# 5 Masses of Compact Galactic Objects with Microlensing

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ABSTRACT

*Using the technique of gravitational microlensing, SIM Lite will conduct a representative census of all compact Galactic objects from brown dwarfs to black holes, including white dwarfs, neutron stars, and main sequence stars. The masses of these objects will be derived from a combination of precise measurement of the tiny astrometric deflections that are generated by all microlensing events with a photometric "microlens parallax" measurement. The latter is possible because SIM Lite will be in solar orbit, which means that it measures a "different" photometric event compared to what is seen from the ground. No other known technique can undertake such a systematic survey, although Gaia and ground-based interferometers might succeed in measuring the masses of some black holes.*

## 5.1 Census of Dark and Luminous Objects

What would an unbiased census of Galactic objects — dark and luminous — reveal? At a minimum, it would yield the frequency of black holes, neutron stars, white dwarfs, and old brown dwarfs, which are either completely dark or so dim that they defy detection by normal methods. It might also find a significant component of the dark matter, although the majority of dark matter cannot be in the form of compact objects (Alcock et al. 2000; Tisserand and Milsztajn 2005). The only known way to conduct such a census is to put a high-precision astrometry telescope in solar orbit.

Masses of astronomical bodies can be measured only by the motions they induce on other objects, typically planets and moons that orbit Solar System bodies and binary companions that orbit other stars. Masses of luminous isolated field stars can be estimated from their photometric and spectroscopic properties by calibrating these against similar objects in bound systems. Hence, photometric surveys yield a reasonably good mass census of luminous objects in the Galaxy.

Dark objects like black holes are another matter. Mass measurements of isolated field black holes can be obtained only by their deflection of light from more distant luminous objects. Indeed, it is difficult to even detect isolated black holes by any other effect. However, to go from detection to mass measurement (and therefore positive identification) of a black hole is quite challenging.

## 5.2 Mass Measurement from the Gravitational Deflection of Light

Gravitational microlensing experiments currently detect about 700 microlensing "events" per year. The vast majority of the "lenses" are ordinary stars, whose gravity deflects (and so magnifies) the light of a more distant "source star." As the source gets closer to and farther from the projected position of the lens, its magnification $A$ waxes and wanes according to the Einstein formula (Einstein 1936):

$$A(u) = \frac{u^2 + 2}{u\sqrt{u^2 + 4}} \quad \text{where} \quad u(t) = \sqrt{u_0^2 + \left(\frac{t - t_0}{t_E}\right)^2} \tag{1}$$

where $u$ is the source-lens angular separation (normalized to the so-called Einstein radius $\theta_E$), $t_0$ is the time of maximum magnification (when the separation is $u_0$), and $t_E$ is the Einstein radius crossing time, i.e., $t_E = \theta_E / \mu$, where μ is the lens-source relative proper motion. The mass $M$ cannot be directly inferred from most events because the only measurable parameter that it enters is $t_E$, and this is a degenerate combination of $M$, μ, and the source-lens relative parallax $\pi_{rel}$:

$$t_E = \frac{\theta_E}{\mu} \quad \text{where} \quad \theta_E = \sqrt{\kappa M \pi_{rel}} \tag{2}$$

and $\kappa = 4G/(c^2\,AU) \sim 8\ \text{mas}/M_\odot$.

It follows immediately that to determine $M$, one must measure three parameters, of which only one, $t_E$, is routinely derived from microlensing events. Another such parameter is $\theta_E$, which could be routinely measured from the image positions, if it were possible to resolve their separation, which is of order 1 mas. A third is the "microlens parallax," $\pi_E$,

$$\pi_E = \pi_{rel} / \theta_E \tag{3}$$

Combining equations (2) and (3),

$$M = \frac{\theta_E}{\kappa \pi_E} \tag{4}$$

implying that the mass can be extracted from $\theta_E$ and $\pi_E$ alone. (See, e.g., Gould 2000.)

