Top-pair production at hadron colliders

Alexander Mitov

Yang Institute for Theoretical Physics
SUNY Stony Brook

Precision in the LHC era

Precision = confidence!

- LO (leading order) = crude estimate of the result
- NLO (next to leading order) = better estimate of the result
  crude estimate of uncertainty
- NNLO = for the first time quantify the uncertainty

Three precision observables have been identified for the LHC:

"The three pillars"

Precision in the LHC era: Drell-Yan

Drell-Yan lepton pair and vector boson production.

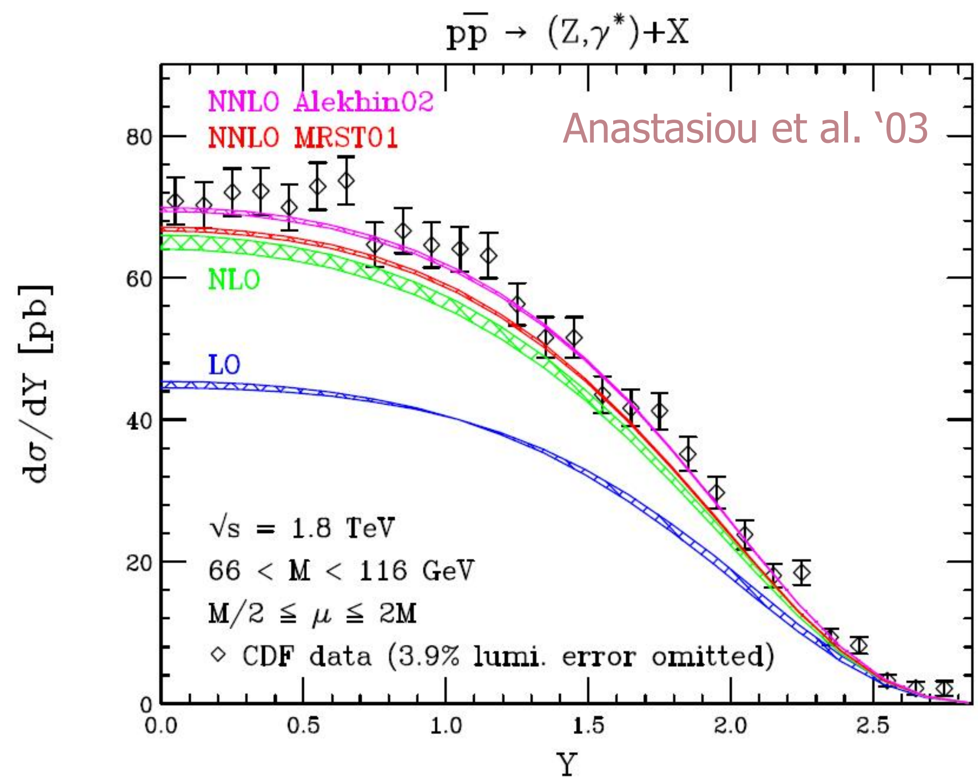
DY is a SM process!

- ✓ luminosity measurement
- ✓ determination of PDF's

NNLO rapidity distribution at the Tevatron.

Scale variation:

