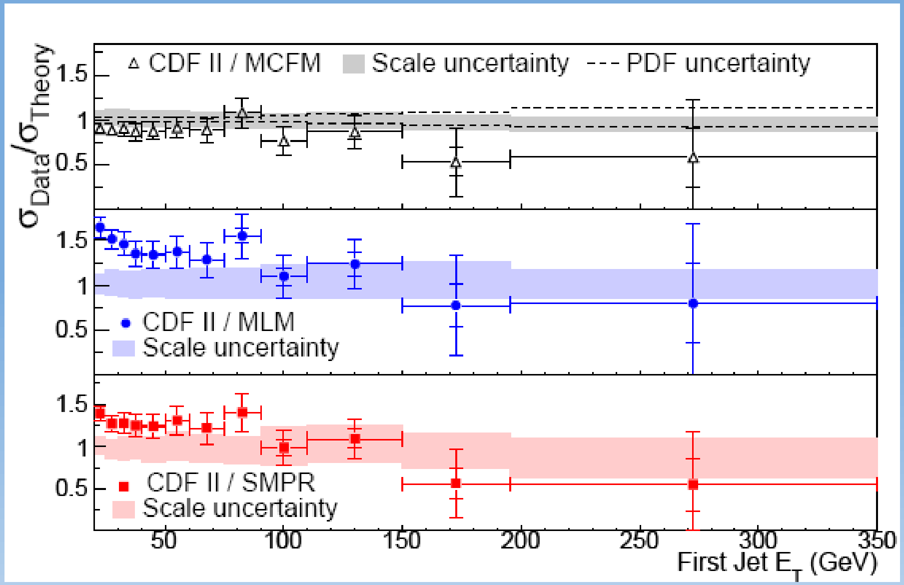
	Cut
Electron Et	20 GeV
Electron eta	1.1
Missing Energy	30 GeV
W Transverse Mass	20 GeV
Jet Et	20 – 25 GeV
Jet eta	2
Delta R	0.4

