A

Appendix A: Signal to Noise Calculations

This appendix details some of the most important factors that affect the capabilities of TPF. The calculations described here provide the basis for the simulations given in Chapter 6. The first set are astrophysical concerns that can be ameliorated, if at all, only by target selection. The remainder are properties of the TPF instrument.

TARGET PROPERTIES

The Target Planet. The target planet is typically taken to be an Earth-diameter blackbody at an orbital radius a=1 AU around a solar type star at distance d=10 pc. Various atmospheric constituents can modify the appearance of the planet and its effective temperature at a particular wavelength. These properties of the planet are summarized in Table A.1.

The Effects of Zodiacal Emission. The emission from zodiacal dust in our own solar system and in the target system is an important source of noise. In the absence of any other information, we adopt the fan-shaped form that characterizes our solar system dust for the structure of the exo-zodiacal cloud, varying only the scaling factor for the optical depth (ρ_0) to account for differing amounts of zodiacal material (Reach *et al*. 1995). Equation 1 is used to evaluate the emission along any line of sight. The temperature structure varies according to the luminosity of, and distance from, the central star. Equation 2 gives the equilibrium temperature of a dust grain located r AU from its central star, where r and z are cylindrical coordinates within the cloud:

$$I_\nu = \int B_\nu(T(r))\rho_0 r^{-\alpha} e^{-\beta(z/r)^\gamma}\, dl \qquad (1)$$

$$T(r) = T_0 r^{-\delta} \qquad (2)$$

Local Zodiacal Cloud. The local zodiacal (LZ) cloud provides the foreground through which TPF must observe. The cloud is characterized by the zodiacal parameters described in Table A.2 where the parameter values are derived from fits to the COBE or IRAS data (Reach *et al*. 1995). The LZ diminishes with distance from the Sun due to decreased dust density and decreased temperature. At short wave-

Table A.1. Properties of Target Planet	
Distance	10 pc
R_{planet}	1 $R_{\oplus}$
L^*	1 L_{sun}
Orbital Distance, a	1 AU
$T_{planet(10\ microns)}$	265 $a^{-0.5}L_*^{0.25}$ K
F_ν(planet)	0.34 μJy @ 12 μm

lengths the drop is very dramatic (a factor of >300 at λ<7 μm) while at longer wavelengths the drop is less pronounced (~150 at 12 μm).

Infrared Cirrus. An additional background against which TPF must observe is the galactic cirrus, which sets a minimum sky brightness even if there were no LZ emission (Bernard *et al.* 1992). A minimum value of the cirrus emission corresponding to I_ν(100 μm) = 1 MJy/sr is included in all calculations, e.g., I_ν(12 μm)~0.08 MJy/sr from cirrus. The confusion noise from structure in the cirrus will not be a problem at the TPF resolution (Gautier *et al.* 1992).

Exo-Zodiacal Emission. The exo-zodiacal cloud contributes photon noise to the total signal starting at the dust sublimation radius and extending out to the edge of the primary beam of a single telescope. A two-dimensional image of the zodiacal cloud is determined by integrating the 3-dimensional dust distribution (Equation 1) for a particular inclination to the line of sight. Since the signal reaching the detector passes through the null pattern of the interferometer, the hot, inner portions of the zodiacal disk are hidden from view and the Poisson-noise producing zodiacal signal is decreased by about a factor of ~3 from the nominal exo-zodiacal flux.

Structured EZ Component. Structured emission in the zodiacal light of the target star is potentially a noise source. A planet must be detected

Table A.2. Properties of Zodiacal Cloud	
ρ_0	1.14×10^{-9} AU^{-1}
α	1.39
β	3.26
γ	1.02
Temperature at 1 AU (T_0)	$286(L/L_o)^{0.25}$
δ	0.42
Local Zodiacal Surface Brightness at 30º, Inclination at 12 μm	19 MJy/sr (@1 AU) 0.23 MJy/sr (@5 AU, including cirrus)
Dust Destruction Temperature	1500 K

Table A3. Interferometer Observing System	
Baseline	75 m
Telescope Apertures	1.8:3.5:3.5:1.8 m (1 AU) 1.1:2.2:2.2:1.1 (5 AU)
Telescope Temperature	40 K(1 AU) 35 K(5 AU)
Telescope Emissivity	0.1
Spectral Resolution	20
Net Efficiency (Optics*Detector*Beam)	0.04
Deepest Null	10^{-5} (1 AU) 10^{-6} (5 AU)
Phase Center Pointing Jitter	0.25 milli-arcsec
Detector Dark Current	5 e^-/s
Detector Read Noise	1 e^-
Flat Field Error	10^{-5} (1 AU) 10^{-4} (5 AU)

against a non-flat field of corrugations in the target field. Structures in our own cloud have roughly <0.1% of the amplitude of the total cloud brightness (as discussed at the Exozodiacal Dust Workshop; Backman *et al*. 1998; See Chapter 5). At levels less than 1% of the total brightness, the structured emission is not a significant contributor to the noise budget. However, large coherent structures, such as wakes and clumps behind planets, can masquerade as planets or serve as markers for their presence.

