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DYNAMICS AND EQUILIBRIUM IN A STRUCTURAL MODEL OF COMMERCIAL AIRCRAFT OWNERSHIP

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"Dynamics and Equilibrium in a Structural Model of Commercial Aircraft Ownership"

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Abstract

This paper develops and estimates a dynamic equilibrium model of the market for commercial aircraft. Airline choices are modeled as the solution to a discrete time dynamic programming problem, where in each period, each airline chooses one or more of various models of new and used aircraft. In equilibrium aircraft prices are such that no airline would benefit from buying, selling, trading or scrapping aircraft. The parameters of the model are estimated by maximum simulated likelihood using a new dataset that contains all aircraft transactions made in the twenty-year period 1978-1997. The transaction data is merged with a dataset containing aircraft prices. The estimated model is used to show that a 10 percent investment tax credit on the purchase of new aircraft has only a small effect on airline behavior and that the demand for new durable goods is more elastic than previous studies have shown. In addition, forcing the modernization of older aircraft causes U.S. airlines to reduce the number of older aircraft they operate by approximately 4 percent, and it is shown that the new aircraft are the poorest substitute for older aircraft.

JEL Classification: L20, C51

Key Words: Dynamic structural model, secondary market equilibrium, durable goods, airline economics, simulation estimation

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1 Introduction

This article examines the effects of durability, secondary markets, and multi-unit ownership on demand for new and used wide-body commercial aircraft. Theoretical models of durable goods markets have shown that a good's durability and resellability can have meaningful effects on its value. Durable goods are often useful for several periods, and therefore, a good's expected viability in the future impacts its value to consumers in the current period. Furthermore, buyers of new and used durable goods are often sellers of used goods as well, and therefore the expected future market prices of durable goods impact the opportunity cost of keeping a used durable in the current period.

Several recent empirical models of demand for durable goods have employed variations of a static discrete-choice logit framework, and have focused on developing methods that allow for very general patterns of substitution across differentiated products and heterogeneity in consumer preferences.[1] However, the static frameworks typically ignore features that are central to many theoretical models if durable goods markets, like intertemporal dependence across consumers' ownership decisions and secondary markets for used goods.[2] The dynamic model developed in this paper allows consumers to consider the effects decisions made in the current period have on the (expected) payoffs they will receive in the future, and assumes that consumers participate as both buyers and sellers in the market for used goods.

Due to computational complexity and/or the lack of good data, estimation of structural dynamic models is rare. Rust (1987) developed a framework for estimating a dynamic model of a single agent's decision to purchase or replace a single durable good (a commercial bus engine).[3] The estimated parameters of the discrete time, discrete choice, dynamic model showed that the agent's behavior was consistent with the solution to a regenerative optimal stopping problem that formalized the trade-off between minimizing expenditures on maintenance and replacements and minimizing the occurrence of break-downs in the service the good provided.

In spite of the large volume of trade in most markets for used durable goods,[4] there have been few empirical studies that account for the effects of used markets on demand for durable goods. In a static frameworks, Manski (1983) and Berkovec (1985) estimated models of the market for

[1] See, for example, Berry, Levinsohn and Pakes (1995), Goldberg (1995), and Petrin (2002).

[2] There is a large body of theoretical work on the impact of good durability and secondary markets for durable goods on demand that is not covered in this paper. See Smith (2005) for a review of this literature.

[3] Kennet (1994) and Cho (2004) used Rust's framework to study the optimal timing of the replacement of aircraft engines and the optimal timing of repair or replacement of mainframe computers respectively.

[4] For the period 1978-1997 approximately 61% of all aircraft sold were used. And active secondary markets are not unique to commercial aircraft. For example, approximately 67% of all automobile sales take place in secondary markets.

new automobiles while controlling for the stock of used automobiles. Both authors developed computable general equilibrium models that controlled for the effects markets for used automobiles have on equilibrium in the market for new automobiles (and visa versa) in each period. Although the modelling approach has several nice properties and is easy to compute, it does not explicitly account for the intertemporal linkages in individual consumer purchase decisions the way the dynamic structural model developed in this work does. In more recent work, Engers, Hartmann and Stern (2005) use a dynamic structural model to quantify the equilibrium effects of lemons in the market for used cars.[5] Similarly to Rust (1987), Engers et. al. use a single agent, single asset framework where car owners are permitted to trade-in the used car they currently own for another new or used car in every period.

Similarly to previous work, the dynamic model developed below allows for intertemporal substitution by consumers and, in addition, expands the set of choices consumers can make in each period. In contrast to owners of cars or boats, owners of goods like buses, airplanes, or computers maximize their expected discounted present value by choosing or maintaining the entire portfolio of several different models and ages of a good in each time period.[6] Specifically, in each period each airline chooses a fleet of aircraft to maximize the expected discounted present value of its profit flows by deciding whether to engage in one or more of the following transactions: selling one or more used aircraft, buying one or more used aircraft, and buying one or more new aircraft. Airline choices are made simultaneously, and equilibrium is attained when, given the allocation of aircraft and aircraft prices, no airline wants to buy, sell or trade any aircraft. It is shown below that, given the specification of the model, as long as transactions costs are low enough and/or there are sufficient taste differences across airlines, a market for both new and used commercial aircraft exists.

The estimates of the structural model show that transaction costs and adjustment costs are substantial. In fact, it is not uncommon for the purchase of a single aircraft to have costs associated with the transaction and fleet adjustment that exceed the aircraft's market value. Additionally, differences in airlines' strategic choices of route structures are reflected in their varying preferences for scale of operation, rate of growth, and the types of aircraft they operate. Specifically, Large U.S. airlines have exhibited the highest rate of growth in the overall use of wide-body aircraft; airlines outside of the U.S. have the greatest relative preference for newer aircraft; and Boeing aircraft are appealing to airlines throughout the world, while Douglas and Lockheed aircraft appeal primarily to U.S. airlines, and Airbus aircraft are most popular outside of the U.S.

[5] Esteban and Shum (2004), and Benkard (2004) develop dynamic models that explicitly consider the impact of used product markets on the strategies employed by manufacturers of durable goods. In contrast to this research, all of the dynamics in Esteban and Shum (2004), and Benkard (2004) are on the supply side of the model.

[6] Hendel (1999) shows that considering the decisions made by multi-unit owners in each period to be independent can systematically bias parameter estimates.

One of the benefits of estimating a structural model is that one can evaluate the effects of changes in market characteristics and policies on equilibrium outcomes. In addition, the evaluation of economic policies that effect the airline industry has intrinsic value. The sale of commercial aircraft generates billions of dollars every year and therefore changes in economic policy that affect the market for commercial aircraft can have profound effects on national and even global economies.[7] The estimates of the structural model developed below are used to analyze the effects of two economic policies on airline investment in new and used aircraft. First, the effects of instituting a 10 percent tax credit for investment in new capital goods are analyzed. The 10 percent investment tax credit is identical to the one removed by the Tax Reform Act of 1986. Second, the effects of implementing stricter safety and/or noise abatement legislation are evaluated.

The results of the first counterfactual experiment show that the implementation of an investment tax credit generates only small increases in total aircraft ownership by U.S. airlines, and slightly decreases aircraft ownership in other parts of the world. Additionally, the policy induces a small amount of substitution toward new aircraft from used aircraft. The experiment indicates that demand for new commercial aircraft is inelastic, although elasticity estimates are quantitatively larger than those from previous studies of durable goods industries. The increase in price elasticity is likely due to the model accounting for both used goods markets and intertemporal substitution by consumers, and both features of the market increase consumers' sensitivity to changes in the price of new goods.

Stricter noise safety and/or noise abatement policy is simulated by imposing a one-time additional cost to operating older aircraft. Newer generations of aircraft are designed to be safer and quieter than previous generations. The design of the experiment mimics the hushkits and improvements airlines purchase to modernize older aircraft. The hushkit mandate reduces the number of older aircraft operated by U.S. airlines. Interestingly, the number of older aircraft operated by airlines outside of the U.S. increases slightly. However, the total number of older aircraft scrapped does increase. New aircraft are shown to be the worst substitute for the oldest vintages of aircraft. Due to the dynamics in the model, U.S. airlines reduce the number of older aircraft they operate well in advance of the mandated policy.

The remainder of the paper is organized as follows. The next section provides a brief history of the market for wide-body commercial aircraft. Section 3 describes the data used to estimate the structural model. Section 4 details the structural model and discusses some of the implications of its specification, including the existence of equilibrium. The model solution and estimation techniques employed in this research are discussed in Sections 5 and 6 respectively. Section 7

[7] The price of a new wide-body commercial aircraft range from \$10 million to over \$100 million in my sample period, and the sale of wide-body commercial aircraft produced \$36 billion in revenue in 1997.

discusses the estimation results, and Section 8 performs two counterfactual simulations using the estimated structural model. Finally, Section 9 concludes.

2 The Market for Wide-body Commercial Aircraft

In 1970, Pan American Airlines introduced the 500-passenger Boeing 747 aircraft as the "wave of the future" for long-distance passenger flights. Although initially other major carriers were more reserved in their optimism for the bigger-is-better notion that was being promoted by Pan Am and Boeing, eventually all major carriers were using wide-body jets to service their longer routes. Unfortunately for wide-body commercial jet producers, the introduction of wide-body jets occurred at a very tenuous time for many airlines. As airlines were introducing additional capacity to carry passengers through wide-body jets, growth in air-travel was stagnating.

After a number of airlines appealed to the Civil Aeronautics Board (CAB), the authority that regulated the setting of fares prior to deregulation, the CAB granted a general increase in airfares (about 6 percent) in the hope that airline revenues would increase. However, after the increase in fares, growth in passenger traffic decreased even more.

Internationally, Pan Am and TWA were feeling the pressure of competition, as low-fare charter services were increasing their market share, especially on the transatlantic routes. After a series of discussions that lasted through 1971, all the major transatlantic carriers agreed to fix fares for the transatlantic route, making the average flight across the Atlantic Ocean about $204. The lower fares introduced intercontinental travel to a whole new social class making transatlantic flights affordable for the middle class.

In spite of the facts that the oil crisis of the early 1970s raised airline fares significantly and decreased the rate of growth in passenger flights, most of the major airlines were able to survive. Air traffic growth slowed throughout the 1970s, but there was never a decline in the number of flights. In fact, the number of passengers continued to grow despite the fare increases, evidence of the public's increasing reliance on flying, now synonymous with jet aircraft, as a routine activity.

Fascination with the Boeing 747 generated enough momentum to produce two additional wide-body jets. The Douglas DC-10 was introduced by American Airlines in 1971, and the Lockheed L-1011 was introduced by Eastern Airlines in 1972. Both aircraft were capable of carrying about 300 passengers and had a range of 2,500 to 3,000 miles. During the period 1978 through 1997, there were a total of ten different wide-body aircraft introduced by four manufacturers: Boeing, Airbus, Lockheed-Martin, and McDonnell-Douglas. Even though there have never been more than four manufacturers of wide-body jets, competition among the manufacturers and from used aircraft sold in secondary markets have made it tough for manufacturers to be profitable. Currently, there are only two manufacturers producing wide-body jets. Lockheed-Martin recorded negative profits in every year its L-1011 was manufactured and finally exited the market in 1985. McDonnell-Douglas, the manufacturer of the DC-10 and MD-11 aircraft, was acquired by Boeing in 1997.

Since their introduction, sales of wide-body commercial aircraft have increased steadily. In 1997, the sale of new wide-body commercial aircraft generated $36 billion in revenue, which accounted for approximately 60% of the revenue and 30% of the number of units sold in the industry. Currently, commercial aircraft are one of the Unites State's largest net exports, accounting for an average of $25-30 billion in trade surpluses in the 1990s. U.S. airline consumption of wide-body jets has been consistent, and there has been significant growth in sales to foreign airlines. In total, over 500 passenger, charter, and freight airlines have, at one time or another, owned a wide-body jet.

In addition to the sale of new wide-body jets, an active secondary market for used wide-body commercial aircraft has developed and grown in the past 30 years. Used wide-bodies made up about 50% of the total number of units sold in 1978 and grew steadily until they made up approximately 85% of units sold in 1997. Growth in the volume of used aircraft sales is due primarily to the market for wide body jets being relatively young and aircraft being extremely durable goods.[8] Therefore, new wide bodies have continued to enter the market and very few used wide-body jets have been scrapped.

The market for used wide-bodies has similarities to and differences from the market for used cars. Like automobile owners, when an airline wishes to sell one or more aircraft, it publishes an advertisement in one of many trade journals and sells the aircraft to the highest bidder. One significant difference between the market for wide-body jets and automobiles is that there is likely to be little or no asymmetric information about the quality of used commercial aircraft. There are a couple of reasons why this conjecture is likely to be true. First, wide-body aircraft are very expensive and obtaining information about their quality is relatively inexpensive. In fact, the data that I use for this project contains information on almost everything one could possibly want to know about an aircraft, from when it was damaged on a landing to when a row of seats was changed. Second, sellers in this market cannot afford to get a reputation for selling poor quality aircraft. Everyone knows who the sellers are, and, in general, commercial aircraft owners sell in secondary markets with greater frequency than do automobile owners. So selling a poor quality aircraft and withholding information has the potential to be particularly costly to airlines.

[8] The average wide-body jet has a life-cycle of 25-35 years.

3 Data

In this section I present an overview of the data. I present some descriptive statistics and information about how the data were compiled and aggregated to estimate the structural model. The aircraft transaction data were provided by Back Aviation Solutions (BAS) and transaction prices and aircraft appraisals were provided by Avmark Incorporated (AI).[9]

3.1 Transaction data

The transaction data include detailed information about every aircraft that was registered to fly in the past 60 years at several points in time. The data provide a rich set of information about each change in aircraft operator or owner for each aircraft in the sample, including the day of the transaction, the type of transaction (e.g. sale, lease, sublease, or retirement), the identities of the buyer and seller of the aircraft, and the condition of the aircraft when it was sold.

