Measuring the Orbits of Exo-Earths in Multiplanet Systems

The Synergy of Direct Imaging and Astrometry

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Abstract

The holy grail of exoplanet searches is an Earth mass planet in the middle of the habitable zone around a nearby star. A single image of such a planet however does not provide evidence that the planet is Earth mass nor that it is in a habitable zone orbit. Mass is the most important parameter of a planet and can only be measured by measuring the motion of the star around the planet-star center of gravity. The planet's orbit however can be measured either by imaging the planet at multiple epochs or by measuring the position of the star at multiple epochs by space based astrometry. The measurement of an exo-planet's orbit by direct imaging is complicated by two factors. One is the inner working angle. A space coronagraph or interferometer, imaging an exo-Earth can separate the light from the planet from the light from the star only when the star-planet separation is larger then the inner working angle. The second complication comes from the fact that the apparent brightness and color of a planet depends on the phase angle, the moon is much brighter at full moon than at half moon. If we have only two images of a multiple planet system it is not possible to assign a dot to a planet based just on photometry and color of the planet. This paper looks at the synergy between astrometric and direct imaging in measuring the orbit of a planet. The measurement of the orbit of a planet requires a moderately large number of images, compromising the ability of some types of coronagraphs (occulters) from searching a large number of stars for exo-Earths.

Introduction

When a potential exo-Earth is detected, the first thing we want to know is, "is this an Earth?" and is it in the habitable zone? Measuring the orbit of a planet in our solar system is pretty straight forward, that's because we can observe the planet over approximately 90% of its orbit. In Figure 1, the large blue circle is the IWA and the yellow arcs are the parts of the orbit when the planet is observable. The planet is not always observable even outside the IWA, such as when the bright side of the planet is facing away from us. With a coronagraph whose IWA is only slightly smaller than the max star-planet separation, some orbital parameters like orbit inclination can't be measured. The planets's apparent brightness can vary by a factor of 3 from the "full moon" phase to the half moon phase. In multiple planet systems two images with one planet in each image leaves open the possibility that there are two separate planets only one of which is outside the IWA at a time.

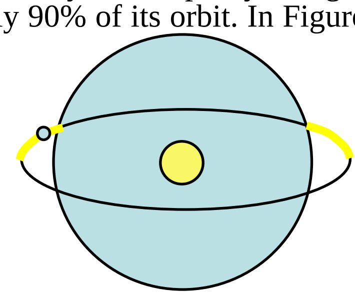

Figure 1: Observable IWA and planet orbit

If the IWA is substantially smaller (e.g. 50%) of the maximum star-planet separation, the planet becomes observable over most of its orbit. In this case it will be possible to look for seasonal variations in brightness. Such variations may be because the surface is a non-lambertian scatterer. Another source of seasonal change may be due to a change in albedo. In the winter time, an ocean's surface may be covered with ice, which has a much higher albedo than liquid surface of the ocean. Seasonal changes in brightness is a double edged sword, it tells us important information about the surface of the planet, but it also complicates the identification of which dot is which planet and the determination of the planet's orbit around the star.

The key to measuring the orbit of the planet is to have many images of the planet at different times of the year. But some types of coronagraphs are seriously limited in its ability to take images at many epochs. This paper attempts to calculate a rough estimate of the number of observations needed for orbit determination with imaging alone and with a combination of imaging and astrometry.

Planetary Orbits, with Imaging and/or Astrometry

When we image a planet as a dot in a sea of speckles, we want to know if this is a potentially habitable planet. We want to know its mass and the semi-major axis of its orbit. A 1 M_{earth} in a 1 AU orbit at 10pc, only astrometry at the sub-microarcsec level can measure the mass of the planet with reasonable precision (+/- 0.3 M_{earth}). But both astrometry and imaging can in theory measure the orbit. Astrometry, because it looks at the star, doesn't have an IWA limitation and the motion of the planet as inferred by its reflex motion on the star can be measured throughout the orbit. We consider 2 different scenarios; Scenario 1) where the planet is first discovered by an astrometric mission, the role of imaging is to a) confirm the discovery, and b) improve on its orbit determination. Scenario 2) is where the imaging mission must both detect and then characterize the discovered planet to the same level of precision as in the precursor astrometric mission followed by the imaging mission.

