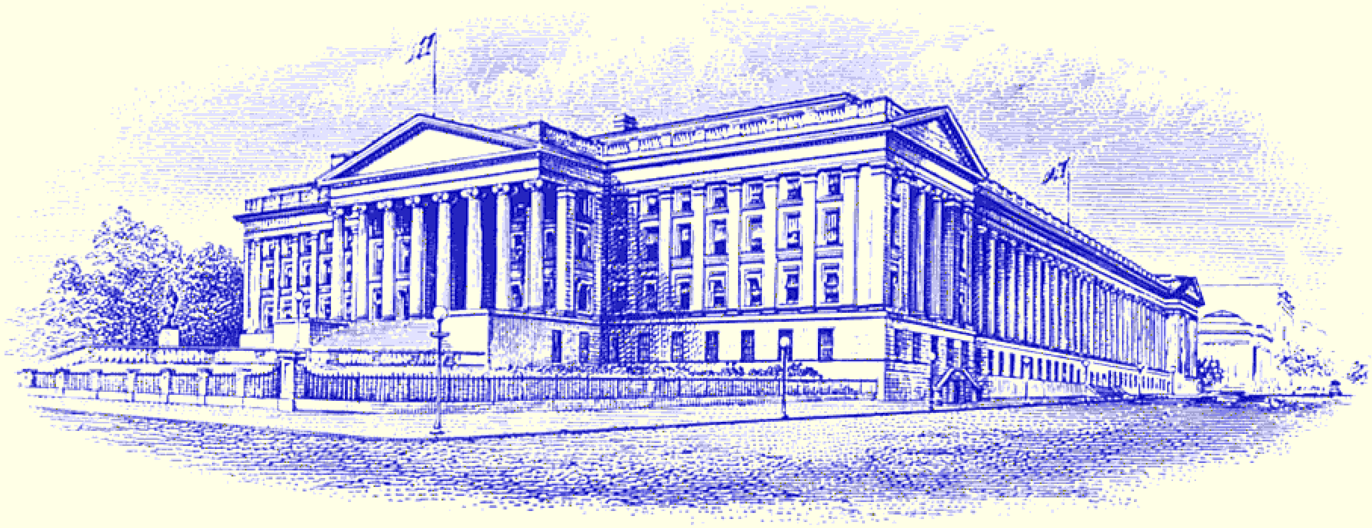
THE CORPORATE BOND YIELD CURVE FOR THE PENSION PROTECTION ACT



James A. Girola

U.S. Department of the Treasury

October 11, 2007

Introduction

This presentation describes the current Treasury methodology for constructing the corporate bond yield curve required by the Pension Protection Act of 2006 (PPA).

Previous versions of this methodology are set out in the Treasury White Paper (February 7, 2005) found at

http://www.treas.gov/offices/economic-policy/reports/pension_yieldcurve_020705.pdf

and the White Paper Update (January 24, 2006) at

http://www.treas.gov/offices/economic-policy/reports/wp_up_x.pdf.

The methodology may be changed in the future, either because market conditions change, or for other reasons.

The Pension Protection Act

- The PPA mandates that Treasury publish a corporate bond yield curve for calculating the present values of pension liabilities and lump sum distributions.
- The methodology chosen for this yield curve must produce a curve that satisfies the general requirements for a reliable yield curve that successfully captures market behavior, as well as the specific requirements of the PPA.
- The following discussion sets out these requirements.

PPA Requirements

- The PPA requires that the yield curve represent the corporate bond market rather than the U.S. Treasury market, as in the typical yield curve. Corporate bonds are much more heterogeneous than Treasuries: many have special features, and criteria must be developed for deciding upon the bond set for the curve.
- The yield curve must be a single blended curve reflecting high quality corporate bonds, i.e., bonds rated AAA, AA, or A. Typical yield curves do not combine different qualities from disparate markets, and so an approach must be developed for the combination.

Yield Curve Requirements: Projection

- An important requirement for any yield curve is that ***the yield curve must be projected for indefinitely long maturities*** beyond 30 years maturity. This is necessary because the yield curve may be used to discount cash flows well beyond 30 years into the future. The methodology must be developed for doing this; the usual yield curve stops at 30 years and contains no provision for projection. The projected discount rates must be reliable and must reflect the behavior of long-term interest rates.
- In the case of pension liabilities, the yield curve must be projected out through 100 years maturity.

Yield Curve Requirements: Production

- Since ***the yield curve must be estimated daily in a production environment***, the yield curve methodology must provide estimates that are robust and stable with respect to perturbations in the bond set while capturing movements in the market. Consequently, the yield curve should evolve smoothly over time.
- While the methodology can be changed as warranted by market conditions or for other reasons, such changes should be infrequent, and the curve should not need to be tuned every day.

Additional Yield Curve Requirements

- In addition to the reasoning behind the yield curve methodology, the results produced by the methodology must be good indicators of market conditions.
- In particular, the yield curve should not be subject to the criticism that it is arbitrary, in the sense that there are equally valid methodologies or equally valid ways of implementing a given methodology that produce different results from the same data. Therefore, there must be a rationale for setting every parameter contained in the methodology, and no parameter should be left to discretion.

Additional Requirements, continued

- Yield curves can exhibit arbitrary behavior when they are derived from overly complicated models that may contain strong assumptions.
- However, stringent assumptions usually do not apply in bond markets, and so such assumptions can give different results. Also, bond data do not show clear enough patterns to distinguish among competing complicated models, so again the models give different results.
- To avoid problems of arbitrariness, the chosen yield curve methodology should aim toward robust and straightforward market averages that provide reliable indicators of central market tendencies.

The White Paper Methodology

This presentation focuses on the current version of the Treasury White Paper methodology, which produces corporate bond yield curves for the PPA representing the market-weighted average quality of the combined high quality bond market.   These yield curves are termed High Quality Market-Weighted (HQM) Corporate Bond Yield Curves.

The White Paper Methodology, continued

- The White Paper methodology computes the yield curve for a point in time from data for a selected set of corporate bonds. The bond set covers the range of maturities up to 30 years and includes the three high quality levels. Criteria for bond selection are discussed later.
- The curve is directly fitted to the bond set without the use of a separate yield curve for Treasury securities. Reliance on a Treasury curve is unnecessary and needlessly complex.
- The yield curve employs a bond pricing model based on a discount function, and extends the model with regression terms for factors not contained in the discount function.

The Discount Function

- The starting point for the White Paper methodology is the discount function.
- The discount function $\delta(\tau)$ gives for each maturity τ (in years) the amount that must be invested to receive \$1 in the future at that maturity. The discount function is valid for the point in time for which the yield curve is being computed.
- The discount function as used here pertains to the market-weighted average credit quality in the high quality market, and gives the present prices of future payments at this quality. The interpretation of the discount function will be made more precise later in the discussion of the regression terms.

The Forward Rate

The forward rate curve $\xi(\tau)$ is defined as the relative curvature of the discount function:

$$\xi(\tau) = -\frac{d\delta(\tau)}{d\tau}\frac{1}{\delta(\tau)}$$

Forward rates are instantaneous future interest rates based on the discount function. The discount function can be stated in terms of the forward rate:

$$\delta(\tau) = \exp(-\int_{z=0}^{\tau} \xi(z)dz)$$

The Spot Rate

The relevant interest rate concept for discounting future cash flows, such as pension benefits, is the spot rate, which is the yield on a bond with a single payment at maturity (zero coupon bond). Therefore, the spot yield curve $y(\tau)$, showing spot yields for all maturities, is the central yield curve concept.

With semiannual compounding, the spot rate (in percent) is related to the discount function as follows:

$$\delta(\tau) = 1/(1+y(\tau)/200)^{2\tau}, \qquad \tau \geq 0.5$$

$$\delta(\tau) = 1/(1+2\tau\, y(\tau)/200), \qquad 0 < \tau < 0.5$$

The Spot Rate, continued

In terms of the discount function, the spot rate is:

$$y(\tau) = 200 \times (1/\delta(\tau)^{\frac{1}{2\tau}} - 1), \qquad \tau \geq 0.5$$

$$y(\tau) = 200 \times (1/\delta(\tau) - 1)/(2\tau), \qquad 0 < \tau < 0.5$$

It can be shown by continuity that $y(0) = 100 \cdot \xi(0)$.

If the forward rate eventually settles down to a constant ξ^*, the spot rate will converge to:

$$y^* = 200 \times (\exp(\xi^*/2) - 1)$$

which is not the same as ξ^* because the spot rate is semiannually compounded.

The Par Rate

- Another yield curve concept is the par yield curve, which gives for each maturity the yield on a bond with that maturity and paying semiannual coupons, and which is trading at par, that is, whose flat price (excluding accrued interest) is 100.
- Par yields are also computed from the discount function. The coupon rate at par is determined with the discount function, and the par yield is the yield for that coupon rate.
- Par yields are indicators of bond market conditions, and therefore they can be used to compare the corporate bond market with the Treasury market.

Bond Price Equation

Using the discount function and forward rate curve, the price for the k^{th} bond out of a set of P bonds can be written as:

$$p_k = \sum_{i=1}^{n_k} \delta(\tau_{ki}) c_{ki} + \sum_{j=1}^{m} \zeta_j x_{kj} + \varepsilon_k$$

$$= \sum_{i=1}^{n_k} \exp\left(-\int_{z=0}^{\tau_{ki}} \xi(z)dz\right) c_{ki} + \sum_{j=1}^{m} \zeta_j x_{kj} + \varepsilon_k$$

where p_k is the price of the k^{th} bond (including accrued interest), the c_{ki} for $i = 1,\ldots,n_k$ are the n_k cash flows from the bond to be received at maturities τ_{ki}, the ζ_j for $j = 1,\ldots,m$ are the m coefficients for the m regression variable values x_{kj}, and ε_k is the random disturbance.

Functional Form of the Discount Function

- In order to estimate the bond price equation, the functional form must be determined for the discount function.
- It is more useful to work with the forward rate curve $\xi(\tau)$, because forward rates all have the same instantaneous maturity and can be compared across maturity ranges.
- Moreover, to make economic sense it is necessary that $\delta(\tau) > 0$ and $\delta(0) = 1$, and any functional form for the forward rate curve satisfies these conditions.
- It is also generally reasonable to impose the condition $d\delta/d\tau < 0$, which says that \$1 received later should have a lower present price. If the functional form for $\xi(\tau)$ can be chosen to be positive, this condition is also satisfied.