The BAS data contains information for all types of aircraft from small private jets to large commercial aircraft. This research is limited to the just over 3500 wide-body commercial aircraft used during the twenty year period 1978-1997. I focus on wide-body jets because (1) a model of the entire commercial aircraft market would be computationally infeasible, and (2) focusing on wide-body commercial jets is consistent with other work in the literature (e.g. Benkard (2004)). Moreover, the market for wide-body commercial jets is appealing because it generates more revenue in both primary and secondary markets than all other types of commercial aircraft combined.

In the theoretical model of airline behavior, airlines make ownership decisions in several discrete time periods. I choose a length of time for each decision period that is both consistent with the theoretical model, and that is tractable from a model solution and estimation standpoint. The BAS data is extremely detailed and provides the day of each transaction. However, the time cost associated with solving and estimating the model increases dramatically as the number of time periods increases. Therefore, I consider only changes in ownership that last a year or longer sales. One year decision periods allow the model to be solved and estimated, and are consistent with other models in the literature on dynamic decision problems. Obviously aggregating time periods in this way disregards any information provided by ownership spells lasting less than a year, and the model's estimates may therefore underpredict changes in aircraft ownership. However, some of the extremely short ownership spells observed in the data do not indicate actual changes in aircraft ownership.[10] Some sell and buy-back transactions may simply be a form of refinancing or outsourcing of resources by the original owner of the aircraft. For example, some of the sell and buy-back decisions occur

[9] I thank Todd Pulvino at the Kellogg Graduate School of Management for providing me with the Avmark data.

[10] Some ownership spells are as short as one day.

seasonally, and it is hard to differentiate the sale and buy-back from the original owner hiring the buyer to fly the planes on a different set of routes for a short period of time.

The data provided by BAS also include measures of several observed attributes of each aircraft in the sample. The attributes included are as general as the aircraft's make and model and as specific as the height of the aircraft's cargo door in centimeters. Airlines consider carefully the technical specifications of an aircraft prior to making a purchase, and monitor each aircraft's condition as it ages. Therefore, it is reasonable to assume that the observed attributes of aircraft influence the value of aircraft to airlines. However, I sacrifice some of the information provided by the detailed data on observed differences across products in order to facilitate model solution and estimation. Specifically, I aggregate the over 3500 aircraft into 20 distinct types, where a aircraft's type is determined by its model and vintage.

3.2 Price data

The transaction data are merged with observed prices and appraisals provided by Avmark Incorporated (AI). The data include actual transaction prices for 1165 deliveries and sales for the period 1978-1993. Prior to 1993, airlines in the United States were required by law to report the price of any new or used aircraft they purchased or sold. The data was originally reported to the Department of Transportation and the Federal Aviation Association and was then compiled by Avmark. In addition to observed transaction prices, the Avmark data also contains appraisal values for aircraft for the period 1978-1997. The appraisals are computed using observed aircraft prices. In fact, Avmark's appraisals for the period 1978-1993 can be duplicated using cell means from the observed price data. Pricing information on particular aircraft is sacrificed for the period 1994-1997 because airlines were not required to divulge the price of aircraft that they purchased or sold during that time.

Observed aircraft prices are used to create a value for every type of aircraft in every period. I refer to this single price for each type of aircraft in each period as a price index. There are two reasons for creating a price index. First, there are several transactions for which I do not observe a price. For example, leases are considered a change in ownership in the theoretical model, but I do not observe a transaction price for ownership changes that involve a lease. Second, aircraft are aggregated into types and all aircraft of the same type are assumed to be identical. Observed prices and appraisal values are used to construct the Specifically, I run a regression of observed log prices on aircraft make, model, age, aircraft characteristics, and a full set of age and time dummies. The fitted values of the regression are then used to create a price for each type of aircraft in each time period. The robustness of the imputation is checked by comparing the prices to aircraft appraisals computed by Avmark Incorporated.

3.3 Descriptive Statistics

Table 1 gives the frequency of each model of aircraft observed in the sample (all tables and figures are in Appendix E. The most frequently observed aircraft in the sample is the Boeing 747 with 1078 aircraft. As was mentioned above, the 747 was also the first wide-body aircraft to enter the market, making its first passenger flight in early 1970 for Pan Am Airlines. The model with the fewest aircraft in the sample, the Boeing 777, was also the most recent introduction, making its first passenger flight in 1995.

Table 2 gives the means of selected aircraft characteristics for aircraft observed in the sample period 1978-1997. The mean length of a wide-body commercial jet is approximately 59 meters. The Boeing 747 is the longest model of aircraft at over 70 meters, whereas some versions of the Airbus A310 were just under 50 meters long. The average aircraft in the sample carries about 410 passengers. The Boeing 747 can be outfitted to carry 600 passengers, whereas the Boeing 767 has a maximum seating capacity of 280 passengers. The mean fuel capacity for aircraft in the sample is 128,000 gallons, and the average range of flight when at full capacity is approximately 5,900 miles. This implies that the average aircraft in the sample requires 100 gallons of fuel to fly just over 4 miles. Being the largest aircraft, the Boeing 747 also has the greatest fuel capacity and range of flight at approximately 400,000 gallons and 9,000 miles respectively.

Figures 1 and 2 describe patterns of aircraft movement from owner to owner observed in the data. Each line in Figure 1 measures the proportion of each make aircraft of a particular age that were sold in secondary markets in each period. Perhaps intuitively, aircraft tend to be sold more frequently as they age. For example, only 1.5% of Boeing aircraft in their fifth year are sold in secondary markets, while 6.4% of Boeing aircraft in their fifteenth year are sold in secondary markets; trends are similar for Airbus, McDonnell-Douglas, and Lockheed-Martin aircraft. Airbus aircraft were sold the most frequently for 13 of the 19 age categories, while Boeing aircraft were sold with the least frequency for ten of the first 11 age categories. Assuming there is no asymmetric information across airlines about the quality of aircraft,[11] there are a few possible explanations for one type of aircraft being sold more frequently than others. Aircraft may be sold more frequently in secondary markets because they physically degrade and become relatively ineffective more quickly than other aircraft. If an aircraft degrades rapidly, it rapidly moves out of the range of desired usefulness to its current owner, and therefore the aircraft will be sold to another airline relatively quickly. Another possible

[11] This assumption is reasonable because airlines can obtain almost costlessly a complete record for any aircraft.

reason for one aircraft being traded relatively often in secondary markets is that the aircraft has low transaction costs (e.g. costs associated with learning to operate the aircraft, cost of adapting the aircraft for different tasks, or a lack of warranties or guarantees). As is discussed in Anderson and Ginsburgh (1994), and Konishi and Sandfort (2002), an aircraft with relatively low transaction costs is likely to be sold with greater frequency in secondary markets.

Figure 2 shows the proportion of aircraft sold in secondary markets that are sold to passenger airlines. As might be expected, in general older aircraft are less likely to be purchased by a passenger airline and are more likely to be purchased by a freight or charter airline. Of Boeing aircraft sold in secondary markets, approximately 84% of the newest age range are sold to passenger airlines, while 70% of the oldest age range are sold to passenger airlines. The drop in percentage of secondary market purchases made by passenger airlines is much more dramatic for Airbus aircraft. Approximately 87% of the newest category of Airbus aircraft sold in secondary markets were sold to passenger airlines, while only 41% of the oldest category of Airbus aircraft sold in secondary markets were sold to passenger airlines. The percentage of McDonnell-Douglas aircraft sold to passenger airlines is very consistent, staying at about 80% for all age categories, while the percentage of Lockheed-Martin aircraft sold to passenger airlines is, in general, lower averaging between 50% and 60%. An aircraft is sold to freight or non-passenger airlines earlier in its life-cycle because (1) freight and non-passenger airlines care less about the aircraft's level of degradation than passenger airlines, and the aircraft physically depreciates more quickly than other aircraft, or (2) the aircraft is more easily converted to serve the purposes of freight or non-passenger airlines (i.e. lower transaction costs).

Figure 3 shows trends in the volume of trade in markets for new and used wide-body commercial aircraft. The growth in the number of new aircraft delivered into the market over time is due primarily to an increase in the number of new aircraft delivered to non-U.S. airlines. The figure shows very clearly that the trend in the number of deliveries of wide-body aircraft taken by US passenger airlines is relatively flat, while the trend in total deliveries of wide-bodies is slightly upward sloping. The trend in secondary market trades per period is steeply upward sloping. The increase in secondary market activity is not surprising because the secondary market for wide-body commercial aircraft is still relatively young (the oldest aircraft in the sample are about 25 years old), which implies that aircraft have continuously entered the market but very few aircraft have been scrapped.

4 Structural Model

At each time t, each airline maximizes the expected discounted present value of its current and future profit flows by choosing a fleet of aircraft to own and operate. An airline may change its fleet by selling one or more used aircraft, buying one or more used aircraft, and/or buying one or more new aircraft. Airline fleets are chosen from an exogenously determined quantity of new aircraft that enter the market in each period, and a stock of used aircraft is carried over from the previous period. Airline preferences vary over the different makes, models, and ages of aircraft; and, in addition, airlines may prefer different scales of operation. All airlines make ownership decisions simultaneously which determines the equilibrium quantity and price for all new and used aircraft. Specifically, airline decisions are consistent with an equilibrium where no airline wants to buy, sell, or trade any of its aircraft given the current prices and allocation of aircraft.

Typically, discrete choice models of demand limit the choices of buyers to purchasing a single unit of one of the products available or choosing an outside option (generally, the outside option is choosing to not purchase any of the available products). An agent that purchases more than one unit of the available products in a single period is assumed to have made each purchase decision independently. The independent discrete choice assumption is made because (1) the modeler believes the assumption is reasonably close to reality, and (2) the assumption greatly reduces the computational burden of estimating the model. However, in the case of commercial aircraft, airlines are known to make multiple ownership decisions simultaneously in each decision period. Therefore, ignoring the multiple-discrete choices made by airlines would discard valuable information and would likely distort estimation results. The model specified here allows agents to choose multiple aircraft in each period. Hendel (1999) refers to a model that allows agents to simultaneously choose multiple products as a multiple-discrete choice model.[12] Allowing airlines to choose combinations of multiple units greatly increases the total number of possible outcomes in each period, and therefore greatly increases the computational complexity of solving and estimating the model. I mitigate much of computational burden created by the increase in the number of choices afforded to airlines, by imposing enough structure on airline value functions to derive an expression for the marginal contribution of each aircraft to the airline's fleet. These marginal values are then used to derive a tractable number of equilibrium conditions which are used to estimate the parameters of the model.[13]

The market for commercial aircraft consists of manufacturers, airlines, and scrappers (who are responsible for disposing of unwanted aircraft). Admittedly, the models assumptions greatly simplify

[12] Similar modelling approaches have been used in other fields. For example, Howell (2004) models the application stage of a students college enrollment decision as a multiple discrete choice over all postsecondary institutions. That is, students are allowed to apply to multiple institutions contemporaneously.

[13] The equilibrium concept used in this paper is similar to the equilibrium assumed by Howell (2004), in her structural model of students' college application and admission decisions.

the behavior of manufacturers and scrappers.[14] However, the focus of this work is on the dynamic equilibrium behavior of airlines, and the simplifications are necessary to make the model tractable.

4.1 Scrappers

Following Rust (1985), I assume that scrappers have an infinitely elastic demand for each type of aircraft at a fixed price $\underline{P}_j$. That is, scrappers are always willing to acquire and scrap any number of aircraft at a price of $\underline{P}_j$. For the empirical work, I assume $\underline{P}_j = 0$ for all j, regardless of the aircraft's type or level of degradation.[15]

4.2 Manufacturers

Let there be an arbitrarily large but finite number of discrete time periods indexed by $t = 1, 2, ...T$. Furthermore, let $m = 1, .., M$ index the different makes of aircraft available in the market, and at each time t, assume there are J_t types of aircraft available, where types are indexed $j = 1, ..., J_t$. I assume all aircraft of the same type (i.e. model and vintage) are identical. The total number of distinct types J_t vary across time periods because new vintages of aircraft enter the market in each period and old vintages of aircraft stay in the market for several periods. I assume that the number of new type j aircraft supplied to the market at time t is exogenously specified. Once new aircraft enter the market, they are allocated to airlines or the scrapper by an efficient auction mechanism.

4.3 Airlines

Used aircraft carried over from the previous period and the exogenously determined quantity of new aircraft supplied to the market comprise the stock of aircraft available to airlines in each period. Each new and used aircraft available in each period is acquired by either an airline or the scrapper. If an aircraft is acquired by an airline, it will again be available to airlines for purchase in the following period. If an aircraft is acquired by the scrapper, it is destroyed and leaves the market forever.

I airlines, indexed $i = 1, ..., I$, are in the market at each time t. The only non-airline consumer of aircraft, the scrapper, is indexed by $i = 0$. In addition, I assume that there are $K < I$ different groups of airlines indexed $k = 1, ..., K$ that have similar preferences for aircraft. Airlines are separated into groups by size and the service they provide (e.g. passenger airline or freight airline).

[14] Benkard (2004) focuses on the strategic interactions of aircraft manufacturers. He uses the estimates of his model to analyze industry pricing, industry performance, and optimal industry policy. Benkard estimates a static discrete choice demand system and treats used aircraft as additional differentiated products.

[15] Assuming a price of zero for scrap aircraft is not a significant deviation from reality. New and used aircraft that are still in use are generally exchanged for millions of dollars, whereas scrappers can typically acquire unwanted aircraft for less than \$100,000.