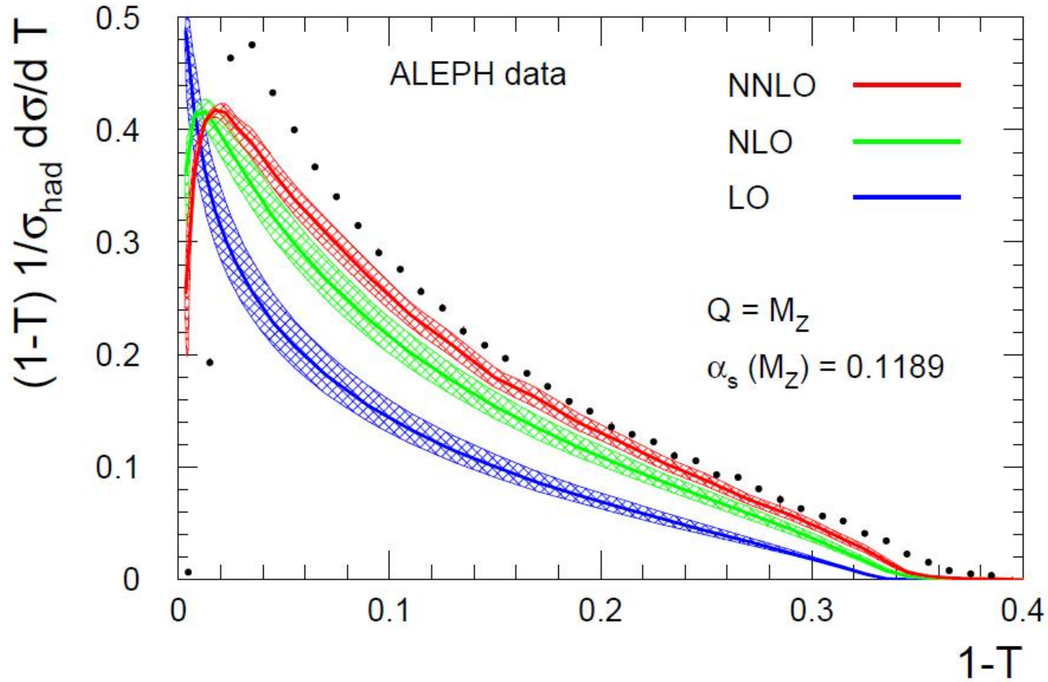
$M/2 < \mu < 2M$



Blast from the past: LEP

Thrust distribution in $e^+e^- \rightarrow$ 3 jets at NNLO
Scale variation $M_Z/2 < \mu < 2M_Z$

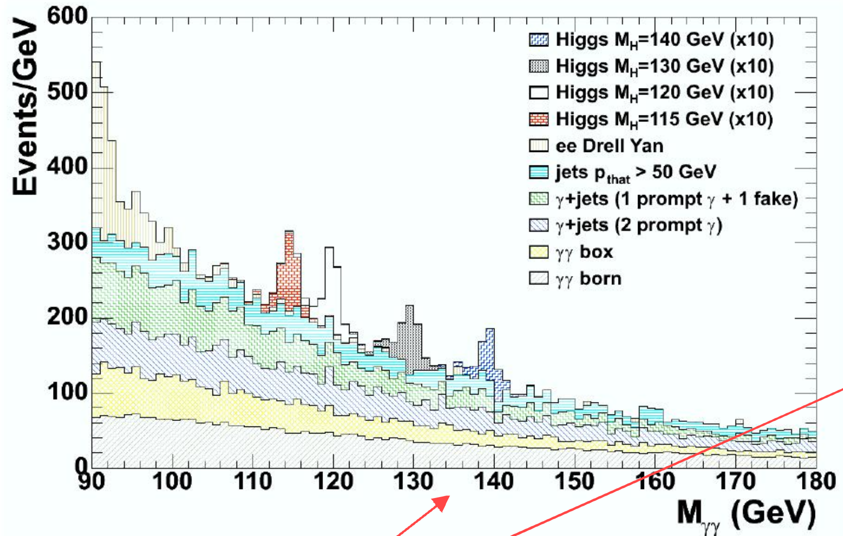
Gehrmann–De Ridder et al. '07

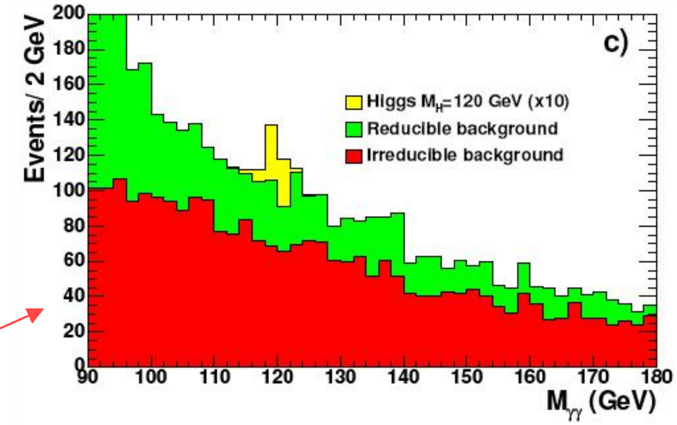


Precision in the LHC era: Higgs

Higgs production. Its discovery is no simple matter!