Astrometric Orbit precision

NASA conducted a double blind study for the astrometric detection of Earth like planets in multiple planet systems[1]. The result of the test was that the presence of multiple planets has a marginal to negligible impact on the astrometric mission's ability to detect terrestrial planets in the habitable zone. One of the side products of that study was a determination of the accuracy of the astrometric orbit at the "edge" of detectability. A mission with a Signal to Noise Ratio (SNR) of 5.8 was deemed necessary to detect planets with a false alarm probability of only 1%. At SNR=5.8, the period of a 1 year planet would have a 1 sigma error of 3% and the mass 1 sigma error of 0.3 M_{earth}. In indirect detection, the semi-major axis of the orbit is derived from its period using Kepler's laws. The orbital phase at mid-mission has an error that was roughly 0.24 radian (+/-14 days in a 365 day year). If the astrometric data preceded the imaging search by 5 years, the uncertainty in the orbital period would cause the orbital phase error bar to grow linearly with time. Five years after the mid-epoch of the astrometry data, the orbital phase uncertainty would be roughly (+/- 50 days, or about 0.85AU).

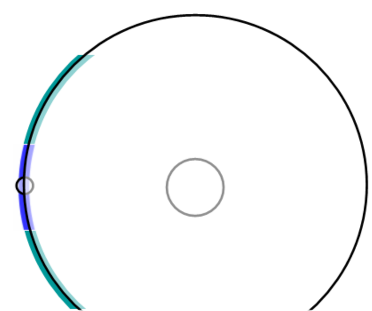

Figure 2: Astrometic Orbit Error

If we start an imaging search of an Earth-Sun clone at 10 pc that was previously found astrometrically, the astrometric error bar 5 yrs after the mean epoch of astrometric measurements would be 0.03AU in the radial direction and about 0.85 AU in the circumferential direction. A single image of the planet taken five or more years after the astrometric data set could dramatically reduce the 1AU error bar in the circumferential direction.

Imaging verification and refinement of the Orbit

If the coronagraph has an IWA that was 0.9 of the max star-planet separation, the planet would be observable for 26 days on either extreme of its orbit. With a 60 day 1 sigma error bar, the probability that the planet would be seen on the first attempt is 18%. Two attempts spaced 26 days apart would increase the probability to 33%. The situation is better for a planet where the IWA was 0.8 of the max star-planet separation. The first visit probability is 25% and the 2 visit probability is 48%. For planets that barely poke their heads beyond the IWA, one needs about 4-6 images to image the planet the first time with near 100% probability.

One image of the planet would significantly narrow the circumferential error bar in the orbit. If the coronagraph was working at 2 λ/D, planet-star separation = 1 AU = 2 λ/D, then a single SNR=5 image would locate the planet to roughly 0.1AU. The major error in the astrometric orbit is the circumferential position of the planet (1 AU) due primarily to the 5 year time delay between the astrometric survey and imaging followup. That error is reduced by a factor of about 10 with one image. The one image would improve the period from 3% to 0.3% and the orbital phase uncertaintity would be 0.1AU at the time the image was taken and degrade to 0.14AU after 5 years after the 1st image..

Planet orbit from Imaging data alone

Astrometry can detect 1 M_{earth} planets in the habitable zone. But the "best" targets for astrometry and coronagraphic imaging are two distinct sets of stars that overlap at the 60~70% level. The most important result from astrometry is the detection and the "non-detection" of planets in the 70% overlap. If astrometry has detected a planet we know that the probability that a planet exists is 99%. But conversely if astrometry sees no periodic signal it can state with > 99% confidence the planet doesn't exist. If 10% of stars have terrestrial planets in the habitable zone, this knowledge saves 90% of valuable mission time in "searching" for the planets and getting its orbit. Without this knowledge we need to image the star-planetary system many times to first detect one or more planets, then confirm the faint dots are not background objects and finally to get their orbits.

An important consideration is that a planet with uniform albedo in reflected light will exhibit factors of 3 changes in apparent brightness from zero phase (full moon) to 90º phase (half moon). If the system has multiple planets, you can't use the apparent brightness (with a SNR=5 image) to identify a planet. Three images of one planet at 3 different epochs are sufficient to determine its orbit. But since we can't use photometry to assign a dot in an image to a specific planet, we have to take all combinations of 3 dots at 3 different epochs, generate an orbit for each 3 dot combination and see if any of them predict the location of an observed planet at a 4th epoch. If the planet is observable only over 10~20% of the orbit, getting an orbit may be impossible in a couple of years of observing.

Some types of coronagraphs, the external occulters, have a very limited number of visits. External occulters use a large ~50m star shake a long distance, ~ 70,000km in front of a telescope to block the starlight. Preliminary mission design of occulter missions allow between 130 to 180 observations over 5 years (Glassman, 2007). These preliminary mission designs however only allowed a few visits to see the planet once, not the much larger number of visits needed to determine an orbit. If the planet is outside the IWA for 33% of the orbit, 12 visits will be needed per star to determine measure the orbit of a potential planet in the habitable zone. With 180 total visits, only 15 stars can be search over a 5 year mission. If the fraction of stars with habitable planets is 10% this search of 15 stars will on average only find 1.5 planets. The probability of finding zero habitable planets is distressingly high for a multi-billion dollar mission.

