Many Body Lattice QCD

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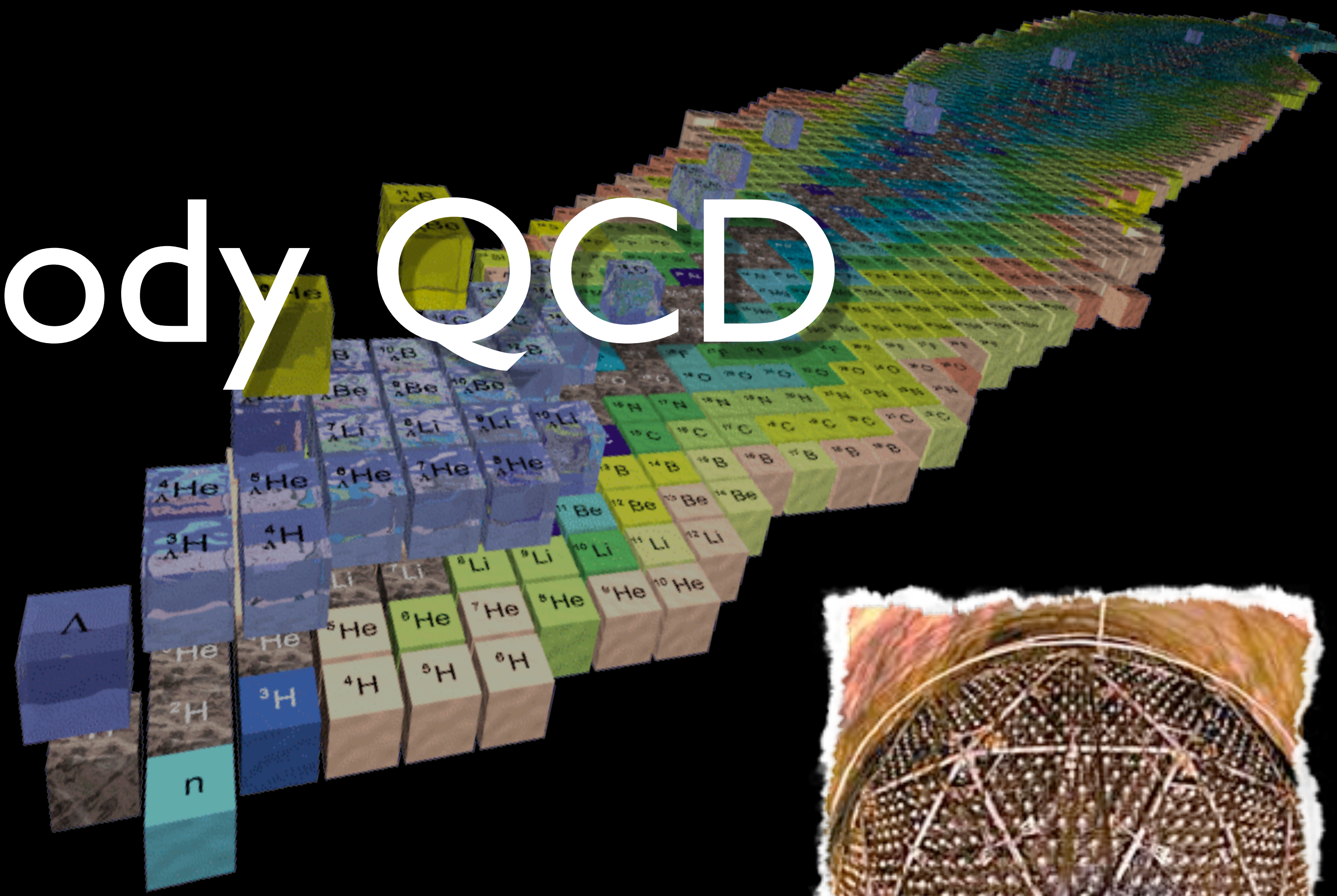
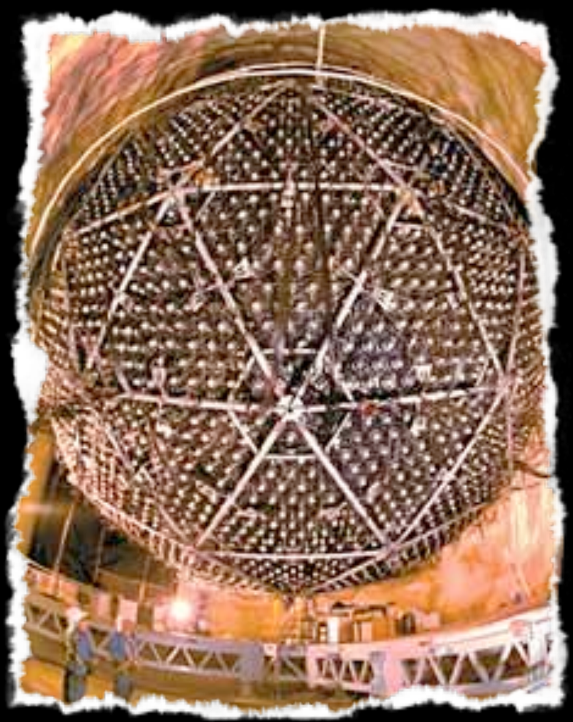
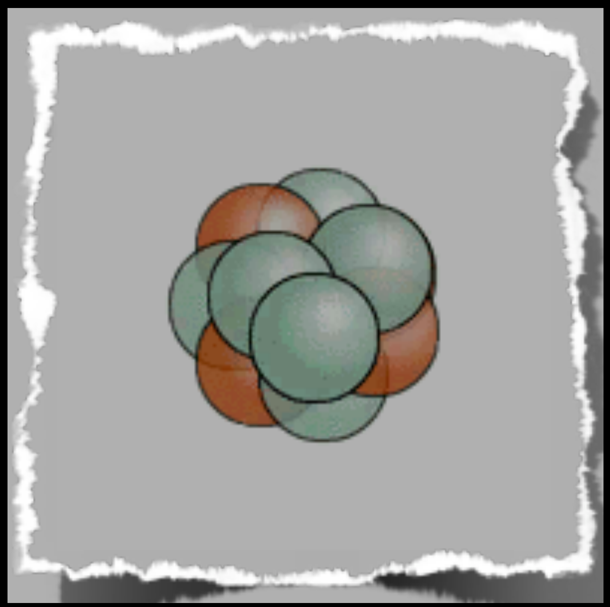
- Multi-hadron systems in QCD
- $n<13$ pions and kaons: meson condensates
  - Two and three body interactions
  - Pion and kaon condensates
  - In-medium screening of the $Q\bar{Q}$ potential

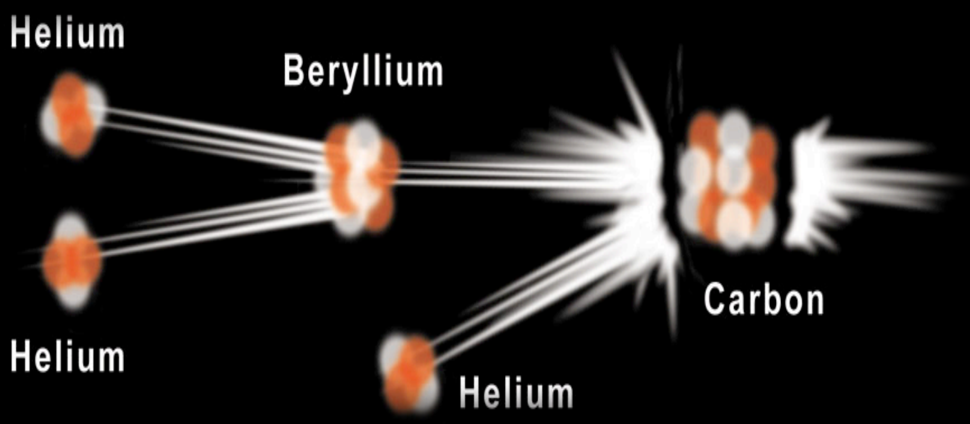
Many body QCD



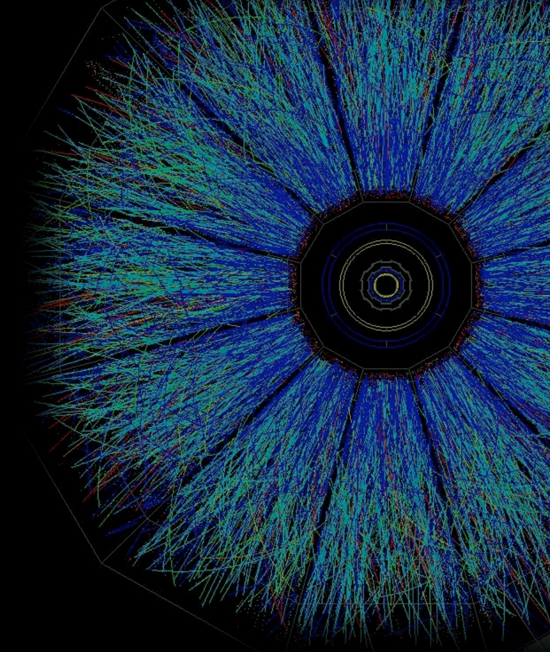
- Nuclei
  - Spectra
  - Interactions and decays
  - m_q dependence



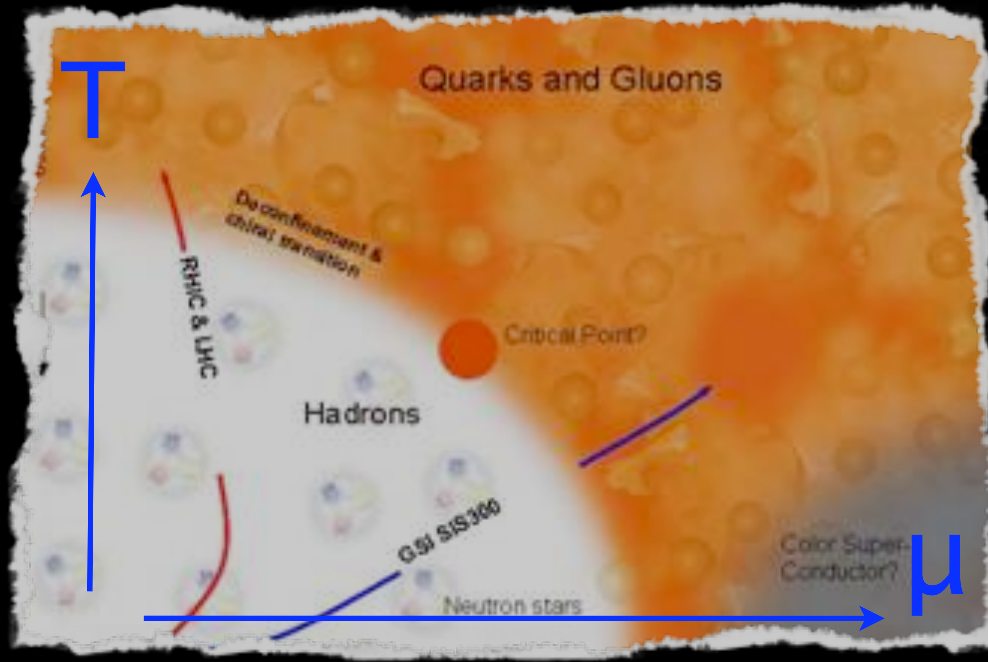


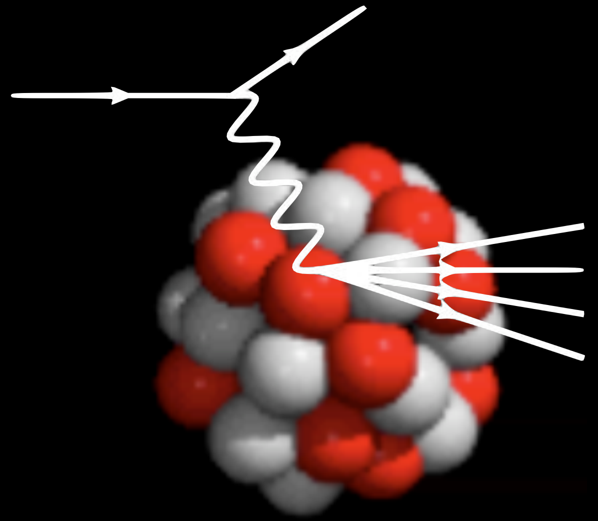


Many body QCD



- π and K condensation
  - EOS in n stars (kaons)
  - Critical point at RHIC
- Three π (and K) interferometry (HBT) at SPS and RHIC
- Medium effects

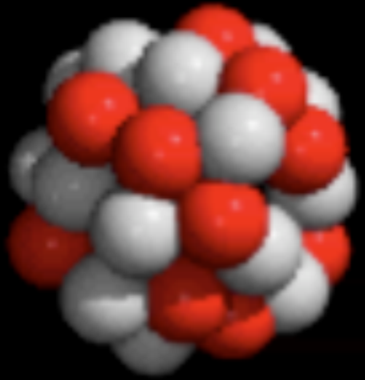




Nuclear physics from LQCD

- Can we compute the mass of ^{208}Pb in QCD?

$$\langle 0|T\, q_1(t)\ldots q_{624}(t)\,\overline{q}_1(0)\ldots\overline{q}_{624}(0)|0\rangle$$



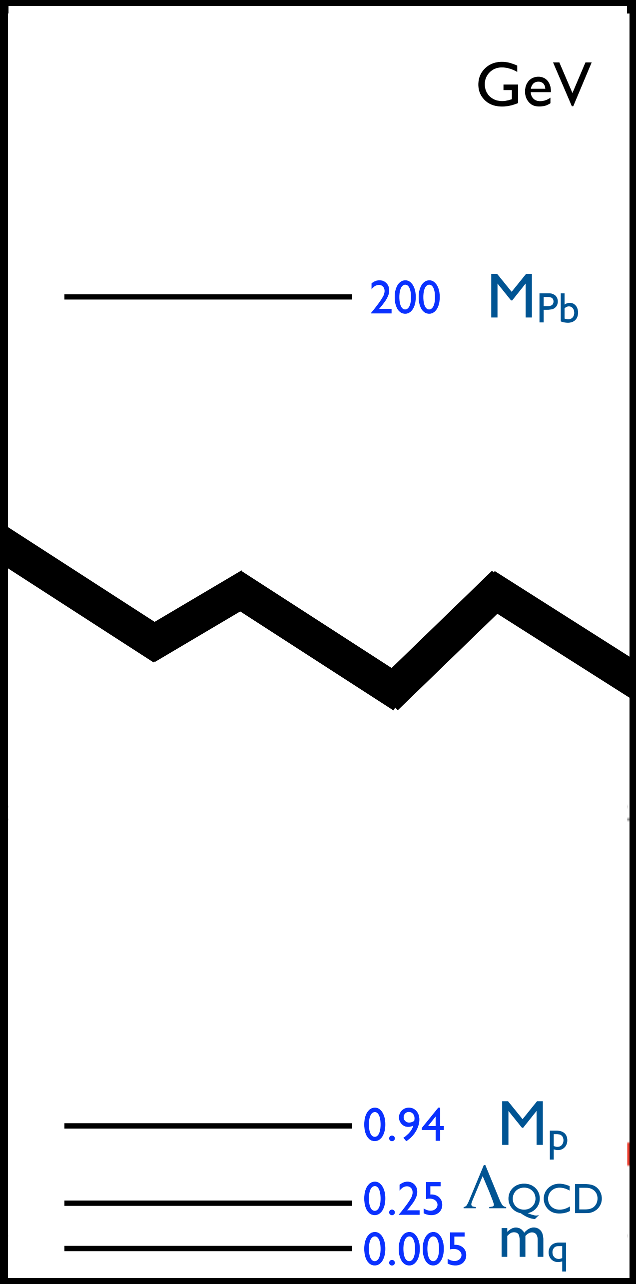
- Long time behaviour gives GS energy

$$\stackrel{t\to\infty}{\longrightarrow} \#\exp(-M_{Pb}t)$$

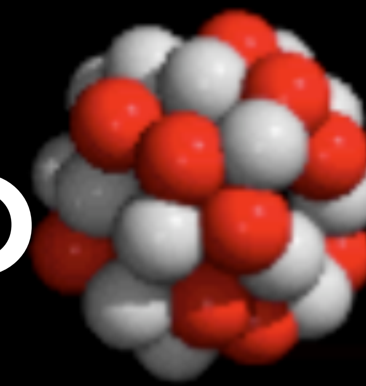
- But...