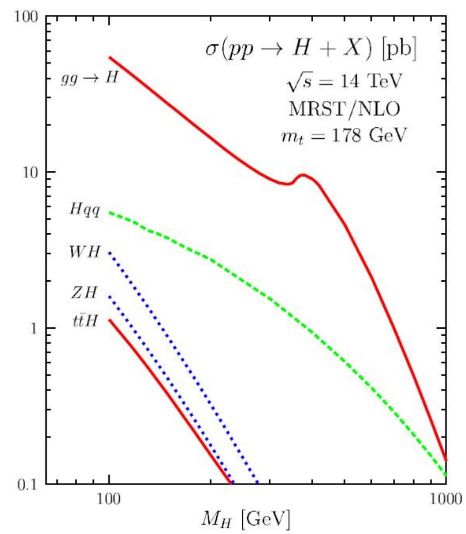
CMS '08

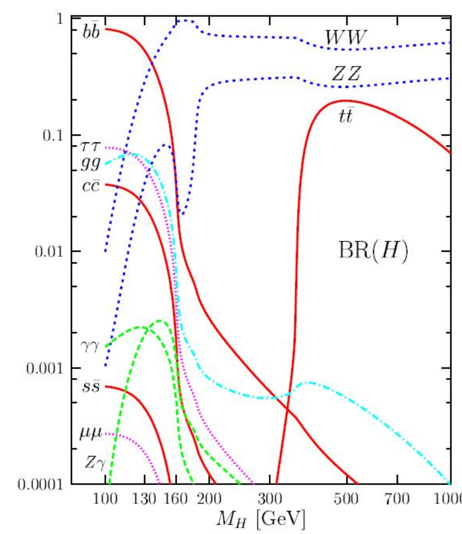




$H \to \gamma\,\gamma$

ATL-PHYS-CONF-2008-006





- ✓ for 1 fb^{-1} integrated luminosity
- ✓ Signal x10

Precision in the LHC era: Top-quark

Top is central for just about everything:

- We want to study it directly (SM)

  but also

- Very important for reducing uncertainties in other observables
  - ✓ constrain gluon PDF at larger x,
  - ✓ reduce uncertainties in anti-correlated observables:
    Examples: singe top and heavy Higgs

CTEQ '08

- Important for Higgs
  - direct background (in $H \rightarrow W^+W^-$),
  - conceptually – large Yukawa coupling!

Continues …

Precision in the LHC era: Top-quark

- Central collider signature for BSM physics:
  - $t\bar{t}$ is a preferred decay mode of TeV KK particles

- Top mass (direct/x-section)

  Alternative determination of M_{top} from cross-section.

  ➔ Great, because we would be confident in ***what*** we measure!