Airline choices at time t are summarized by the vector $q_{it} = (q_{i1t}, ..., q_{iJ_tt})$, where q_{ijt} is the quantity of type j aircraft that airline i owns at time t. Airline i may alter q_{it} by buying aircraft, selling aircraft, and/or scrapping aircraft. There is a finite quantity of type j available at time t, $\overline{q}_{jt}$. Therefore, the set of feasible airline choices are all possible permutations of the vector $(q_{i1t}, ..., q_{iJ_tt})$ such that $0 \leq q_{ijt} \leq \overline{q}_{jt}$. Airlines choose a fleet of aircraft to maximize the expected discounted present value of their profit flows.

4.3.1 Profit flow equations

The profit flow generated by airline i at time t is a function of the state vectors s_{it} and ξ_{it}. s_{it} is a vector of observed state variables which include airline i's fleet choice in the previous period $q_{it-1} = \left(q_{i1t-1}, ..., q_{iJ_{t-1}t-1}\right)$, aircraft prices $P_t = (P_{1t}, ..., P_{J_tt})$ in the current period, and the time t. Each q_{it-1} was determined the previous period, and P_t is determined by equilibrium (the equilibrium of the model is described in detail below). $\xi_{it} = (\eta_t, \varepsilon_{i1t}, ..., \varepsilon_{ijt}, ..., \varepsilon_{iJ_tt})$ is a vector of random state variables, where η_{kt} is a unobserved airline group-specific random variable which captures the effect of time trends and shocks on the demand for air-travel,[16] and ε_{ijt} is a unobserved shock to the match i, j at time t. ξ_{it} captures all influences on airline profit not observed by the econometrician. Agents in the model know the current realization of ξ_{it}, but know only the distribution of future realizations $\xi_{it+1}, \xi_{it+2}, ...$. A detailed description of the joint distribution of the components of ξ_{it} is given in the estimation section.

Airline i's profit flow at time t is determined by the attributes of the individual aircraft they own, scale effects, transaction costs, and adjustment costs. X_{jt} is a N-dimensional vector of aircraft j's observed attributes in period t, where the elements of X_{jt} are limited to variables that are deterministic and have known paths (e.g. seating capacity and age). The effect of X_{jt} on airline i's profit is a function of a parameter vector representing airline i's group-specific preferences. Airline profit flow is a quadratic function of the total number of aircraft airline i operates at time t. The behavior of airlines observed in the data is only individually rational and consistent with the assumptions of the model if the discounted value of airline profit exhibits decreasing returns to scale at the relevant margin. Buyers of aircraft incur a transaction cost for each aircraft they purchase. The transaction cost varies across aircraft make and captures costs associated with reconfiguring the aircraft, learning to operate the aircraft, and/or forgone warranties or guarantees. Additionally, airlines incur adjustment costs that are quadratic in the difference between q_{ijt} and q_{ijt-1} for all j. Adjustment costs capture the cost of changing an airlines route structure. Transaction and

[16] I assume that different groups of airlines have different preferences for different types of aircraft because they construct their routes in significantly different ways to maximize their objective function. Similarly, time-specific shocks to the demand for air-travel differ across groups of airlines because, for example, freight airlines may be impacted by the introduction of alternative shipping mechanisms that do not affect passenger airlines.

adjustment costs explain the patterns of aircraft purchases and sales observed in the data. Rust (1985) shows that, in the absence of market frictions, agents replace their entire stock of assets in every period. As a result, a model of demand for commercial aircraft that ignores market frictions is likely to over-predict the number of changes airlines make to their fleets in every period. Finally, a airline group-specific time trend is included to capture the growth in the demand for travel on routes serviced by wide-body aircraft (and therefore the demand for wide-bodies).[17] Differences in rates of growth across the different groups of airlines reflect differences in strategies employed by airlines when developing their route structure. In particular, it is possible that one group of airlines has grown by acquiring more international routes, while another group of airlines has expanded mostly in the number of domestic routes they operate.

Using the variables described above, airline profit flow is specified as

$$\begin{aligned}\pi\left(q_{it}, s_{it}, \xi_{it}\right) &= \sum_{j=1}^{J_t}\left\{q_{ijt}\left(X_{jt}\gamma_k + \eta_{kt} + \varepsilon_{ijt}\right)\right. &(1)\\ &\quad \left.+1\left(q_{ijt} > q_{ijt-1}\right)\lambda_m\left(q_{ijt} - q_{ijt-1}\right)\right\}\\ &\quad -c\left(q_{ijt} - q_{ijt-1}\right)^2 - \left(q_{ijt} - q_{ijt-1}\right)P_{jt} - \delta_k\left(\sum_{j=1}^{J_t} q_{ijt}\right)^2 &(2)\end{aligned}$$

Airline i's preferred scale of operation is shifted by the airline group-specific shock η_{kt}, and the idiosyncratic error ε_{ijt}. γ_k is a N-dimensional vector measuring the intensity of a group-k airline's preferences for the observed attributes of aircraft. λ_m is a transaction cost incurred by consumers when purchasing an aircraft. The m subscript indicates that transaction costs may differ across different aircraft makes. The indicator function $1(.)$ equals one if its argument is satisfied and zero otherwise, so that transaction costs are incurred only if the airline increases the number of type j aircraft it owns in period t. c measures the penalty an airline pays when making adjustments its fleet in each period. The penalty is increasing in the magnitude of the changes the airline makes. Finally δ_k measures the rate at which marginal returns to a airline from group k's profit is decreasing in the number of aircraft the airline operates. $\gamma_k, \lambda_m, c, \delta_k$ and the parameters of the joint distributions of η_{kt} and ε_{ijt} are estimated.

[17] The average size of airline fleets has grown in each year of the sample. One possible explanation for this growth is that the market for wide-body commercial aircraft has not yet matured. In particular, the first wide-body aircraft was delivered in 1969 and new wide-body aircraft have been entering the market ever since, but since aircraft have useful lives of 25-35 years very few aircraft have been scrapped during the same time period. Additionally, during the sample period population growth, a growing familiarity with flying, the globalization of the economy, etc. have caused a steady increase in the demand for air travel.

4.3.2 Value Function

Airline i's value function at time t is the expected discounted present value of the profit flow it receives in the current and future periods. That is,

$$V\left(q_{it}, s_{it}, \xi_{it}\right) = E_t \left\{ \max_{q_{it+1},\ldots} \sum_{s=t}^{T} \beta^{s-t} \pi\left(q_{is}, s_{is}, \xi_{is}\right) \right\}, \tag{3}$$

where $0 < \beta < 1$ is a common discount factor. The expectation is taken over the joint distribution of ξ_{it} for all future time periods. The value function given by equation (3) can be expressed in Bellman equation form as

$$V\left(q_{it}, s_{it}, \xi_{it}\right) = \pi\left(q_{it}, s_{it}, \xi_{it}\right) + \beta E_t \left\{ \max_{q_{it+1} \in C(s_{it+1})} V\left(q_{it+1}, s_{it+1}, \xi_{it+1}\right) \right\} \tag{4}$$

where the last term on the right-hand side is the expected value of airline i's best choice in the next period and $C\left(s_{it}\right)$ is airline i's set of feasible fleet choices given the observed state. The choice set $C\left(s_{it}\right)$ contains only a finite number of choices since at equilibrium prices each airline wants only a finite number of aircraft. That is, if there is excess demand for a particular type of aircraft at some price P_{jt}, P_{jt} is too low to be an equilibrium price.

4.4 Equilibrium

Conditional on the value functions described above, equilibrium is defined by a aircraft allocation and prices where no airline wants buy, sell, or trade an aircraft. Each airline maximizes its value function by acquiring the airline fleet that maximizes its value function given aircraft prices, and prices are determined by an equilibrium in the actions of all firms. Specifically, airlines' optimization problems are solved simultaneously, which implies a mapping between airline quantity choices and aircraft prices, and the equilibrium quantities and prices of the model are a fixed point to this mapping. More formally, equilibrium is defined as follows.

Definition 1 *At each time t, a vector of aircraft prices $P_{1t}^*, \ldots, P_{jt}^*, \ldots, P_{J_t t}^*$ and a vector of airline quantity choices $q_t^* = \left(q_{11t}^*, \ldots, q_{ijt}^*, \ldots, q_{IJ_t t}^*\right)$ are an equilibrium if*

$$q_{it}^* = \arg\max_{q_{it}} V\left(q_{it}, s_{it}, \xi_{it}\right) \quad \text{for all } i, \tag{5}$$

and the market clearing conditions

$$\begin{aligned} \text{if } P_{jt}^* &> 0 \text{ then } \sum_i q_{ijt}^* = \overline{q}_{jt} \quad \text{or} \\ \text{if } P_{jt}^* &= 0 \text{ then } \sum_i q_{ijt}^* \leq \overline{q}_{jt} \end{aligned} \tag{6}$$

are satisfied for all aircraft types j.

Equation (5) says that, in equilibrium, all airlines must be maximizing their value function. The market clearing conditions given by (6) say that if prices are positive the supply of type j aircraft must exactly equal the demand for type j aircraft, and establishes a lower bound of zero for all aircraft prices. If the final equation is satisfied with strict inequality, then the excess aircraft are acquired and destroyed by the scrapper.

4.5 Model Implications

This section presents propositions that assert some properties of the value functions that ultimately ensure the existence of the equilibrium defined above. The following assumptions are necessary for the model to have the properties described below.

$A1$: The joint distribution of the errors of the model is continuous.

$A2$: The common discount factor β is positive, constant over time, and less than one.

Assumption $A1$ ensures for each vector of aircraft prices, there is a combination of errors $\xi_{it} = (\eta_{kt}, \varepsilon_{i1t}, ..., \varepsilon_{ijt}, ..., \varepsilon_{iJt})$ that corresponds to every feasible combination of aircraft the airline could own, and that the boundaries that separate adjacent fleets are well-defined in terms of the possible ξ_{it} (this point is discussed in greater detail below). Assumption $A2$ ensures that the value functions are finite.

Additionally, to prove the propositions of this section, it is useful to define the marginal value of a type j aircraft to airline i's fleet as the difference in airline i's value function with q_{ijt} type j aircraft and airline i's value function with $q_{ijt} - 1$ type j aircraft, while leaving the quantity of all other types of aircraft unchanged. Define e_j to be a vector with a one in the jth position and zeros everywhere else. Then the marginal value of a type j aircraft to airline i at time t is expressed

$$\begin{aligned} M_{ijt}(q_{it}) &= V(q_{it}, s_{it}, \xi_{it}) - V(q_{it} - e_j, s_{it}, \xi_{it}) \qquad (7) \\ &= \mu_k + \eta_{kt} + X_{jt}\gamma_k + \varepsilon_{ijt} - 1(q_{ijt} > q_{ijt-1})\lambda_m - \\ &\quad (2q_{ijt} - 2q_{ijt-1} - 1)c - P_{jt} - \left[2\left(\sum_{j=1}^{J_t} q_{ijt}\right) - 1\right]\delta_k \\ &\quad + \beta E_\xi \left\{ \max_{q_{it+1} \in C_i(s_{it+1})} V[q_{it+1}, s_{it+1}, \xi] - \max_{q'_{it+1} \in C_i(s'_{it+1})} V[q'_{it+1}, s'_{it+1}, \xi] \right\}, \end{aligned}$$

where s'_{it+1} and q'_{it+1} are respectively the observed state and expected best choice in period $t+1$ conditional on airline i choosing $q_{it} - e_j$ (instead of q_{it}) at time t. Note that the marginal value of each type of aircraft to airline i at time t depends on the entire vector of aircraft the airline owns, q_{it}.

It is easy to see that the marginal value of an aircraft is increasing in the group-specific return shifter μ_k, the aggregate shock η_{kt}, and the idiosyncratic airline-aircraft match ε_{ijt}; and it is also straight-forward to see that the marginal value of an aircraft is decreasing in its price P_{jt} and the non-linear scale effect δ_k. However, since both current and future profit flows depend on q_{ijt}, and q_{ijt} indirectly influences future choices $q_{i1t+1},...,q_{ijt+1},...,q_{iJt+1}$, it is not as obvious that the marginal value of an aircraft is decreasing in q_{ijt}. The first proposition shows that, given the specification of the model, the value function is finite at mean values of ξ_{it}, and that the marginal value of aircraft is decreasing in the number of each type of aircraft the airline owns.

Proposition 1 *$V(q_{it}, s_{it}, 0)$ is finite, and $M_{ijt}(q_{it}, s_{it}, \xi_{it}) > M_{ijt}(q_{it} + e_j, s_{it}, \xi_{it})$ for all j, q_{it}, s_{it}, and ξ_{it}.*

Proof. All proofs are in Appendix A ■

Proposition 1 states that the value function is finite for finite values of q_{it}, s_{it} and ξ_{it}. Additionally, the fact that the marginal value of aircraft is decreasing implies that, given the observed state vector, s_{it}, and the vector containing all other random components of the model, $\xi_{i-jt} = (\eta_{kt}, \varepsilon_{i1t}, ...\varepsilon_{ij-1t}, \varepsilon_{ij+1t}, ..., \varepsilon_{iJt})$, there exists a reservation value ε^*_{ijt} that separates the higher values of ε_{ijt} where airline i chooses q_{ijt} aircraft j in period t from the lower values of ε_{ijt} where i chooses $q_{ijt} - 1$ aircraft j. That is,

for each observed state, s_{it}, and vector of all other random components, $\xi_{i-jt} = (\eta_{kt}, \varepsilon_{i1t}, ...\varepsilon_{ij-1t}, \varepsilon_{ij+1t}, ..., \varepsilon_{iJt})$, there exist reservation values ε^*_{ijt}, where if $\varepsilon_{ijt} > \varepsilon^*_{ijt}$ then $V\left(q_{it}, s_{it}, \xi_{i-jt}, \varepsilon_{ijt}\right) > V\left(q_{it} - e_j, s_{it}, \xi_{i-jt}, \varepsilon_{ijt}\right)$, and if $\varepsilon_{ijt} < \varepsilon^*_{ijt}$ then $V\left(q_{it}, s_{it}, \xi_{i-jt}, \varepsilon_{ijt}\right) < V\left(q_{it} - e_j, s_{it}, \xi_{i-jt}, \varepsilon_{ijt}\right)$ for all i, j, t and q_{it}.