Nuclear physics is hard!

- Contractions: $(A+Z)!(2A-Z)!$
- Signals for very massive states
- Small energy splittings
- Numerical precision
- Statistical noise: exponentially increases with *A*

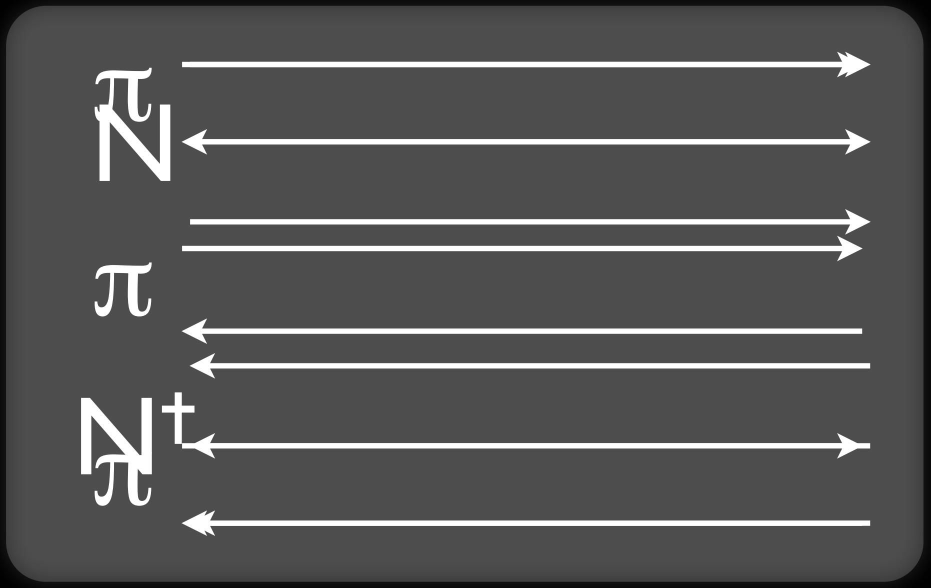


Signal-to-noise ratio



- QCD functional integrals done by importance sampling: propagators
- Variance in correlator determined by

$$\sigma^2\langle C\rangle = \langle C\, C^\dagger\rangle - |\langle C\rangle|^2$$



- For nucleon:

$$s/n \sim \exp[-(M_N - 3/2 m_\pi)t]$$

- For nucleus A:

$$s/n \sim \exp[-A(M_N - 3/2 m_\pi)t]$$

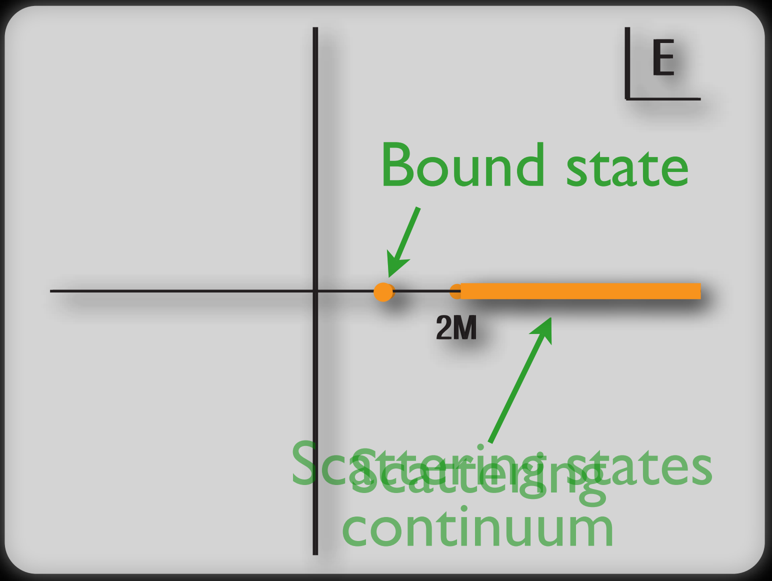
[Lepage '89]

Phys Rev D76:074507, 2007
Phys. Rev. Lett. 100:082004, 2008
Phys Rev D77:057502, 2008
Phys Rev D78:014507, 2008
Phys Rev D78:054514, 2008
Phys. Rev. Lett. 102:032004, 2009
\+ ...

Multi-meson systems

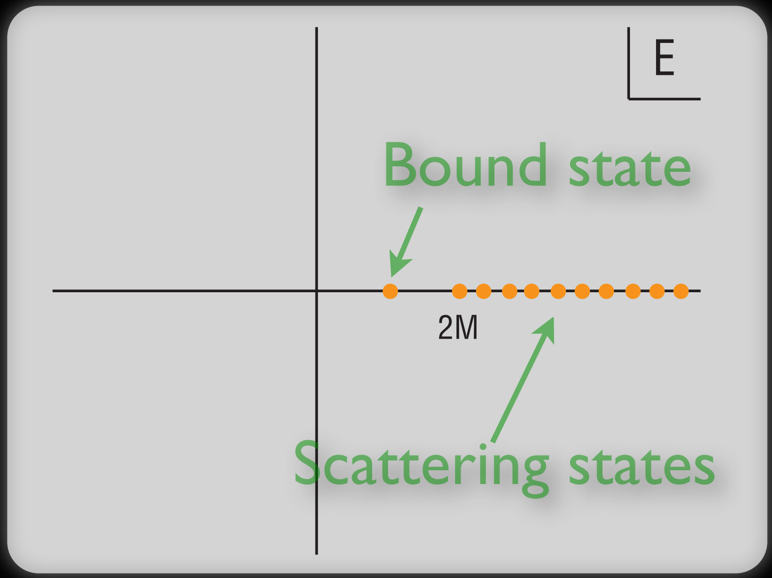
Hadron scattering

- Maiani-Testa: *extracting multi-hadron S-matrix elements from Euclidean lattice calculations of corresponding Green functions is impossible*
- Lüscher: volume dependence of two-particle energy levels ⇒ scattering phase-shift up to inelastic threshold



Hadron scattering

- Maiani-Testa: *extracting multi-hadron S-matrix elements from Euclidean lattice calculations of corresponding Green functions is impossible*
- Lüscher: volume dependence of two-particle energy levels ⇒ scattering phase-shift up to inelastic threshold
- Exact relation provided $r \ll L$
- Used for $\pi\pi$, KK, NN, ΛN



Bosons in a box

[Beane, WD & Savage; WD & Savage]

- Large volume expansion of GS energy of *n* meson system to $1/L^7$
  - 2 & 3 body interactions (*N* body: $L^{-3(N-1)}$)
  - Relativistic up to particle production
  - *n*=2: reproduces expansion of Lüscher
- Can include higher PW, higher body, excited states

[Bogoliubov '47][Huang, Yang '57]

Bosons in a box

[WD+Savage arXiv:0801.0763]

$$\Delta E_n = \frac{4\pi\bar{a}}{M\,L^3}\,{}^nC_2\left\{1-\left(\frac{\bar{a}}{\pi L}\right)\mathcal{I}+\left(\frac{\bar{a}}{\pi L}\right)^2\left[\mathcal{I}^2+(2n-5)\mathcal{J}\right]\right.$$
$$-\left(\frac{\bar{a}}{\pi L}\right)^3\left[\mathcal{I}^3+(2n-7)\mathcal{I}\mathcal{J}+\left(5n^2-41n+63\right)\mathcal{K}\right]$$
$$+\left(\frac{\bar{a}}{\pi L}\right)^4\left[\mathcal{I}^4-6\mathcal{I}^2\mathcal{J}+(4+n-n^2)\mathcal{J}^2+4(27-15n+n^2)\mathcal{I}\,\mathcal{K}\right.$$
$$\left.\left.+(14n^3-227n^2+919n-1043)\mathcal{L}+16(n-2)\left(\mathcal{T}_0+n\mathcal{T}_1\right)\right]\right\}$$
$$+\,{}^nC_3\frac{1}{L^6}\hat{\eta}_3^L+{}^nC_3\,\frac{6\pi\bar{a}^3}{M^3L^7}\,(n+3)\,\mathcal{I}\;+\;\mathcal{O}\left(L^{-8}\right)$$

$$\bar{a}=a+\frac{2\pi}{L^3}a^3\,r$$

Scattering length

$$\hat{\eta}_3^L=\bar{\eta}_3^L\left[1-\frac{6\bar{a}}{\pi L}\mathcal{I}\right]+\frac{72\pi\bar{a}^4 r}{ML}\mathcal{I}$$

Three body interaction

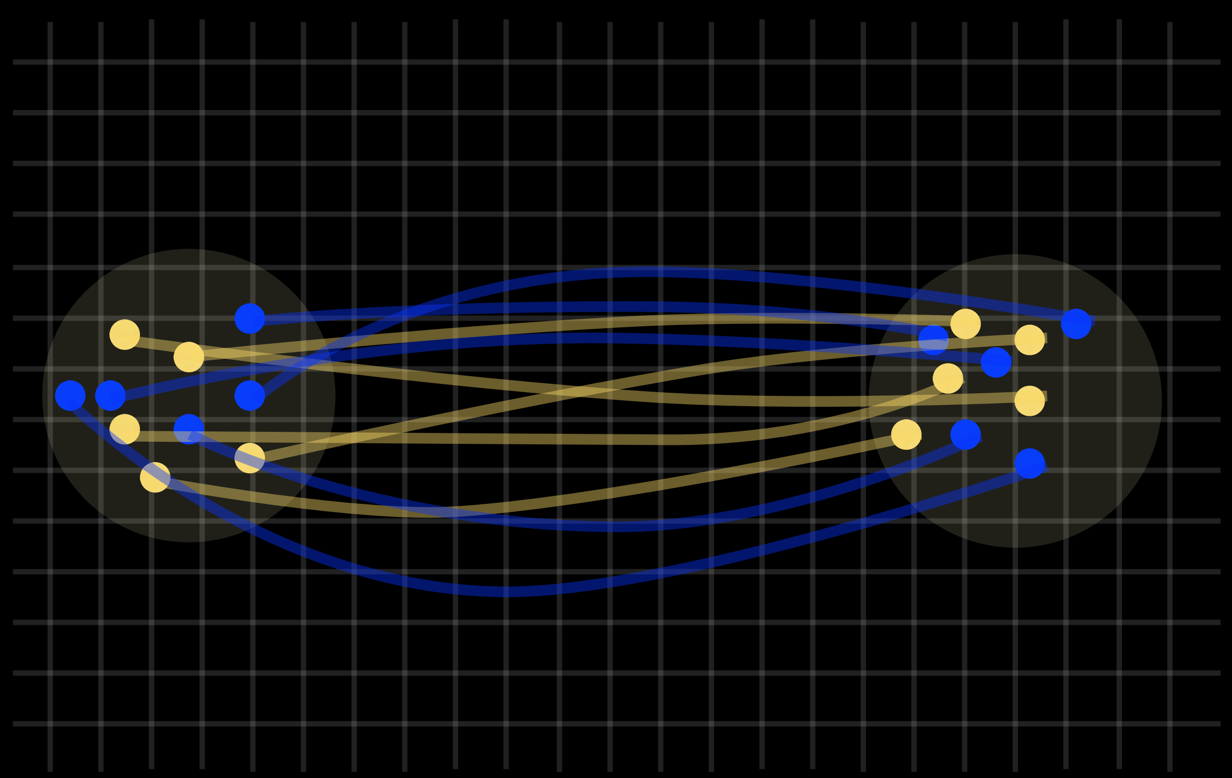
$\mathcal{I},\mathcal{J},\ldots$

geometric constants

Many mesons in LQCD

- Consider *n* π^+ correlator (m_u=m_d)

$$C_n(t) = \left\langle 0 \left| \left[\sum_{\mathbf{x}} \bar{d}\gamma_5 u(\mathbf{x},t)\bar{u}\gamma_5 d(0,0) \right]^n \right| 0 \right\rangle$$
$$\rightarrow A e^{-E_n t}$$



Many mesons in LQCD

- Consider *n* π^+ correlator (m_u=m_d)

$$C_n(t) = \left\langle 0 \middle| \left[\sum_{\mathbf{x}} \overline{d}\gamma_5 u(\mathbf{x},t)\overline{u}\gamma_5 d(0,0) \right]^n \middle| 0 \right\rangle$$
$$\rightarrow Ae^{-E_n t}$$

- $n!^2$ Wick contractions: $(12!)^2 \sim 10^{17}$

$$C_3(t) = \mathrm{tr}\left[\Pi\right]^3 \; - \; 3\ \mathrm{tr}\left[\Pi\right]\mathrm{tr}\left[\Pi^2\right] + \; 2\ \mathrm{tr}\left[\Pi^3\right]$$

$$\Pi = \sum_{\mathbf{x}} \gamma_5 S(\mathbf{x},t;0)\gamma_5 S^\dagger(\mathbf{x},t;0)$$

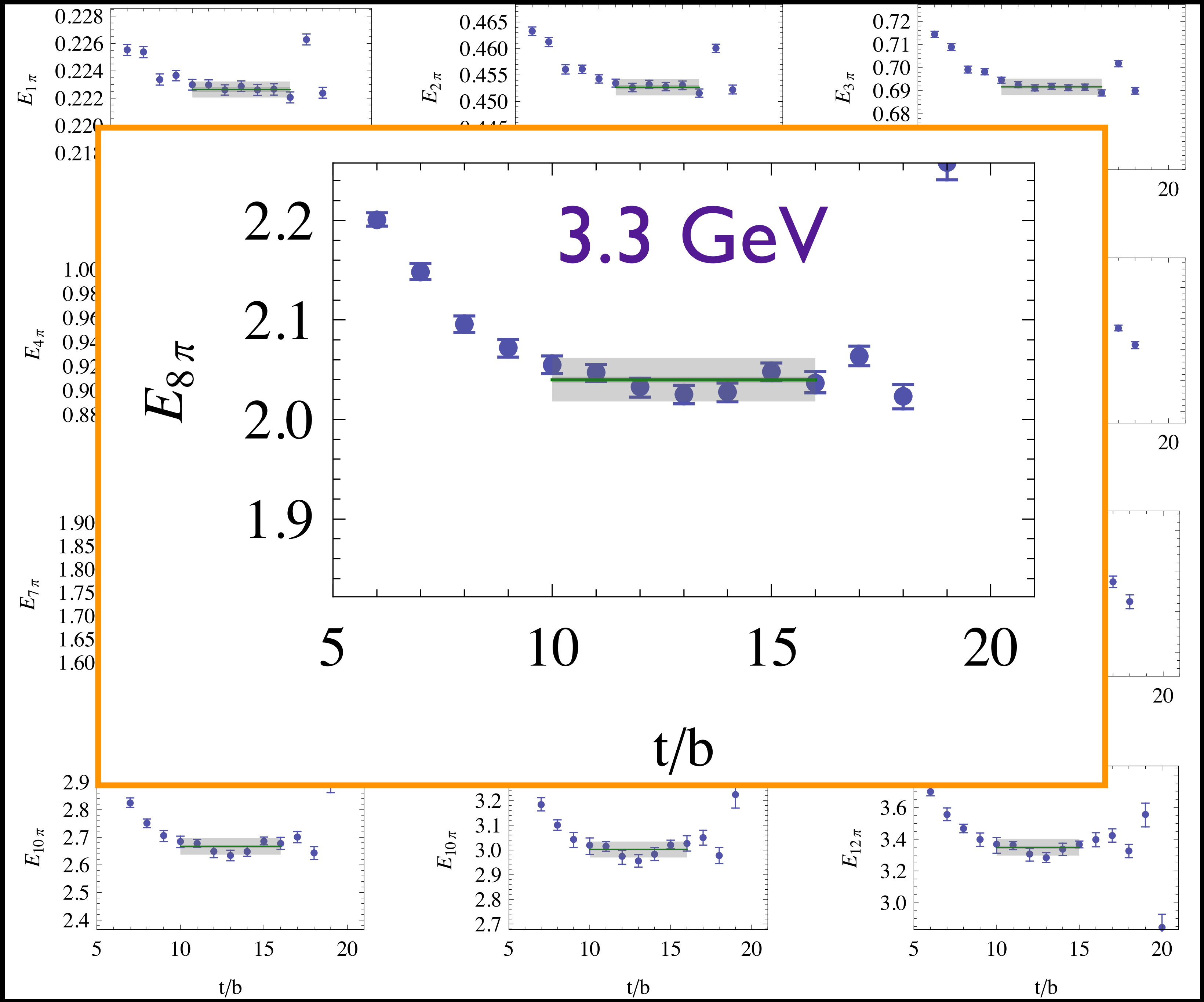
- Maximal isospin: only a single quark propagator