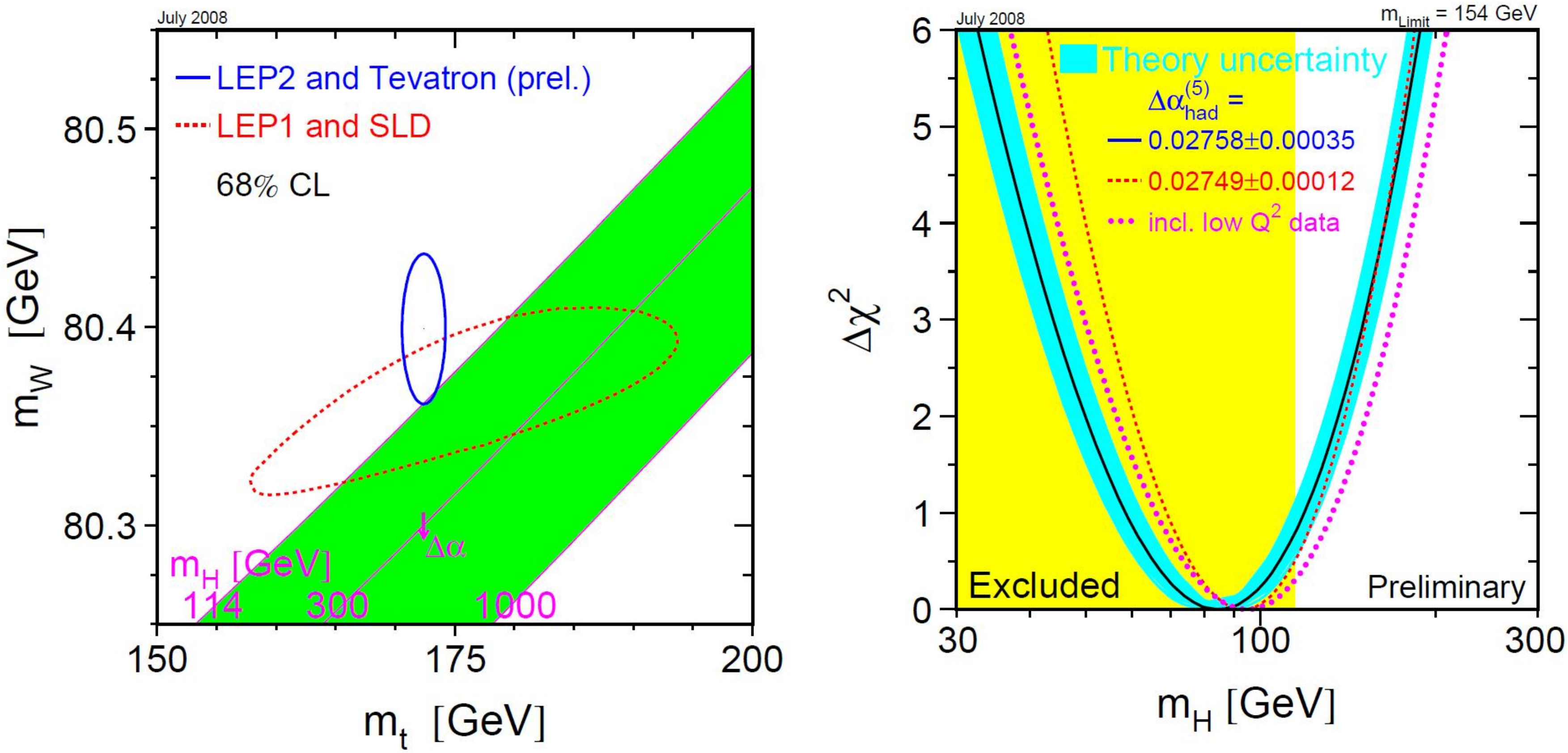
Continues …

Precision in the LHC era: Top-quark

Places where the top mass is crucial: - Higgs mass
- Higgs-inflation

$M_{\rm t} \;=\; 172.4 \pm 1.2\ \mathrm{GeV}/c^2$ CDF+D0 '08

Precision Electroweak Measurements and Constraints on the Standard Model arXiv:0811.4682v1 [hep-ex]



Precision in the LHC era: Top-quark

- Top is the most challenging collider signal:
  - 2-to-2 reaction (unlike Drell-Yan and Higgs),
  - involves four colored particles: color correlations
  - massive: its mass cannot be approximated as very small or very large
- The above have led to many theoretical developments
  - prediction of massive amplitudes at high energy,
  - exponentiation, resummation and singularities of massive amplitudes.
- These results can be applied to many more places (jets, tt-Higgs, massless amplitudes at higher orders, N=4 SYM etc.)

Precision in the LHC era: Top-quark

An incredible, real life example for

why precision and detailed understanding are very important

The history of the top-pair cross-section calculation:

- Calculated at NLO ~ 20 years ago (numeric approx.)
- NLL resummation was done ~ 10 years ago

☺ It seemed: "top was a done deal" ;

☺ It was even called "a plain vanilla" .

But let's have a closer look :

Precision in the LHC era: Top-quark

The previous results were based on numerical fits (high quality 1%)

Nason, Dawson, Ellis '88
Beenakker et al '89

Only now the exact result could be calculated (2 months ago)

M. Czakon, AM '08

First exact result beyond LO in a 2-to-2 reaction with masses

Compare the new analytic result with the earlier numerical ones.

Of special interest is the threshold expansion $\beta \to 0$ (i.e. $4m^2 \to s$) :

$$\begin{aligned} f_{gg}^{(1)}(\beta) &= \frac{1}{4\pi^2} f_{gg}^{(0)}(\beta) \Bigg(\Bigg(C_F - \frac{(N^2-4)\,C_A}{2(N^2-2)} \Bigg) \frac{\pi^2}{2\beta} + 2C_A \log^2(8\beta^2) - \frac{(9N^2-20)C_A}{N^2-2} \log(8\beta^2) \\ &+ C_A \Bigg(\frac{21N^2-50}{N^2-2} - \frac{(17N^2-40)\,\pi^2}{24\,(N^2-2)} + \frac{(N^2-4)\log 2}{N^2-2} - 2\log^2 2 \Bigg) \\ &+ C_F \Bigg(-5 + \frac{\pi^2}{4} \Bigg) + \mathcal{O}(\beta) \Bigg) \,. \end{aligned}$$

It is possible to predict (through *resummation*) the LL,NLL,… behavior of the cross-section close to threshold. Helpful in establishing the theoretical uncertainties.