Maturity Ranges

- In determining the functional form, it is useful to recognize that corporate bond maturities tend to divide into ranges. The ranges reflect the fact that many trades in the market have preferred maturity habitats. The forward rates in each range are related.
- The maturity ranges can change with significant changes in bond market conditions. For recent years, the White Paper methodology has delineated the ranges for corporate bonds by the maturity points 0, 1.5, 3, 7, 15, and 30 years.
- The range 0 to 3 years reflects short-term trading, and there is enough activity that it should be further divided in two with a point at 1.5 years.

Maturity Ranges, continued

- The range 3 to 7 years contains bonds of somewhat longer term centering around 5 years. And the range 7 to 15 years contains the bulge of bonds frequently seen around 10 years.
- Finally, the last range 15 to 30 years reflects the longest bonds. There is not enough information in current long-term bonds to determine whether to break down this range further, so all the longer bonds are kept together in one range.

Averages in the Ranges

- Given the ranges, the forward rate in each range could be approximated by its average. However, it is desirable to allow some movement of the forward rate within each range, especially in the last range, and also to derive a forward curve that is smooth over all maturities.
- The way to do this that does not depart too far from averages is to use a cubic polynomial for each range, and to string the polynomials together smoothly as a cubic spline with knots at the 6 maturity points. Smoothness means that at each of the interior knots 1.5, 3, 7, and 15, the spline is made to be continuous with continuous first and second derivatives.

Averages in the Ranges, continued

- This discussion shows that the use of a cubic spline for the forward rate arises naturally from averaging within maturity ranges together with smoothness.
- The choice of fixed knots for the spline based on maturity ranges significantly increases the stability of the yield curve over time.

Constraint in the Nearest Range

- Because there are no bonds with zero maturity, the forward curve has no anchor at the zero point in the first maturity range.
- For this reason it makes sense to impose an approximate linear constraint on the movement of the forward rate at the earliest maturities. This is accomplished by constraining the second derivative to zero at the beginning:

$$\frac{d^2\xi(0)}{d\tau^2} = 0$$

The Long-Term Forward Rate

- Special constraints are needed for the last maturity range 15 to 30 years, because this range has to lead into the projected forward rate beyond 30 years that is used for the projections of the yield curve.
- Because it is usually not possible to estimate accurately the movements in the forward rate beyond maturity 30 years, the projected forward rate is fixed at a constant, reflecting its long-term average.
- Moreover, the forward curve should flatten out at year 30 to connect smoothly with this long-term constant. This leads to the first constraint for the last maturity range:

$$\frac{d\xi(30)}{d\tau} = 0$$

The Long-Term Forward Rate, continued

- One approach to estimating the forward rate beyond 30 years maturity focuses on the typical drop-off in yields that occurs a few years before 30 years, and projects out this decline. The forward rate given by this approach generally falls to a very low level at long maturities.
- This approach is dubious because it allows the yield behavior for a few years before 30 to determine forward rates for many decades beyond 30 years.
- And as a practical matter, the low forward rates that it projects would imply relatively low spot rates and consequently relatively high discounted present values of pension obligations beyond 30 years.

The Long-Term Forward Rate, continued

- More significantly, this approach places insufficient importance on risk and term premiums at far maturities.
- In contrast, the White Paper methodology emphasizes the fact that the forward rate is the marginal return on lengthening the maturity of a loan, and as such is heavily influenced by term premiums, which reflect risks that cannot be hedged.
- Accordingly, aversion to risk has the effect of pushing up forward rates, and this suggests that the long-term forward rate may remain at a relatively high level.

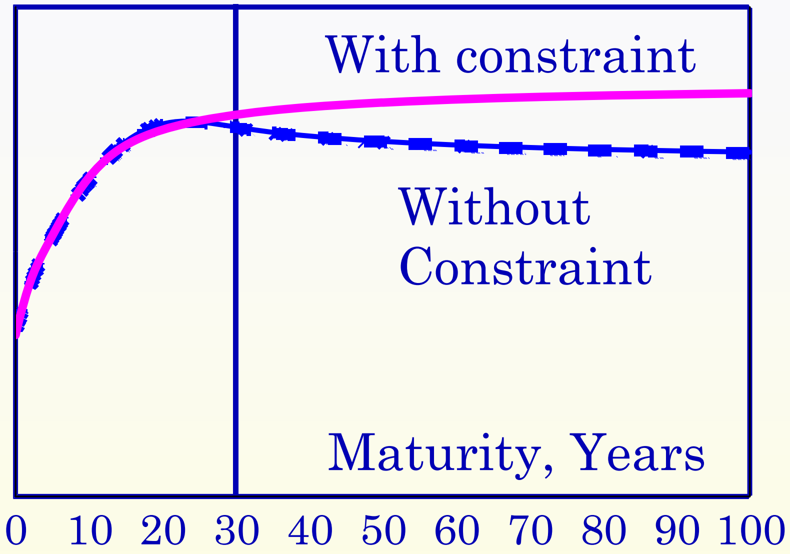
The Long-Term Forward Rate, continued

- Based on these risk considerations, the forward rate at maturity 30 years and beyond can be assumed to be determined on average by the factors that affect the forward rate in the 15- to 30-year maturity range, since that range is sufficiently distant to reflect long-term attitudes toward risk.
- Therefore, the constant long-term forward rate from maturity 30 forward is assumed to be the average forward rate in the 15- to 30-year range. This implies the following additional constraint on the forward rate curve:

$$\frac{\int_{z=15}^{30} \xi(z)dz}{15} = \xi(30)$$

The Long-Term Projection

- The effect of the last constraint on spot yield curves is shown in the side picture, which depicts a typical situation. The constraint averages out the drop-off in yields frequently seen in a small maturity range right before year 30.



- In sum, the constraint fixes the forward rate at 30 years at its average, with the result that the yield curve at far maturities around 30 years and beyond reflects average market yields.

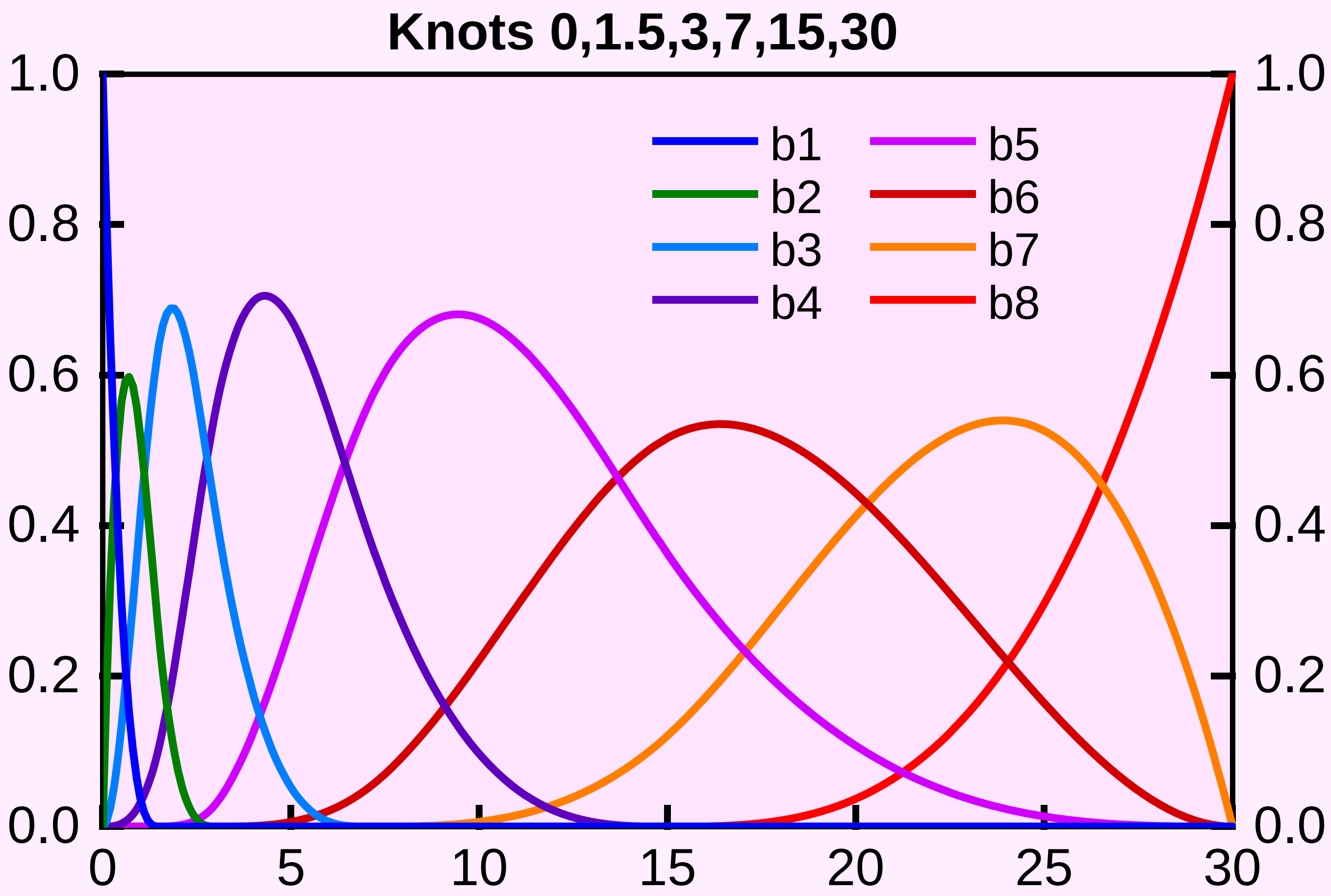
The Spline for Estimation

For the purposes of estimation, the cubic spline for the forward rate curve $\xi(\tau)$ can be written as a linear combination of 8 B-splines, $b_1(\tau),\ldots,b_8(\tau)$.