The probability that a star has a habitable terrestrial planet may be a relatively low 10%. But that's because such planets represent a very fraction of all possible planets, 1~10 Mearth versus 1~3000 Mearth and 0.7~1.5AU versus 0.05AU to 20AU for orbital radius. If a star has just one planet, a single image or 2 images of the system doesn't provide conclusive information that the star doesn't have a terrestrial planet in the habitable zone. Consequently one has to continue taking images of the system until there is conclusive evidence that a terrestrial planet is either present or absent. If a star has several Neptune sized planets but no Earths in the habitable zone, one needs to take a dozen images before the absence of any Earths is conclusive.

A further complication of an occulter type coronagraph is the temporal sampling of the images. An occulter can take an image of a star/planets only when the object is 90deg +/- 10deg from the Sun. Sun light can't hit the side of the occulter the telescope is looking at because the scattered sunlight would be much too bright. The occulter has to be edge on as seen from the Sun to minimize thermal effects on the precise shape of the occulter. As a consequence, a star/planet system in general is observable only twice per year ~6 months apart. This limitation is rather severe if we are looking for planets around G stars whose orbital period is roughly 1 year. It may be impossible for an external occulter to measure the orbit of exo-Earth with a 1 year period, because we need 4 images of the Earth in different parts of its orbit and but the observing cadence limits us to just 2 different orbital positions, even in the absence of an IWA limitation.

Observational requirements, number of target stars

The Exo-planet Task Force (EXOPTF) report recommended that an astrometric mission be able to survey 60~100 nearby stars for Earths. We feel that this is an appropriate number for a direct imaging mission as well.

Until Kepler data is analyzed and followed up, no one has any data on what fraction of stars have terrestrial planets in the habitable zone, and we are left to speculate. If we look at the fraction of stars that have Jovian planets, we find that the number of planets (per unit mass) increases dramatically at low masses. But a plot of density vs log Mass and log Period show the density only slowly varying with logM and LogP. From periods of 3 days to 3 years and from 0.3 M_{jup} to 10 M_{jup}, about 15% of stars have planets and about 10% of stars that have planets have multiple Jovian planets. If we used this density in logM and log P and extrapolated to terrestrial planets about 1% of stars would have terrestrial planets in the habitable zone. The reason for the small number is the small volume of phase space of the habitable zone. More recently, the Swiss RV team has predicted that between 3 days and 3 months and 5~50 Earths masses, up to 30% of stars have one such Neptune/super earth type planet. This is a dramatic increase in the density of planets. When extrapolated to terrestrial planets of 1~10 M_{earth} in the habitable zone, ~ 10% of stars are expected to have such planets.

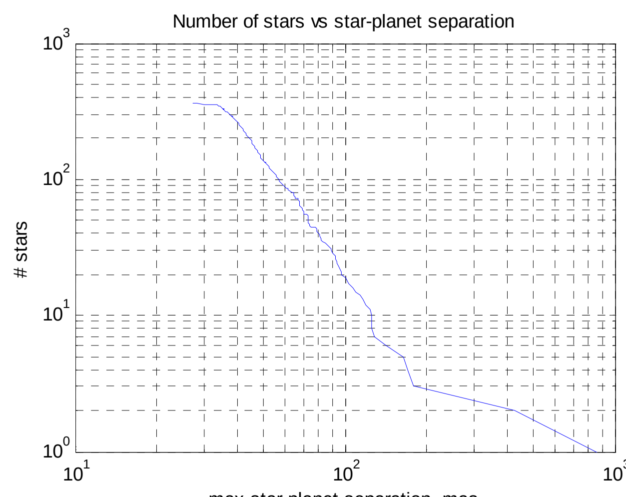

Figure 3: Numers of stars vs. planet separation

While we won't have data on the prevalence of Earths in the habitable zone until Kepler data has been analyzed, an assumption of 10% seems a reasonable guess given current knowledge. A coronagraph capable of detecting an Earth in the habitable zone of 60 nearby stars seems like a reasonable "minimum" capability for a mission designed to characterize the spectra of an exo-Earth in the habitable zone. An Earth at 1 AU from the Sun has a contrast of 1.2e-10 when the planet is at 90deg phase angle (1/2 moon). We can select candidate stars by assuming a 1 M_{earth} planet in the mid-habitable zone,1 AU*sqrt(Luminosity), satisfying the following criteria. 1) star planet contrast < 8e-11 at 90deg phase, 2) brighter than 7 magnitude, 3) < 30pc away. Figure 4 shows the number of target stars versus max star-planet separation. There are a total of around 360 stars that fit the above three criteria. The curve follows the power law #stars = Sep^-3 as expected once we get past alpha cen A and B, two stars very close to the Sun where a terrestrial planet would be markedly easier to image. The list stops at 360 because of the 7 mag and 8e-11 contrast cut offs.