Precision in the LHC era: Top-quark

Extraction of the constant C_3 in the threshold limit:

It is so important, there is an established notation for it ☺

$$C_A\left(\frac{21N^2-50}{N^2-2}-\frac{(17N^2-40)\,\pi^2}{24\,(N^2-2)}+\frac{(N^2-4)\log 2}{N^2-2}-2\log^2 2\right)+C_F\left(-5+\frac{\pi^2}{4}\right)$$

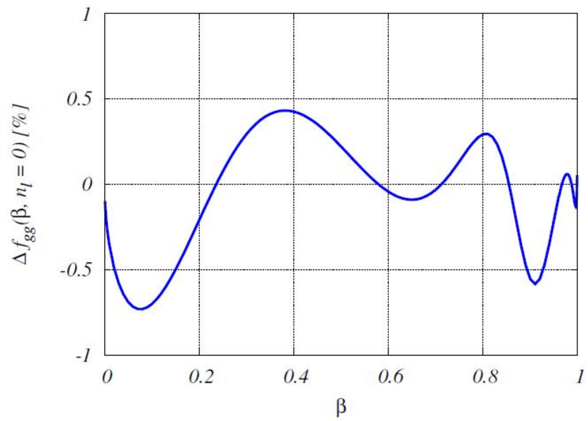
$$=\frac{1111}{21}-\frac{283\pi^2}{168}+\frac{15\log 2}{7}-6\log^2 2\simeq 34.88\,,$$

Czakon, AM '08

$$\frac{768\pi}{7}a_0^{gg}\simeq 37.25\,.$$

Nason, Dawson, Ellis '89

- X-section better than 1%
- But the constant C_3 is 7% different !
- Turns out, it is all consistent …



Very significant (and unexpected) effect for threshold resummation!

Precision in the LHC era: Top-quark

From resummation, the following 2 loop logs can be predicted:

$$\sigma_{gg}(\beta) = \sigma_{gg}^{\mathrm{Born}}(\beta) + \frac{\alpha_s}{4\pi}\sigma_{gg}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \sigma_{gg}^{(2)} + \mathcal{O}(\alpha_s^3)$$

$$\sigma_{gg}^{(2)} = \sigma_{gg}^{\mathrm{Born}}(\beta)\left(4608\log^4\beta + 1894.9\log^3\beta - 3.4811\log^2\beta + \mathcal{O}(\log\beta)\right)$$

Moch Uwer '08

It turns out the coefficient of $\ln^2(\beta)$ is of the form: $-14306.9505 + 384C_3$

where: $C_3 = 37.23$ As extracted from NDE '89 and used in ALL resummation literature

$C_3 = 34.88$ The exact value just recently derived Czakon, AM '08

Therefore the coefficient of $\ln^2(\beta)$ becomes -912.35

i.e. a change by a factor of 260 !

Note: the reason is pure numerics!

This is a 5% modification to σ, comparable with the current conservative estimates (Cacciari et al '08) of the uncertainty and rather large when compared to (Moch, Uwer '08).

Precision in the LHC era: Top-quark

The changes discussed so far are purely due to numerics.

However: there is another modification compared to earlier literature

Exponentiation in Mellin space: $f(N) = \int_0^1 \rho^{N-1} f(\rho) d\rho$, $\rho = 4m^2/s$

$$\sigma_{ij}^{\mathrm{TOT}}(N) = \sigma_{ij,\mathbf{1}}(N) + \sigma_{ij,\mathbf{8}}(N) \qquad \sigma_{ij,\mathbf{I}}(N) = \sigma_{ij,\mathbf{I}}^{\mathrm{Born}}(N)\, \sigma_{ij,\mathbf{I}}^{\mathrm{H}}\, \Delta_{ij,\mathbf{I}}(N)$$

We were the first to point out σ^H depend on the color state of the heavy quark pair. We calculated the two coefficients.