The B-splines are derived by expanding the knots to the vector (0,0,0,0,1.5,3,7,15,30,30,30,30); each of the B-splines is derived from a sequence of 5 of these knots: b_1 is derived from (0,0,0,0,1.5), b_2 is derived from (0,0,0,1.5,3), and so on, with b_8 derived from (15,30,30,30,30).

The chart on the next page depicts the 8 B-splines.

B-SPLINES



Knots 0,1.5,3,7,15,30
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
30
b1
b2
b3
b4
b5
b6
b7
b8

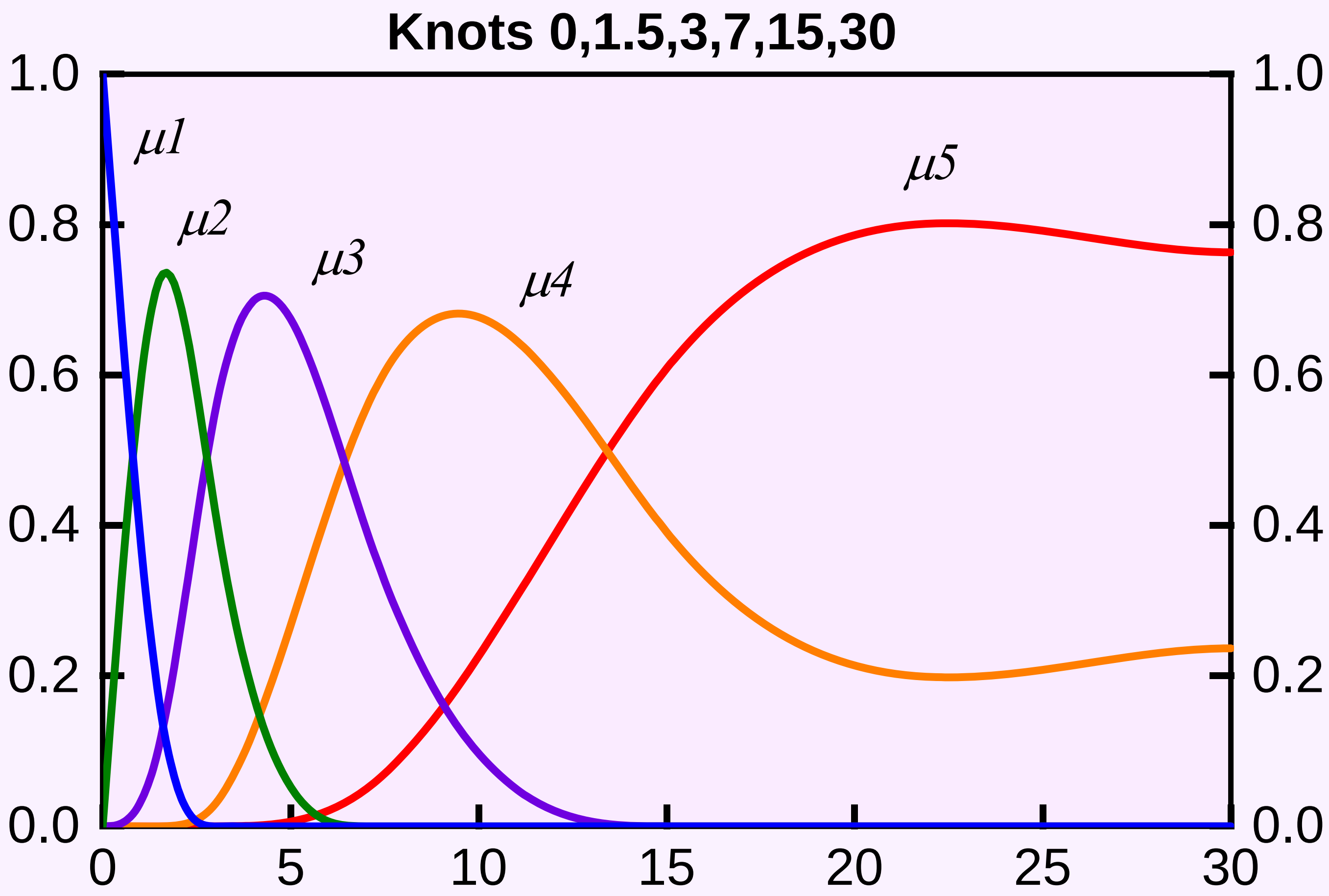
The Spline for Estimation, continued

The next step is to impose the 3 constraints on the 8 B-splines. The result is a set of 5 nonnegative cubic splines $\mu_\alpha(\tau)$, for $\alpha = 1,\ldots,5$. The constraints can be represented by the following matrix (written to 2 decimal places):

$$\mathrm{M} = \begin{pmatrix} 1 & 0.67 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.33 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0.24 & 0.24 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0.76 & 0.76 \end{pmatrix} \times \mathrm{B}$$

where M is the column vector $(\mu_1(\tau),\ldots,\mu_5(\tau))'$, and B is the column vector $(b_1(\tau),\ldots,b_8(\tau))'$. The cubic splines $\mu_\alpha(\tau)$ are plotted in the next chart.

BASIS SPLINES WITH CONSTRAINTS



The Spline for Estimation, continued

Therefore, the forward rate curve $\xi(\tau)$ including the 3 constraints can be written as a linear combination of the $\mu_\alpha(\tau)$ with 5 coefficients β_α to be estimated:

$$\xi(\tau) = \sum_{\alpha=1}^{5} \beta_\alpha \mu_\alpha(\tau)$$

Defining

$$\nu_\alpha(\tau) = \int_{z=0}^{\tau} \mu_\alpha(z)dz$$

the discount function is written as

$$\delta(\tau) = \exp(-\sum_{\alpha=1}^{5} \beta_\alpha \nu_\alpha(\tau))$$

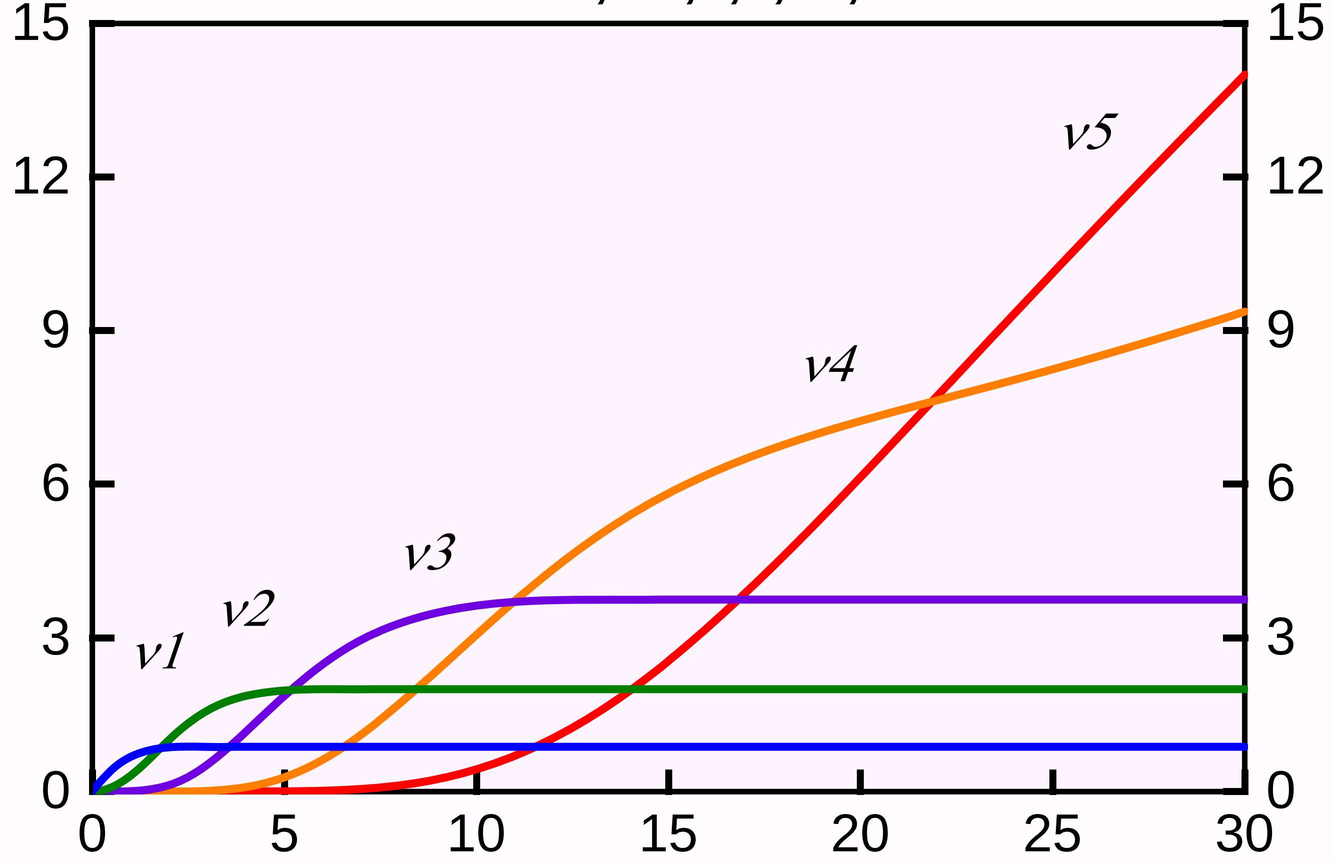
The Spline for Estimation, continued

If the estimates of the 5 parameters β_α are strictly positive, as they have been in recent years, the resulting discount function fulfills the conditions discussed earlier, in that it is positive and declining with a value of 1 at $\tau = 0$.

The next chart plots the 5 integrated functions $v_\alpha(\tau)$.

INTEGRATED BASIS SPLINES WITH CONSTRAINTS

Knots 0,1.5,3,7,15,30



Regression Variables

- In addition to the 5 parameters β_α, the coefficients ζ_j on the regression variables x_j must be estimated. The White Paper methodology currently includes 2 regression variables. These variables could change in the future.
- The 2 variables are used to combine the three bond qualities into a single pricing model; therefore, they implement the PPA requirement that a single yield curve be produced for the combined high quality corporate bond market.