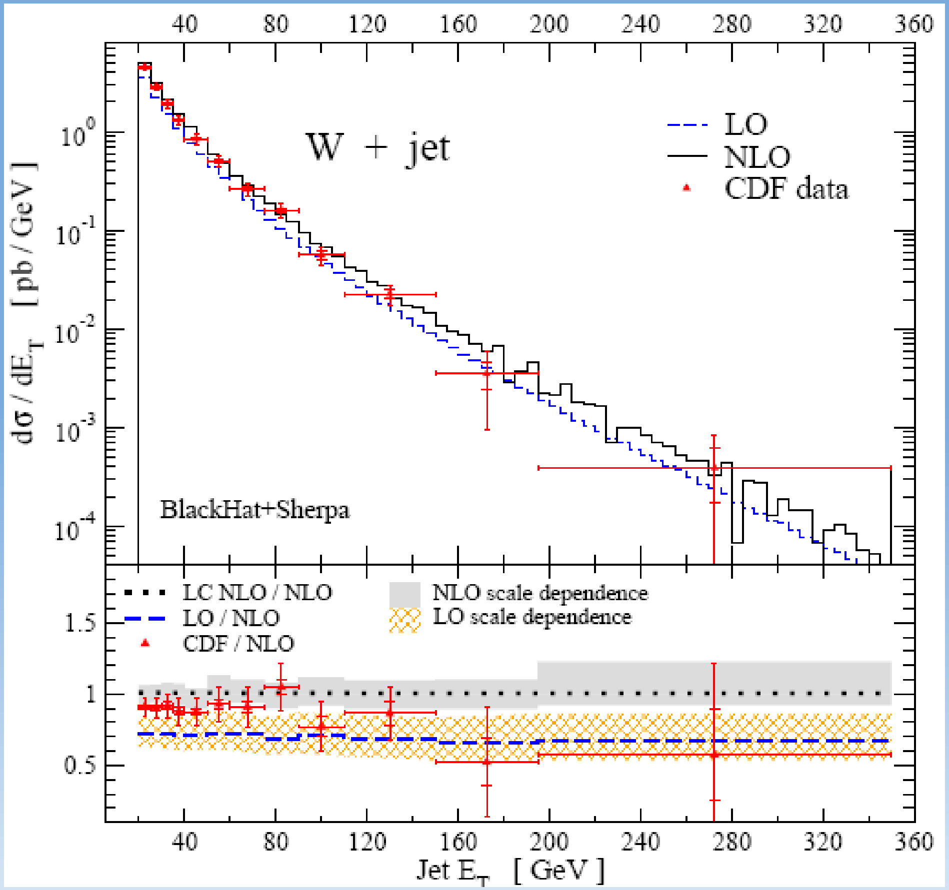
We employ the SISCone Jet Algorithm

Salam, Soyez arXiv:0704.0292

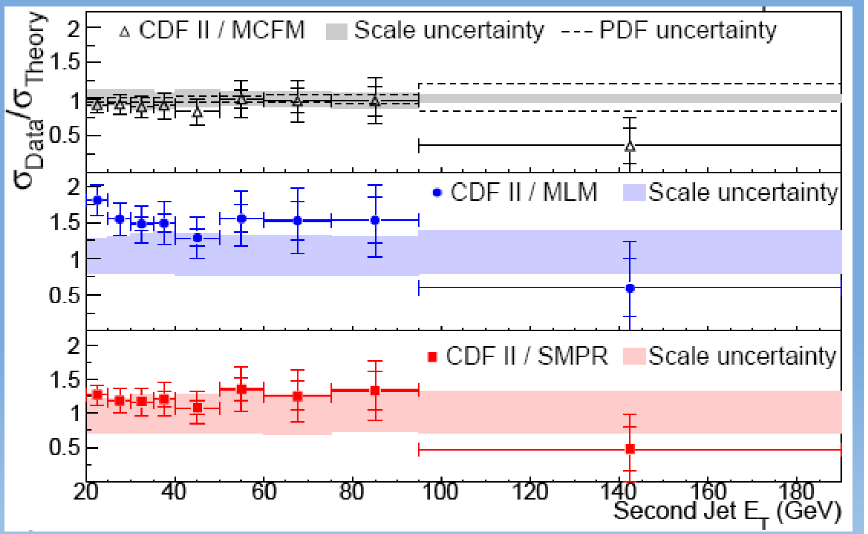
CTEQ pdfs, and a dynamical factorization/renormalization scale (sqrt(Mw^2+pt_W^2)) for comparison with data