This result facilitates the derivation of the likelihood function used to estimate the parameters of the model. Specifically, the equilibrium conditions of the model can be expressed as inequalities in terms of the ε_{ijt}, and the probability of observing an equilibrium outcome are computed by integrating over the joint distribution of ξ_{it}.

Given the number of choices available to airlines in each period, searching over all possible quantity vectors to find the quantity vector that satisfies equation (5) for each airline requires the evaluation of $V(q_{it}, s_{it}, \xi_{it})$ at a extremely large number of points. Furthermore, the model must be solved several times to estimate the parameters. Therefore, searching for equilibrium in this manner is computationally infeasible. To mitigate some of the computational burden associated with solving the model, I use the model's theoretical properties to reduce the number of quantity vectors that must be considered in each period to find equilibrium. The following proposition states that, given the specification of the model, if an airline's value function does not increase by choosing any of the vectors of aircraft that are adjacent to the vector of aircraft it currently holds, then there are no alternative fleets of aircraft that are an improvement over its current fleet. That is, if an airline choosing quantity vector q_{it}^* does not want to add an aircraft to its fleet, subtract an aircraft from its fleet, or swap one aircraft out of its fleet for another aircraft, there are no alternative quantity vectors that airline i prefers to q_{it}^*.

Proposition 2 *Given equilibrium prices, P_t^*,*

$$q_{it}^* = \arg\max_{q_{it}} V\left(q_{it}, s_{it}, \xi_{it}\right)$$

if and only if

$$M_{ijt}\left(q_{it}^*\right) > 0 \quad \forall j, \tag{8}$$

$$M_{ijt}\left(q_{it}^* + e_j\right) < 0 \quad \forall j, \tag{9}$$

and

$$M_{ijt}\left(q_{it}^*\right) - M_{ij't}\left(q_{it}^* + e_{j'} - e_j\right) \quad > \tag{10}$$

$$0 \; \forall j, j' \;\; \text{and} \;\; j \;\; \neq \;\; j'. \tag{11}$$

The conditions given by equations (8) and (9) can be thought of as a discrete-choice analog to a set of first order conditions for airline i's optimization problem at time t. The last condition states formally the revealed preference argument that the last aircraft that an airline acquires must be more beneficial than any alternative aircraft. This condition together with the conditions of Proposition 1 are a discrete-choice analog to the an airline satisfying its second order conditions. An important implication of Proposition 2 is that, given a vector of equilibrium prices P_t^*, satisfaction of the conditions on q_{it}^* given by equations (8), (9), and (10) for all i is both necessary and sufficient for $q_t^* = \left(q_{1t}^*, ..., q_{It}^*\right)$ to be an equilibrium as defined above.

4.5.1 Existence of Equilibrium

Given the assumptions of the model, a equilibrium in aircraft prices and airline quantity vectors exists. Equilibrium in each period relies on the ability of airlines to compute the discounted expected value of any fleet of aircraft in the current period. This implies that airlines can compute expected equilibrium outcomes in future periods given each possible choice it makes in the current period.

Equilibrium in the quantities of aircraft airlines choose in each period depend on the appropriate vector of aircraft prices. That is, given prices, if each airline chooses its best fleet of aircraft, and the number of aircraft demanded does not equal to the number of aircraft supplied for each type of aircraft, then the market has not yet reached equilibrium.[18] A vector of aircraft prices exists that are consistent with a equilibrium in aircraft quantities.

The following proposition states formally the assertions of the existence of equilibrium. Proof of the proposition is by construction. Specifically, I will show that there is a auction mechanism that generates an allocation of aircraft and a vector of aircraft prices that is an equilibrium with the properties given in Proposition 2. In each period, aircraft quantities and prices are determined using the following simple auction mechanism.

To initialize, in each period (t subscripts are suppressed) start with a finite number of aircraft ,$\overline{q}_j$, of each aircraft type j available to airlines, set all prices $P_j(0) = 0$, and order aircraft arbitrarily from $j = 1, ..., J$.

Starting with iteration $k = 1$.

1. Given the current vector of prices $P(k-1)$, allow airlines to choose their highest valued airline fleets. This can be done by allowing each airline to choose aircraft one at a time in order of their highest marginal contribution to the airline's fleet.

2. If $\sum_{i=1}^{I} q_{ij}(k) = \overline{q}_j$, or $P_j(k) = 0$ and $\sum_{i=1}^{I} q_{ij}(k) < \overline{q}_j$, for all j then go to 5. Otherwise, for all j, starting with aircraft $j = 1$, if $\sum_{i=1}^{I} q_{ij}(k) > \overline{q}_j$, increase $P_j(k)$ until $\sum_{i=1}^{I} q_{ij}(k) = \overline{q}_j$ (the fact that the quantity demanded will be reduced each time price is increased follows from Proposition 1). Alternatively, if $\sum_{i=1}^{I} q_{ij}(k) < \overline{q}_j$ leave $P_j(k) = P_j(k-1) = 0$.

3. $P(k)$ is the new vector of prices.

4. Set $k = k + 1$ and go to 1.

5. $P(k)$ is the vector of equilibrium prices and $q(k)$ is the vector of equilibrium quantities.

Proposition 3 - Existence *Given assumptions A1 and A2 and Proposition 1, a equilibrium in airline quantity choices and aircraft prices that satisfies the conditions of Proposition 2 exists.*

Proof of the proposition follows from the properties of the linear operator

$$q_{it}^{*} = \arg\max_{q_{it}} \left[\pi(q_{it}, s_{it}, \xi_{it}) + \beta E_t \left\{ \max_{q_{it+1} \in C(s_{it+1})} V\left(q_{it+1}, s_{it+1}, \xi_{it+1}\right) \right\} \right]$$

[18] The notable exception to this statement is the case of aircraft with a price of zero. When the price of an aircraft is zero, the demand for that type of aircraft may exceed the supply of that type of aircraft, in which case the excess aircraft are scrapped.

and the pricing mechanism described above, which lead to the four lemmas written below. Proofs of the lemmas are in the appendix.

Lemma P3-1 *The bid prices implied by the algorithm are non-negative and finite.*

Prices are non-negative by the assumption that aircraft can always be scrapped for free. The fact that prices are finite follows from the assumptions that (1) each airline chooses the aircraft that maximize its value function, (2) there is free disposal of aircraft, and (3) airlines' value functions are finite.

Lemma P3-2 *If* $\sum_{i=1}^{I} q_{ij}(k) = \overline{q}_j$, *given the vector of prices* $\{P_1(k), ..., P_j(k),$

$P_{j+1}(k-1), ..., P_J(k-1)\}$, *then* $\sum_{i=1}^{I} q_{ij}(k) \geq \overline{q}_j$ *given prices* $\{P_1(k), ..., P_j(k),$
$P_{j+1}(k), ..., P_J(k)\}$, *where* $P_j(k) \geq P_j(k-1)\, \forall j$.

Lemma P3-2 asserts that aircraft are gross substitutes for one another. That is, when the price of one aircraft type goes up the demand for another type will not go down. Type j' aircraft contribute to the marginal value of type j aircraft only through its effect on the aggregate number of aircraft airline i owns. When the number of type j' aircraft airline i owns is decreased, the total number of aircraft airline i owns is decreased as well, which cannot decrease the marginal value of type j aircraft to airline i. This proposition ensures that prices are non-decreasing for all types of aircraft in successive bidding rounds.

Lemma P3-3 *Prices will eventually stop increasing at allocation where* $\sum_{i=1}^{I} q_{ij}(k) = \overline{q}_j$ *and* $P_j(k) \geq 0$ *or* $\sum_{i=1}^{I} q_{ij}(k) < \overline{q}_j$ *and* $P_j(k) = 0$ *for all aircraft types* j.

Lemmas P3-1 and P3-2 imply that prices are non-decreasing and finite, which leads to the result given in Lemma P3-3. The following lemma implies that a equilibrium aircraft allocation and prices will occur in a finite number of bidding rounds, and completes the proof of the proposition. The final lemma stated below, ties the existence result to the equilibrium conditions described in Proposition 2.

Lemma P3-4 *The equilibrium allocation and prices found using the above auction algorithm has the properties described in Proposition 2.*

The proof of the final lemma completes the proof of the proposition.

4.5.2 A Note on Uniqueness

Given a starting point, the algorithm described above will always locate the same equilibrium. However, I have been unable to show that there are not other allocations of aircraft and aircraft prices that are an equilibrium as defined above that would result if aircraft were ordered differently to start the algorithm.

5 Model Solution

5.1 Computing the value functions

This section discusses in detail the stochastic algorithm used to solve the dynamic model described above. A common way of solving dynamic decision problems is to designate a final decision period and solve the model backwards recursively using the value function equations

$$V\left(q_{it}, s_{it}, \xi_{it}\right) = \pi\left(q_{it}, s_{it}, \xi_{it}\right) + \beta E_t \left[\max_{q_{it+1} \in C(s_{it+1})} \left\{V\left(q_{it+1}, s_{it+1}, \xi\right)\right\}\right]. \tag{12}$$

Equation (12) implies that to find its best choice in period t airline i must compute the expected value of its best choice in period $t+1$ for all feasible q_{it}. To facilitate exposition, I define the following notation

$$EMAX\left(q_{it}\right) = E_t \left[\max_{q_{it+1} \in C(s_{it+1})} \left\{V\left(q_{it+1}, s_{it+1}, \xi\right) | q_{it}\right\}\right].$$

Computing $EMAX\left(q_{it}\right)$ at each possible q_{it} is computationally expensive for two reasons. First, $EMAX\left(q_{it}\right)$ is, in general, a multivariate integral. Therefore, depending on the distribution of the errors, evaluating $EMAX\left(q_{it}\right)$ numerically for a given q_{it} can be difficult or impossible. Second, $EMAX\left(q_{it}\right)$ must be computed for every feasible q_{it}, and for any reasonable specification of the model, the number of possible q_{it} will be quite large. In addition, the computational burden of solving the dynamic model backwards recursively grows exponentially as the number of decision periods grows since $V\left(q_{it+1}, s_{it+1}, \xi\right)$ relies on the computation of $EMAX\left(q_{it+1}\right)$, and $V\left(q_{it+2}, s_{it+2}, \xi\right)$ depends on $EMAX\left(q_{it+2}\right)$ and so on. In the literature on discrete choice dynamic decision problems, the increase in the computational burden of solving a dynamic model that is due to an increase in the number of possible states is known as the 'curse' of dimensionality. It is well documented that the curse of dimensionality can make solving dynamic decision problems infeasible. Given the number of choices that airlines are allowed in each period, solving model presented in this paper using standard backward solution methods is not possible.

Recent works by Rust (1997) and Pakes and McGuire (2001) develop methods that use simulation to break the curse of dimensionality. The basic idea of a stochastic algorithm is to compute future outcomes several times for randomly simulated draws of the errors of the model, and treat the resulting outcomes as possible realizations of the future. Then the possible realizations of the future at each state are averaged to approximate the expected value of optimal decisions in the future. The Pakes and McGuire algorithm has two key features that reduce the computational burden of solving the dynamic equilibrium model presented in this paper. First, the expected future value terms given by $EMAX(q_{it})$ are never actually computed. Instead, the $EMAX(q_{it})$ terms are approximated using the average of equilibrium outcomes of the model that occur at simulation draws of the random components of the model. Second, given an airline's current guess of the $EMAX$ term for each state and a draw of the random components of the model, the algorithm solves for a single sequence of states in periods $t = 1, 2, ...$ Solving the model at only a single sequence of states has two advantages: (1) each iteration of the algorithm can be performed very quickly, and (2) the choices that are more likely to occur given the parameters of the model will occur with greater frequency than other choices, and therefore the approximations of the $EMAX\left(q_{it}\right)$ will be more accurate at the "important" q_{it}. The computational benefits of the stochastic algorithm are not without costs. The stochastic algorithm is not as precise as backward solution methods. However, the precision of the approximation of the $EMAX$ terms increases with number of times the model is solved, and since the number of times the model must be solved to obtain a accurate approximation of the $EMAX$ terms is not necessarily related to the size of the state space, the stochastic algorithm may mitigate the curse of dimensionality entirely.

In this application, a vector of approximations of the $EMAX(q_{it})$ terms are kept in storage and updated at each iteration of the algorithm. At each iteration of the algorithm, the most recent approximation of the $EMAX(q_{it})$ terms and a simulation draw of the random vectors $\xi_1, ..., \xi_T$ are used to solve for equilibrium in periods $t = 1, ..., T$. Then the approximations of the $EMAX\left(q_{it}\right)$ at the q_{it} that occurred in equilibrium at times $t = 1, ..., T-1$ are updated by averaging the value functions computed at times $t = 2, ..., T$ with the most recent approximations of the $EMAX(q_{it})$ terms. The algorithm can be thought of as "what would happen if airlines, given their best guess about the future and a draw of the random vectors $\xi_1, ..., \xi_T$, simultaneously choose an optimal sequence of airline fleets $q_{i1}, ..., q_{iT}$." Then each airline takes the outcome that occurs and averages the information with its previous knowledge to improve its guess about the future.

As was mentioned above, the equilibrium of the model must be computed several times at each time t to get accurate approximations of the $EMAX$ terms, and in this research, the model's equilibrium also found by an iterative procedure. That is, the model solution algorithm is a nested algorithm, where the "inner" loop of the algorithm takes airlines' most recent approximations of the $EMAX$ terms at time t and a draw of the random components of the model and solves for equilibrium, and the "outer" loop of the algorithm takes the value functions from the solution to the inner loop and averages them with previous outcomes to update airlines' guesses of the $EMAX$ terms. The nested algorithm converges when airlines' approximations of the $EMAX$ terms are changing within some prespecified tolerance level. The next section discusses the algorithm used to solve for equilibrium at each time t. Appendix B provides additional details of both the "inner" loop and the "outer" loop of the nested algorithm used to solve the model.