Regression Variables, continued

- The variables are chosen so that the discount function pertains to the quality level that is the market-weighted average of the three qualities, with weights given by the par amounts outstanding of the respective qualities.
- The greater the effect of the variables in adjusting the prices of bonds in a particular quality class, the less the discount function is representative of that quality. So to get a market-weighted discount function, the amount of adjustment should vary inversely with the size of the quality class.
- As a result, the spot curve derived from the discount function also pertains to market-weighted average quality.

Definition of the Regression Variables

- Let Γ_{AAA}, Γ_{AA}, and Γ_{A} be the total par amounts outstanding in the bond set for AAA, AA, and A bonds, respectively. Define:

$$\omega_1 = \frac{\Gamma_{AA}}{\Gamma_{AAA} + \Gamma_{AA}}$$

$$\omega_2 = \frac{\Gamma_{A}}{\Gamma_{AAA} + \Gamma_{AA} + \Gamma_{A}}$$

Definition of the First Regression Variable

- The first variable x_1 is defined as follows: for each AAA bond, x_1 is $\omega_1\tau_{AAA}$, where τ_{AAA} is the bond's maturity, for each AA bond, x_1 is $(\omega_1\text{-}1)\tau_{AA}$, where τ_{AA} is the bond's maturity, and for each A bond, x_1 is zero.
- The larger the weight of AA bonds ω_1, the greater is the price adjustment through x_1 for AAA bonds relative to AA bonds, and so the closer is the discount function to a AA discount function relative to AAA.

Definition of the Second Regression Variable

- The second variable x_2 is defined analogously: for each AAA bond, x_2 is $\omega_2\tau_{AAA}$, where τ_{AAA} is the bond's maturity, for each AA bond, x_2 is $\omega_2\tau_{AA}$, where τ_{AA} is the bond's maturity, and for each A bond, x_2 is $(\omega_2\text{-}1)\tau_A$, where τ_A is the bond's maturity.
- The larger the weight of A bonds ω_2, the greater is the adjustment for AAA and AA bonds relative to A bonds, and the closer is the discount function to a A discount function relative to AAA and AA.

Market Weighting

- These 2 variables together cause the discount function to be set in between the AAA and AA markets based on their market sizes, and then to be moved further so that it ends up in between the AAA-AA market and the A market, again based on market weights. The resulting discount function therefore represents market-weighted average quality across the three quality levels AAA, AA, and A.

Regression Coefficients

- The coefficients ζ_1 and ζ_2 show the price difference between AAA and AA bonds, and AAA-AA and A bonds, respectively.
- Specifically, since x_1 is scaled by maturity, ζ_1 gives the average amount per year of maturity that investors are willing to pay for the insurance against risk provided by the AAA rating relative to AA. Analogously, ζ_2 is the average amount for AAA-AA bonds relative to A.
- Although more complicated functions of maturity could be defined for these variables, straightforward linear scaling by maturity helps capture average market behavior in a stable fashion over time.

Estimation

The final price equation for estimation is written as follows for the k^{th} bond:

$$p_k = \sum_{i=1}^{n_k} \exp\left(-\sum_{\alpha=1}^{5} \beta_\alpha \nu_\alpha(\tau_{ki})\right) c_{ki} + \sum_{j=1}^{m} \zeta_j x_{kj} + \varepsilon_k$$

with notation as before. In the current White Paper approach, there are 7 coefficients to be estimated, 5 spline coefficients, the β_α, and 2 regression coefficients, the ζ_j.

Estimation is by nonlinear least squares, which enables the usual covariance matrix to be computed for the regression variables. The next sections discuss the bond data and weighting scheme.

Data

- The PPA requires that the corporate bond yield curve be derived from a set of high quality corporate bonds. The set must accurately represent the high quality corporate bond market.
- In contrast to Treasury coupon issues, corporate bonds come in many varieties, not all of which provide useful information for the curve. So it is necessary to make a selection from the universe of all high quality bonds, based upon several types of characteristics.
- Since the curve shows rates of return over different time spans, each selected bond must generally provide an unambiguous stream of fixed future cash flows in return for an explicit price.

Data, continued

- Therefore, the basic type of selected bond is analogous to conventional Treasury coupon issues: a bond that pays a fixed semiannual nominal coupon denominated in U.S. dollars until maturity, when the principal is returned.
- Bonds that differ from the basic type are generally excluded. Bonds with floating coupons provide little if any information about future rates of return, since their own interest payments are changing over time. Convertible bonds do not have a clear price for cash flows because the price depends on equities.
- The chosen bonds must be issued by corporations. Asset-backed bonds are excluded, as well as bonds issued by U.S. sponsored agencies.

Data, continued

- The bond set covers maturities up through 30 years. Currently, bid prices are used for the bonds. Maturities below 1 year can be filled in by Federal Reserve commercial paper rates.
- To ensure sufficient liquidity, each bond must meet a minimum size threshold in terms of par amount outstanding. The current minimum is $250 million.
- At present, callable bonds are excluded, unless the call feature is make whole. Putable bonds and bonds with sinking funds are also excluded.
- The number of corporate bonds that fulfill these specifications is sufficiently large that there is no problem in computing the yield curve.

Weighting

- Before estimation, the White Paper methodology applies weights to the bond data in two stages.
- In the first stage, equal weights are assigned to any commercial paper rates in the data set, and the par amounts outstanding of the bonds are rescaled so that their sum equals the sum of the commercial paper weights.
- Then the data are multiplied by the square root of the commercial paper weights and the rescaled par amounts, which implies that the squared residuals of the bonds in the least squares fit are weighted by par amount outstanding.

Weighting, continued

- The purpose of the first stage is to give greater weight to larger bonds because they are more liquid and more important in the market.
- Commercial paper rates are assigned a high weight so that they can anchor the curve at the short end.
- In the second stage of weighting, for bonds with duration greater than unity, the weighted bond data are divided by the square root of (Macaulay) duration.
- The second stage corrects for heteroscedasticity: bonds with higher duration are more volatile, and the disturbance terms of such bonds have higher variance.

Software

- The model derived from the White Paper methodology is estimated using software developed at the Treasury Department.
- The software provides the capability to estimate general yield curves by splining the forward rate curve.
- As discussed, the spline on the forward curve is built up from B-splines, and arbitrary linear constraints can be imposed on the B-spline coefficients. The 3 constraints used here are examples of linear constraints.

Software, continued

- A special feature of the software is that the coefficients of the (possibly constrained) B-splines can also be constrained to be nonnegative, thus ensuring that the resulting discount function is nonincreasing. This feature has not been necessary in recent data for the corporate bond yield curve, but it can be useful with particularly intractable data sets.
- The software also enables arbitrary linear regression variables to be added to the yield curve, as is done for the corporate curve.
- The software takes account of coupon and principal payment dates that fall on weekends or holidays, and moves them forward to the next business day.

Results

- The following charts present sample results for the High Quality Market-Weighted (HQM) Corporate Bond Yield Curve computed by the White Paper methodology for the business day Wednesday, June 20, 2007.
- In the bond set for this date, there are 81 AAA rated bonds, 417 AA bonds, and 953 A bonds, for a total of 1,451 bonds. There are 12 commercial paper rates.
- The numbers of bonds whose maturities fall into the 5 ranges 0-1.5, 1.5-3, 3-7, 7-15, and 15-30, are, respectively, 99, 237, 523, 315, and 277.

Results, continued

- The 5 β_k spline coefficients estimated for this date in percentage terms are:

β_1	5.396
β_2	5.404
β_3	5.973
β_4	6.666
β_5	6.769

Results, continued

- The estimates of the regression coefficients ζ_j for this date in basis points are as follows:

Variable	Coefficient	T-Ratio
x_1	6.8	4.45
x_2	9.7	13.64

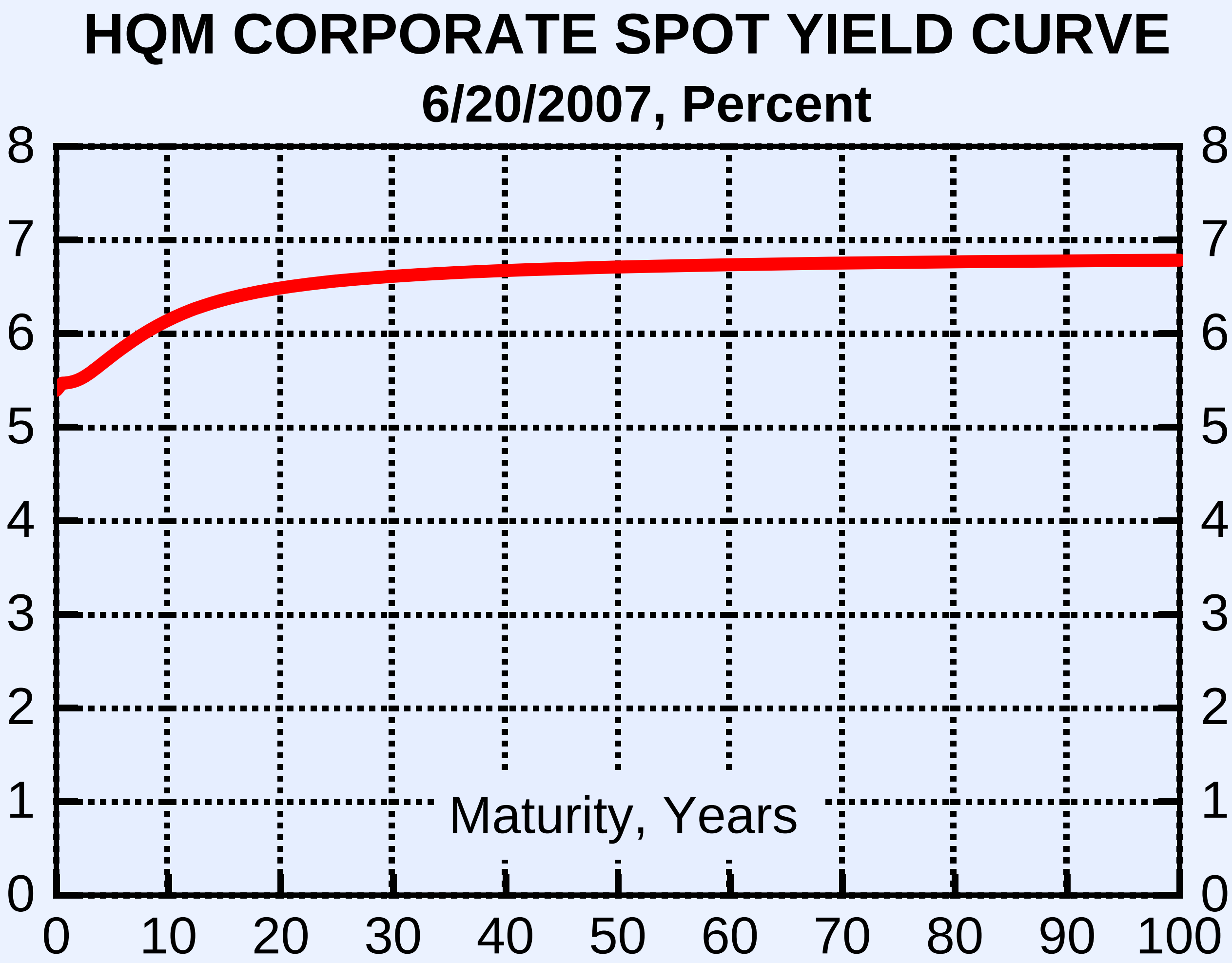
- These coefficients say that the price of a AAA bond was 6.8 basis points higher per year of maturity than the price of a AA bond with the same characteristics, and the price of a AAA-AA bond was 9.7 basis points higher per year of maturity than an A bond.