Change in the gg Sudakov resummed X-section: compare to Bonciani et al '98

$$\begin{aligned}
\sigma_{gg}^{\mathrm{H\,(BCMN)}} &= 1 + \frac{\alpha_s}{\pi}\, 14.39 + \mathcal{O}(\alpha_s^2)\,, \\
\sigma_{gg}^{\mathrm{H\,(BCMN)}}\Big|_{C_3\ \mathrm{exact}} &= 1 + \frac{\alpha_s}{\pi}\, 12.04 + \mathcal{O}(\alpha_s^2)\,, \\
\sigma_{gg,\mathbf{1}}^{\mathrm{H}} &= 1 + \frac{\alpha_s}{\pi}\, 9.16 + \mathcal{O}(\alpha_s^2)\,, \\
\sigma_{gg,\mathbf{8}}^{\mathrm{H}} &= 1 + \frac{\alpha_s}{\pi}\, 13.19 + \mathcal{O}(\alpha_s^2)\,,
\end{aligned}$$

- C_3 numerics: -5%,
- color singlet channel: -12%,
- color octet channel: -3%,

Conclusions

- Great time to work in high energy physics!
- LHC will revolutionize our knowledge of the world
  - Discover the Higgs, or its relatives ☺
  - Discover New Physics: SUSY, extra dimensions
  - Improve understanding about dark matter and early Universe.
- Successful LHC program requires precision

  to be confident in what we measure and how we interpret it.
- The challenges ahead are great, and we are more motivated than ever!

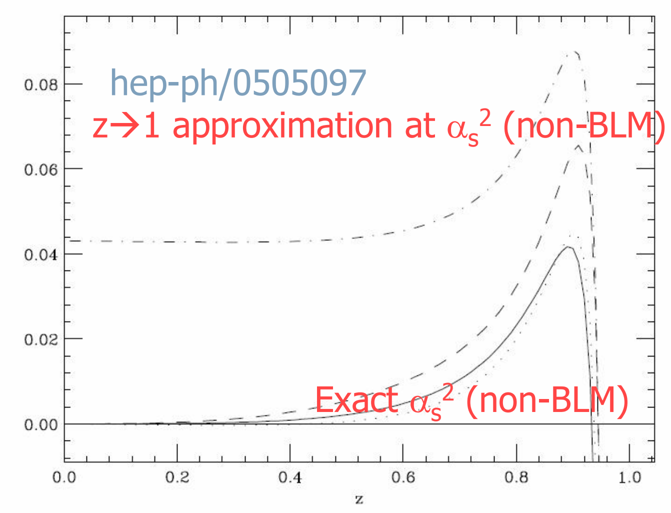
Stay tuned!

What one *needs* to know about top production?

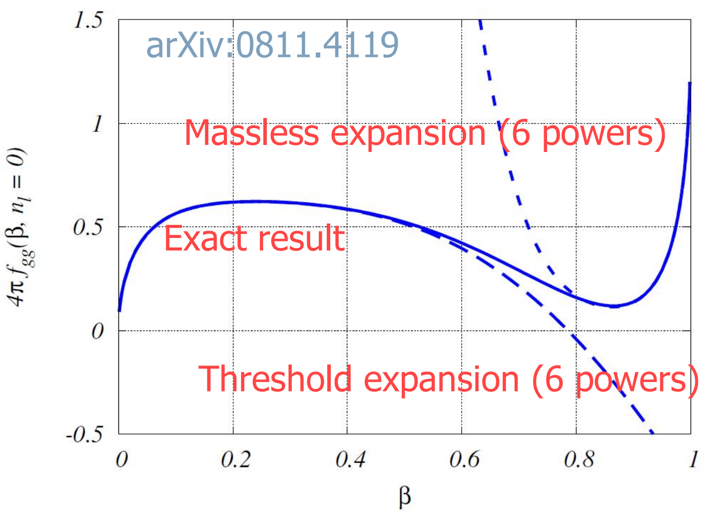
- Understanding true scale uncertainty requires full NNLO calculation !
- The appropriate observable is the total inclusive cross-section.
- Some NNLO terms can be obtained by truncating all-order resummation.
  - is this a systematic approximation?

In general, this is a poor approximation to fixed order calculations:

Photon spectrum in B→ s+γ:



Top X-section: NLO correction



Back to Phenomenology

What our results actually imply?

Conclusion #1: the earlier FO NLO calculations are of high quality

However: in the last 10 years or so, all "improvements" have been done by soft gluon resummation (since no new FO results)

Idea of threshold resummation: towers of $\ln(\beta)$ terms in the total inclusive cross section $\sigma_{TOT}(\beta)$ can be predicted to all orders in the coupling.

Conclusion #2: the resummation results are altered by our work