Impact of IWA on orbit determination

While there are 20 candidate stars whose max star-planet separation is 100mas or larger, in practice if the IWA is only slightly smaller than the max star-planet separation measurement of its orbit is impossible. Figure 5 shows the contrast and star-planet separation for an Earth-Sun system at 10pc as the planet orbits the star. If the IWA is 0.7 smaller than the maximum star-planet separation and the minimum detectable contrast is 8e-11, 1.5X smaller than the contrast of an Earth at 90deg phase angle, the planet would be observable over 32% of its orbit. This plot was generated assuming the planet was a lambertian scatterer with 0.3 albedo and no seasonal dependent albedo. The orbit was a circular orbit at 1AU at 80deg inclination. Figure 6 plots the average number of images needed to see the planet 4 times as a function of IWA/(max star-planet separation). This can be combined with figure 1, to calculate the number of images needed to measure the orbit of the habitable zone planets around the nearest stars.

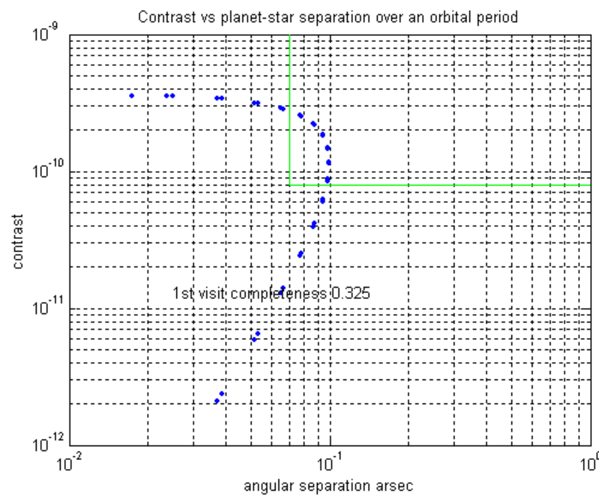

Figure 5: Contrast and separation over an orbit

Summary and Conclusions

Taking an image of a planet does not tell us whether the planet is terrestrial, between 1~10 Mearth, or whether the planet is in the habitable zone. The planet's orbit can be measured by space astrometry, or by direct imaging or a combination of the two. If imaging is combined with astrometry, one image of the planet is all that is needed to confirm the orbit. If the planet only has imaging data, 10 to 20 images are needed to determine that one of the planets orbiting the star is in the habitable zone, or none of the planets orbiting the star are in the habitable zone. A large number of images are needed even when the star does not have a planet in the habitable zone. The need for a large number of images at different epochs has the largest impact on coronagraphic instruments that use an external occulter. Even large occulter missions in the ~5 Billion category may be limited to ~180 visits where only ~15 stars may be searched for Earth clones. Such missions are not viable unless some other mission or technique has previously found the Earths and measured their orbits.

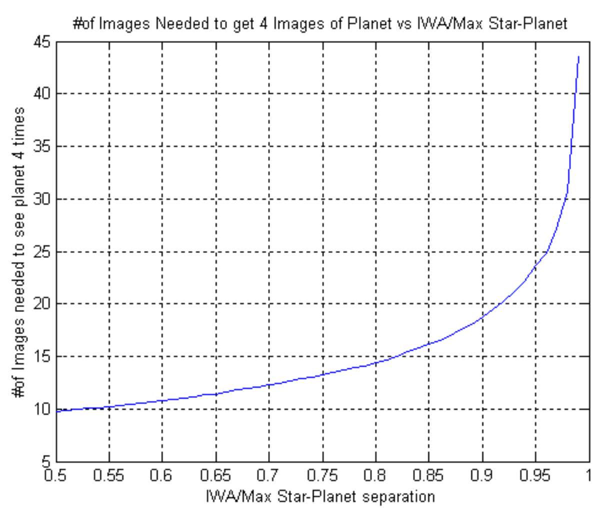

Figure 6: Number of visits needed to measure an orbit

Acknowledgments

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References and Further Reading:

[1] http://planetquest.jpl.nasa.gov/SIM/researchOpps/roDoubleBlind
Glassman, T., et. Al. at http://newworlds.colorado.edu/docs/index.htm