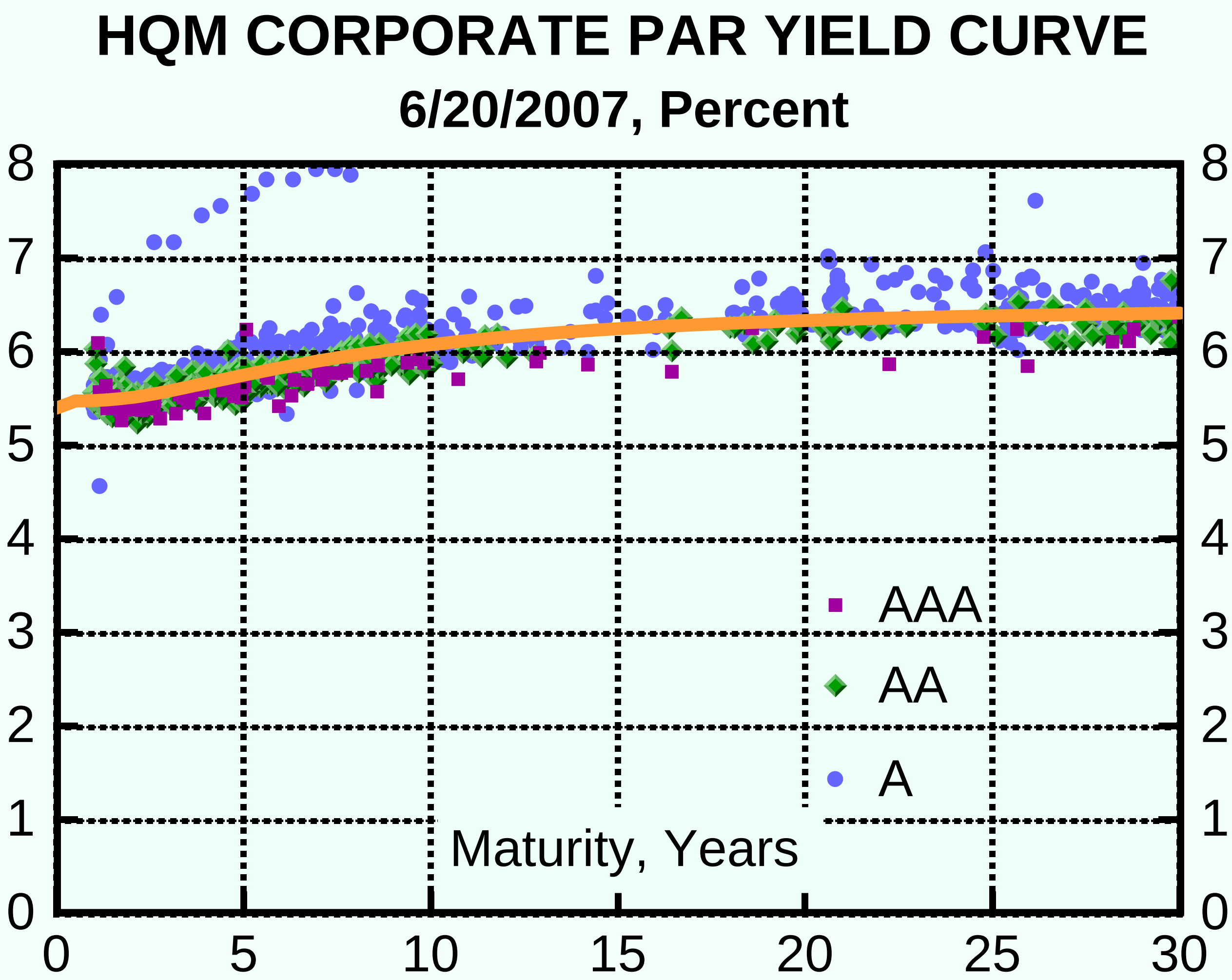
Results, continued

- As a measure of fit, the mean absolute error between actual and fitted prices for this date was 0.78 percentage point.
- The mean absolute price error relative to actual price (including accrued interest) was 0.8 percent.
- This is a close fit in view of the diverse characteristics of corporate bonds.

Results, continued

- The next two charts depict the spot and par HQM yield curves for this day.
- The spot curve is projected out 100 years, and slopes gently upward.
- The par chart includes the yields to maturity for all the bonds in the bond set, and shows that the HQM par curve falls within the set of yields.

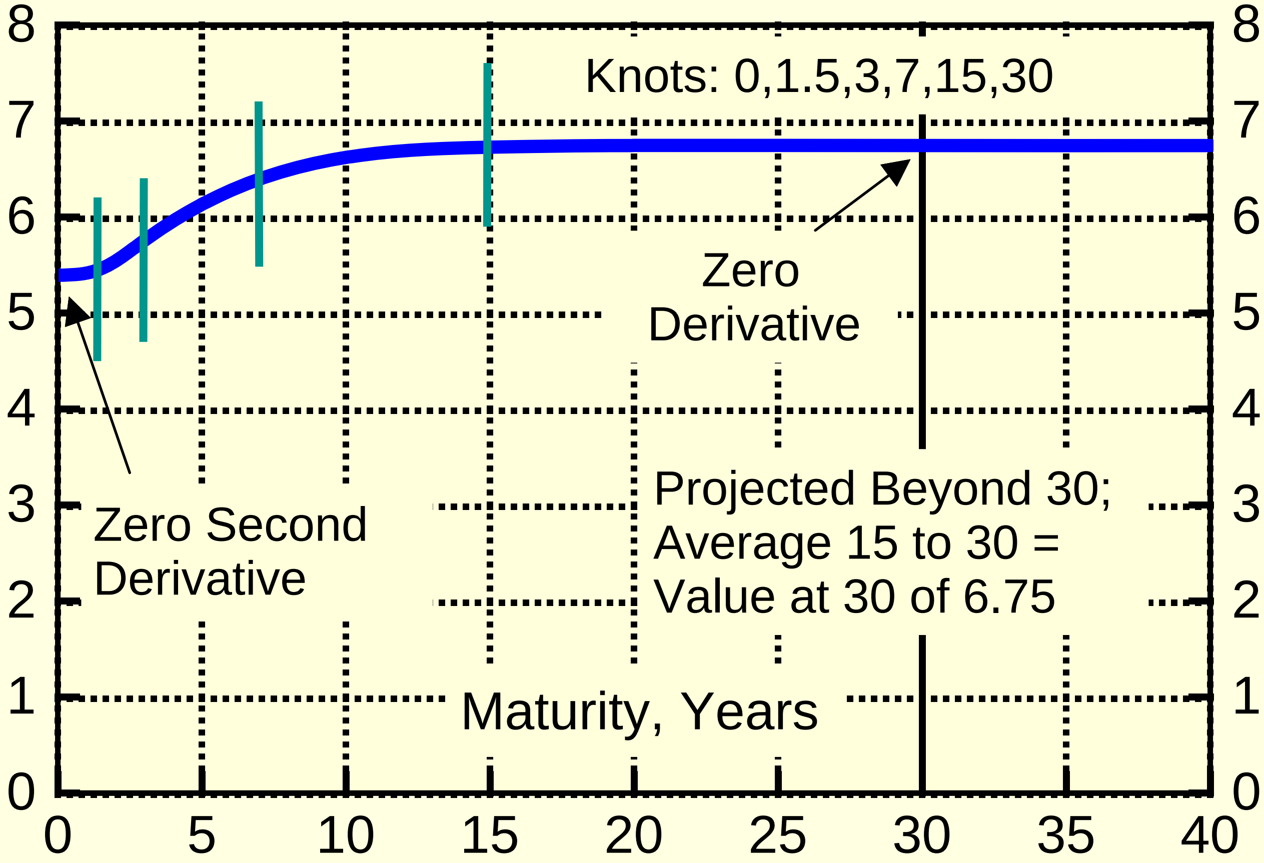

HQM CORPORATE SPOT YIELD CURVE
6/20/2007, Percent
Maturity, Years
0 1 2 3 4 5 6 7 8
0 10 20 30 40 50 60 70 80 90 100


HQM CORPORATE PAR YIELD CURVE
6/20/2007, Percent
8
7
6
5
4
3
2
1
0
0
5
10
15
20
25
30
AAA
AA
A
Maturity, Years

Results, continued

- The next chart shows the forward rate curve for this date extended out to 40 years.
- The chart illustrates the cubic equations within the maturity ranges indicated by the vertical lines. The curve is fixed at 30 years maturity and beyond at the 15–30 year forward rate average of 6.75 percent.