Background Confusion Noise. Extrapolation of 12 µm star and galaxy counts (Chapter 8) indicate that confusion noise will not be an issue at the high spatial resolution of TPF.

INTERFEROMETER PROPERTIES

The interferometer taken as a reference system for this study has properties listed in Table A.3. The OASES system (Angel and Woolf 1997) is but one of many possible nulling configurations in which light from pairs of telescopes are combined (nulled) and then pairs of interferometers are combined for a still higher order of rejection. The separation between the telescopes and the amplitudes of the signals from the telescopes are matched to produce a particular null. The OASES system utilizes a ratio of telescope diameters of 1:2:2:1 or 1:3:3:1 to give a deep null (10^{-6}) suitable for operation in low background conditions. The breadth of the OASES null ($\propto\theta^6$) is suitable for fixed baselines that cannot be tuned for stars at different distances.

The interferometer projects a nulling interference pattern onto the sky. Let $\Theta(r,\theta)$ be the fringe pattern of the interferometer, where θ is the orientation of the fringe pattern on the sky and r is the radial distance from the center. The nulling pattern for the 4-telescope OASES interferometer arranged in a 1:2:2:1 configuration and overall dimension B is given by

$$\Theta(r,\theta) = 4 \sin^2\phi \sin^4(\phi/2) \text{ with } \phi=(\pi rB/\lambda)\cos\theta \qquad (3)$$

A perfect interferometer would produce fringes with perfect contrast. We clip the null to have a maximum depth of 10^{-6} for the 5 AU system at 10^{-5} for the 1 AU system. At the closer distance, the higher local background reduces the requirements for the deepest possible null. As described below, some interferometer configurations allow for rapid chopping of the nulled signal to enhance detectability of a planetary signal in the presence of various detector or thermal drifts.

Leakage Signal. The amount of star light coming through the null is given by

$$Q_{\text{leak}} = A_{\text{tel}}N_{\text{tel}}\, \Delta\nu\eta\tau(h\nu)^{-1} \iint B_{\nu,*}(r,\theta)\, \Theta(r,\theta) r dr d\theta \qquad (4)$$

integrated over the stellar radius. $A_{\text{tel}}N_{\text{tel}}$ is the total collecting area, corrected for the reduced signals from the outer telescopes in the 1:2:2:1 or 1:3:3:1 configurations; where Ω_{planet} is the solid angle subtended by the planet; $\Delta\nu$ is the bandwidth of the observation; η is the product of the optical detector quantum and beam efficiencies; τ is the integration time; $h\nu$ is the quantum of energy at the observing frequency; and $\Theta(r,\theta)$ is the fringe pattern of the interferometer. The technological challenge of nulling represents the heart of TPF and is discussed in Chapters 9 to 12.

Leakage Jitter. An additional noise source comes from wandering of the null across the star due to variations in pointing of the phase center of the interferometer. As the phase center of the starlight drifts within the null pattern, the amount of leakage varies. Calculations show that the noise associated with this jitter is negligible for values < 1 milliarcsec. This noise source depends on the angular diameter of the star, the breadth of the null, and the stability of the system. The interferometer will be stabilized at 2 µm where 1 milliarcsec corresponds to 1/100 of the point spread function and typically to 1/10 of a fringe spacing. The high signal-to-noise ratio expected from target stars (>1000:1 in a few milliseconds) is consistent with this stability requirement.

Telescope Properties. The telescope and optics temperatures are set by the need to avoid any contribution to instrumental noise by the telescope. The temperature depends on the wavelength and local zodiacal foreground. At 1 AU a telescope temperature of 40 K produces <10% degradation of the sensitivity at 17 µm. At 5 AU, the telescope temperature must be < 35 K to avoid influencing the 17 µm sensitivity. The overall system efficiency includes reflections off many optical surfaces, transmission through filters and beamsplitters (0.12), detector quantum efficiency (0.5), and beam efficiency due to taking only the central part of the primary beam (0.6). For operation at 1 AU and at low (R~20) spectral resolution, SIRTF detector performance is adequate. Detector properties expected as part of the development of NGST may allow increases in spectral resolution without incurring a read-noise penalty.