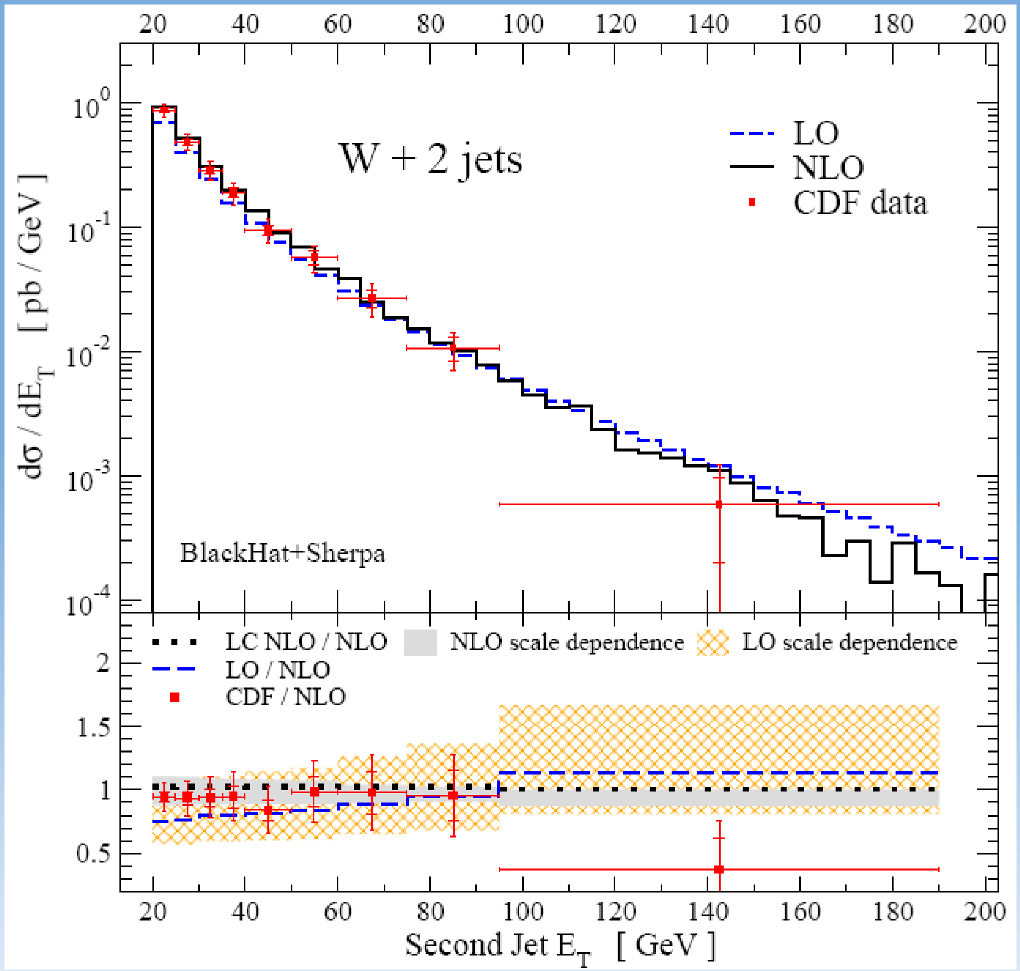
W+ jet +X at the Tevatron



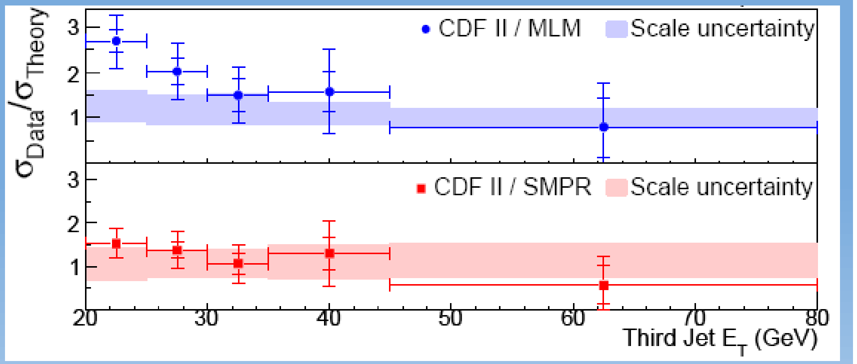


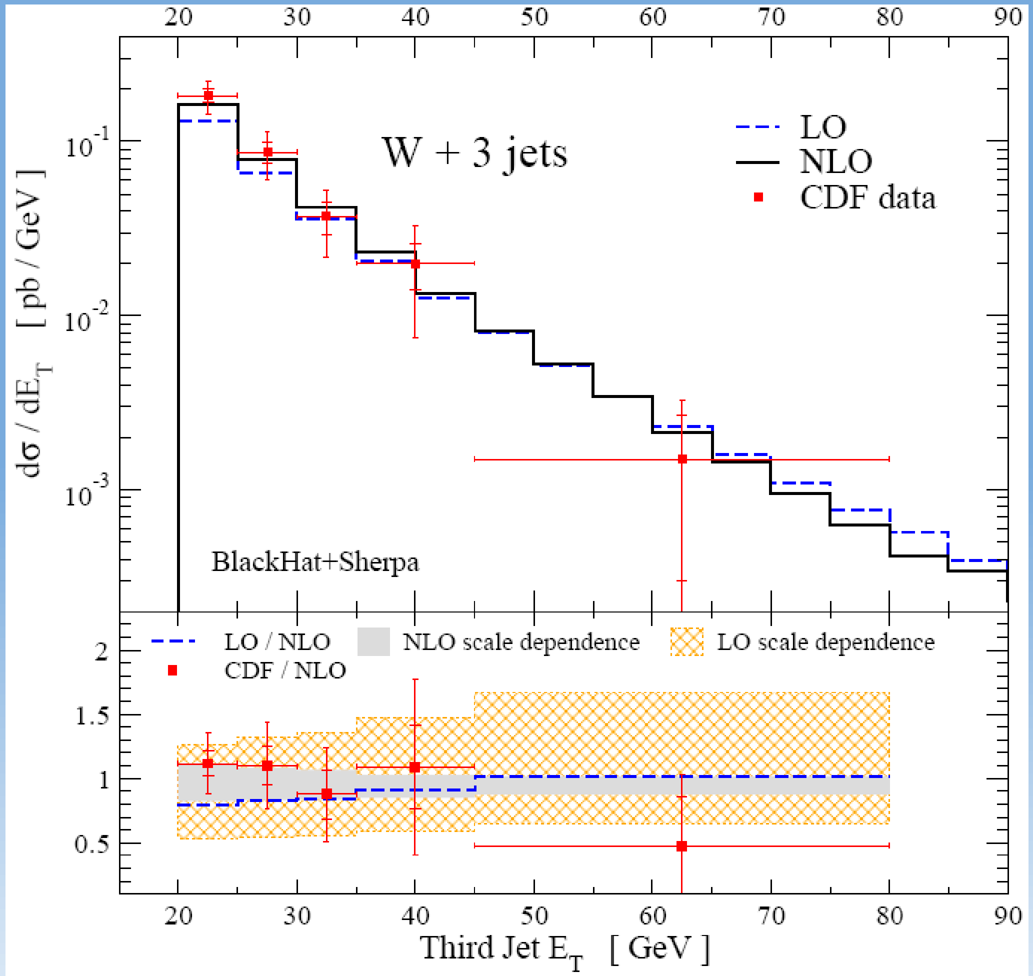