Lattice details

- Calculations use MILC gauge configurations
  - L=2.5 fm, a=0.12 fm, *rooted* staggered
  - also L=3.5 fm and a=0.09 fm
- NPLQCD: domain-wall quark propagators
  - $m_\pi \sim$ 291, 318, 352, 358, 491, 591 MeV
  - 24 propagators / lattice in best case
- I_z=n=1,...,12 pion and (S=n) kaon systems

n-meson energies

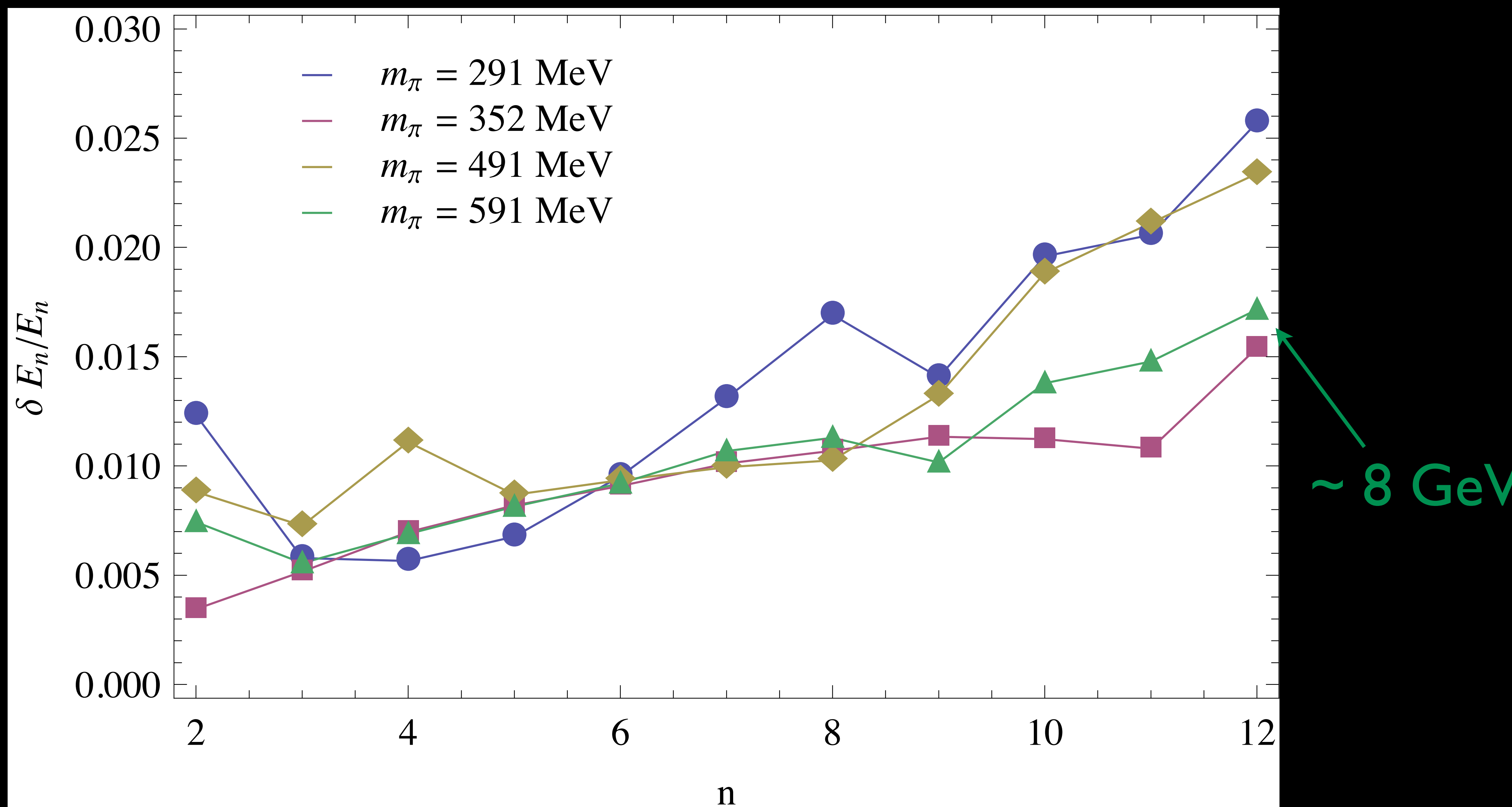
- Effective energy plots: $\log[C_n(t)/C_n(t+1)]$



m_π = 352 MeV

n-meson energies

- Clean signals for n=1,...,12



Bosons in a box

- Multiple extractions of $\overline{a}$ and $\hat{\eta}_3^L$

$$\Delta E_n = \frac{4\pi\overline{a}}{M\,L^3}\,{}^nC_2\Big\{1 - \left(\frac{\overline{a}}{\pi\,L}\right)\mathcal{I} + \left(\frac{\overline{a}}{\pi\,L}\right)^2\left[\mathcal{I}^2 + (2n-5)\mathcal{J}\right]$$

$$-\left(\frac{\overline{a}}{\pi\,L}\right)^3\left[\mathcal{I}^3 + (2n-7)\mathcal{I}\mathcal{J} + \left(5n^2 - 41n + 63\right)\mathcal{K}\right]$$

$$+\left(\frac{\overline{a}}{\pi\,L}\right)^4\Big[\mathcal{I}^4 - 6\mathcal{I}^2\mathcal{J} + (4+n-n^2)\mathcal{J}^2 + 4(27-15n+n^2)\mathcal{I}\,\mathcal{K}$$

$$+(14n^3 - 227n^2 + 919n - 1043)\mathcal{L} + 16(n-2)\left(\mathcal{T}_0 + n\mathcal{T}_1\right)\Big]\Big\}$$

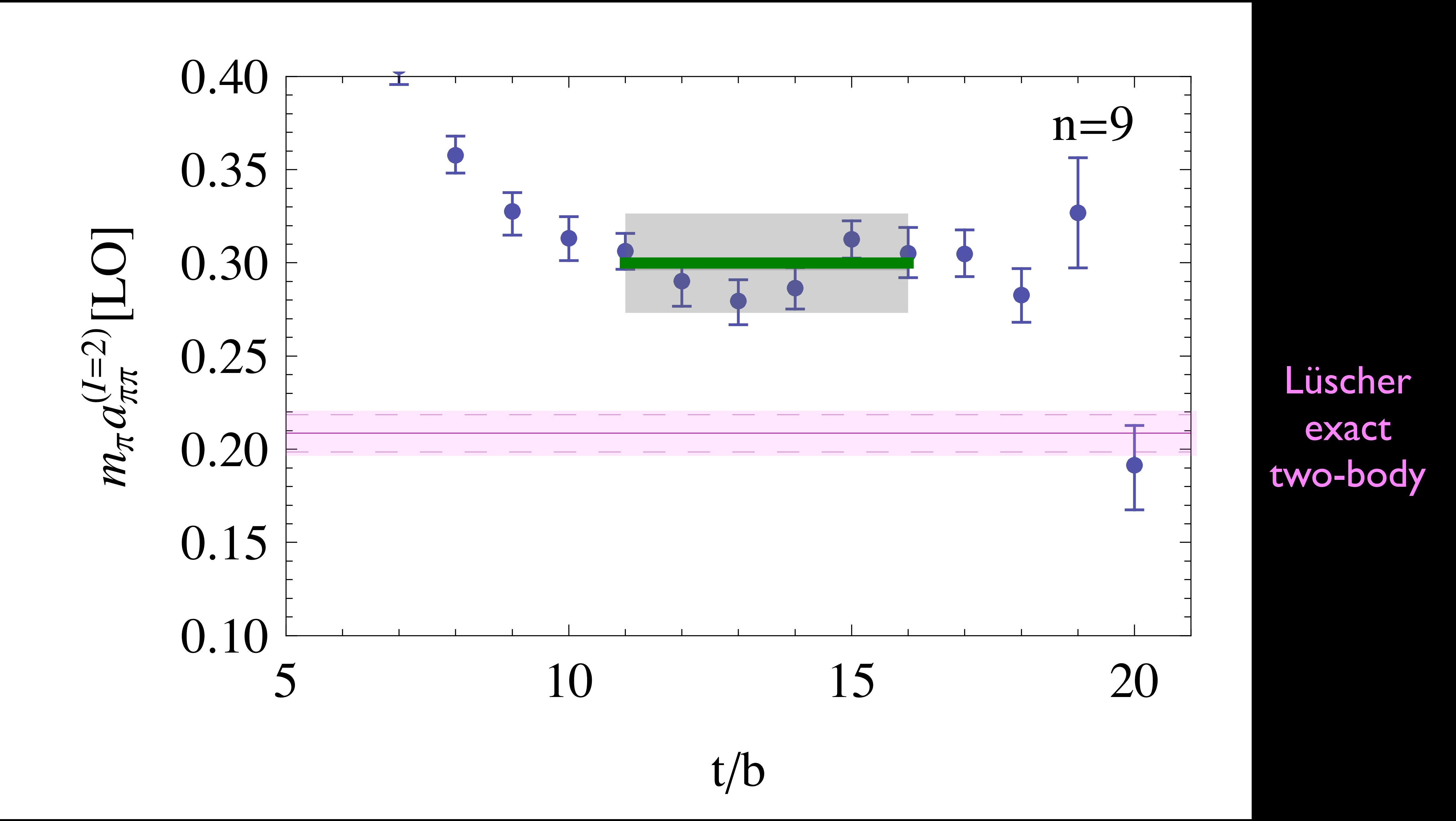
$$+{}^nC_3\frac{1}{L^6}\hat{\eta}_3^L + {}^nC_3\,\frac{6\pi\overline{a}^3}{M^3L^7}\,(n+3)\,\mathcal{I} \;+\; \mathcal{O}\left(L^{-8}\right)$$

$$\overline{a} = a + \frac{2\pi}{L^3}a^3\,r$$

$$\hat{\eta}_3^L = \overline{\eta}_3^L\left[1 - \frac{6\overline{a}}{\pi\,L}\mathcal{I}\right] + \frac{72\pi\overline{a}^4 r}{M\,L}\mathcal{I}$$