Just as $\theta_E$ is the Einstein radius projected onto the plane of the sky, $\tilde{r}_E$ is the Einstein radius projected onto the observer plane and is related to the microlens parallax $\pi_E$ by $\tilde{r}_E \equiv AU/\pi_E$. Just as $\theta_E$ could in principle be measured by resolving the two images on the sky, $\pi_E$ could be routinely measured by simultaneously observing the event from two locations separated by a distance on the order of $\tilde{r}_E$ (Refsdal 1996; Gould 1995). "Routine" measurement of both $\pi_E$ and $\theta_E$ is essential. As of today, there have been a few dozen measurements of these parameters separately (e.g., Poindexter 2005), but only one very exceptional microlensing event for which both were measured together with sufficient precision to obtain an accurate mass (Gould, Bennett, and Alves 2005).

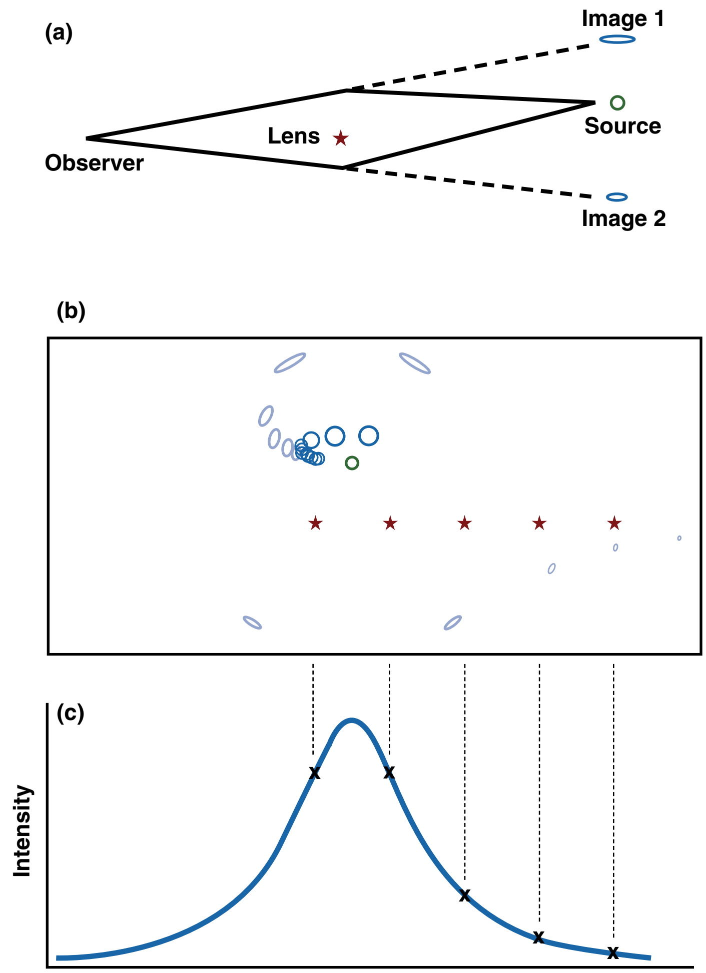
In fact, such routine measurement is possible by placing an accurate astrometric and photometric telescope in solar orbit. For current microlensing experiments carried out against the dense star fields of the Galactic bulge, $\pi_{rel}$ ~40 μas, so for stellar masses, $\theta_E$ ~500 μas and $r_E$ ~10 AU. Hence, a spacecraft in solar orbit would be an appreciable fraction of an Einstein radius from Earth, so the photometric event described by Equation (1) would look substantially different than it would from the ground. From this difference, one could infer $r_E$ (and so $\pi_E$).

Determining $\theta_E$ is more difficult. As mentioned above, this would be straightforward if one could resolve the separate images, but to carry this out routinely (i.e., for small as well as large values of $\theta_E$) would require larger baselines than are likely to be available in next-generation instruments. Rather, one must appeal to a more subtle effect, the deflection of the centroid of the two lensed images. This deflection is given by (Miyamoto and Yoshii 1995; Hog, Novikov, and Polanarev 1995; Walker 1995):

$$\Delta\theta = \frac{\mathbf{u}}{u^2 + 2}\theta_E \tag{5}$$

Simple differentiation shows that this achieves a maximum at $u = \sqrt{2}$, for which $\Delta\theta = \theta_E / \sqrt{8}$, roughly 1/3 of an Einstein radius. Hence, if the interferometer can achieve an accuracy on the order of 10 μas at the time when this deflection is the greatest, then $\theta_E$ can be measured to a few percent.