W+2 jets + X at the Tevatron





W+3 jets + X at the Tevatron





On to the LHC

Preliminary

SISCone

$$E_{CM} = 14 \text{ TeV}$$

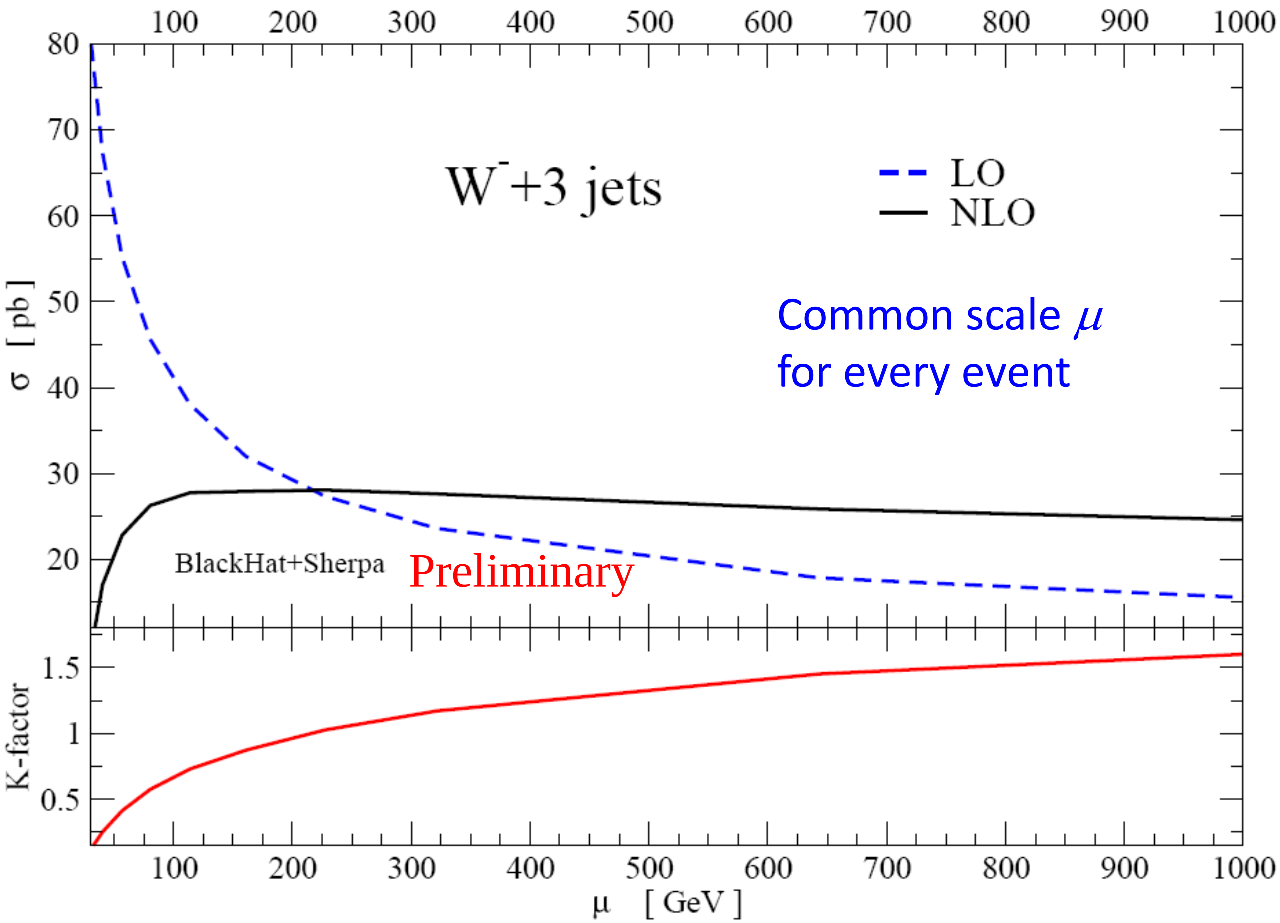
Cuts:

$E_T^e > 20 \text{ GeV}$ $\quad$ $E_T^{\text{jets}} > 30 \text{ GeV}$

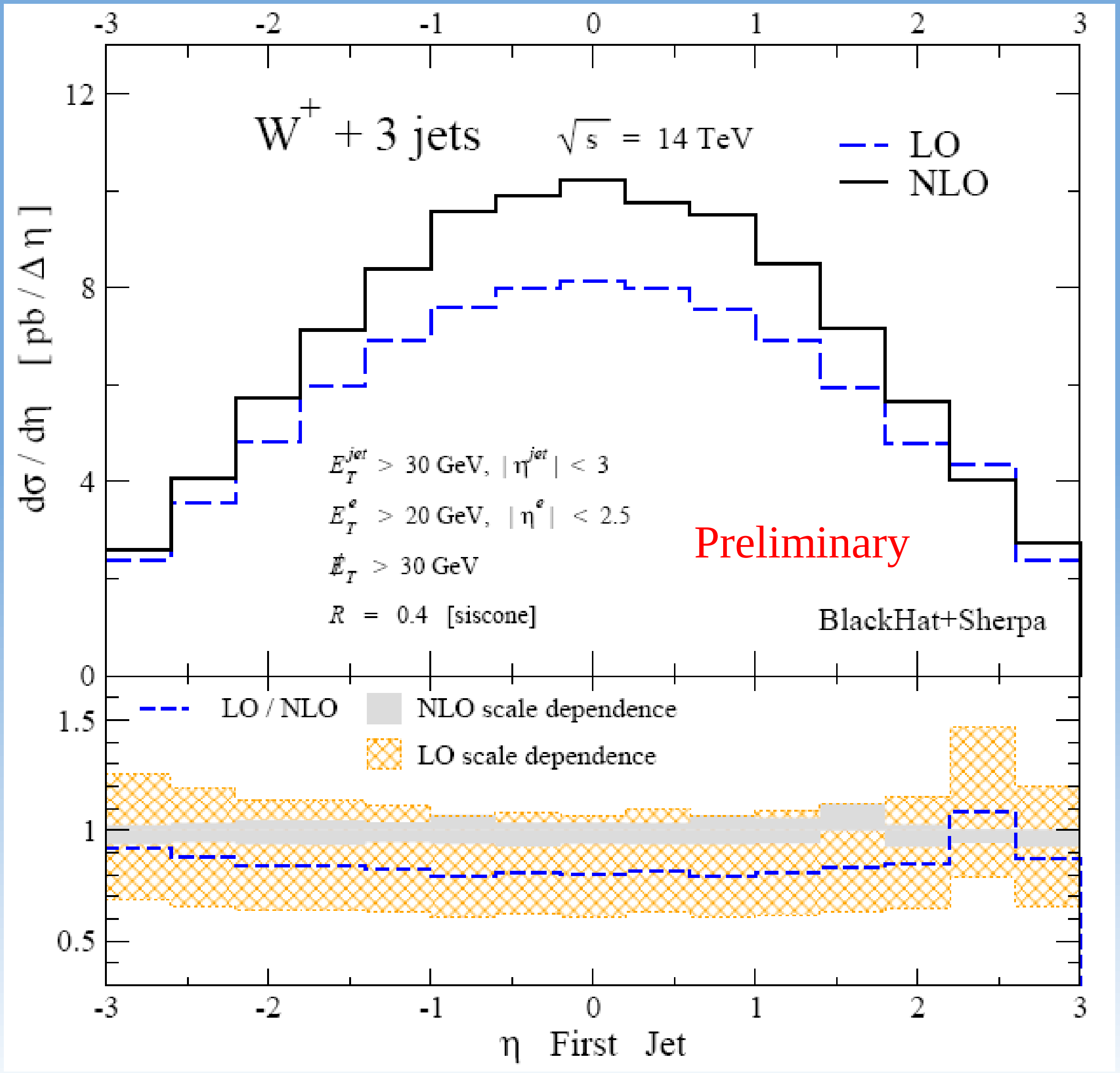
$|\eta^e| < 2.5$ $\quad$ $\not{E}_T > 30 \text{ GeV}$