Signal to Noise Calculations. The signal from the various sources of radiation are defined as follows in terms of photo-electrons detected by TPF in an integration time, τ. The signal depends on the location of the planet relative to the position of the interferometer pattern projected onto the sky. The planet signal is modulated by the fringe pattern as the interferometer rotates around the line of sight to the star.

$$Q_{\mathrm{planet}}(r,\theta) = \Theta(r,\theta)\, B_\nu(\nu, T_{\mathrm{planet}})\Omega_{\mathrm{planet}} A_{\mathrm{tel}} N_{\mathrm{tel}} \Delta\nu\eta\tau(h\nu)^{-1} \qquad (5)$$

The background signals come from the local, Q_{LZ}, and exo-zodiacal, Q_{EZ}, dust clouds.

$$Q_{LZ} = A_{\mathrm{tel}} N_{\mathrm{tel}} \Delta\nu\eta\tau(h\nu)^{-1}\, I_\nu(LZ)\iint \Theta(r,\theta) r\,dr\,d\theta \qquad (6)$$

where the integral extends to the edge of the primary telescope beam, $r_{\mathrm{max}} = 0.66\ \lambda/D$. This choice of r_{max} optimizes the signal to noise for a background-limited measurement of a point source. Similarly, for the exo-zodiacal emission:

$$Q_{EZ} = A_{\mathrm{tel}} N_{\mathrm{tel}} \Delta\nu\eta\tau(h\nu)^{-1}\iint I_\nu(r,\theta)\Theta(r,\theta) r\,dr\,d\theta \qquad (7)$$

The minimum level of noise arising from these photo-electrons is equal to the square root of the counts, $Q_{\mathrm{tot}} = (Q_{LZ} + Q_{EZ} + Q_{\mathrm{dark}} + Q_{\mathrm{planet}})^{1/2}$. Another source of noise comes from instabilities in the observing system in the presence of large background signals. This "flat field" noise can be modeled as being linearly proportional to the total signal, $flat{*}Q_{\mathrm{tot}}$, and represents a systematic noise level that cannot be improved with further integration time. Ground-based infrared systems typically operate at 10^{-6}-10^{-7} of the large atmospheric background using rapid chopping (~30 Hz). Space systems typically operate at 10^{-3}-10^{-4} of the low space background with no or only slow (<1 Hz) chopping. The requirements on background cancellation are quite different for the 1 AU case where the local zodiacal background is relatively high. Chopping, discussed in Chapter 6, may be required to make the system work at 10^{-5} of the background at 1 AU.

Detector Noise. Detector properties are given by the read noise, RN, and dark current, $Q_{\mathrm{dark}} = i_{\mathrm{dark}}\tau$. Linearity and stability must be optimized for a 1 AU system operating in the relatively high local background. Chopping at ~1 Hz is possible without noise penalty for devices with RN<10 e^-.

Table A.4. Signal and Noise Sources at 1 and 5 AU		
Signal (Photo-Electrons) R=20, $\tau=10^5$ s at 12 µm	2 m (5 AU)	3.5 m (1 AU)
Planet @ 10 pc	0.008×10^6	0.025×10^6
Exo-Zodiacal Background	0.71×10^6	2.15×10^6
Local Zodiacal Background	0.10×10^6	8.56×10^6
Nulled Star Leakage	0.04×10^6	1.16×10^6
Dark Current	0.50×10^6	0.50×10^6
Total Counts	1.35×10^6	12.4×10^6
√(Counts)	1.16×10^3	3.52×10^3
Flat*Counts	0.14×10^3	0.12×10^3
Noise	1.17×10^3	3.52×10^3
SNR (Signal/Noise)	7.0	7.1

Total Noise. The total noise is the quadratic sum of all the individual components:

$$Q_{\text{noise}}^2 = Q_{\text{leak}} + Q_{LZ} + Q_{EZ} + Q_{\text{planet}} + RN^2 + Q_{\text{dark}} + (flat{*}Q_{\text{tot}})^2. \quad (8)$$

which is to be compared with the exo-planet signal given in Equation (4) to give

$$SNR = Q_{\text{planet}}/Q_{\text{noise}} \quad (9)$$

Table A.4 compares the noise and signals for the 1 and 5 AU systems described in Table A.3 for a wavelength of 12 µm. The 1 AU system imposes less stringent requirements on detector and nulling performance, as well as easier implementation and operational constraints. The two accomplish roughly the same sensitivity although with a very different balance of backgrounds, signal levels, and star leakage.

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