There are some subtleties as well as some challenges. Satellite measurements of $\tilde{r}_E$ are subject to a four-fold discrete degeneracy, which can only be resolved by appealing to higher-order effects (Gould 1995). It is not enough to measure the centroid location to determine the astrometric deflection: one must also know the undeflected position to which the measured position is to be compared, and this can only be found by extrapolating back from late -time astrometry. And the precision of the mass measurement depends directly on the signal-to-noise ratio of the underlying photometric and astrometric measurements. This is important because space-based astrometric telescopes are likely to be photon challenged and so to require relatively bright (and hence rare) microlensing events to provide accurate mass measurements. Gould and Salim (1999) carried out detailed simulations based on the characteristics of SIM Lite and concluded that with about 1200 hours of mission time, it would be possible to make 5 percent mass measurements for about 200 microlenses. Most of these lenses will be stars, but at least a few percent are likely to be black holes, and several times more are likely to be other dark or dim objects like neutron stars, old white dwarfs, and old brown dwarfs. Since such a census is completely new, it may also turn up unexpected objects. (The measurement of masses from microlensing is illustrated in Figures 5-1 and 5-2.)



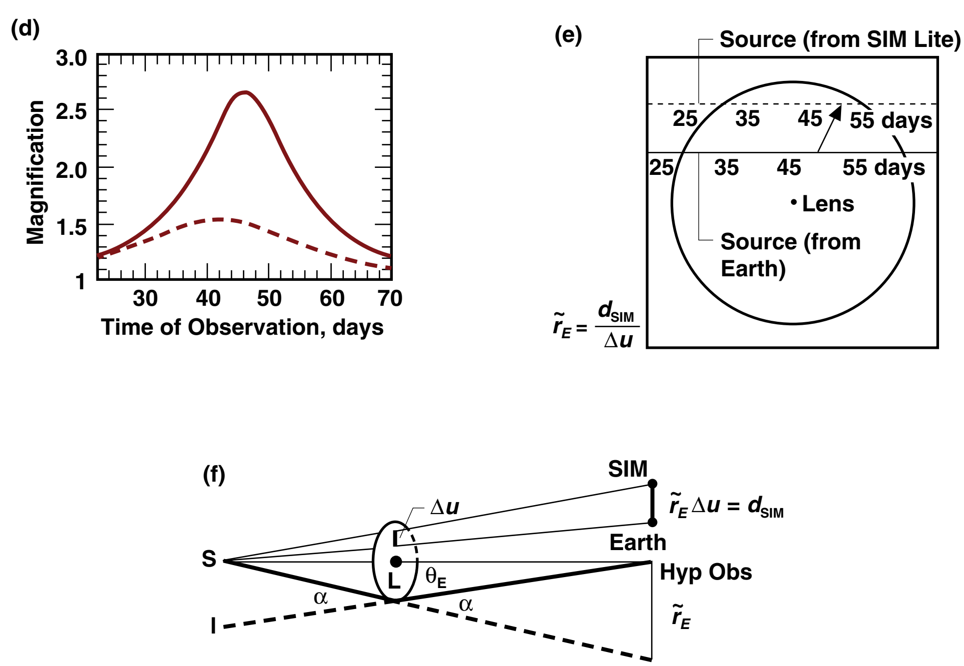




Figure 5-1. How SIM Lite measures the mass of a dark object. (a) Side view of the geometry of a microlensing event, across the line of sight. The lens passes near the line of sight between an observer and a distant source and produces two images. (b) High-magnification sky view, along the line of sight. Starting from the left, the gravitational lens (red star) passes the background source (green circle), splitting its light into two magnified images (cyan) that trace separate paths above and below the lens. Their combined light (blue circle), measured astrometrically by SIM Lite and photometrically [depicted schematically in (c)] by SIM Lite and on the ground, is displaced from the source (green circle) by an amount that varies according to Equation (5), reaching a maximum deviation of $\theta_E/\sqrt{8}$, where $\theta_E$ is the angular Einstein radius. Hence, by measuring this deflection, SIM Lite determines $\theta_E$. (c) The light curve as observed by SIM Lite or from the ground (scaled for their different lines