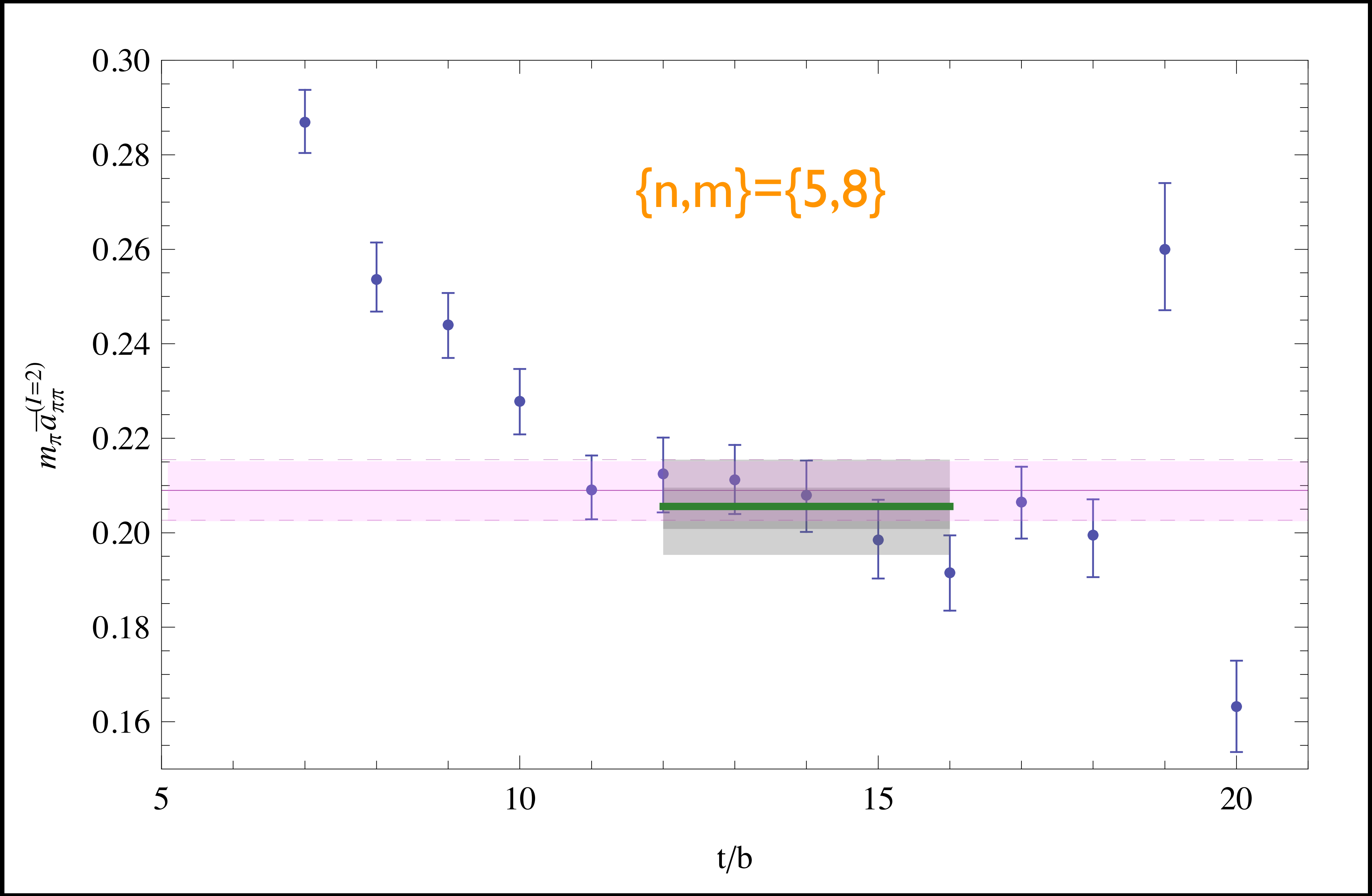
Pion scattering

- Extractions of $m_\pi a$ from four orders in L



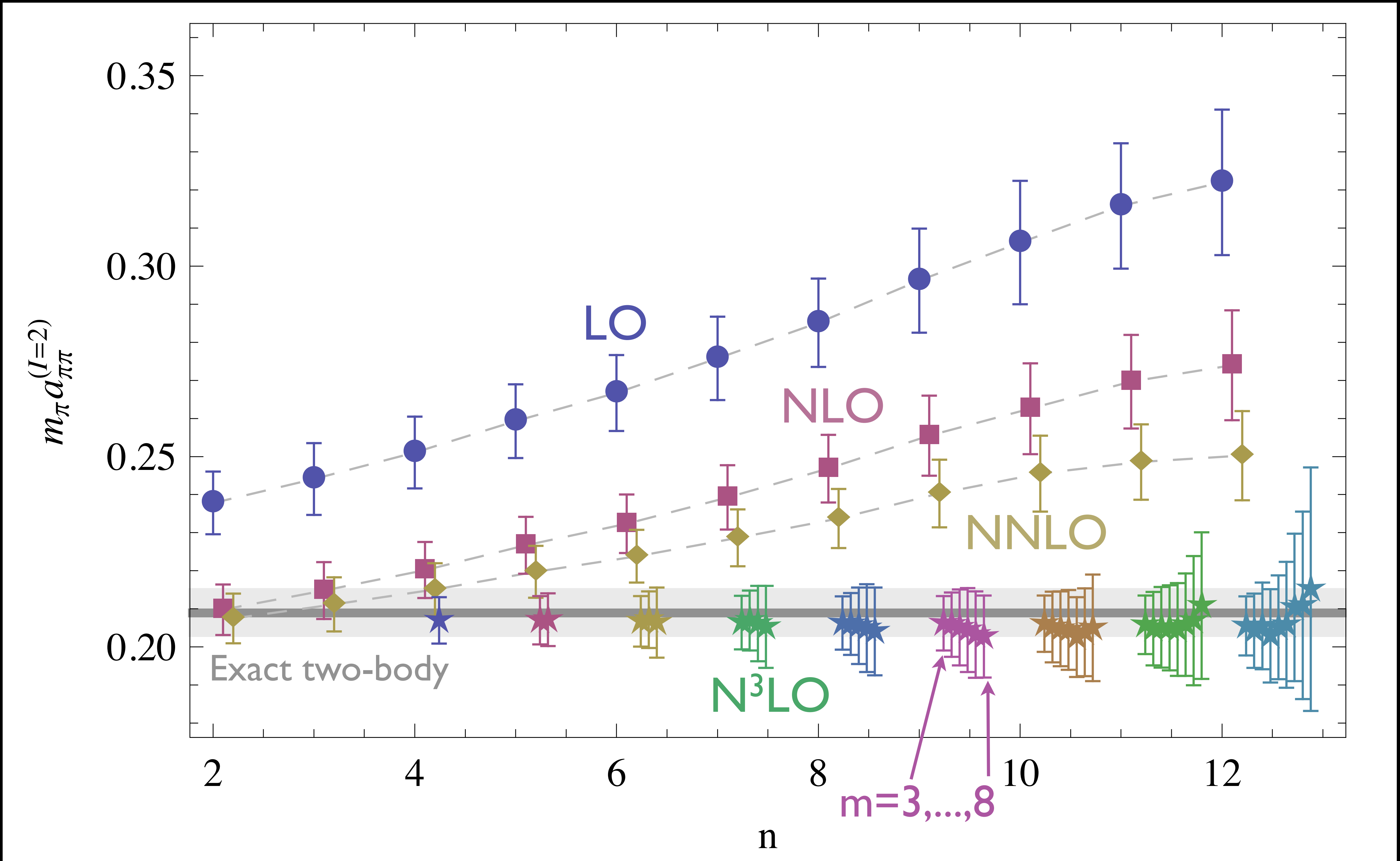
N^3LO

- Two energies to cancel 3-body: 45 combinations



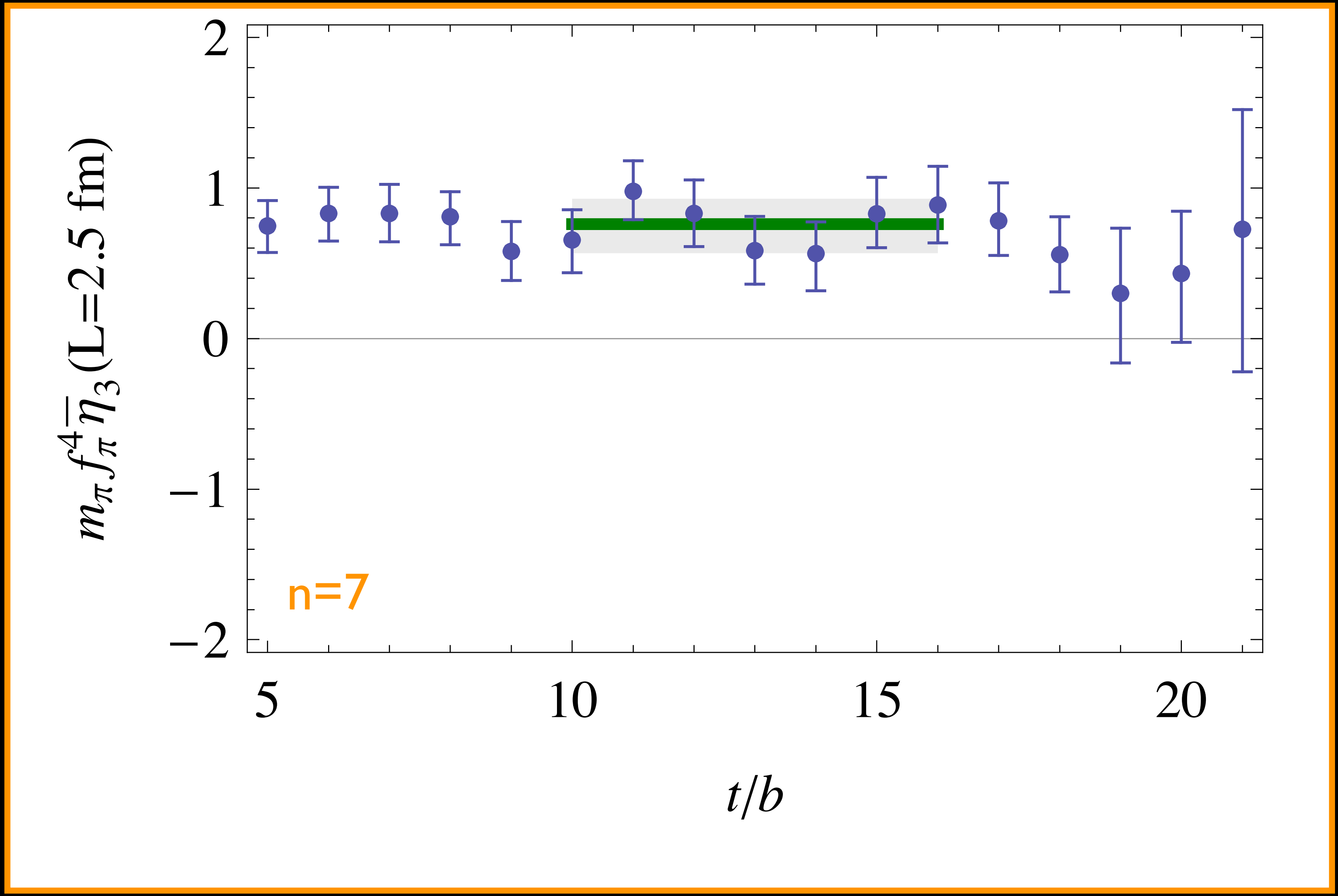
Lüscher exact two-body

Pion scattering



Expansion shows no sign of breakdown?

$\pi^+\pi^+\pi^+$ interaction

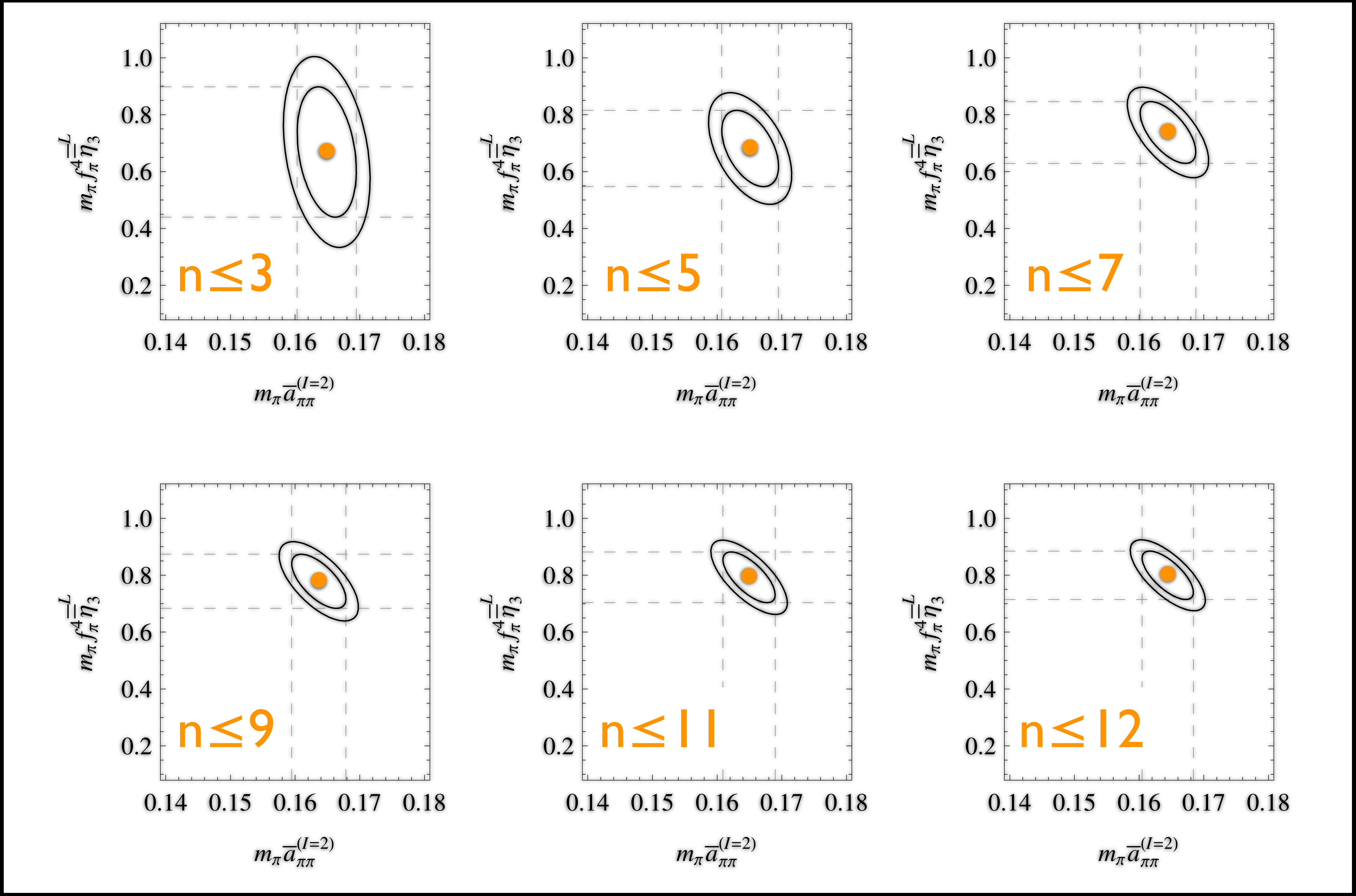


m_π = 352 MeV

n correlations

- Fit effective energies to extract two parameters: a, η_3 from 1/L expansion
- Use 12 eff. energies in *n*-*t*-correlated analysis
  - Large correlation matrix: correlated χ^2
  - Reduces uncertainties as *n* pion correlators "explore more of the lattice"