5.2 Solving for Equilibrium

Given the most recent approximation of the $EMAX$ terms and a draw of the random components $\xi_1, ..., \xi_T$, the equilibrium of the model at each time t is found using a algorithm very similar to the simple auction mechanism described just before Proposition 3 above. Bertsekas, Castanon, and Tsaknakis (1993) prove that there exists a forward and reverse auction algorithm that is equivalent to the above algorithm, and arrives at equilibrium much more quickly. The state at time t is a function of the equilibrium outcome at time $t-1$, and the state at time $t+1$ is a function of the equilibrium outcome at time t, etc. In the algorithm, airlines increase their bids to obtain the aircraft they desire the most (this is called a forward step), and then, when necessary, the price of an aircraft that is dropped by its owner may be reduced to attract a new owner (this is called a reverse step). The details of how the bids for aircraft are determined and equilibrium is found using the forward and reverse auction algorithm are relegated to Appendix B.

6 Estimation

The parameters of the structural model are estimated by maximum simulated likelihood estimation using data on observed airline ownership choices from Back Aviation Solutions and prices from Avmark Incorporated. Construction of the likelihood function follows directly from the solution of the model and the distributions assumed for the structural errors. Simulation methods are then used to evaluate the high dimensional integrals of the likelihood function.

The first part of this section defines the error distributions assumed for η_{kt} and ε_{ijt}. Next, I provide a informal discussion of how the data identifies the parameters of the structural model. Then, the outcome probabilities that compose the likelihood function are derived. Finally, I discuss the Maximum Simulated Likelihood estimation algorithm used to estimate the parameters of the model.

6.1 Error distributions

η_{kt} a common shock that influences the average size of group-k airlines' fleets at time t. η_{kt} can be attributed to growth in the economy, war, the threat of terror, etc. As was mentioned above, different groups of airlines may have different growth rates in the segment of their fleet operated by wide-bodied aircraft because of the way they've chosen to structure their routes. In this initial specification of the model, I assume

$$\eta_{kt} = \alpha_k t + \phi D_{1986,t},$$

where the α_k are estimated as fixed parameters, and $D_{1986,t}$ is a dummy variable that equals one if the purchase took place in or after 1986 so that ϕ captures any additional effects the Tax Reform Act of 1986 may have had on airlines' investment behavior. Alternative distributional assumptions, such as the addition of a aggregate random effect, could very easily be employed in future specifications. ε_{ijt} is an idiosyncratic shock to the productivity of a particular airline-aircraft match in time t. I assume that

$$\varepsilon_{ijt} \sim iidN\left(0, \sigma_{\varepsilon}\right)$$

across airlines, aircraft and time.

6.2 Identification

I now provide an informal discussion of how variation in transaction and price data identify the parameters of the structural model. The set of parameters to be estimated is

$$\theta = \left(\mu_1, ..., \mu_K, \gamma_1, ..., \gamma_K, \alpha, \delta_1, ..., \delta_K, \lambda_1, ..., \lambda_4, c, \sigma_{\varepsilon}^2, \alpha_1, ..., \alpha_K, \phi\right)$$

and the data used to estimate the parameters is given by

$$\{q_{111}, ..., q_{ijt}, ..., q_{IJT}; X_{11}, ..., X_{jt}, ..., X_{JT}; \\ P_{11}, ..., P_{jt}, ..., P_{JT}; 1, ..., t, ..., T\}.$$

Referring to the profit flow equation (1), the α_k are identified by within-group variation in the average size of airline fleets over time, while the airline specific parameter μ_k is identified by variation in fleet-size across airlines within each time period, ϕ is identified by variation in the composition of fleets before and after the Tax Reform Act of 1986. The parameters δ_k measure the rate at which marginal returns to aircraft is decreasing and is identified by variation in fleet size. The airline-type preference parameters γ_k are identified by covariation in the observed characteristics of aircraft, ownership choices of different types of airlines, and aircraft prices. The parameters λ_m measure transaction costs, and are identified by variation across different aircraft makes in the length of time an aircraft is held between sales. c, which measures adjustment costs is identified by covariation in the number of adjustments an airlines make to a particular model of aircraft in its fleet and the overall number of changes airlines make to their fleet in each period.

The discount factor β is weakly identified by changes in investment patterns over time since it is assumed that airline preferences do not change across time periods. However, I do not attempt to identify β, but instead assume it is constant and equal to 0.9. Transaction costs and adjustment costs are separately identified non-parametrically, the assumptions that transaction costs enter the profit flow equation linearly and adjustment costs enter quadratically are made to capture the facts that purchasing new aircraft is costly, but making large adjustments to the composition of an airline fleet requires the airline to acquire or sell gates and routes to accommodate its new fleet structure.

6.3 The Likelihood Function

The estimation method assumes that observed prices and quantities are consistent with the definition of the market equilibrium given by equations (5) and (6). Given aircraft prices and quantities, the likelihood of the observed outcome in each period can be expressed in terms of the necessary and sufficient conditions for marginal aircraft given in Proposition 2. As can be seen in the profit flow equation (1), airline profit flow is additively separable in the errors ε_{ijt}, i.e. $\pi\left(q_{it}, s_{it}, \varepsilon_{it}\right) = \overline{\pi}\left(q_{it}, s_{it}\right) + \sum_j q_{ijt}\varepsilon_{ijt}$. Therefore, the marginal value of aircraft j to airline i's fleet, given by equation (7) is also additively separable in ε_{ijt} and can be expressed $M_{ijt}\left(q_{it}\right) = \overline{M}_{ijt}\left(q_{it}\right) + \varepsilon_{ijt}$. The following terms are defined to facilitate the construction of the outcome probabilities that are used to derive the likelihood function:

$$\begin{aligned} \Delta^{+}_{ijt} &\equiv -\overline{M}_{ijt}\left(q_{it} + e_j\right), && (13) \\ \Delta^{-}_{ijt} &\equiv -\overline{M}_{ijt}\left(q_{it}\right) \quad \text{and} && (14) \\ \Delta_{ij'jt} &\equiv \overline{M}_{ijt}\left(q_{it}\right) - \overline{M}_{ij't}\left(q_{it} + e_{j'} - e_j\right). && (15) \end{aligned}$$

Using the definitions (13), (14) and (15), the conditions given by equations (8), (9) and (10) in

Proposition 2 can be rewritten in terms of the errors ε_{ijt} as

$$\begin{aligned}\varepsilon_{ijt} &> \Delta^{-}_{ijt}, &(16)\\ \varepsilon_{ijt} &< \Delta^{+}_{ijt}, &(17)\\ \varepsilon_{ijt} &< \Delta_{ijj't} + \varepsilon_{ij't} \text{ for all } i, j, j' \text{ and } t. &(18)\end{aligned}$$

Ordering aircraft from (1) to (J) (where the parentheses indicate that an aircraft's type index j is not necessarily the same as its position in the ordering of aircraft)[19] Combining the conditions given by equations (16), (17), and (18), the upper and lower bounds of each ε_{ijt} are given by

$$\begin{aligned}\overline{b}_{ijt} &= \min\left(\Delta^{+}_{ijt}, \varepsilon_{i1t} + \Delta_{ij1t}, ..., \varepsilon_{ij-1t} + \Delta_{ijj-1t},\right. &(19)\\ &\qquad \left.\varepsilon_{ij+1t} + \Delta_{ijj+1t}, ..., \varepsilon_{iJt} + \Delta_{ijJt}\right) \text{ and}\\ \underline{b}_{ijt} &= \max\left(\Delta^{-}_{ijt}, \varepsilon_{i1t} - \Delta_{i1jt}, ..., \varepsilon_{ij-1t} - \Delta_{ij-1jt},\right.\\ &\qquad \left.\varepsilon_{ij+1t} - \Delta_{ij+1jjt}, ..., \varepsilon_{iJt} - \Delta_{iJjt}\right).\end{aligned}$$

The probability that a vector of airline quantity choices $q^*_{it} = (q^*_{i1t}, ..., q^*_{iJt})$ are optimal for airline i at time t can be expressed in terms of the errors $\varepsilon_{i(j)t}$, and the bounds $\overline{b}_{i(j)t}$ and $\underline{b}_{i(j)t}$ as

$$Q_{it} = \Pr\left[\underline{b}_{ijt}\left(q^*_{it}\right) < \varepsilon_{ijt} < \overline{b}_{ijt}\left(q^*_{it}\right) \quad \forall j\right],$$

which, given that the ε_{ijt} are $iidN\left(0, \sigma^2_{\varepsilon}\right)$, can be written

$$Q_{it} = \int \cdots \int \prod_{j=1}^{J} 1\left(\underline{b}_{ijt}\left(q^*_{it}\right) < \varepsilon_{ijt} < \overline{b}_{ijt}\left(q^*_{it}\right)\right) \frac{1}{\sigma^2_{\varepsilon}} \phi\left(\frac{\varepsilon_{ijt}}{\sigma_{\varepsilon}}\right) d\varepsilon_{ijt}, \tag{20}$$

where $1(.)$ is an indicator variable that equals one if its argument is satisfied and zero otherwise, and ϕ is the standard normal *pdf*.

Since the errors ε_{ijt} are independent across airlines the log-likelihood of observing a sequence of equilibrium outcomes in periods $t = 1, ..., T$, can be calculated as the sum over i of the natural logarithm of the outcome probabilities given by equation (20), or

$$L(\theta) = \sum_{t=1}^{T} \sum_{i=1}^{I} \ln\left[Q_{it}\right], \tag{21}$$

where $\theta = (\mu_k, \gamma_k, \sigma_{\varepsilon}, \lambda_m, \delta_k, c, \alpha_k, \phi)$ is the vector of parameters to be estimated.

6.4 Maximum Simulated Likelihood

A frequency simulator of Q_{it} as it is expressed in (20) can be written

$$\widehat{Q}_{it}(\theta) = \frac{1}{R} \sum_{r=1}^{R} \prod_{j=1}^{J} 1\left(\underline{b}_{ijt}\left(q^*_{it}, \varepsilon^r_{it}\right) < \varepsilon^r_{ijt} < \overline{b}_{ijt}\left(q^*_{it}, \varepsilon^r_{it}\right)\right),$$

[19] The method used to order aircraft is discussed in detail in the appendix that describes the importance sampling algorithm used to compute the integrals of the likelihood function.

where $\varepsilon_{it}^{r} = (\varepsilon_{i1t}^{r}, ..., \varepsilon_{iJt}^{r})$ is a vector of simulation draws from the $iidN(0, \sigma_{\varepsilon})$ distribution. The model can be estimated using a frequency simulator, but there are a few significant costs to doing so. First, given the way the model is constructed it is likely (especially for early guesses of the parameters) that many vectors of random simulation draws of the errors will be inconsistent with equilibrium, and therefore many draws of the errors may be necessary to produce an accurate simulator of Q_{it}. A second problem with a frequency simulator is that it is not continuous in the parameters of the structural model. That is, for changes in the parameters of the model the frequency simulator of outcome probabilities may make discrete jumps.

To avoid the problems associated with using a frequency simulator, I construct a importance sampling simulator that focuses its attention on the regions of the error distribution that are consistent with equilibrium, and is continuous in the parameters of the model. The simulator developed in this research is similar to the well known GHK simulator (Geweke (1991), Hajivassiliou and McFadden (1990), and Keane (1990)). Details of how Q_{it} is simulated, and properties of the simulator are given in Appendix C.

The maximum simulated likelihood estimator replaces Q_{it} with the simulated probability $\widehat{Q}_{it}$, to get

$$\widehat{L}(\theta) = \sum_{t=1}^{T} \sum_{i=1}^{I} \ln\left(\widehat{Q}_{it}\right). \tag{22}$$

maximum simulated likelihood estimators are consistent but biased for a finite number of simulation draws because although $E\left(\widehat{Q}_{it}\right) = Q_{it},\ E\left(\ln \widehat{Q}_{it}\right) \neq \ln Q_{it}$. Despite the biased evaluation of $\ln(Q_{it})$, Borsch-Supan and Hajivassilou (1993) use Monte Carlo experiments to show that maximum simulated likelihood estimators that use importance simulation techniques, such as the well known GHK simulator or the Stern (1994) simulator, perform well relative to alternative estimators.

7 Results and Model Fit

This section presents parameter estimates for the structural model developed in Section 6. After the parameter estimates are discussed, the structural model's fit to the data is analyzed. In addition to performing chi-square goodness-of-fit tests, several figures that illustrate the model's ability fit to the data are provided.

7.1 Parameter estimates

Estimation results are given on Tables 3 and 4. Recall that aircraft prices are measured in the natural log of real U.S. dollars. Therefore, the impact of all parameters on airlines' value functions are either discussed in terms of the percentage impact they have on the value of airline fleets or are translated into real dollars. The first set of parameter estimates, presented on Table 3, are parameters that are common across the different types of airlines, including parameters measuring industry-wide time trends and market frictions. Table 4 presents parameters that determine an airlines' preferred scale of operation and fleet composition. The standard errors of the parameter estimates are in parentheses. Almost all of the parameter estimates differ from zero at a high level of statistical significance.[20]

Referring to Table 3, ϕ is a dummy variable that equals one in periods after 1986.[21] The estimate of ϕ does not significantly differ from zero, which indicates that there is not a significant difference in the rate of airline growth before and after 1986. This does not necessarily imply that the Tax Reform Act of 1986 had no effect on investment, but if the tax change did discourage investment in new aircraft, as is commonly believed, the dampening effect was offset by other unobserved factors. The effects of the Tax Reform Act of 1986 are analyzed in greater detail in the next section.

The standard deviation of the airline-aircraft-time specific error, σ_ε, has a estimate of 3.0268, which translates into roughly \$21 million. This amount is a little less than the average transaction price of aircraft sold during the sample period.