of sight). The marks on the curve correspond to the positions of the lensing object (red stars). (d) Photometric microlensing events seen from Earth and SIM Lite differ, with different maximum magnifications (and intensities [(c)]) and different times of maximum. (e) From these differences, one infers that the event as seen from Earth has a smaller impact parameter (since it was more highly magnified) and passed the lens later (since it peaked later). Hence, one can infer the offset $\Delta\mathbf{u}$ between the two trajectories in units of the Einstein ring. (f) The scalar separation $\Delta u$ is equal to the ratio of the SIM Lite–Earth distance to the "projected Einstein radius," $\tilde{r}_E$, i.e., the radius of the Einstein ring projected onto the observer plane. Hence, photometric measurements from SIM Lite and Earth yield $\tilde{r}_E\,\theta_E$. Mass is then given by $M = (c^2 / 4G)\tilde{r}_E$, while source–lens relative parallax is $\pi_{rel} = (AU / \tilde{r}_E)\theta_E$. (Adapted from Gould and Salim 1999.)

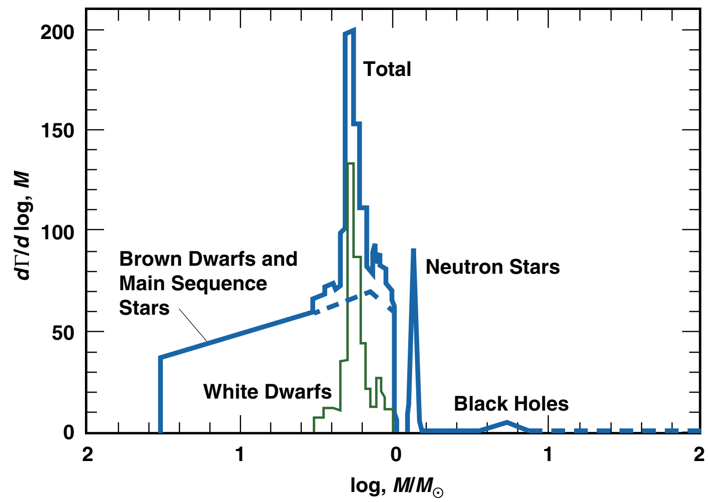



Figure 5-2. Rate of microlensing events as function of lens mass as predicted by the model of Gould (2000), including those due to brown dwarfs, white dwarfs, neutron stars, and black holes, as well as main sequence stars. The last are reasonably well-constrained from optical/IR surveys, but distribution functions of the dim/dark objects are unknown. Only SIM Lite in solar orbit can measure them.

## 5.3 Performance of Gaia and Ground-Based Instruments

Since Gaia is scheduled to be launched before SIM Lite, it is appropriate to ask whether these measurements could be carried out by Gaia, or whether Gaia could somehow leverage its vastly larger number of targets to compensate for its inferior astrometric precision. And, of course, one should also ask how much of this program could be carried out from the ground.

Neither Gaia nor ground-based interferometers can address the integrated problem of measuring the entire mass spectrum of compact objects from brown dwarfs to black holes. However, both could make some progress on the more limited (but very interesting) problem of measuring the frequency of black holes.

With regard to the full mass spectrum, Gaia has two problems. First, Gaia will be in an L2 rather than a solar orbit, and it therefore cannot be used as one of two platforms (the other being Earth) from which to measure the microlens parallax. Of course, this applies still more strongly to ground-based observations of any type.