HQM CORPORATE
FORWARD RATE CURVE
6/20/2007, Percent

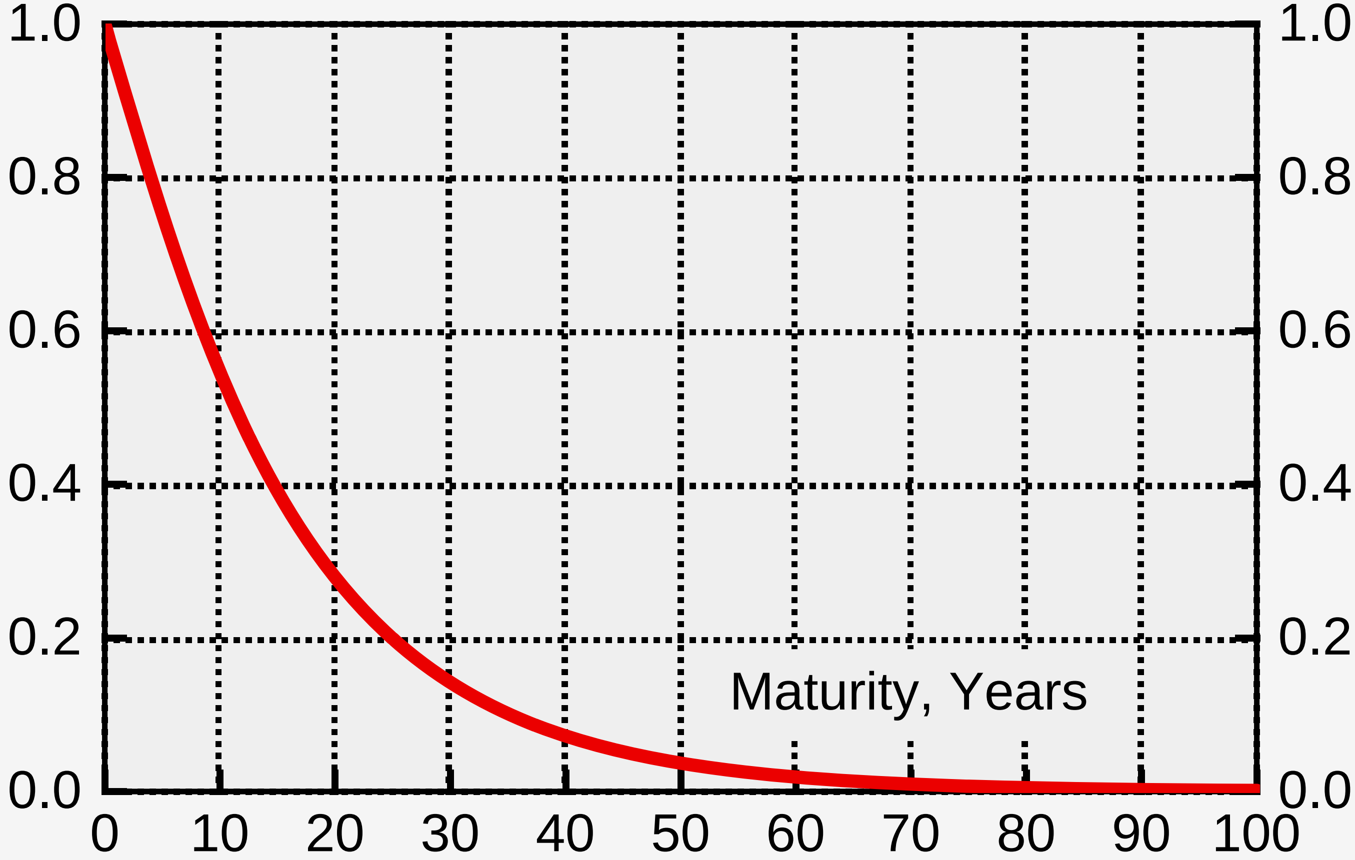


Results, continued

- And the following chart plots the discount function for this date, corresponding to the forward curve in the previous chart.
- The discount function equals unity at maturity zero and is positive and declining, eventually converging to zero.

HQM CORPORATE DISCOUNT FUNCTION

6/20/2007

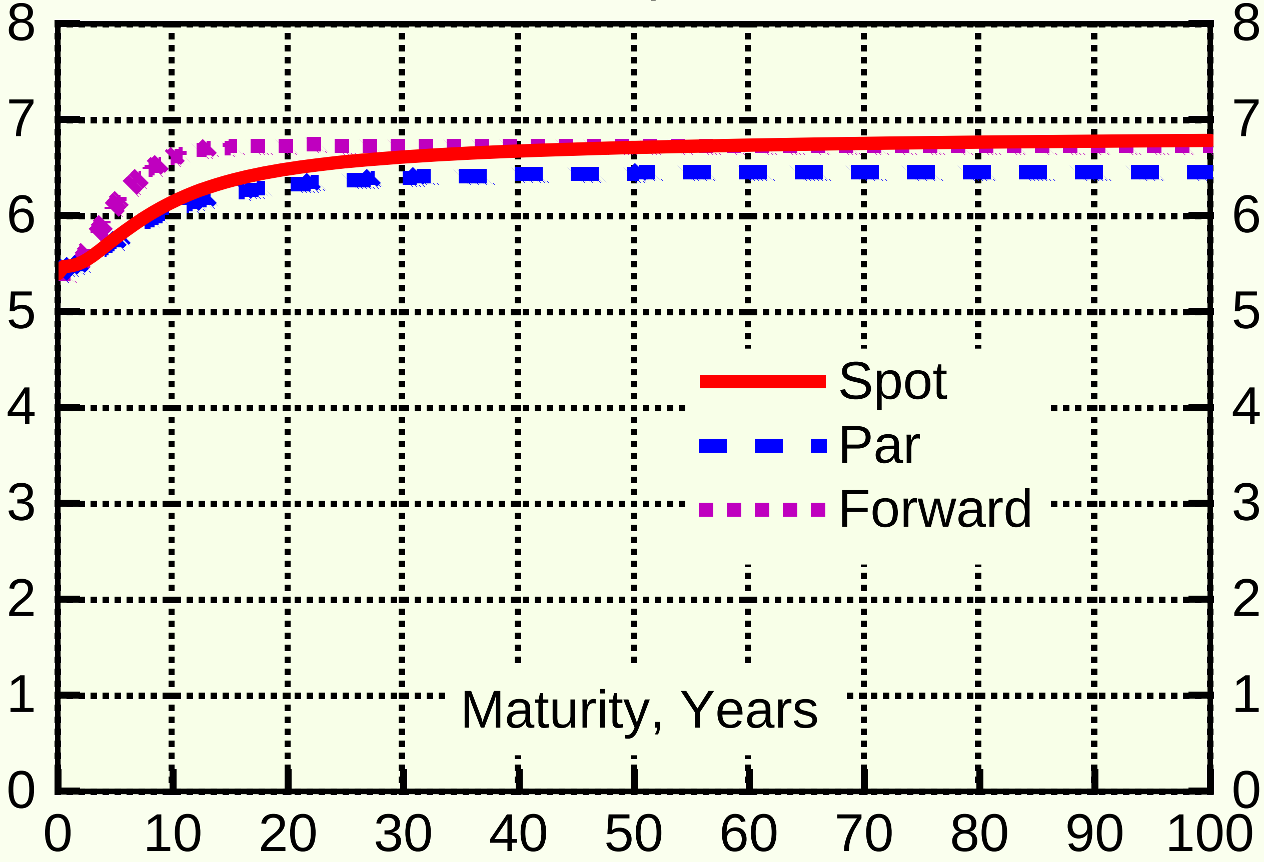


Results, continued

- The next chart pulls together and compares the spot, par, and forward HQM curves for June 20, 2007.
- The chart illustrates the smooth projection of all three curves out 100 years.