$|\eta^{\text{jets}}| < 3$ $\quad$ $M_T^W > 20 \text{ GeV}$

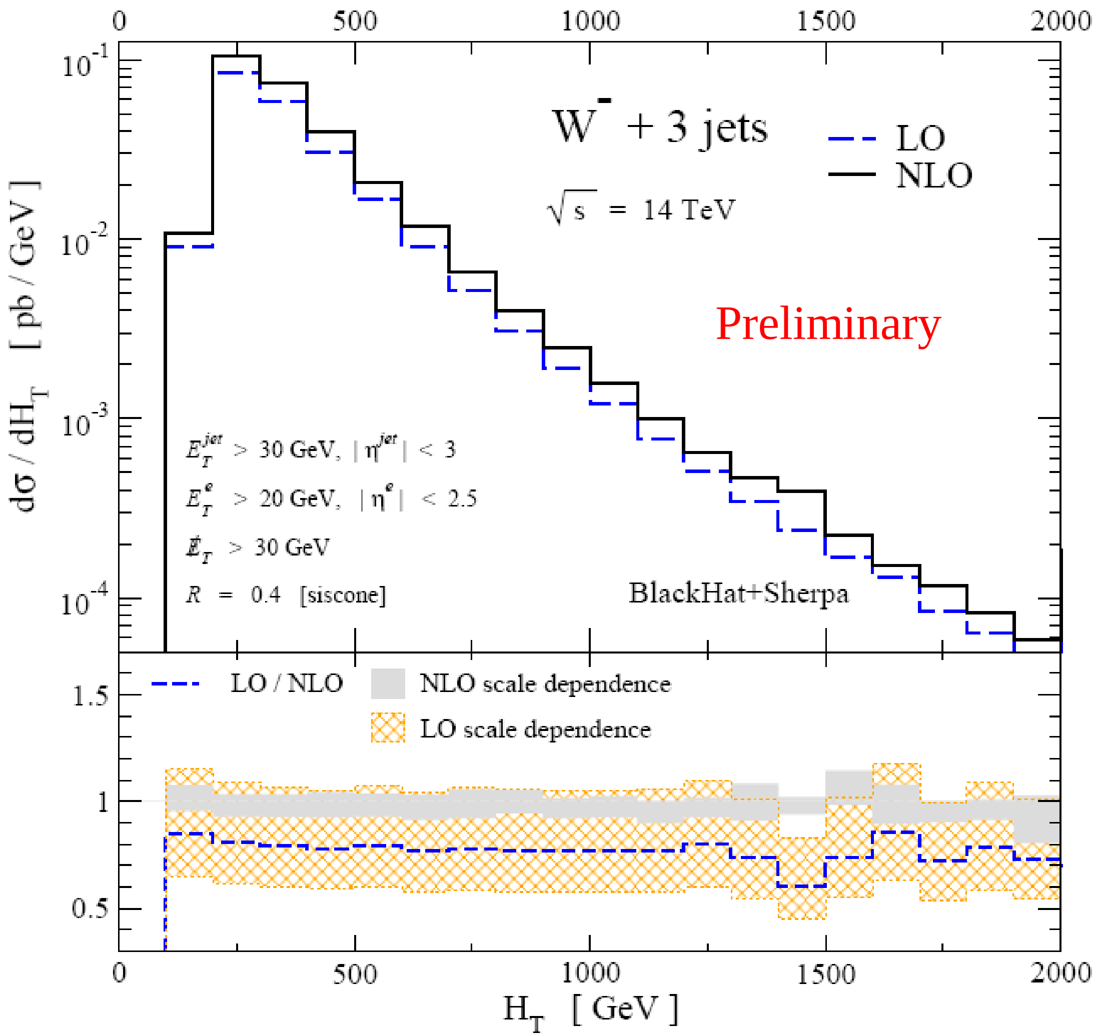
LHC total cross section



First jet eta distribution

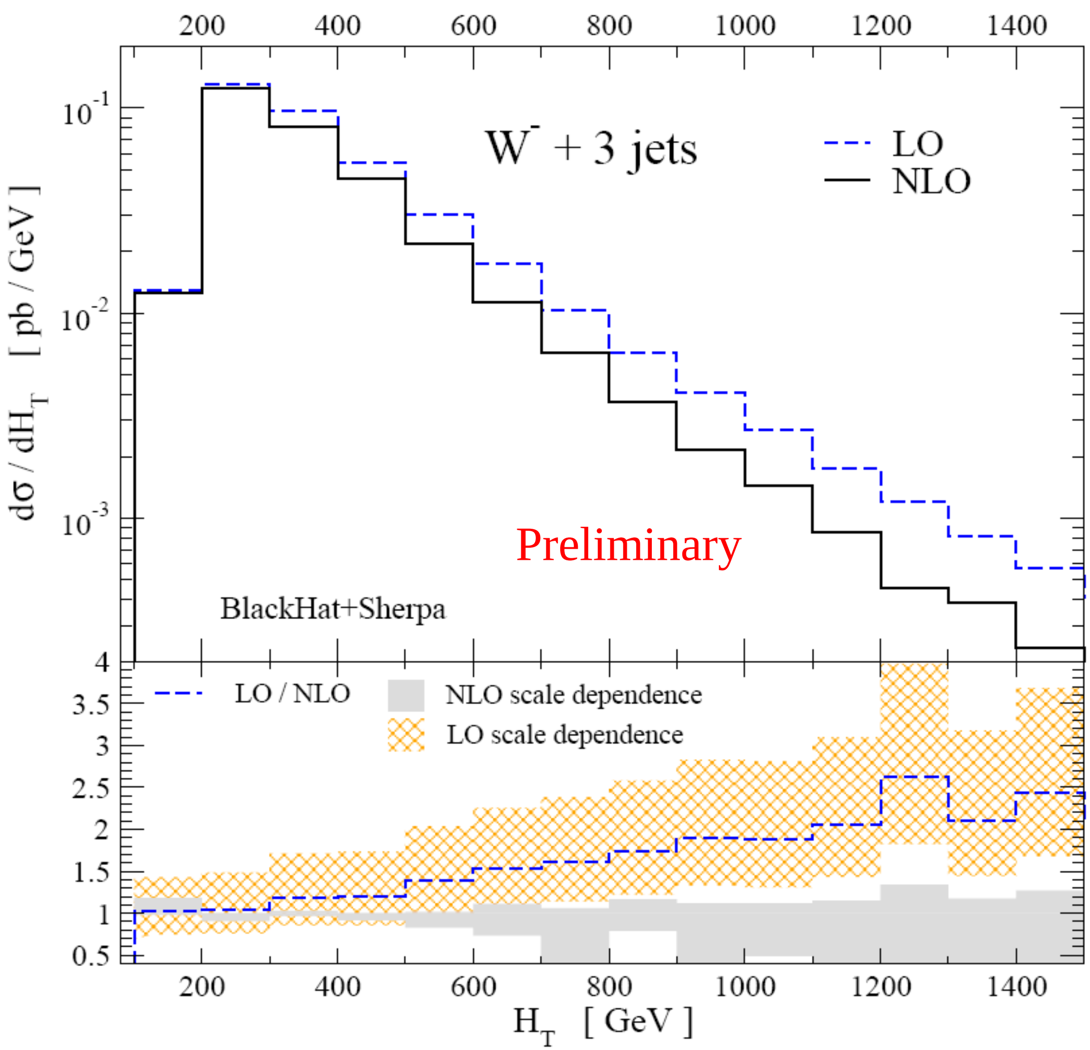


$H_T = \sum_j E_{T,j}^{\text{jet}} + E_T^e + \not{E}_T$ distribution



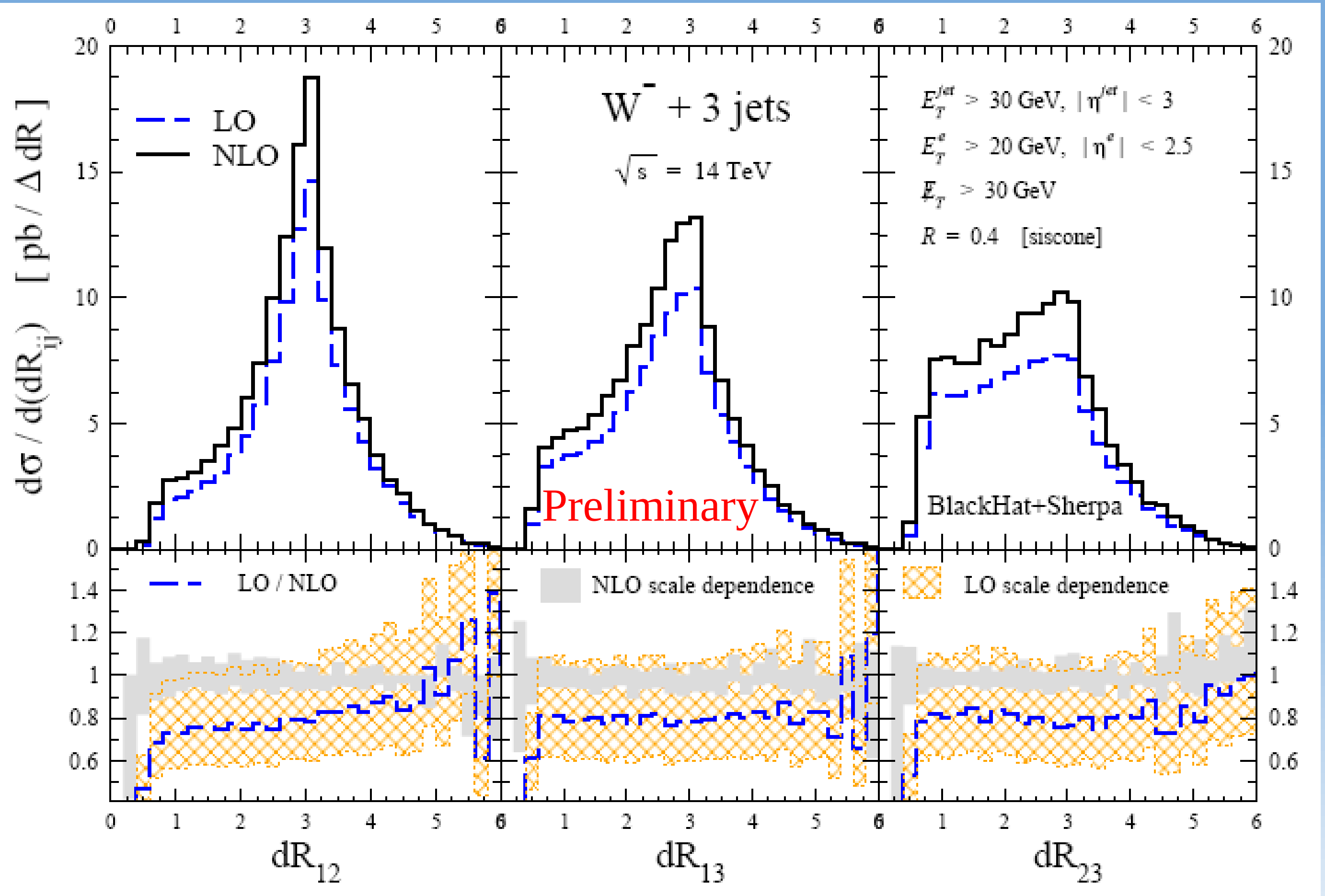
mu = H_T

$H_T = \sum_j E_{T,j}^{\text{jet}} + E_T^e + \not{E}_T$ distribution



$$\mu = \sqrt{M_W^2 + p_T^2(W)}$$

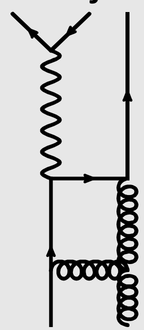