Even though the composition of airline fleets changes over time, there is a significant amount of persistence in aircraft ownership. In fact, it is not unusual for an aircraft to stay with the same owner for more than 15 years. The λ parameters, which measure transaction costs incurred by buyers when a sale is made, and c, which measures the cost to an airline of adjusting the composition of its fleet, capture the persistence in aircraft ownership observed in the data. The magnitude of the estimates of the cost parameters imply that there are significant frictions in the market for commercial aircraft. The estimates of the λ are similar across the different makes of aircraft, and the estimates indicate that when a airline purchases a new \$100 million aircraft, the cost of reconfiguring the aircraft, learning to operate the aircraft, etc. are about \$62 million. Adjustment costs, c, on the other hand, multiply the sum of the squares of the differences from one period to the next in the number of aircraft of each type an airline owns. Therefore, if an airline increases or decreases the number of a particular type of aircraft it operates by only one unit from one period to the next, the marginal value of each aircraft of that type decreases by approximately 13 percent. However, if the increase or decrease in the quantity of aircraft the airline owns is larger, for example four units, the marginal value of each aircraft of that type is decreased by more than 50 percent.

Heterogeneity in airlines' choices of route structures is reflected in their preferred scales of op-

[20] Typically, t-statistics in non-linear structural models are quite large. See, for example, Rust (1987) or Brien, Lillard and Stern (2005).

[21] There was a major change in the U.S. tax code in 1986, which is believed to have affected investment in capital goods like aircraft. This tax reform, known as the Tax Reform Act of 1986, is discussed in greater detail in the next section.

eration and tastes for the different types of aircraft. For example, some airlines choose to offer a large number of routes to Europe, while others focus their attention on routes that service Asia or the Middle East, and the different route structures require different quantities of aircraft with different attributes. There are three types of airlines considered in this work, large U.S. passenger airlines,[22] small U.S. passenger airlines,[23] and all other airlines. The three types are referred to as Large U.S. Airlines, Small U.S. Airlines, and "Other" Airlines hereafter. Referring to Table 4, the parameters α_k multiply a linear time trend, where 1978 is period one, 1979 is period two, etc. In the profit flow equation (1), the interaction $\alpha_k t$ multiplies the total number of aircraft an airline operates in the current period, so that the marginal value of each aircraft a airline operates is increased by $\alpha_k t$ at time t. For Large U.S. Airlines, the coefficient estimate of 0.2663 implies that, *ceterus paribus*, the marginal value of aircraft to airlines increased approximately 26.6% percent in each period. For Small U.S. Airlines and "Other" Airlines the increase in value aircraft value is much smaller, approximately 7.3% and 9.5% respectively. Obviously, this increase in aircraft value over time does not translate directly into a increase in the market value of aircraft because the effect is mitigated to some degree by a contemporaneous increase in the average size of airline fleets.

Table 4 also shows that the different types of airlines also have significantly different preferences for scale of operation and the types of aircraft they prefer to operate. The parameter δ_k, which captures the rate of decreasing returns for each of the different types of airlines, enters the airlines profit flow equation negatively and multiplies the square of the total number of aircraft the airline owns. Therefore, if, for example, a Large U.S. Airline owns 100 wide-bodied aircraft, the marginal value of each of the aircraft the airline owns is decreased by 1.95. Translated into millions of U.S. dollars, if the marginal value of an aircraft in the airline's fleet is $100 million without scale effects, its marginal value is about $14 million after imposing the effect δ_k. The scale effects are much more severe for Small U.S. Airlines. For example, a Small U.S. Airline would only have to own 18 and 19 aircraft for the marginal value of each of the aircraft it owns decreased by the same amount, 2.434. Naturally, "Other" Airlines, which encompass all other airlines in the world, have scale effects that are much less severe. "Other" Airlines could own about 10,200 aircraft before the marginal value of each of the aircraft is decreased by 1.95.

The parameters $\gamma_1 - \gamma_3$ capture each airline's relative preference for different vintages of aircraft. Aircraft manufactured prior to 1978 are the comparison group. Small U.S. airlines place about the same value on a newer aircraft as they do on a comparably equipped older model. Large U.S. Airlines, on the other hand, value newer aircraft more than comparably equipped older aircraft. For example, if the marginal value of a aircraft in the oldest category is $25 million to a Large U.S. Airline,

[22] The large U.S. scheduled airlines are Delta, United, and American.

[23] The small U.S. passenger airlines are TWA, Eastern, Northwest, Continental, Piedmont, and U.S. Air.

then, *ceterus paribus*, the marginal values of a comparably equipped aircraft manufactured from 1978-1984, 1985-1991, and 1992-1997 are \$29.6 million, \$30.8 million, and \$32.4 million respectively. Even more striking is the premium airlines in other parts of the world place on newer aircraft. If the marginal value of a aircraft to "Other" Airlines in the oldest category is \$25 million, then *ceterus paribus* the marginal values of a comparably equipped aircraft manufactured from 1978-1984, 1985-1991, and 1992-1997 are \$55 million, \$64.5 million, and \$222.6 million respectively. The estimates of the vintage parameters reported above are consistent with the delivery trends displayed in Figure 3. Specifically, as can be seen in the figure, the trend in the number of new aircraft delivered to airlines in the U.S. is relatively flat, while the trend in new aircraft deliveries to airlines in other parts of the world is upward sloping. Therefore, a greater proportion of new aircraft are delivered to "Other" Airlines as time goes on, which is consistent with their relatively high preference for the newer generations of aircraft.

The parameters $\gamma_4 - \gamma_9$ measure airlines' relative preferences for different models of aircraft. It is obvious by looking at the estimates that the different types of airlines prefer different models of aircraft. Large U.S. Airlines have a relatively high preference for the Boeing 767 and the McDonnell-Douglas DC-10; while Small U.S. Airlines prefer the Boeing 747, the DC-10, and the Lockheed L-1011; and "Other" Airlines have a strong preference for Airbus aircraft. Comparing preferences for the same model of aircraft across the different types of airlines is complicated by airlines' varying trends in growth over time, scale effects, and preferences for different vintages of aircraft. For example, neglecting the airline's scale, transaction price, transaction costs, and adjustment costs, the value of adding a Boeing 767 produced in the period 1978-1984 to its fleet in 1997 is 5.16 to a Large U.S. Airline's value function, while the same model and vintage adds 4.39 to the value function of "Other" Airlines . In contrast, the same model produced in the period 1992-1997 adds 5.25 to a Large U.S. Airline's value function, and 5.79 to the value function of "Other" Airlines in 1997. The parameter estimates of airline preferences are broadly consistent with the relative popularity of each model of aircraft (see Table 1). For example, the fact that the Boeing 747 and 767 aircraft have been owned and operated with great frequency world-wide is consistent with airlines' relatively high preference for these models, while the fact that Douglas DC-10 is very popular among U.S. airlines is also supported by the parameter estimates. The Douglas DC-10 and the Lockheed L-1011 have similar technical specifications, but according to the parameter estimates, the DC-10 was much more popular among the big three U.S. carriers, Delta, United, and American, while the L-1011 was popular mainly among the Small U.S. carriers. The fact that Large U.S. and international carriers have grown at a faster rate than have Small U.S. carriers provides one possible factor in the eventual demise of the Lockheed L-1011.

7.2 Model fit

In this subsection, I test how well the equilibrium quantities of aircraft generated by the model match those observed in the data. After discussing the statistical significance of the model's fit, I provide a brief discussion of some figures that show that even though the predictions of the model are often rejected by the data statistically, the model does a reasonable job of predicting airline decisions.

The first test statistic is computed using the total number of aircraft owned by each type of airline in each period. There are K types of airlines that can possibly acquire aircraft in each period. Let q_{kt}^o be the observed quantity of aircraft owned by type k airlines at time t, and let q_{kt} be the quantity of aircraft owned by type k airlines at time t predicted by the model. The test statistic

$$W_t = \sum_{k=1}^{K} \frac{(q_{kt}^o - q_{kt})^2}{q_{kt}} \tag{23}$$

has a chi-square distribution with $K-1$ degrees of freedom.

Table 5 presents one chi-square statistic, W_t, for each of the twenty time periods 1978-1997. The null hypothesis is that the quantities predicted by the model are equal to the quantities observed in the data. Since there are only three types of airlines, each statistic W_t given in equation (23) is distributed $\chi^2(2)$. Therefore, if W_t is above the critical value of 5.99 the data rejects the model at the 5 percent level of significance. As can be seen in the table, the null hypothesis is rejected in sixteen of the twenty time periods.

Next, I compute the same test statistic disaggregated by model. There are M models of aircraft available at time t. Therefore, I compute $M * T$ test statistics

$$W_{mt} = \sum_{k=1}^{K} \frac{(q_{kmt}^o - q_{kmt})^2}{q_{kmt}}$$

each of which has a chi-square distribution with $K-1$ degrees of freedom.

In the interest of brevity, I report the results for only one model of aircraft, the Boeing 747. Similarly to Table 5, Table 6 presents twenty chi-square statistics. And again, since there are only three types of airlines, each statistic W_{mt} given in equation (23) is distributed $\chi^2(2)$. Similarly to the results discussed above, the null hypothesis that the simulated and observed quantities are the same is rejected in thirteen of the twenty time periods.

Figures 4-6 show that even though the model is often rejected by the data in statistical tests, the quantities predicted by the model are qualitatively similar to the quantities observed in the data. Figure 4 shows the predicted and observed values of the total quantity of aircraft operated by each type of airline in each period. As can be seen in the figure, because the trend in growth in airline fleets is assumed to be linear, the predicts a very flat trend of growth fro Small U.S. airlines, and therefore under-predicts the number of aircraft owned in the first several periods and over-predicts the number of aircraft owned in the last few periods. Finally, the model tends to fit the quantity choices of Large U.S. and “Other” Airlines quite well.

Figures 5 and 6 show the predicted and observed quantities owned in each time period of the Boeing 747 and the Douglas DC-10 respectively. Unlike Figure 4, the model’s mistakes in predicting the ownership behavior of Small U.S. airlines observed in Figures 5 and 6 is a little more sporadic. For both models of aircraft, the models predictions follow a over-under-over-under pattern over the sample period. Again, the model tends to fit the quantity choices of Large U.S. and “Other” Airlines quite well.

8 Counterfactual Simulations

Structural modelling and estimation provides the researcher with the necessary tools for performing experiments to answer hypothetical questions about agent behavior and potential market outcomes. The estimated parameters are primitives to a economic model of agent behavior, which allows the researcher to evaluate the total effect of a counterfactual change in policy or market structure as opposed to relying on analyses that are based on nonstructural estimation. For example, one might take a positive parameter estimate on the dummy variable “1986” interacted with age in a log-price regression to mean that the relative (to a new aircraft) value of a ten year-old aircraft increased ten percent after the tax reform. However, this conclusion is unlikely to be correct because the tax change impacted the market equilibrium, and the price regression captures only the partial effect of the tax change on prices (holding quantities constant). In contrast, the counterfactual experiments performed below analyze the equilibrium changes in quantity and price that occur given a change in tax policy. In the remainder of the section, the parameter estimates of the model are used to investigate the effects on the market equilibrium of (1) instituting a 10 percent investment tax credit like the one repealed by the Tax Reform Act of 1986, and (2) imposing stricter safety and/or noise abatement policy.

8.1 Tax reform

Prior to 1986 airlines received a tax credit on purchases of new aircraft equal to ten percent of the aircraft’s purchase price.[24] The investment tax credit was designed to increase investment in new

[24] There were several other reforms made to the corporate tax code in 1986. Most noteably, the corporate tax rate was reduced from 46% to 35%.

durable assets. This would suggest that the elimination of the tax credit would cause investment in new aircraft to slow. In addition, since new and used aircraft are substitutes, one would expect holdings of used aircraft to increase more than they otherwise would have when the investment tax credit was removed. To assess the effects of reinstituting the investment tax credit repealed by the Tax Reform Act of 1986, I use the parameter estimates of the model to compare airline quantity choices five periods into the future under two regimes: (1) no change in tax policy, and (2) the implementation of a investment tax credit of 10 percent paid to U.S. airlines on the purchase of new aircraft.

Table 7 summarizes the results of the counterfactual experiment. The tax credit induces only small increases in total aircraft ownership among U.S. airlines and induces small decreases in aircraft ownership among "Other" Airlines. Looking at airlines' aircraft ownership choices across the different vintages of aircraft, one change stands out. U.S. airlines increase the number of 1978-1991 vintage aircraft that they own, and the "Other" Airlines reduce the number of aircraft they own of the same vintage. This result, together with the slight increase in overall ownership by U.S. airlines, implies substitution effects between new and used aircraft are small, and that the tax credit produces a small increase in the relative purchasing power of U.S. airlines.

Large U.S. airlines have an average price elasticity of demand of approximately -0.75 for 1998-2002 vintage aircraft. The inelastic demand implied by the structural model is consistent with previous literature on capital good investment (see for example Auerbach and Hassett (1990, 1992)). However, perhaps due to the specification of a dynamic model and the explicit inclusion of markets for used aircraft, the elasticities implied by the experiment are quantitatively larger than previous studies.

8.2 Safety restrictions and noise abatement

Some industry experts claim that increasingly strict safety, noise abatement and environmental policies have greatly influenced the development and implementation of technological advances in aircraft. From collision avoidance equipment and stronger fuselage materials to quieter engines, research and development that improves an aircraft's safety and reduces its noise output impacts almost every component of an aircraft, and older aircraft are more likely to not meet increasingly strict safety or noise abatement policies than are new aircraft. And since policies have continued to get more strict,[25] airlines have been forced to overhaul or scrap their older aircraft.