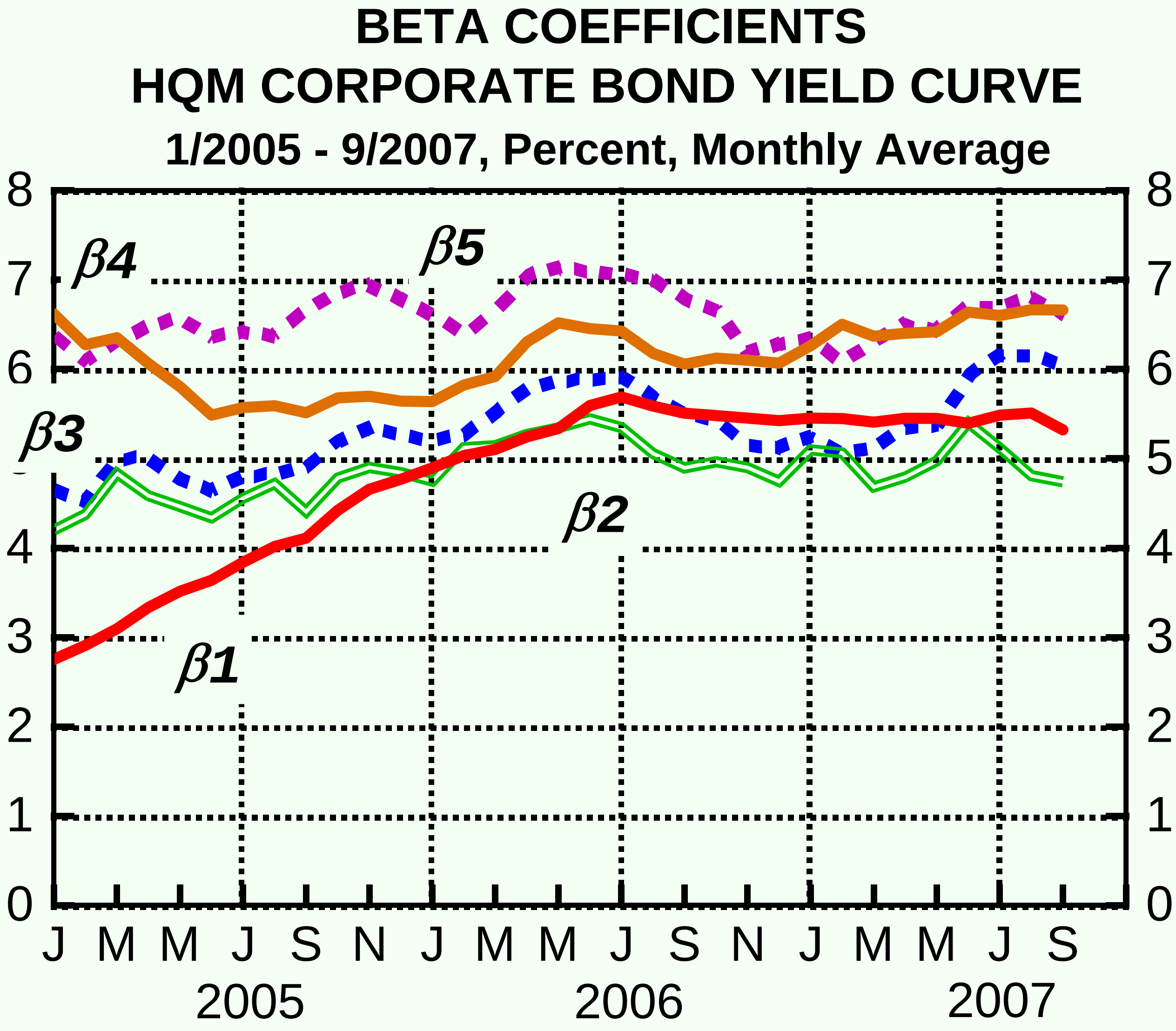
HQM CORPORATE YIELD CURVES

6/20/2007, Percent



Results over Time

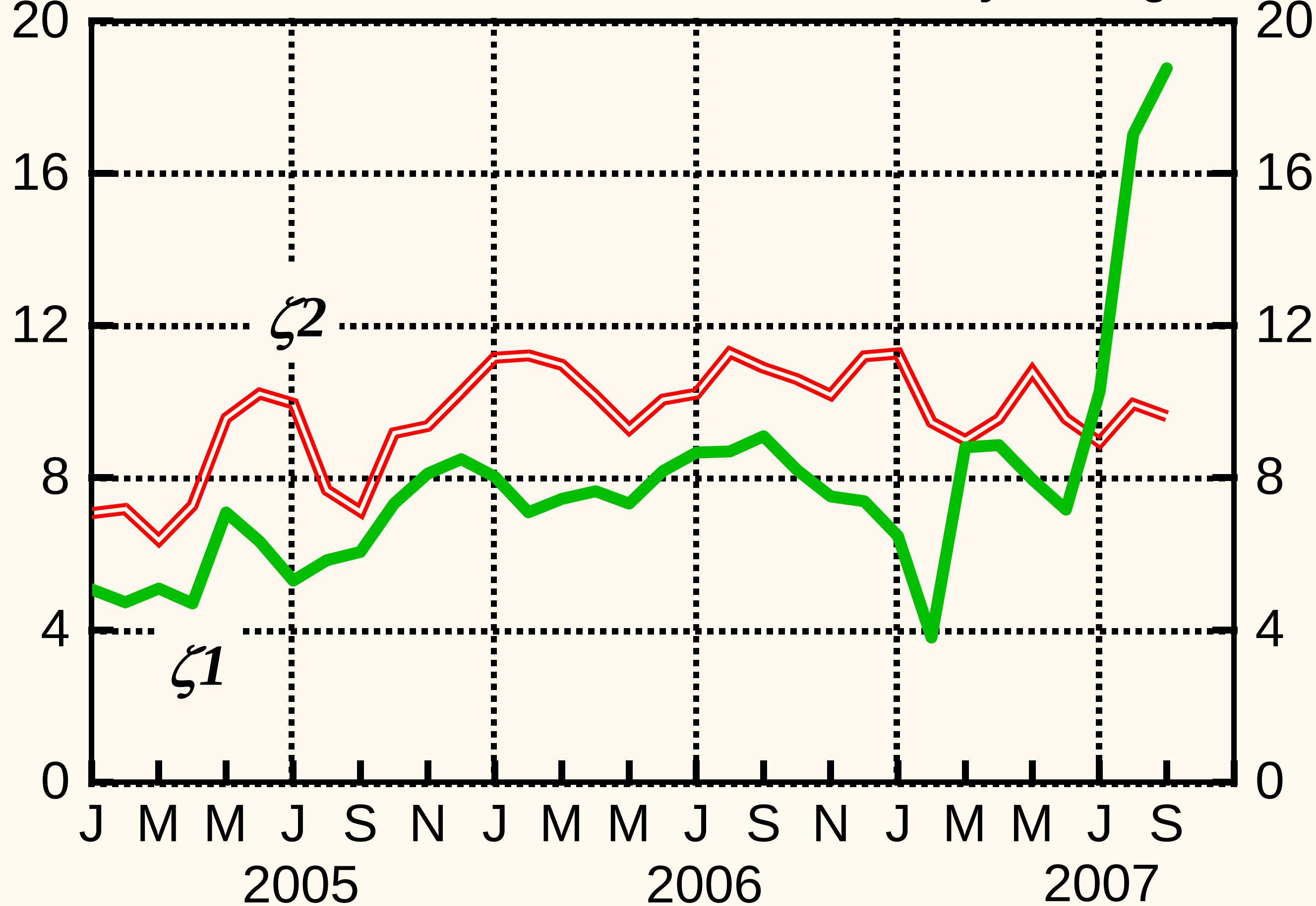
- The next charts show select features of the White Paper methodology over time. The data in the charts are derived from monthly averages of results for the HQM yield curve estimated for the 683 business days from January 2005 through September 2007.
- The first chart depicts the β_α coefficients over this time span, where β_1 is the coefficient on the lowest maturity basis spline, up through β_5, which is the coefficient on the last basis spline.
- The chart shows that β_1 and β_2 generally rose over this period.


BETA COEFFICIENTS
HQM CORPORATE BOND YIELD CURVE
1/2005 - 9/2007, Percent, Monthly Average
8
7
6
5
4
3
2
1
0
β4
β5
β3
β2
β1
J M M J S N J M M J S N J M M J S
2005
2006
2007

Results over Time, continued

- The next chart depicts the 2 regression coefficients ζ_1 and ζ_2 for quality levels over this time period.
- The chart shows that ζ_1 rose a bit over the period, and spiked up recently because of financial market turmoil. The coefficient ζ_2 also increased on balance.

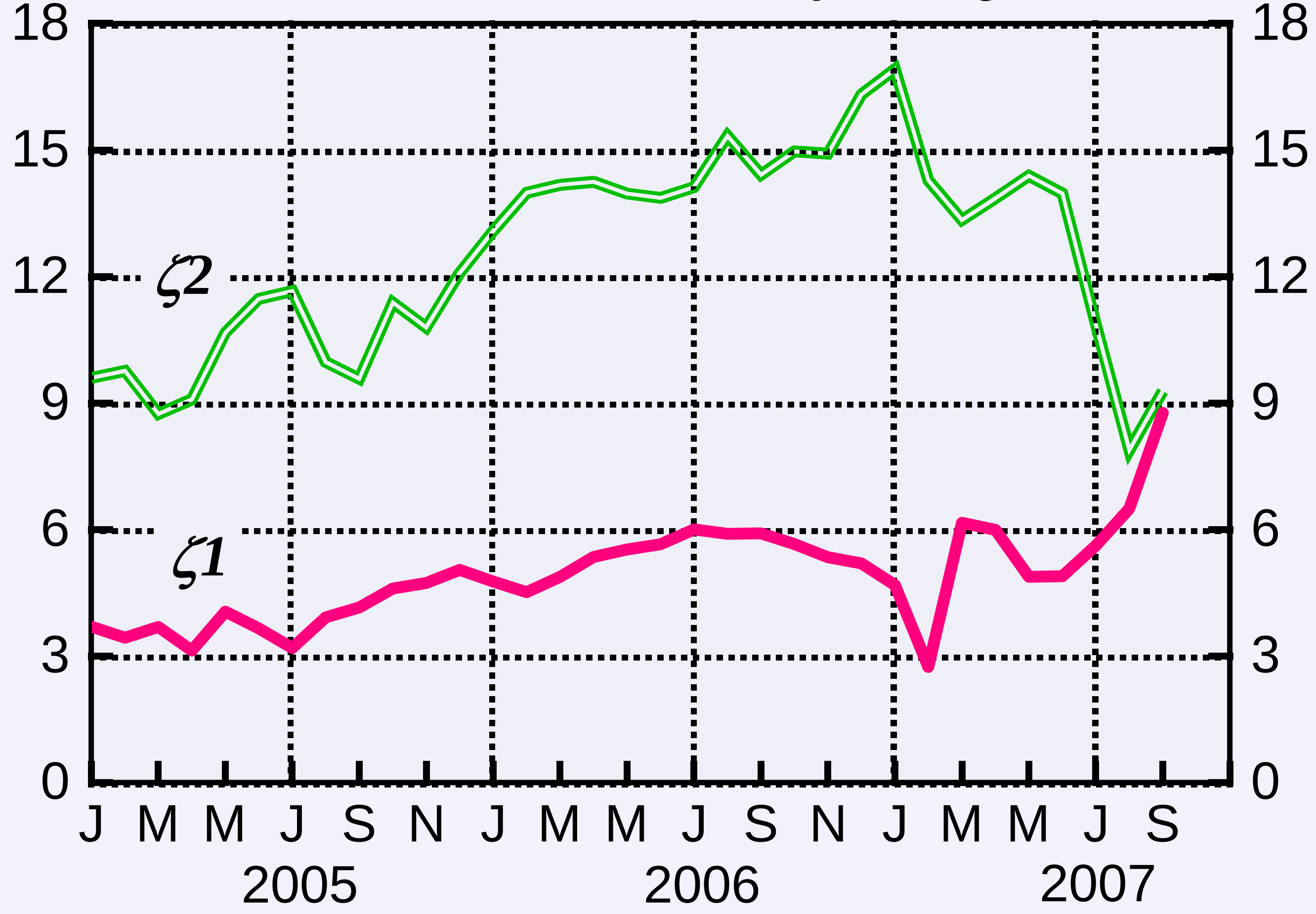
REGRESSION COEFFICIENTS
HQM CORPORATE BOND YIELD CURVE
1/2005 - 9/2007, Basis Points, Monthly Average