n correlations

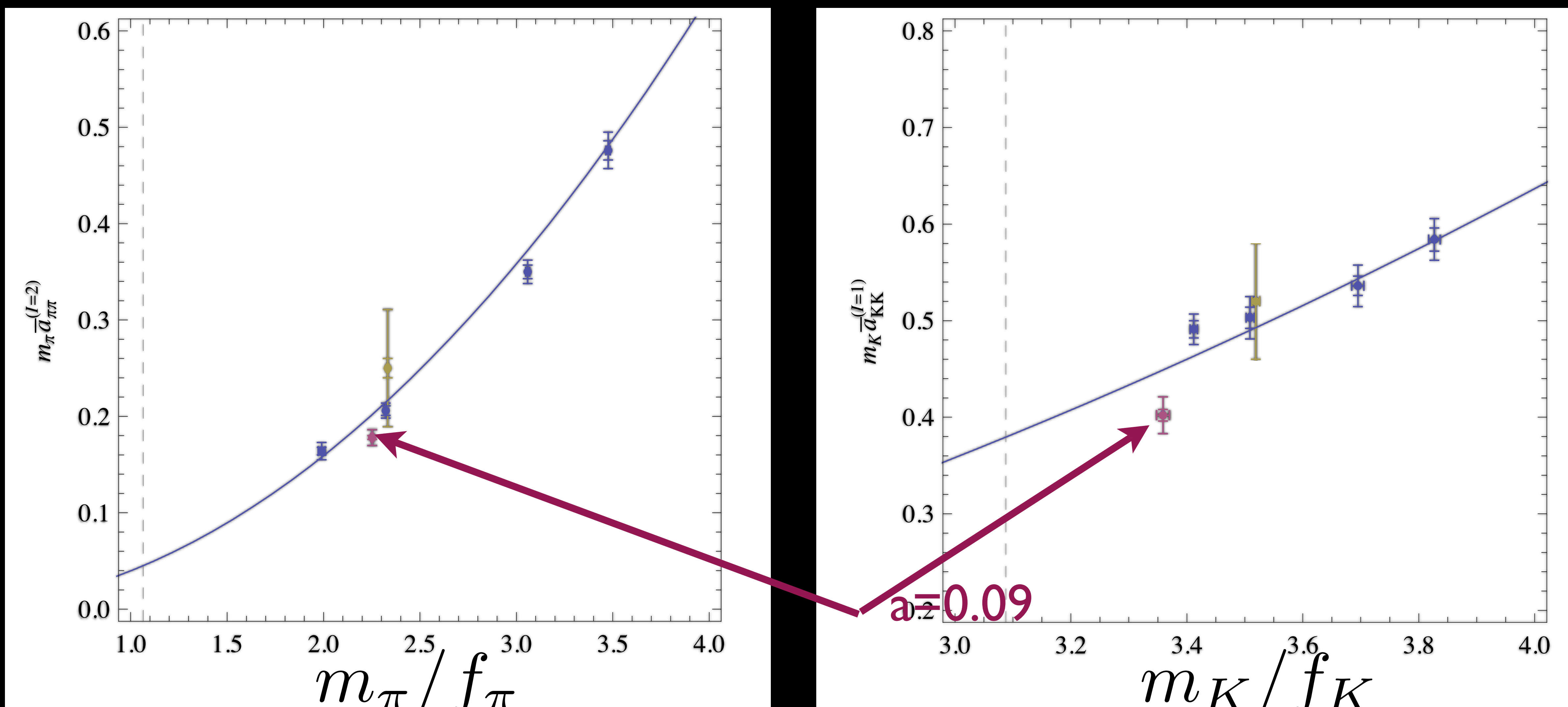


Three-body

Two body

$2\pi^+$ and $2K^-$ interaction

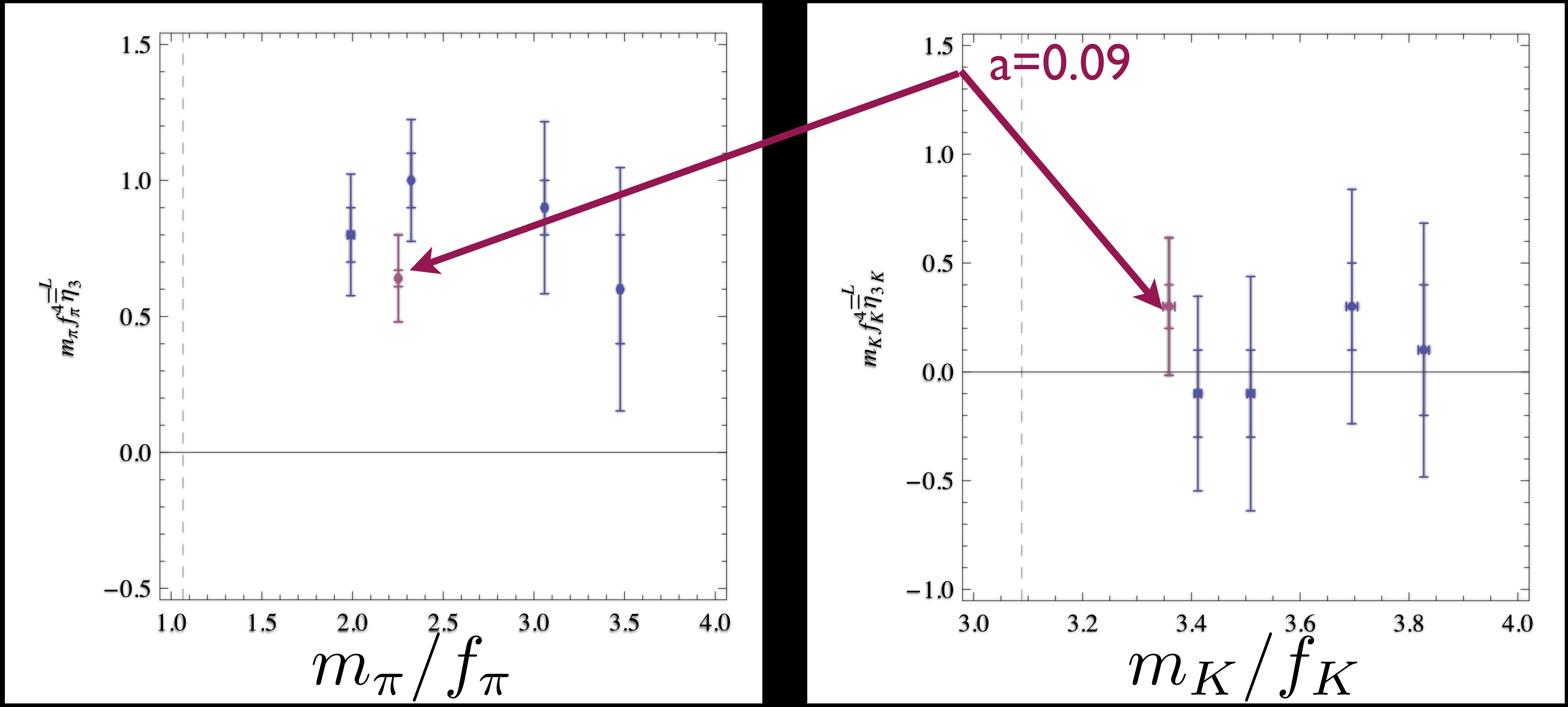
- Scattering lengths



curves: Weinberg

$3\pi^+$ and $3K^-$ interaction

- First QCD three body interaction



Naïve dimension analysis: 1

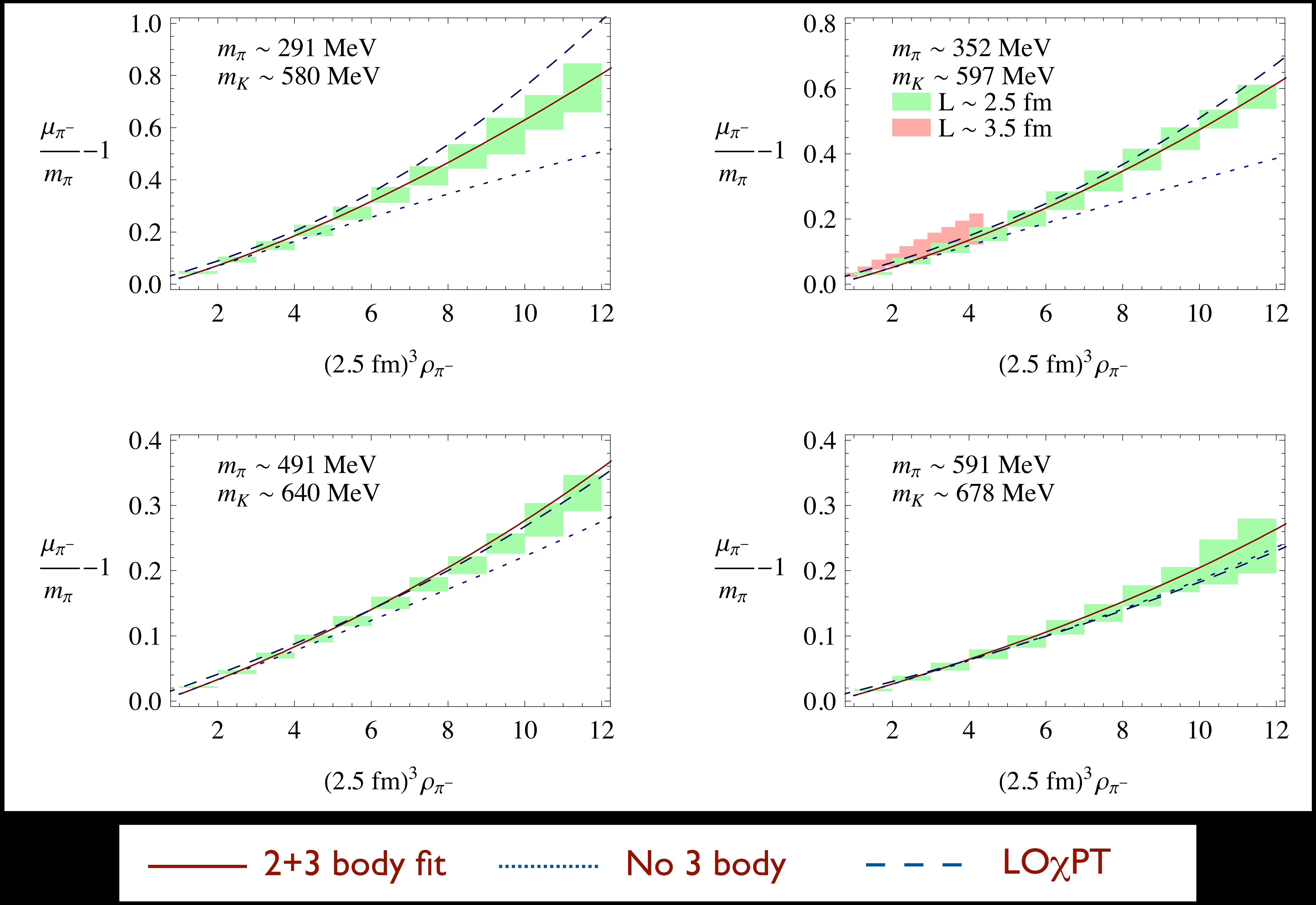
Equation of State

- 1/L expansion: analytic form of EOS
- Chemical potential

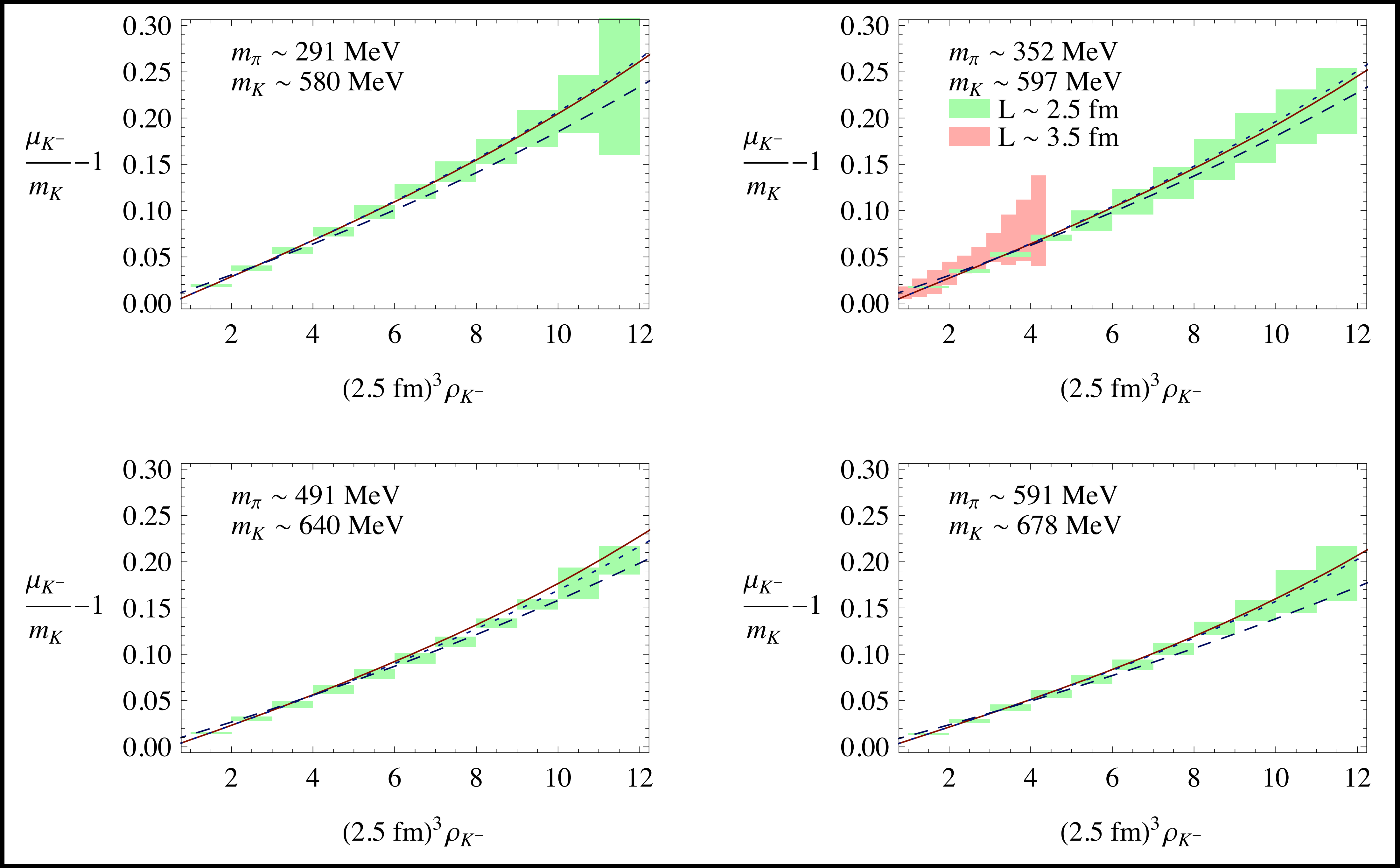
$$\mu = \left.\frac{d\ E}{d\ n}\right|_{V\,const}$$

- $\mu(\rho)$ numerically using finite difference
- Compare with LOχPT [Son & Stephanov]