The counterfactual experiment presented here evaluates the hypothesis that a policy that forces airlines to modernize older aircraft alters the equilibrium quantities of all new and used aircraft

[25] There have been four increasingly strict noise abatement policies instituted in the U.S. in the past 20 years. Source - U.S. Department of Transportation.

airlines own. To imitate the effect of stricter safety, noise, and environmental policies, I enforce a hypothetical policy that in order for airlines to continue operating aircraft over 19 years of age they must pay a one time \$2 million fee. The fee mimics hushkitting and other improvements that can be made to modernize older aircraft.[26]

Table 8 gives the results of the experiment, in which all aircraft over the age of 19 must be modernized. It is evident from the table that the policy decreases the number of older aircraft operated by U.S. airlines, but slightly increases the number of older aircraft operated by “Other" airlines. By the end of five years of the policy, the number of the oldest vintage of aircraft operated by U.S. Airlines has been reduced by approximately 4 percent, and the “Other” Airlines have increased the quantity of aircraft they operate of the same vintage by under 2 percent. This result indicates that new and used aircraft are closer substitutes for U.S. airlines than they are for airlines in other parts of the world.

Ownership of aircraft manufactured in the period 1978-1984 declines not only because of the actual implementation of the policy but also because of airlines’ anticipate the policy and know the aircraft will soon be worth less in the secondary market. Therefore, airlines that would, in the absence of the policy, sell older aircraft in the secondary market at a higher price, are now induced to sell or scrap the aircraft a few periods in advance of when they otherwise would. Large U.S. Airlines even start to sell or scrap 1985-1991 vintage aircraft in 2002 in anticipation of their future mandated hushkitting. The “Other” Airlines take advantage of the U.S. Airlines’ fire sale of older aircraft and increase their ownership of 1985-1991 vintage aircraft.

Small U.S. Airlines partially replace the older aircraft they scrap by increasing the number of newer aircraft, those manufactured after 1985-1997, they own. Large U.S. Airlines, on the other hand, replace most of the older aircraft with aircraft produced 1992-1997, and finally, the “Other” Airlines, with a few exceptions, actually decrease the number of newer versions of aircraft they own. It is interesting to note that as a result of the policy, all airlines reduce the number of the newest vintage of aircraft they own. This indicates that new aircraft are the worst substitutes for aircraft produced before 1985.

9 Conclusions

This article studied purchasing and selling behavior in primary and secondary markets for commercial aircraft. The focus of this work was to develop a framework for analyzing demand behavior that allowed for both intertemporal dependence and intratemporal dependence in markets for durable

[26] Hushkits for widebody aircraft typically run \$500,000 to \$4 million - source Metropolitan Airports Commission in cooperatioin with ACI - NA.

goods. The data used to estimate the structural model came from two private sources. The data on aircraft ownership and aircraft characteristics came from Back Aviation Solutions, and the transaction price data came from Avmark Incorporated.

Data on the locations (owners) of aircraft in the current period, the locations of aircraft in the previous period, and the market price of aircraft in the current period were used to identify the parameters of the model. Airline value functions and the equilibrium conditions of the model were used to derive the likelihood of events observed in the data as a function of the parameters of the model. The model's specification allowed airlines to have different preferences for scale of operation and the age and model of aircraft owned.

The parameter estimates of the structural model indicated that there are significant transaction and adjustment costs that affect transaction patterns in primary and secondary markets for commercial aircraft. These costs explain the observed persistence in aircraft ownership over time. Differences in the length, frequency, and passenger volume of the routes the airlines choose to service are reflected in the heterogeneity in airline preferences for the size and composition of their airline fleets. The estimated model showed that Boeing aircraft are relatively popular to all types of airlines, while Douglas and Lockheed models mainly appeal to U.S. airlines, and Airbus aircraft are most popular among European and Asian airlines.

The parameters of the structural model were used to perform two counterfactual experiments. First, the impact of reinstituting a 10 percent income tax credit on the purchase of new aircraft by U.S. airlines was analyzed. The hypothetical tax credit used in the experiment was identical to the one that was removed by the Tax Reform Act of 1986. The results of the experiment showed that the implementation of the tax credit would have a small impact on equilibrium quantities. Specifically, there would be only a small amount of growth in the size airline fleets and a small amount of substitution toward new aircraft by U.S. airlines as a result of the policy. In addition, the estimated demand elasticities averaged about -.75 for Large U.S. airlines, which is slightly higher than elasticities computed in previous studies of markets for durable capital goods.

Next, I performed an experiment illustrating the effects of stricter noise abatement and/or safety policies. The results of the experiments showed that mandating the modernization of older aircraft reduces the size of the number of older aircraft operated by U.S. airlines. Interestingly, airlines in other parts of the world actually increase the number of older aircraft they operate as a result of the policy indicating that new and used aircraft are closer substitutes to U.S. airlines.

In future work in markets where competing firms purchase durable assets, I plan to develop a model that allows for more strategic interaction the buyers. Firms that buy capital and labor often appear to compete with other firms by buying or maintaining ownership of assets to keep them from the competition. The results of such behavior, like buying a store-front and leaving it closed to keep a competitor from reaping the benefits of owning it, are pure rent-seeking. Firms that own durable goods may engage in similar behavior by buying or maintaining ownership of a good and storing it so that their competitors cannot use it. By developing a model of behavior that allows for this type of strategic behavior by firms, I hope to identify the social costs of such behavior.

In addition, this research develops methodologies for estimating a model of dynamic - equilibrium purchasing and selling behavior in the market for commercial aircraft. However, the modelling and estimation techniques can be applied much more broadly. Commercial and residential real estate markets are comprised of a limited number of durable assets that are often sold several times and appreciate in value over time. Rare consumer durables (e.g. paintings) are also likely candidates for the methodologies developed in this research. In addition, labor markets that contain a relatively small supply of workers with unique talents, like athletes in professional sports leagues or college professors, would be an intriguing application of the above methodologies. Each of the aforementioned markets are characterized by goods or labor that are useful for several periods, and purchase and sales decisions of each potential owner that depend on the purchase and sales decisions of other owners in the market. The methodologies developed in this work can be used to estimate models of behavior in these and similar markets, and the estimated models can be used to analyze potential changes in economic policy or market structure.

A Proofs

Proof of Proposition 1.

$$\begin{aligned} V\left(q_{it}, s_{it}, 0\right) &= \sum_{j=1}^{J_t} \{q_{ijt} X_{jt} \gamma_k + \eta_{kt} \\ &\quad -1\left(q_{ijt} > q_{ijt-1}\right) \lambda_m \left(q_{ijt} - q_{ijt-1}\right)\} \\ &\quad -c\left(q_{ijt} - q_{ijt-1}\right)^2 - \left(q_{ijt} - q_{ijt-1}\right) P_{jt} - \delta_k \left(\sum_{j=1}^{J_t} q_{ijt}\right)^2 \\ &\quad +\beta E_t \left[\max_{q_i' \in C\left(s_{it+1}\left(q_{it}\right)\right)} \left\{V\left(q_i', s_{it+1}\left(q_{it}\right), \xi\right)\right\}\right] \end{aligned} \tag{24}$$

Each of the terms in the first three lines of equation (24) are finite. Next, consider the final term

$$\begin{aligned} & E_t \left[\max_{q_i' \in C\left(s_{it+1}\left(q_{it}\right)\right)} \left\{V\left(q_i', s_{it+1}\left(q_{it}\right), \xi\right)\right\}\right] \\ = & \int \cdots \int \max_{q_i' \in C\left(s_{it+1}\left(q_{it}\right)\right)} \left\{V\left(q_i', s_{it+1}\left(q_{it}\right), \xi\right)\right\} f\left(\xi\right) d\xi, \end{aligned} \tag{25}$$

where $f(.)$ is the joint density of ξ_{it+1}, which is the product of independent normal densities. Since $\max_{q_i' \in C\left(s_{it+1}\left(q_{it}\right)\right)} \left\{V\left(q_i', s_{it+1}\left(q_{it}\right), \xi\right)\right\}$ grows linearly in each element of ξ_{it+1} and $f(.)$ declines faster than exponentially in each element of ξ_{it+1}, equation (25) is finite. Thus, $V\left(q_{it}, s_{it}, 0\right)$ is also finite.

Now, assume there exists a time t' after which airlines make the same choice in every period, implying

$$\begin{aligned} V\left(q_{it'}, s_{it'}, \xi_{it'}\right) &= \pi\left(q_{it'}, s_{it'}, \xi_{it'}\right) + B \\ &= \left(\mu_k + \eta_{kt'}\right) \left(\sum_{j=1}^{J_t} q_{ijt'}\right) - \delta_k \left(\sum_{j=1}^{J_t} q_{ijt'}\right)^2 + \\ &\quad \sum_{j=1}^{J_t} q_{ijt'} \Big[\left(X_{jt'} \gamma_k + \varepsilon_{ijt'}\right) - c\left(q_{ijt'} - q_{ijt'-1}\right)^2 \\ &\quad - \left(q_{ijt'} - q_{ijt'-1}\right) P_{jt'}\Big] + B, \end{aligned}$$

$$\begin{aligned} V\left(q_{it'}, s_{it'}, \xi_{it'}\right) &= \sum_{j=1}^{J_t} \{q_{ijt'} \left(X_{jt'} \gamma_k + \eta_{kt'} + \varepsilon_{ijt'}\right) \\ &\quad -1\left(q_{ijt'} > q_{ijt'-1}\right) \lambda_m \left(q_{ijt'} - q_{ijt'-1}\right)\} \\ &\quad -c\left(q_{ijt'} - q_{ijt'-1}\right)^2 - \left(q_{ijt'} - q_{ijt'-1}\right) P_{jt} - \delta_k \left(\sum_{j=1}^{J_t} q_{ijt'}\right)^2 + B. \end{aligned} \tag{26}$$

Note that

$$\begin{aligned}
M\left(q_{it'}+e_j,s_{it'},\xi_{it'}\right)-M\left(q_{it'},s_{it'},\xi_{it'}\right) &= \left[1\left(q_{ijt'}+1>q_{ijt'-1}\right)-1\left(q_{ijt'}>q_{ijt'-1}\right)\right]\lambda_m-2\left(\delta_k+c\right)\\
&\leq -1\left(q_{ijt'}>q_{ijt'-1}\right)\lambda_m-2\left(\delta_k+c\right)<0,
\end{aligned}$$

and $\pi\left(q_{it'},s_{it'},\xi_{it'}\right)$ is concave in $q_{ijt'-1}$ since

$$\frac{\partial^2\pi\left(q_{it'},s_{it'},\xi_{it'}\right)}{\partial^2 q_{ijt'-1}}=-2c.$$

Moving backward to period $t'-1$,

$$\begin{aligned}
V\left(q_{it'-1},s_{it'-1},\xi_{it'-1}\right) &= \sum_{j=1}^{J_t}\left\{q_{ijt'-1}\left(X_{jt'}\gamma_k+\mu_k+\eta_{kt'-1}+\varepsilon_{ijt'-1}\right)\right. \\
&\quad \left.-1\left(q_{ijt'-1}>q_{ijt'-2}\right)\lambda_m\left(q_{ijt'-1}-q_{ijt'-2}\right)\right\}\\
&\quad -c\left(q_{ijt'-1}-q_{ijt'-2}\right)^2-\left(q_{ijt'-1}-q_{ijt'-2}\right)P_{jt}-\\
&\quad \delta_k\left(\sum_{j=1}^{J_t}q_{ijt'-1}\right)^2+\beta E_{t'-1}\left[\max_{q_{it'}\in C\left(q_{it'-1}\right)}\pi\left(q_{it'},s_{it'},\xi_{it'}\right)\right]
\end{aligned}\tag{27}$$

and

$$\begin{aligned}
& M\left(q_{it'-1}+e_j,s_{it'-1},\xi_{it'-1}\right)-M\left(q_{it'-1},s_{it'-1},\xi_{it'-1}\right)\\
\leq\ & -1\left(q_{ijt'-1}>q_{ijt'-2}\right)\lambda_m-2\left(\delta_k+c\right)+\\
& \beta E_{t'-1}\left[\begin{array}{c}\max_{q'_{it'}\in C\left(q_{it'-1}+e_j\right)}\pi\left(q'_{it'},s'_{it'}\left(q_{it'-1}+e_j\right),\xi_{it'}\right)+\\ \max_{q''_{it'}\in C\left(q_{it'-1}-e_j\right)}\pi\left(q''_{it'},s''_{it'}\left(q_{it'-1}-e_j\right),\xi_{it'}\right)-\\ 2\left(\max_{q_{it'}\in C\left(q_{it'-1}\right)}\pi\left(q_{it'},s_{it'}\left(q_{it'-1}\right),\xi_{it'}\right)\right)\end{array}\right]\\
<\ & 0
\end{aligned}$$

because $E_{t'-1}\left[\begin{array}{c}\max_{q'_{it'}\in C\left(q_{it'-1}+e_j\right)}\pi\left(q'_{it'},s'_{it'}\left(q_{it'-1}+e_j\right),\xi_{it'}\right)+\\ \max_{q''_{it'}\in C\left(q_{it'-1}-e_j\right)}\pi\left(q''_{it'},s''_{it'}\left(q_{it'-1}-e_j\right),\xi_{it'}\right)-\\ 2\left(\max_{q_{it'}\in C\left(q_{it'-1}\right)}\pi\left(q_{it'},s_{it'}\left(q_{it'-1}\right),\xi_{it'}\right)\right)\end{array}\right]<0$, since $\pi\left(q_{it'},s_{it'},\xi_{it'}\right)$ is concave in $q_{it'-1}$. Furthermore, $V\left(q_{it'-1},s_{it'-1},\xi_{it'-1}\right)$ is concave in $q_{ijt'-2}$ since

$$\frac{\partial^2 V\left(q_{it'-1},s_{it'-1},\xi_{it'-1}\right)}{\partial q_{ijt'-2}^2}=-2c<0.$$