Second, the astrometric precision required for reliable identification of "typical" lenses is substantially higher than will be achieved by Gaia. From equation (2), one finds that for $M = 1M_\odot$ and $\pi_{rel} = 20$ µas (typical of bulge lenses), $\theta_E = 400$ µas. The maximum deflection of the centroid of light from the true source position is therefore $\Delta\theta_{max} \sim 140$ µas, so that a $3\sigma$ detection requires a precision of $\sigma \sim 45$ µas. Of course, Gaia is expected to achieve this precision for many stars, but this is the *mission accuracy*. What is required here is to measure the $\theta_E / \sqrt{8}$ "excursion" while it is actually happening, i.e., over roughly four Einstein crossing times, $4t_E$ ($2t_E$ while approaching and $2t_E$ while receding), typically 120 days. For these short intervals, Gaia precision will be degraded by a factor ~ $\sqrt{5\ years/120\ days} \sim 4$. Since even the brightest sources (indeed the ones that SIM Lite would observe) will be V ~ 17, the required precision is a factor of several beyond Gaia's capability.

Given its small field of regard (and so shortage of stable reference stars), ground-based interferometry would most likely tackle the $\theta_E$ measurement by a different route: detection of the two images, which are separated by $2\ \theta_E \sim 800$ µas. By comparison, at $\lambda = 2$ µm, a 100 m baseline implies a diffraction limit of 4 µas. Hence, for example, the Keck interferometer could not be used. Of course, longer-baseline interferometers are already in use. But here we have to remember that the brighter "typical" sources will be K = 13. While there will be (and already are) a few highly magnified events per season that are amenable to interferometric measurement, their frequency is far too small to permit a systematic survey of compact matter. And, again, even if these measurements were possible, they would yield only $\theta_E$ and not $\pi_E$ (and so the mass and distance).

However, among the dark objects, black holes are probably the most interesting, and because black holes are substantially more massive than typical stellar lenses, black hole events are longer and so are more susceptible to microlens parallax measurements from the ground (Bennett, Anderson, Bond, Udalski, and Gould 2006; Poindexter 2005). This obviates (or partially obviates, see below) the need for a satellite in solar orbit.

Moreover, since they are more massive than typical objects, black holes have larger Einstein rings, which dramatically improves the prospects for measuring them using either Gaia or ground-based interferometers. At the same $\pi_{rel}$ = 20 μas, a "typical" black hole of mass $M \sim 6\ M_\odot$ would have $\theta_E \sim 1$ mas and so $\Delta\theta_{max} \sim 350$ μas, so that a 3σ detection would only require a precision of σ = 120 μas. And, the larger $\theta_E$ implies a longer $t_E$, typically 75 days, which reduces the degradation factor on Gaia astrometry from a factor 4 to a factor 2.5. That is, a "typical" black hole would only require a "mission precision" of σ ~ 120 μas / 2.5 = 50 μas. If Gaia meets its design goals, this will be achievable for the brighter sources.

An important collateral point is that such Gaia measurements would break a common (and usually crippling) degeneracy in the ground-based microlens parallax measurements. Unlike trigonometric parallax, microlens parallax is a vector, $\boldsymbol{\pi}_E$, whose direction is that of the lens source relative proper motion and whose amplitude is, of course, $\pi_E$. For typical events, $\boldsymbol{\pi}_E$ is not measurable at all because these events are too short. For very long events, it is completely measurable (e.g., Poindexter 2005), but for intermediate-length events, such as those expected for typical black holes, one component of $\boldsymbol{\pi}_E$ (the one parallel to the projected position of the Sun–Earth axis) can be tightly constrained, while the orthogonal component is almost completely unconstrained (Gould, Miralda-Escud'e, and Bahcall 1994). See, e.g., Jiang et al. 2003; Ghosh et al. 2003. By measuring the *direction* of the centroid displacement (which can be determined with roughly the same fractional precision as the amplitude of displacement) one can determine the *direction* of $\pi_E$ and so break the degeneracy.

The extra factor of 2.5 in $\theta_E$ for black holes comes close to reaching the diffraction limit of ground-based interferometers, so that it may be possible for them to partially resolve the two images. However, this assumes that they will reach flux thresholds that are substantially more than an order of magnitude fainter than current limits.

Neither Gaia nor ground-based interferometers have any hope of a complete census of compact objects (stars, black holes, neutron stars, white dwarfs, and brown dwarfs). However, Gaia probably could do a reasonable black hole census, and ground-based interferometry could make some inroads in this direction, although it could not do a black hole census.

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