Isospin Chemical Potential

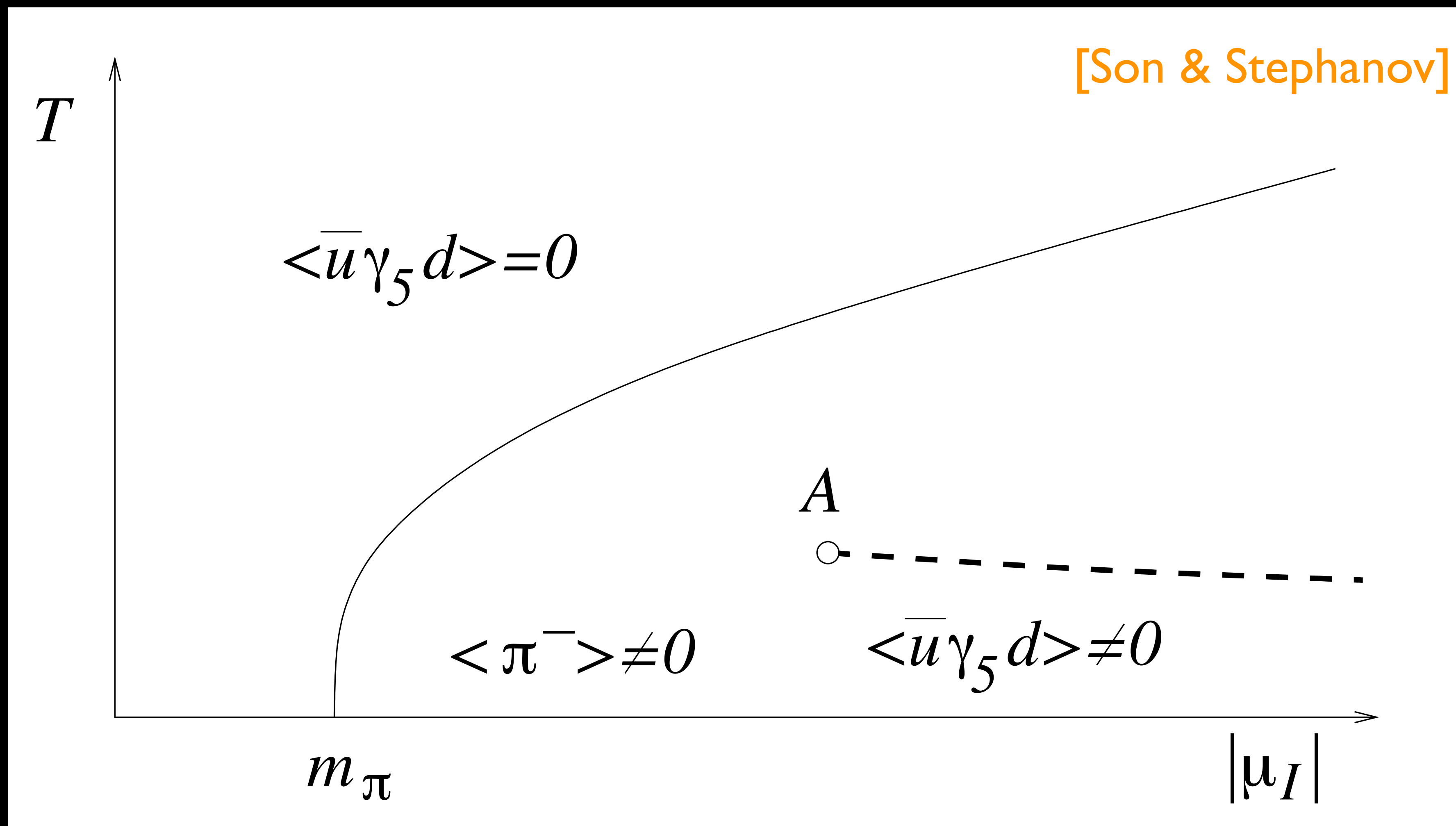


Kaon Chemical Potential



- supports KN analysis of K^- condensation in n-stars

μ_I-T phase diagram

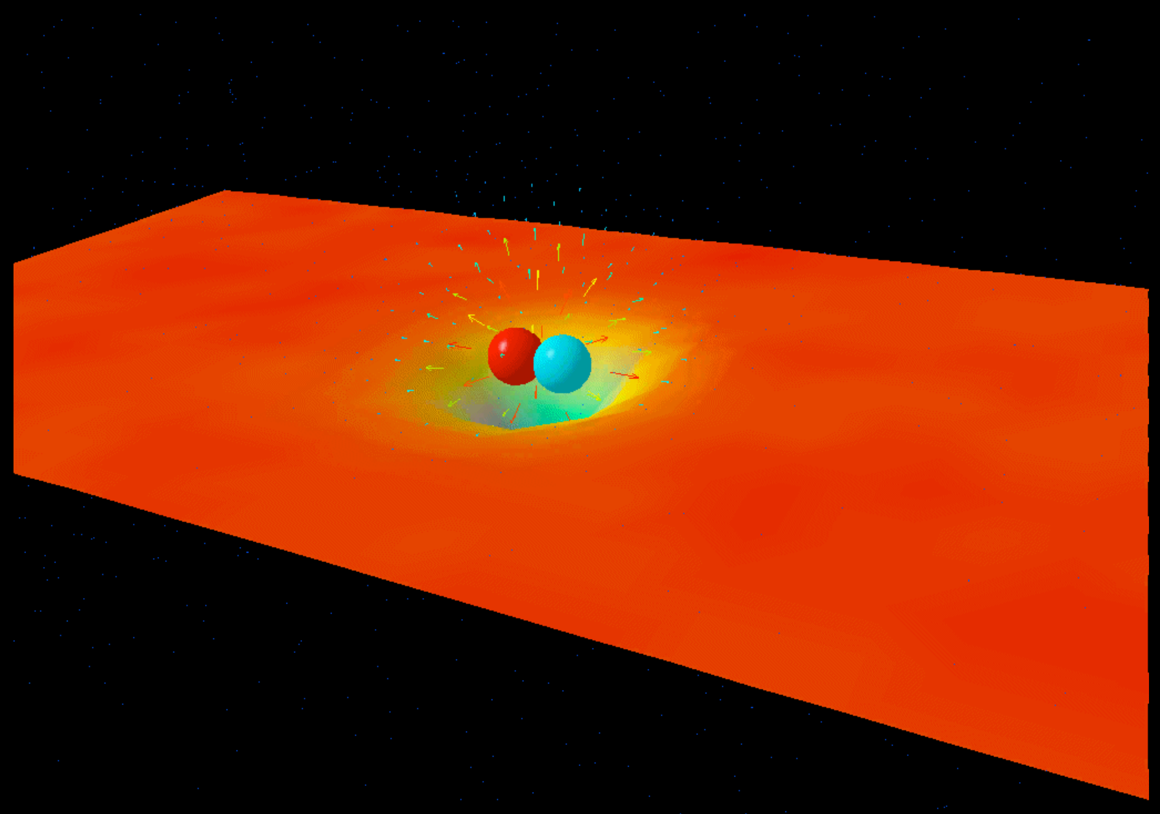


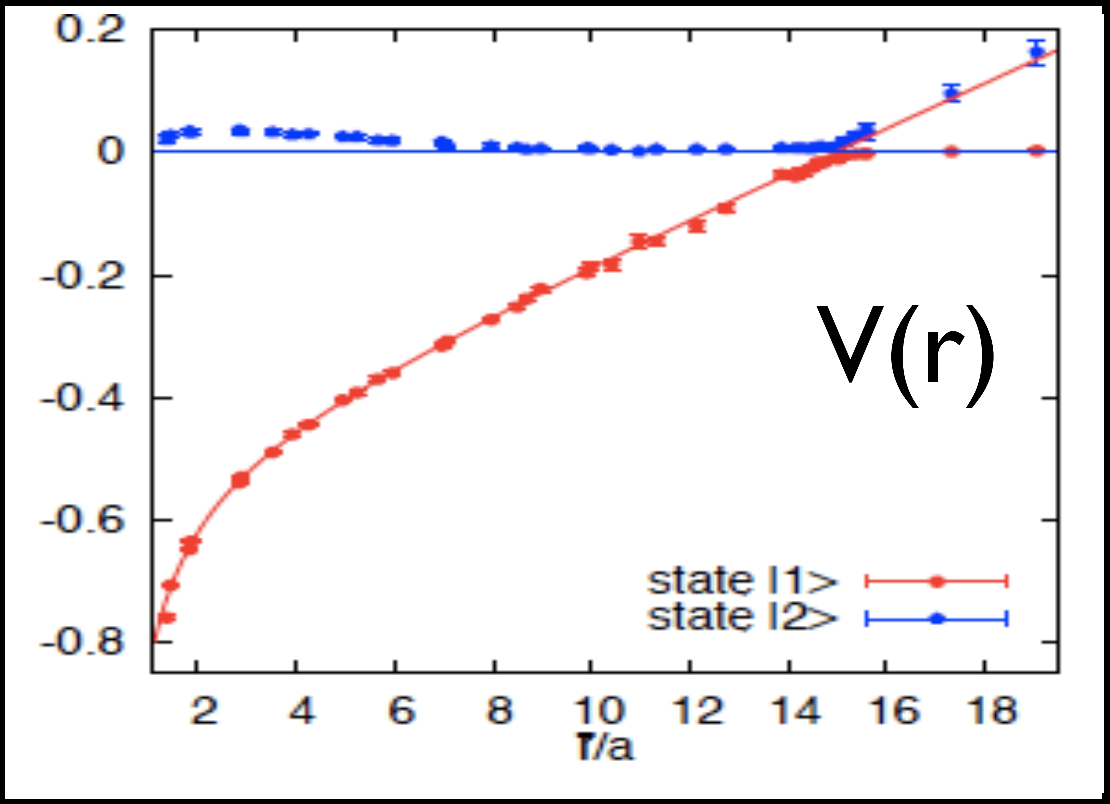
- BEC/BCS transition ?
- Vector meson condensate ?

Colour screening by pions

[WD+ M Savage PRL 09]

- Static quark potential





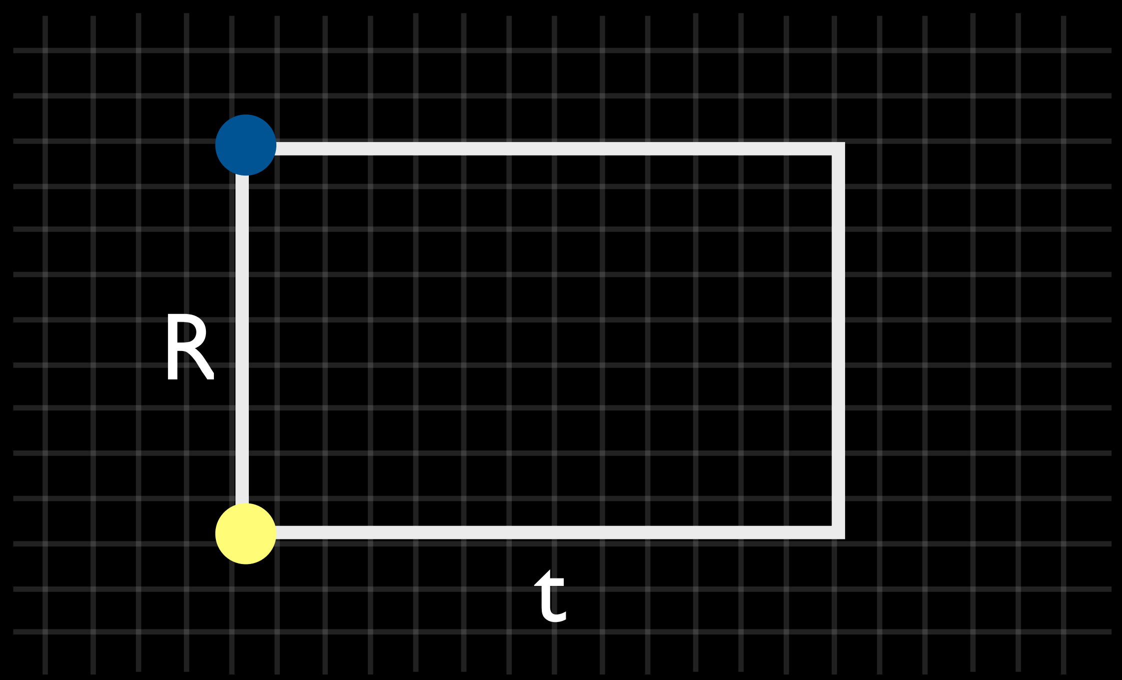
- Screening: evidence for quark-gluon plasma

[Leinweber][Bali *et al.*][Petreczky/Petrov]

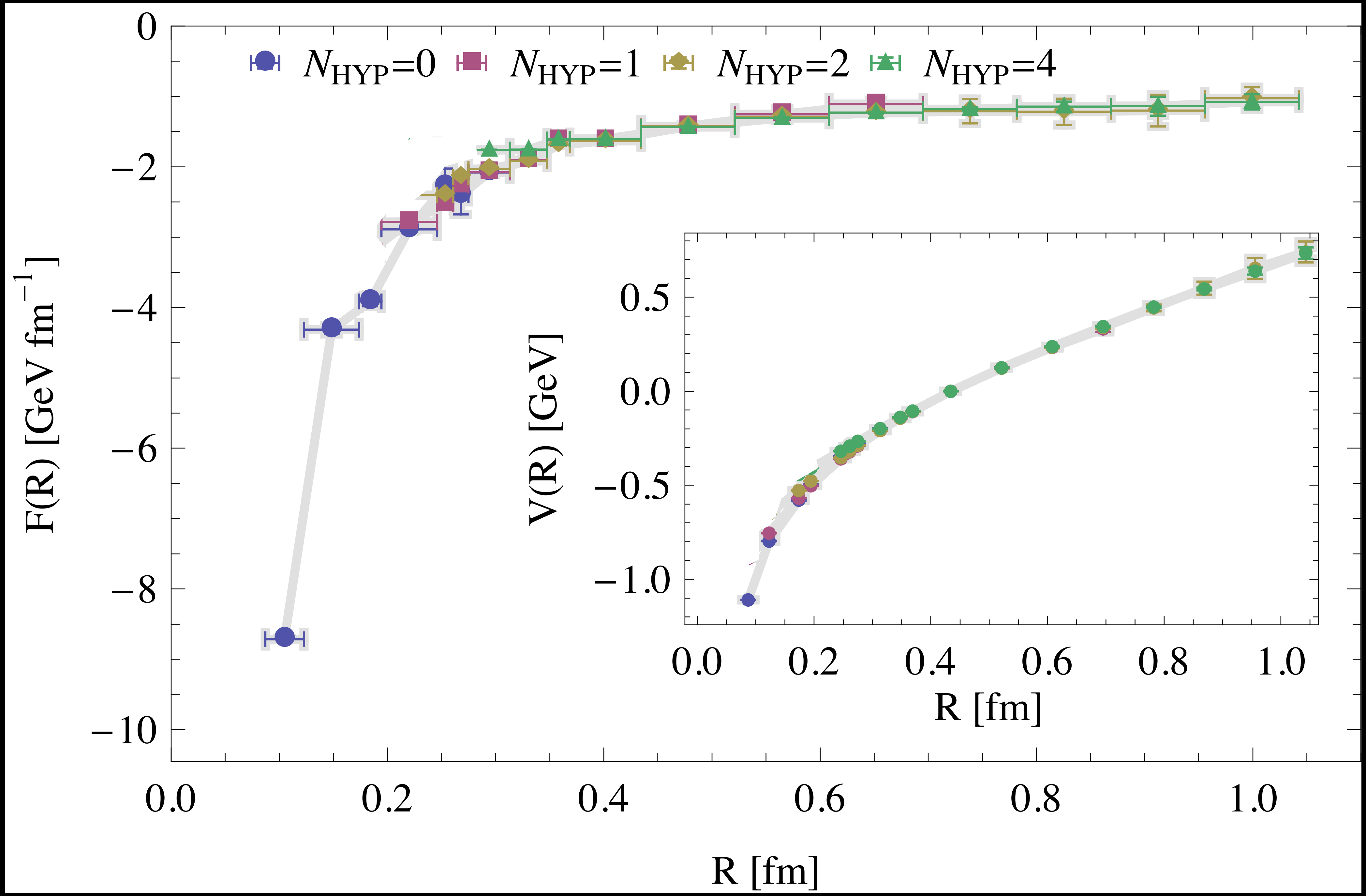
Correlation fucntions

- Static quark potential

$$C_W(R,t_w,t) = \left\langle 0 \right| \sum_{\mathbf{y},|\mathbf{r}|=R} \mathcal{W}\left(\mathbf{y}+\mathbf{r},t;\mathbf{y},t_w\right) \left| 0 \right\rangle$$
$$\longrightarrow Z\,\exp\left[-V(r)(t-t_0)\right]$$

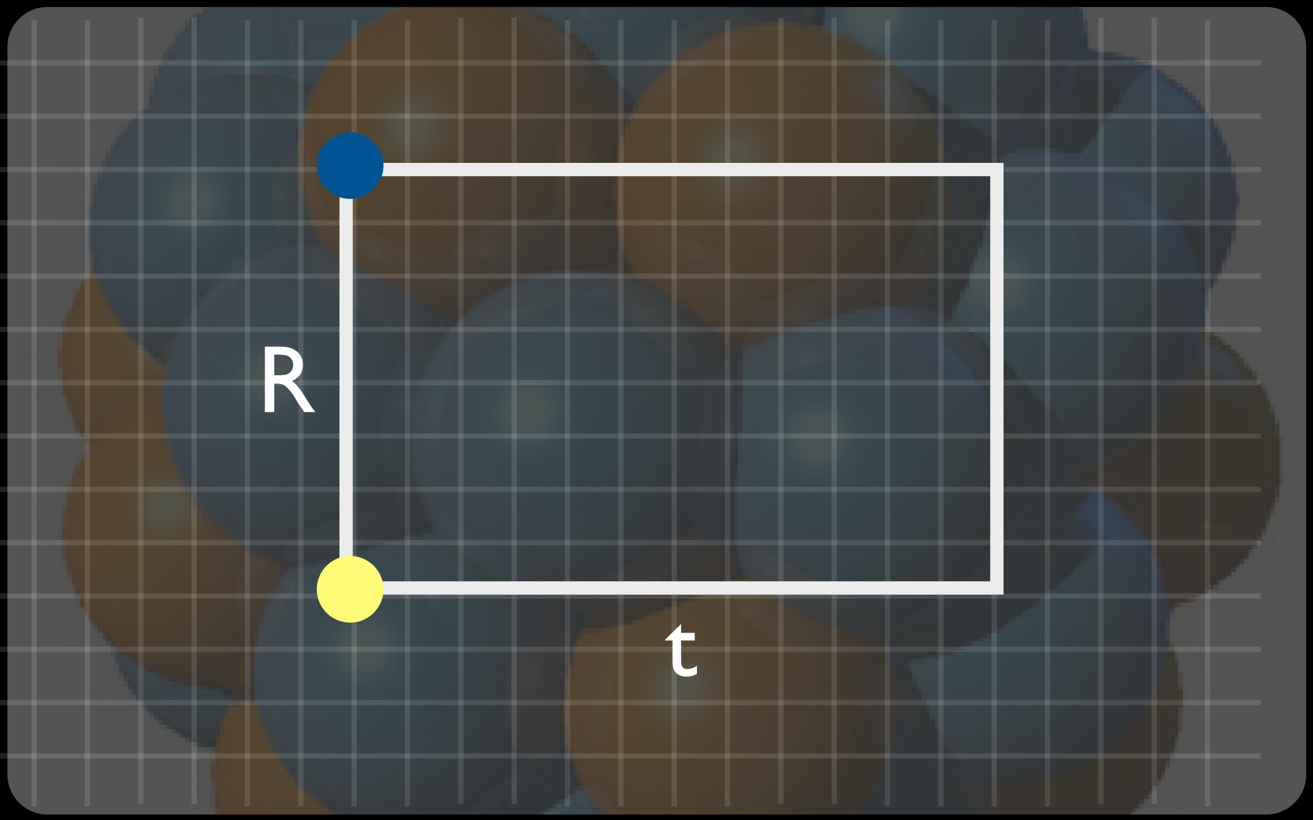


Vacuum $Q\bar{Q}$ force



Color screening

- Static quark potential



- Modified by condensate? Hadronic screening?