Results over Time, continued

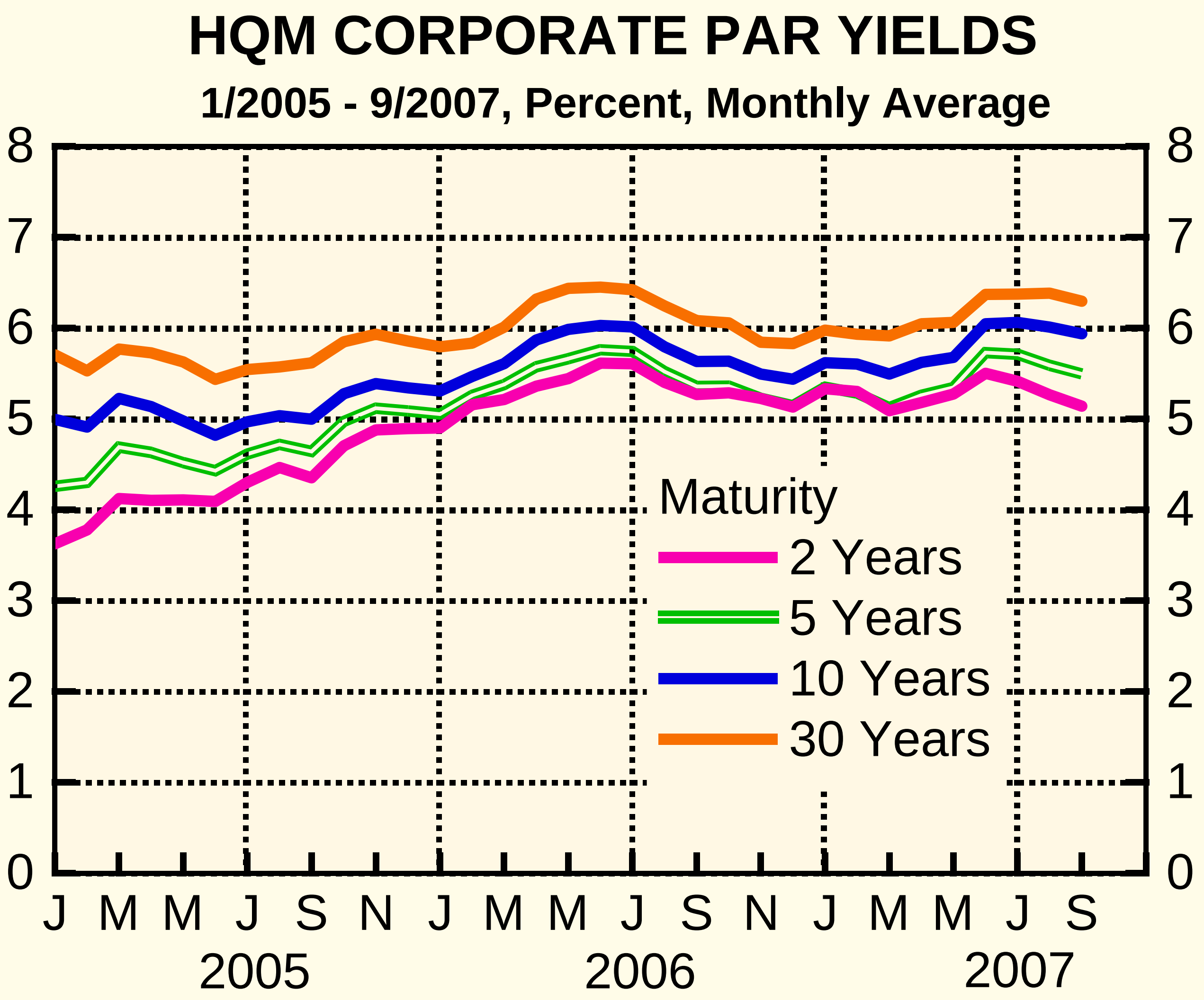
- And the following chart depicts the t-ratios for the regression coefficients over this time span.
- The chart shows that the coefficients were very statistically significant throughout the period.

REGRESSION T-RATIOS
HQM CORPORATE BOND YIELD CURVE
1/2005 - 9/2007, Monthly Average



Results over Time, continued

- The next chart plots monthly average par yields over the same period, at the four maturities of 2 years, 5 years, 10 years, and 30 years.
- The chart indicates some flattening of the par curve over this period.

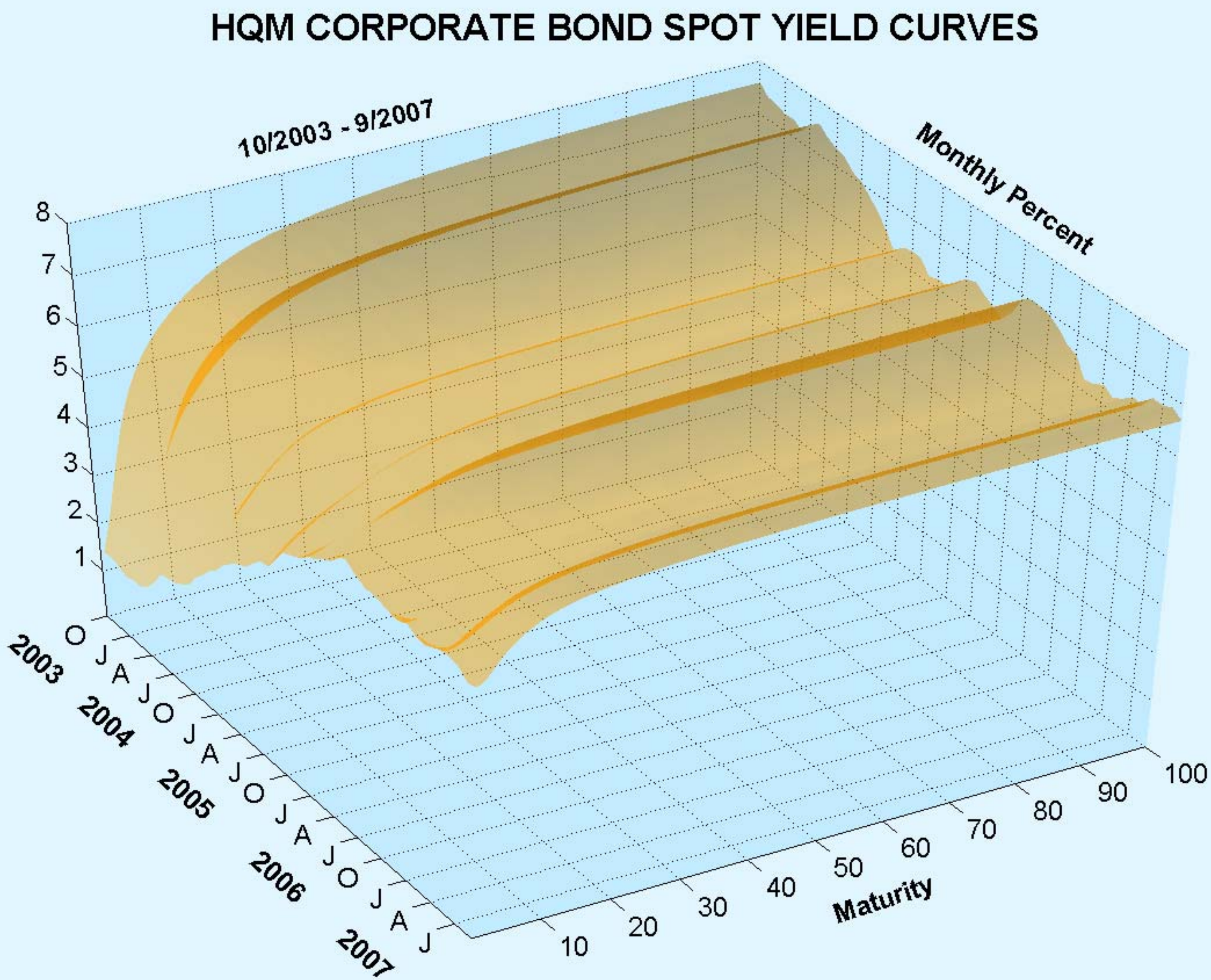

HQM CORPORATE PAR YIELDS
1/2005 - 9/2007, Percent, Monthly Average
Maturity
2 Years
5 Years
10 Years
30 Years
0
1
2
3
4
5
6
7
8
J M M J S N J M M J S N J M M J S
2005
2006
2007

Results over Time, continued

- All these charts show that the 7 estimated coefficients in the White Paper HQM methodology have evolved smoothly over this period.
- The final chart is a three-dimensional chart that plots monthly averages of the HQM spot yield curves for the four years October 2003 through September 2007. The monthly average spot curve at each month is shown as a front-to-back slice with the spot rate on the vertical axis and maturity on the axis extending into the slide away from the viewer.

Results over Time, continued

- The chart shows that these curves have evolved very smoothly throughout the period while picking up market movements. At 100 years maturity, the spot rates start over 7-1/2 percent in late 2003 and end around 6.7 percent in September 2007. At 30 years maturity the rates end at about 6-1/2 percent.
- At the short end, the spot rates rise over time from about 1-1/4 percent to about 5-1/2 percent and then drop off very recently, reflecting the Federal Reserve tightening that began in the middle of 2004 and the easing in September 2007. These movements mirror the behavior of the short-term β coefficients.



HQM CORPORATE BOND SPOT YIELD CURVES
10/2003 - 9/2007
Monthly Percent
8
7
6
5
4
3
2
1
2003
O
J
A
J
2004
O
J
A
J
2005
O
J
A
J
2006
O
J
A
J
2007
10
20
30
40
50
60
70
80
90
100
Maturity

Conclusion

This presentation describes the Treasury White Paper methodology for constructing the High Quality Market-Weighted (HQM) Corporate Bond Yield Curve.

The White Paper methodology meets the yield curve requirements, and provides smooth and robust yield curve results over time that capture market behavior.