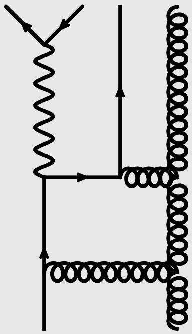
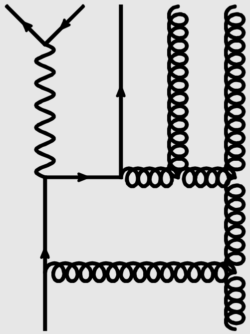
Jet dR Distributions



Conclusions & Outlook

- On-shell methods have opened a new gate to computational power in QFTs
- BlackHat has proven good precision and scaling properties for 1-loop amplitudes
- Together with SHERPA we have presented first NLO results, within a leading color approximation of the virtual pieces, for a process with 6 legs at LO
- W+3 predictions agree well with CDF data, scale uncertainty greatly reduced!
- Look forward for more studies of relevant processes for hadron colliders!

So ***Unitarity Techniques*** Have Their Share in History:

	W+1jet  ~15 Years → Required new techniques	W+2 jets  ~15 Years → Required more new techniques	W+3 jets 
Amplitudes :	Early 80's [Ellis, Martinelli, Petronzio]	1997 [Bern, Dixon, Kosower],	2008
NLO Corrections :	Mid 80's [Arnold, Ellis, Reno],	MCFM 2002 [Campbell, Ellis]	2009

EXTRA SLIDES.