[WD+ M Savage PRL 09]

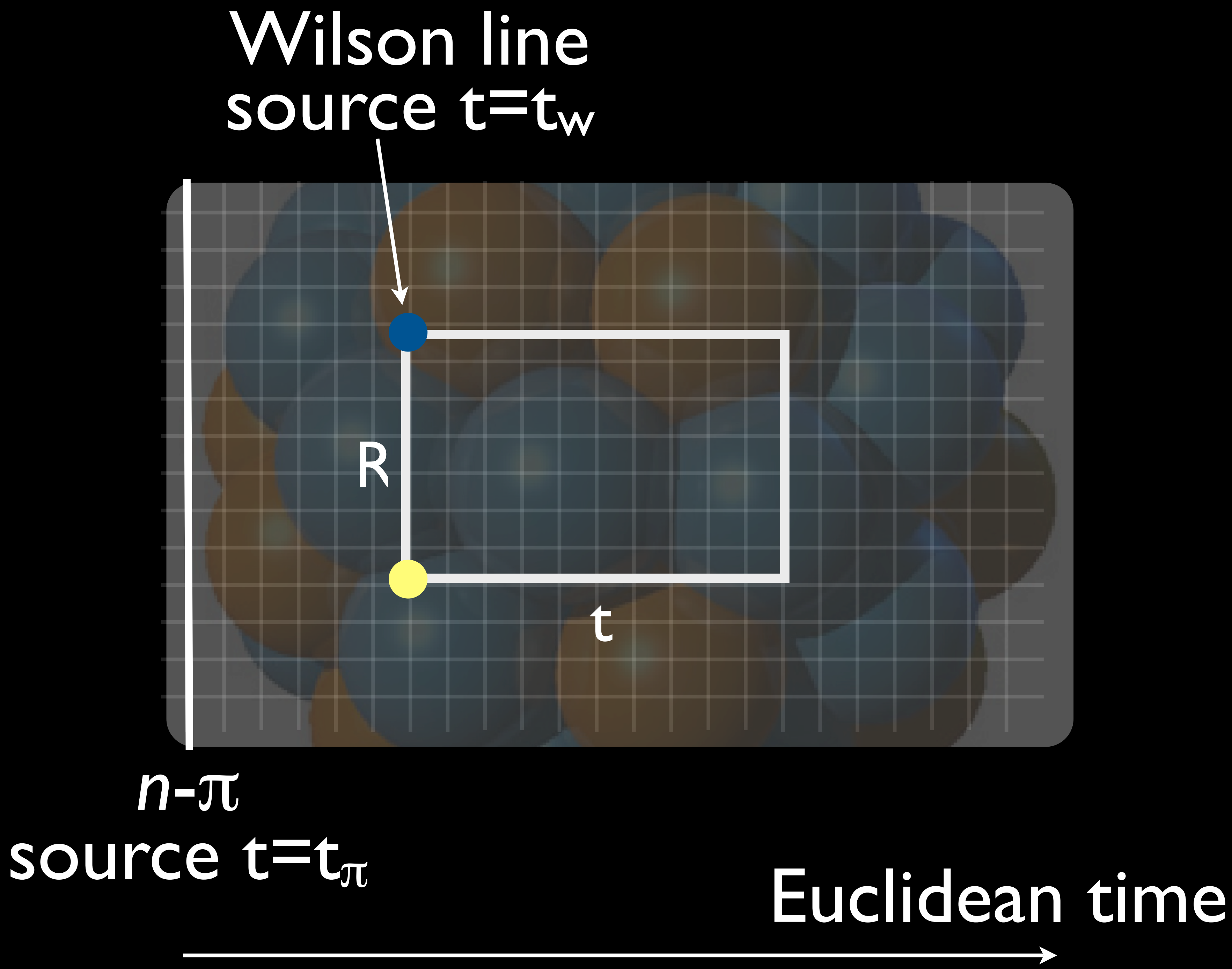
Correlation fucntions

$$C_W(R,t_w,t) = \left\langle 0 \right| \sum_{\mathbf{y},|\mathbf{r}|=R} \mathcal{W}\left(\mathbf{y}+\mathbf{r},t;\mathbf{y},t_w\right) \left| 0 \right\rangle$$
$$\longrightarrow Z \exp\left[-V(r)(t-t_0)\right]$$

$$C_n(t_\pi,t) = \left\langle 0 \right| \Big[\sum_{\mathbf{x}} \chi_{\pi^+}(\mathbf{x},t)\chi^\dagger_{\pi^+}(0,t_\pi) \Big]^n \left| 0 \right\rangle$$
$$\longrightarrow Z' \exp[-E_{n\pi}(t-t_\pi)]$$

$$C_{n,W}(R,t_\pi,t_w,t) = \left\langle 0 \right| \Big[\sum_{\mathbf{x}} \chi_{\pi^+}(\mathbf{x},t)\chi^\dagger_{\pi^+}(0,t_\pi) \Big]^n \times \sum_{\mathbf{y},|\mathbf{r}|=R} \mathcal{W}\left(\mathbf{y}+\mathbf{r},t;\mathbf{y},t_w\right) \left| 0 \right\rangle$$

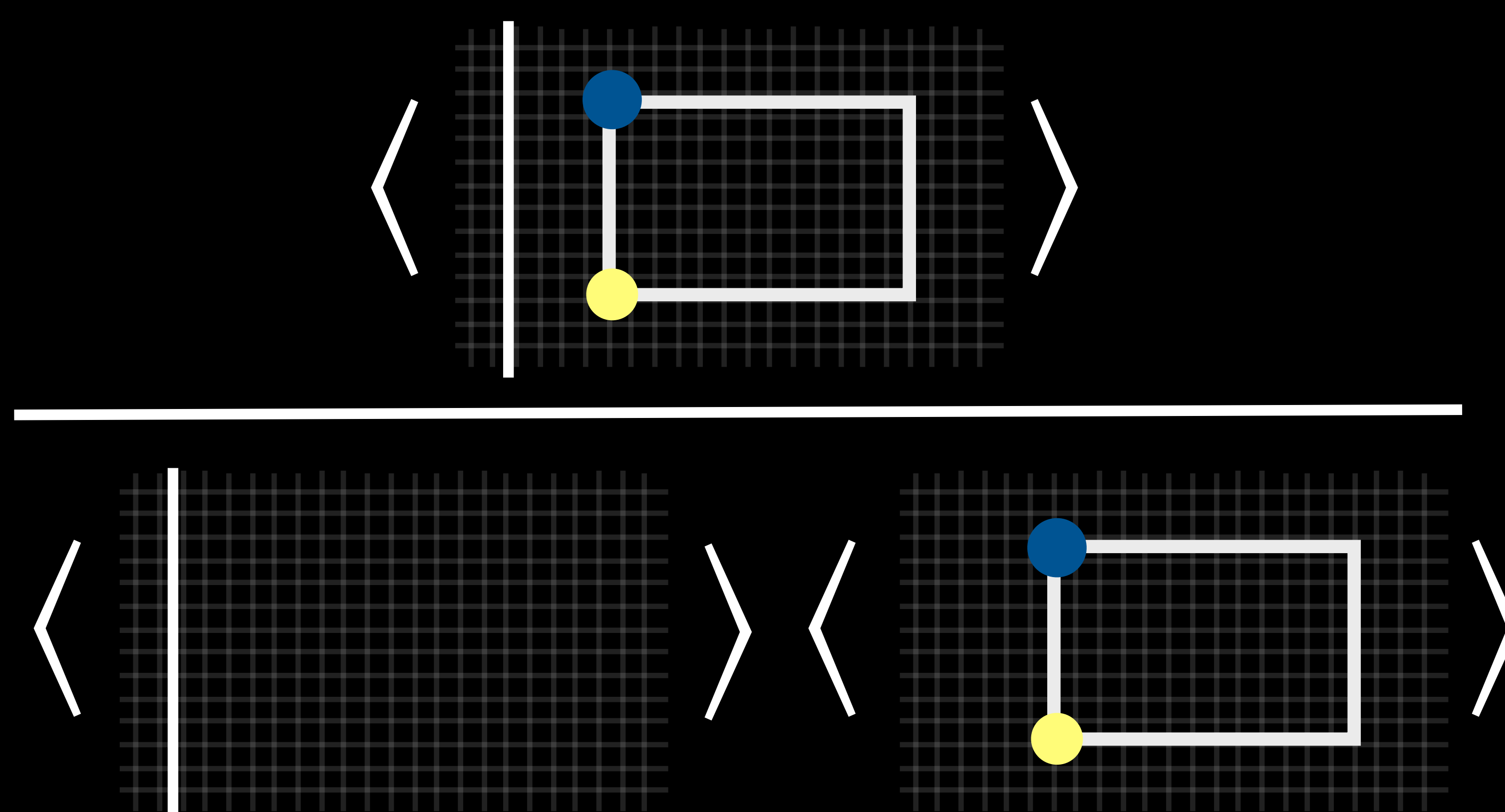
In pictures



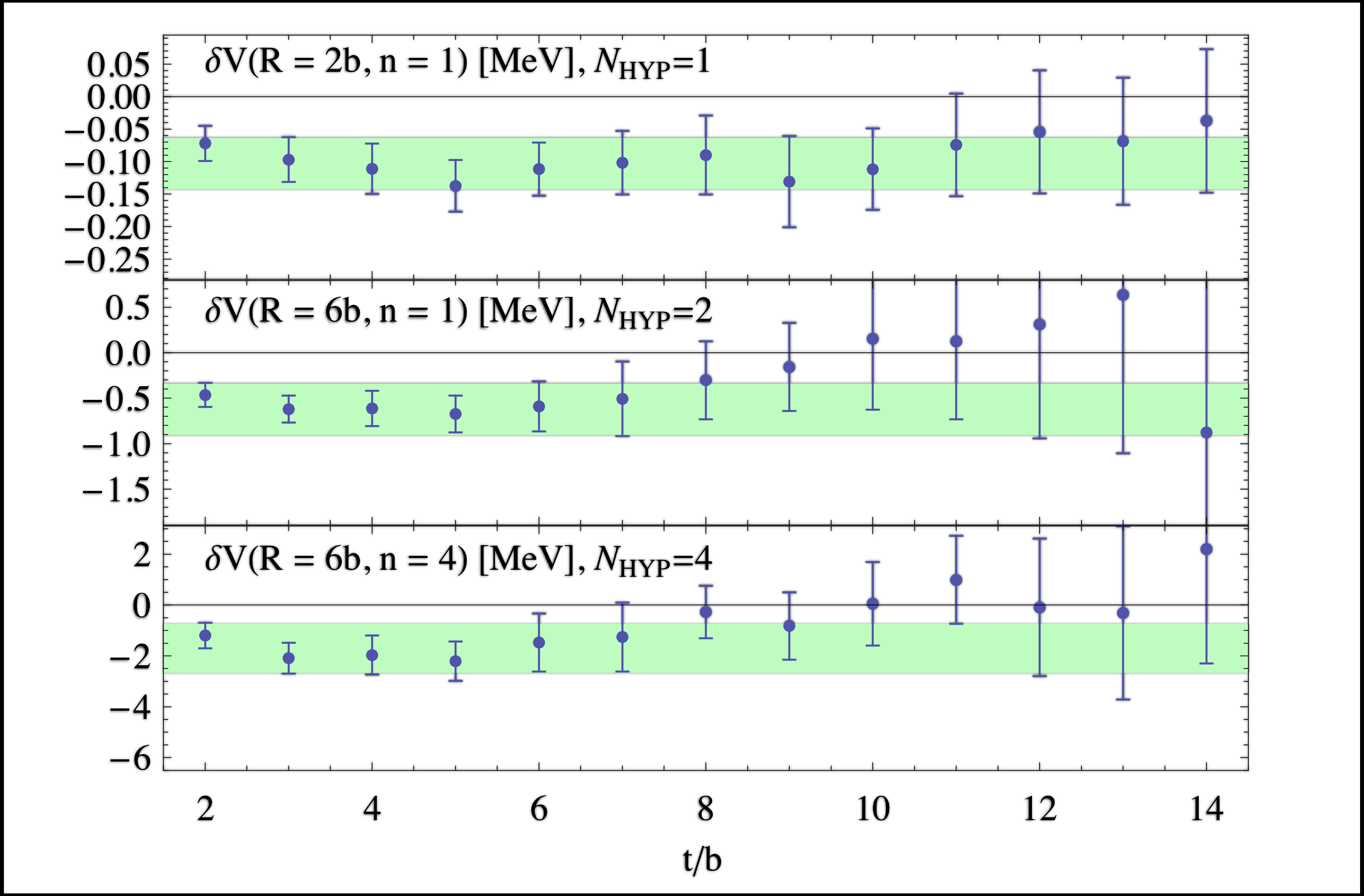
In medium effects

$$G_{n,W}(R, t_\pi, t_w, t) = \frac{C_{n,W}(R, t_\pi, t_w, t)}{C_n(t_\pi, t)\; C_W(R, t_w, t)}$$

$$\longrightarrow \#\ \exp\left[-\delta V(r, n)(t - t_0)\right]$$

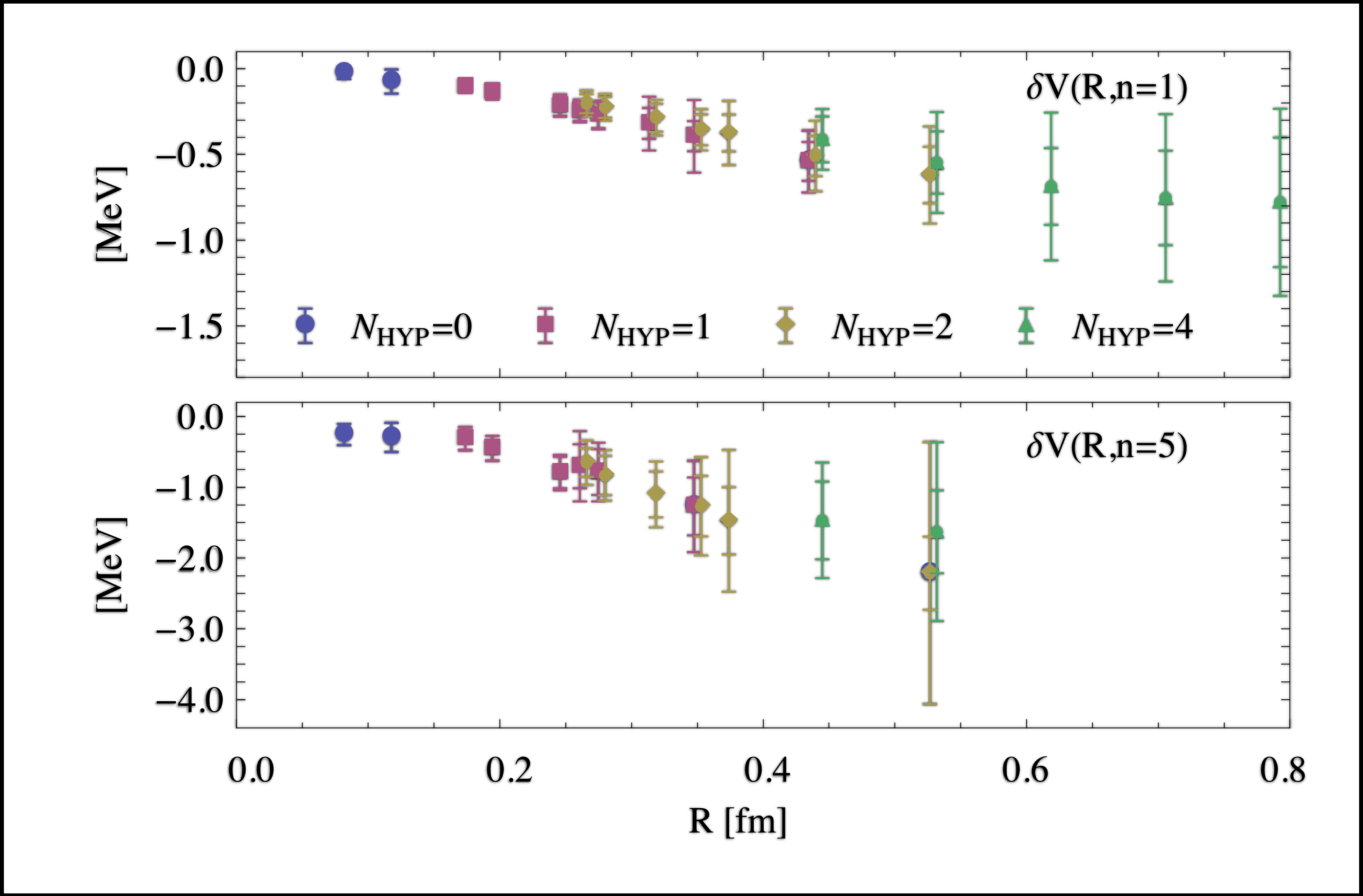


Effective δV plots



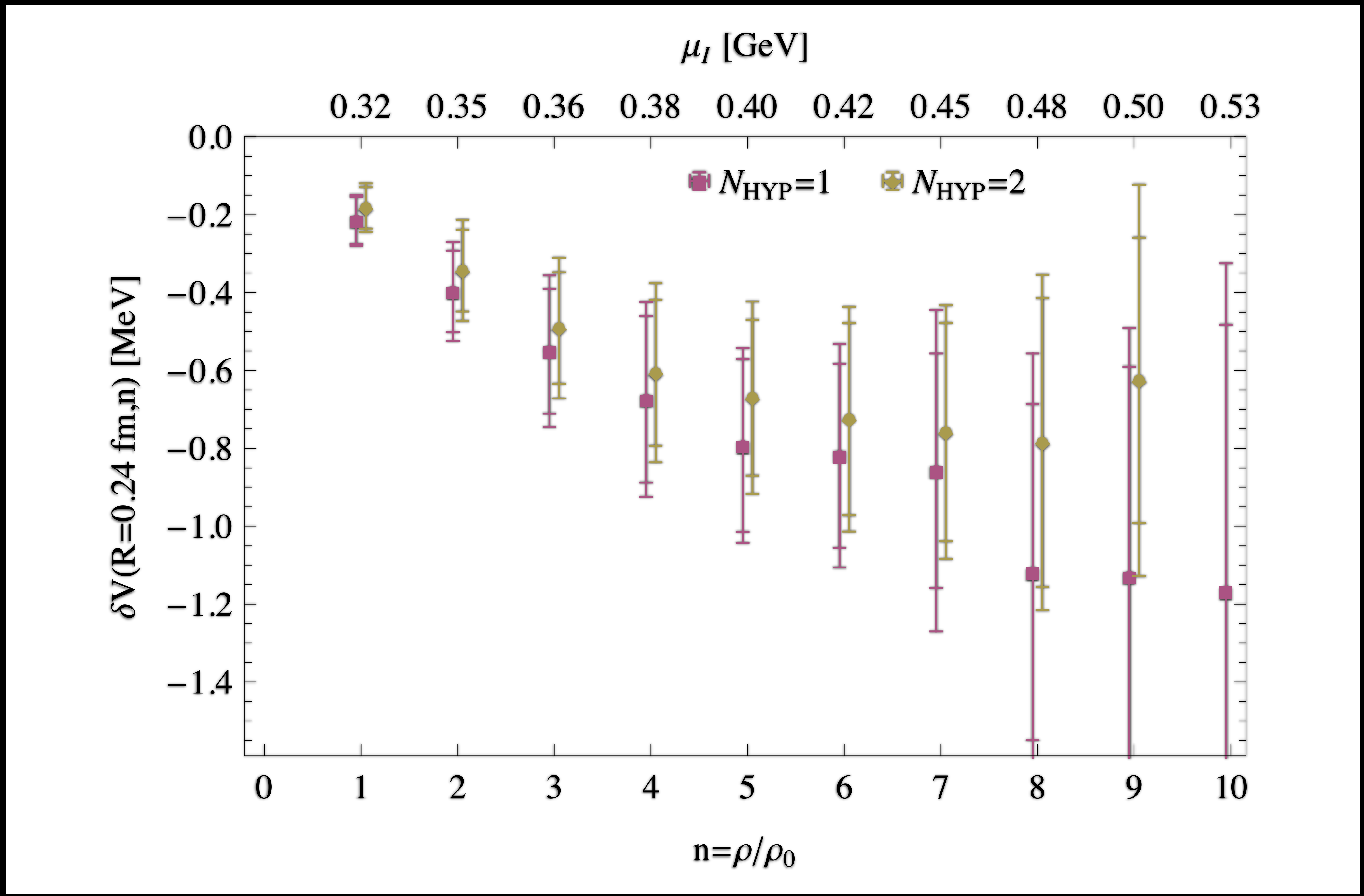
- DWF on MILC: a=0.09 fm, 28^3x96, m_π=318 MeV

δV(R, n=1 & 5)



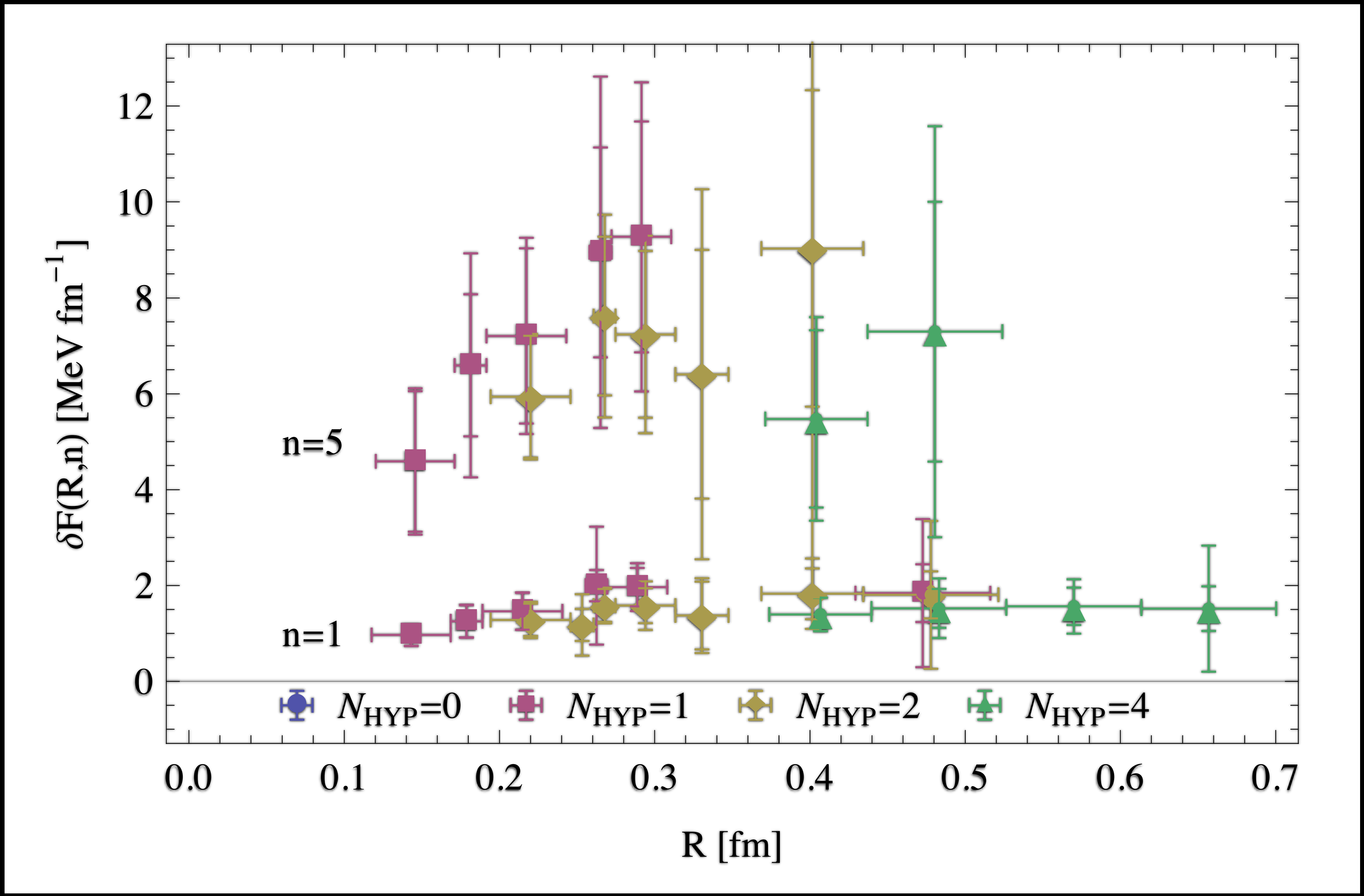
- 100 keV effect at small n, small R

δV(R=0.24fm, n)



- Approximately linear in isospin density

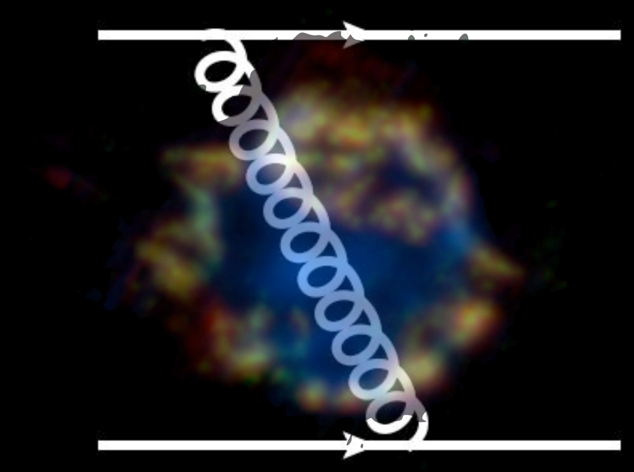
δF(R, n=1 & 5)



- Constant at large R, $\sim R^2$ at small R ??

Pion screening

- r independent shift in $Q\bar{Q}$ force
  - Dielectric medium inside flux tube
  - Small effect: $\delta F(n=1)/F = 0.002$ at large R
- Relevance to J/ψ suppression @ SPS/RHIC?

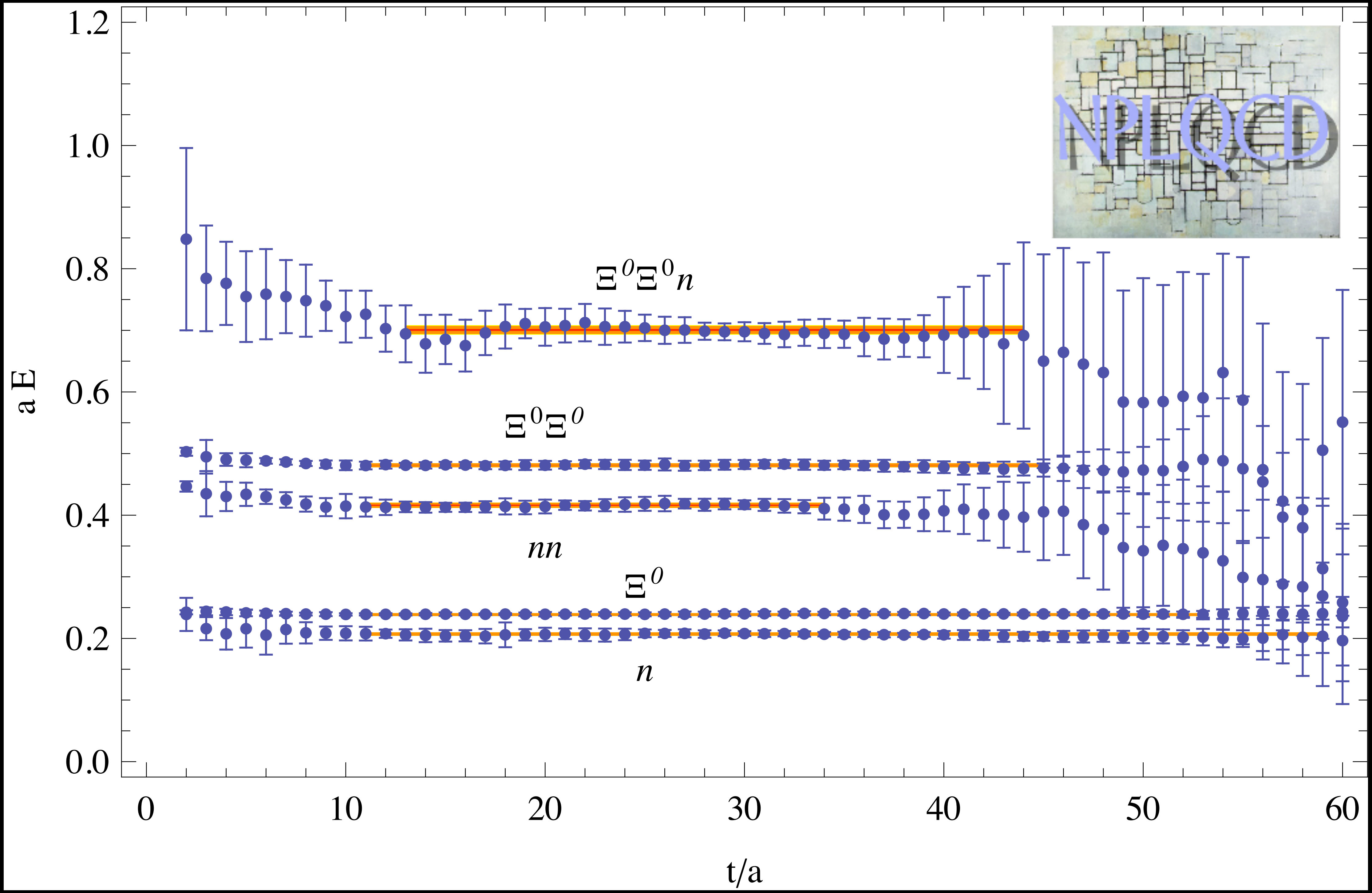


Baryons Strike Back

- Just how hard are nuclei?
- Simple three-baryon system: $\Xi^0 \Xi^0 n$
- Unphysical mass, single L and a
- Extreme statistics = 300K measurements

$\Xi^0\Xi^0 n$: effective energy

PRELIMINARY



Summary

- Numerical meson condensates
  - Two and three meson interactions
  - Equation of state of meson gases
  - Effects: color screening
- Progress can be made in multi-baryon systems

FIN