Finally, at some arbitrary time $t < t'$

$$\begin{aligned} V\left(q_{it}, s_{it}, \xi_{it}\right) &= \sum_{j=1}^{J_t} \left\{ q_{ijt}\left(X_{jt}\gamma_k + \mu_k + \eta_{kt} + \varepsilon_{ijt}\right) \right. \\ & \left. -1\left(q_{ijt} > q_{ijt-1}\right)\lambda_m\left(q_{ijt} - q_{ijt-1}\right) \right\} \\ & -c\left(q_{ijt} - q_{ijt-1}\right)^2 - \left(q_{ijt} - q_{ijt-1}\right)P_{jt} - \delta_k\left(\sum_{j=1}^{J_t} q_{ijt}\right)^2 + \\ & \beta E_t\left[\max_{q_i' \in C\left(s_{it+1}\left(q_{it}\right)\right)}\left\{V\left(q_i', s_{it+1}\left(q_{it}\right), \xi\right)\right\}\right] \end{aligned} \tag{28}$$

and

$$\begin{aligned} & M\left(q_{it} + e_j, s_{it}, \xi_{it}\right) - M\left(q_{it}, s_{it}, \xi_{it}\right) \\ \leq\ & -1\left(q_{ijt} > q_{ijt-1}\right)\lambda_m - 2\left(\delta_k + c\right) + \\ & \beta E_t\left[\begin{array}{c} \max_{q'_{it+1} \in C\left(q_{it}+e_j\right)} V\left(q'_{it+1}, s'_{it+1}\left(q_{it}+e_j\right), \xi_{it+1}\right) + \\ \max_{q''_{it+1} \in C\left(q_{it}-e_j\right)} V\left(q''_{it+1}, s''_{it+1}\left(q_{it}-e_j\right), \xi_{it+1}\right) - \\ 2\left(\max_{q_{it+1} \in C\left(q_{it}\right)} V\left(q_{it+1}, s_{it+1}\left(q_{it}\right), \xi_{it+1}\right)\right) \end{array}\right] \\ <\ & 0 \end{aligned}$$

because $E_t\left[\begin{array}{c} \max_{q'_{it+1} \in C\left(q_{it}+e_j\right)} V\left(q'_{it+1}, s'_{it+1}\left(q_{it}+e_j\right), \xi_{it+1}\right) + \\ \max_{q''_{it+1} \in C\left(q_{it}-e_j\right)} V\left(q''_{it+1}, s''_{it+1}\left(q_{it}-e_j\right), \xi_{it+1}\right) - \\ 2\left(\max_{q_{it+1} \in C\left(q_{it}\right)} V\left(q_{it+1}, s_{it+1}\left(q_{it}\right), \xi_{it+1}\right)\right) \end{array}\right] < 0$, since $V\left(q_{it+1}, s_{it+1}\left(q_{it}\right), \xi_{it+1}\right)$ is concave in q_{ijt}. ■

Proof of Proposition 2. Conditions (8),(9) and (10) are by definition necessary for q^*_{it} to be optimal. Condition (8) and Proposition 1 imply that i would not benefit from subtracting any quantity of aircraft. Condition (9) and Proposition 1 imply that i would not benefit from adding any quantity of aircraft. Finally, assume q^*_{it} satisfies the conditions given by (8),(9) and (10). Now, suppose there is an alternative fleet of aircraft $q^*_{it} + \sum_{j=1}^{J_t} a_j e_j$ such that

$$q^*_{it} + \sum_{j=1}^{J_t} a_j e_j = \arg\max_{q_{it}} V\left(q_{it}\right),$$

such that $a_{j'}$ is a negative integer and $a_{j''}$ is a positive integer for some j' and j''. First note that given (8),(9) and the conditions of Proposition 1, if

$$\sum_{j=1}^{J_t} a_j e_j > 0$$

then

$$M_{j''}\left(q^*_{it} + \sum_{j=1}^{J_t} a_j e_j\right) < 0$$

and if

$$\sum_{j=1}^{J_t} a_j e_j < 0$$

then

$$M_{j'}\left(q_{it}^* + \sum_{j=1}^{J_t} a_j e_j\right) > 0.$$

So consider any vector $q_{it}^* + \sum_{j=1}^{J_t} a_j e_j$ where

$$\sum_{j=1}^{J_t} a_j e_j = 0.$$

If $q_{it}^* + \sum_{j=1}^{J_t} a_j e_j$ is optimal, it must be true that $M_{j''}\left(q_{it}^* + \sum_{j=1}^{J_t} a_j e_j\right) >$ $M_{j'}\left(q_{it}^* + \sum_{j=1}^{J_t} a_j e_j + e_{j'} - e_{j''}\right)$. However, since $M_{j'}(q_{it}^*) > M_{j''}(q_{it}^* - e_{j'} + e_{j''})$, and under the conditions of Proposition 1 $M_{j'}\left(q_{it}^* + \sum_{j=1}^{J_t} a_j e_j + e_{j'} - e_{j''}\right) > M_{j'}(q_{it}^*)$ and $M_{j''}(q_{it}^* - e_{j'} + e_{j''}) >$ $M_{j''}\left(q_{it}^* + \sum_{j=1}^{J_t} a_j e_j\right)$; $q_{it}^* + \sum_{j=1}^{J_t} a_j e_j$ cannot be optimal. Since this is true for all possible a, q_{it}^* must be optimal given satisfaction of (8),(9) and (10). ■

Proof of Proposition 3 **Proof of Lemma P3-1.** This is an obvious result of Proposition 1 and the assumption that airlines are profit maximizing.

Proof of Lemma P3-2. This follows from the properties of the value function. When the price of a type j aircraft goes up the marginal value of type j aircraft goes down. This implies that airlines will want to reduce the quantity of type j aircraft they own if they change the quantity at all. Since the marginal value of each type of aircraft is decreasing in the total number of aircraft the airline owns, this implies that the marginal value of all other types of aircraft will increase. Therefore, no airline will reduce the number of any other type j' aircraft they demand when the price of type j aircraft is increased. ■

One important result of this lemma is that prices will never be reduced once they are increased.

Proof of Lemma P3-3. This lemma is a straightforward application of Tarsky's fixed point theorem. That is, the set of possible prices for every type of aircraft is bounded on the interval $\left[0, \overline{P}\right]$, and the auction mechanism described in the text is a function that maps bid prices $P_j(r)$ into $P_j(r+1)$ such that $P_j(r+1) \geq P_j(r)$. ■

Proof of Lemma P3-4. It is obvious by the way airlines' highest valued fleets are constructed that the selected fleets satisfy the conditions of Proposition 2. ■ ■

B Model Solution Algorithms

B.1 $EMAX$ Updates - "Outer Loop"

At each iteration r of the outer loop of the model solution algorithm, the approximations $\widehat{EMAX}(q_{it}, r)$ of $EMAX(q_{it})$ is updated for all q_{it}. The outcomes q_{it}^r for periods $t = 2, ..., T$ that occur in the inner loop of the algorithm are the outcomes of the model given the most recent approximations $\widehat{EMAX}(q_{it}, r)$ for all q_{it}, and a draw of the random components $\xi_1^r, ..., \xi_T^r$. The value function of the equilibrium outcome at time $t+1$ given q_{it}^r is averaged with previous outcomes of the algorithm to obtain the updated value of $EMAX(q_{it})$. That is,

$$\widehat{EMAX}(q_{it}^r, r+1) = \frac{(n(q_{it}^r) - 1) * \widehat{EMAX}(q_{it}^r, r) + V\left(q_{it+1}^r, s_{it+1}^r, \xi_{it}^r | q_{it}^r\right)}{n(q_{it}^r)}$$

where the state s_{it+1}^r depends on the vector q_{it}^r, and $n(q_{it}^r)$ is the number of times a outcome q_{it}^r has occurred (up to and including iteration r). If, on the other hand, a vector q_{it} does not occur in the inner loop of the algorithm, the approximations of $EMAX(q_{it})$ do not change so that

$$\widehat{EMAX}(q_{it}, r+1) = \widehat{EMAX}(q_{it}, r).$$

The algorithm described above has converged when for the vector $q = (q_{11}, ..., q_{it}, ..., q_{IT})$ the distance measure $\left\|\widehat{EMAX}(q_t, r+1) - \widehat{EMAX}(q_t, r)\right\|$ is less than a predetermined threshold $\epsilon > 0$.

B.2 Equilibrium - "Inner Loop"

t subscripts are suppressed because the algorithm that determines equilibrium aircraft prices and quantities is the same for all t. To compute the price in bidding round b of a type j aircraft, the algorithm calculates the "buy threshold" (i.e. maximum willingness-to-pay) for an additional type j aircraft for each airline, and also the "keep threshold" (i.e. marginal value to airlines of keeping) for currently held type j aircraft, given the current prices and quantities. If the maximum (over airlines) buy threshold exceeds the minimum keep threshold for type j aircraft, a type j aircraft is sold from the airline with the minimum keep threshold to the airline with the maximum buy threshold, and the price of type j aircraft is set equal to the minimum keep threshold plus a arbitrarily small bid increment. If, on the other hand, no type j aircraft are sold, the price of a type j aircraft does not change.

Airline i's buy threshold for a type j aircraft is denoted B_{ij}, and is equal to the maximum of the current price of a type j aircraft plus the larger of (1) the marginal value of adding a type j aircraft to its current fleet or (2) the marginal value of swapping another type of aircraft for a type

j aircraft. That is,

$$B_{ij}(b) = \max\{M_{ij}(q_i(b) + e_j), M_{ij}(q_i(b) + e_j - e_{j'})\} + P_j(b-1).$$

The only caveat to this computation is that the buy threshold of the scrapper is always zero.

The amount an airline is willing to pay to keep a type j aircraft, denoted K_{ij}, is equal to the amount the airline loses if you take away a type j aircraft plus the current price of type j aircraft. That is,

$$K_{ij}(b) = M_{ij}(q_i(b)) + P_j(b-1).$$

Keep thresholds are only calculated for those airlines that own at least one type j aircraft.

A sale of a type j aircraft occurs if and only if at least one airline has a buy threshold that is greater than another airline's keep threshold. That is, if

$$\max_{i:q_{ij}<\overline{q}_j}\{B_{ij}(b)\} > \min_{i:q_{ij}>0}\{K_{ij}(b)\}$$

then a type j aircraft is sold from the airline with the minimum keep value to that airline with the maximum buy threshold. The price of a aircraft j in bidding round b is given by

$$\begin{aligned} P_j(b) &= P_j(b-1) \quad \text{if} \quad \max_{i:q_{ij}<\overline{q}_j}\{B_{ij}(b)\} < \min_{i:q_{ij}>0}\{K_{ij}(b)\} \quad \text{and} \\ P_j(b) &= \min_{i:q_{ij}>0}\{K_{ij}(b)\} + \tau \quad \text{if} \quad \max_{i:q_{ij}<\overline{q}_j}\{B_{ij}(b)\} > \min_{i:q_{ij}>0}\{K_{ij}(b)\}, \end{aligned}$$

where $\tau > 0$ is an arbitrarily small bid increment.

B.2.1 Reverse Step:

An airline determines its buy threshold for a type j aircraft by determining how much it would be willing to pay to add a type j aircraft to its current fleet, and how much it would be willing to pay to get rid of a type j' aircraft from its current fleet and acquire a type j aircraft, for some j'. In the case where the airline with the highest buy threshold for type j aircraft is determined by a airline that is best off swapping out a type j' aircraft for a type j aircraft, the type j' aircraft becomes unowned. In this case the algorithm immediately reduces the price of type j' aircraft until the aircraft is acquired by an airline or is scrapped. Specifically, if

$$\max_{i:q_{ij}<\overline{q}_j}\{B_{ij}(b)\} = M_{ij}(q_i(b) + e_j - e_{j'}) + P_j(b-1) \text{ for some } i,$$

then the price of j' is lowered to the maximum marginal value across airlines of adding a type j' aircraft less an arbitrarily small amount. That is,

$$P_{j'}(b) = \max\left[\max_i\{M_{ij}(q_i(b) + e_{j'}) + P_{j'}(b-1)\} - \epsilon, 0\right].$$

At each time t, the algorithm for finding equilibrium prices and quantities can be written concisely as follows.

Initialization: Set $b = 1,\ j = 1,\ q_t(1) = q_{t-1},$ *and* $P_{jt}(1) = 0$ *for all* $j.$

(1) *For* j, b *compute* $C_{ijt}(b)$ *and* $K_{ijt}(b)$.

(2) *If*

$$\begin{aligned} \max_{i:q_{ijt}<\overline{q}_{jt}} \{B_{ijt}(b)\} &> \min_{i:q_{ijt}>0} \{K_{ijt}(b)\} \qquad \text{and} \\ M_{ijt}(q_{it}(b) + e_j - e_{j'}) + P_{jt}(b-1) &= \max_i \{B_{ijt}(b), 0\} \quad \text{for some } i \end{aligned}$$

remove one type j *aircraft from the airline with the minimum keep threshold and give it to the airline with the highest buy threshold, set*

$P_{jt}(b) = \min_{i:q_{ijt}>0}\{K_{ijt}(b)\} + \tau,$ *and go to reverse auction step* (3). *If, on the other hand*

$$\begin{aligned} \max_{i:q_{ijt}<\overline{q}_{jt}} \{B_{ijt}(b)\} &> \min_{i:q_{ijt}>0} \{K_{ijt}(b)\} \qquad \text{and} \\ M_{ijt}(q_{it}(b) + e_j) + P_{jt}(b-1) &= \max_{i:q_{ijt}<\overline{q}_{jt}} \{B_{ijt}(b), 0\} \quad \text{for some } i \end{aligned}$$

remove one type j *aircraft from the airline with the minimum keep threshold and give it to the airline with the highest buy threshold, set*

$P_{jt}(b) = \min_{i:q_{ijt}>0}\{K_{ijt}(b)\} + \tau,$ *and go to step* (4). *Finally, if*

$$\max_i \{B_{ijt}(b), 0\} \le \min_{i:q_{ijt}>0} \{K_{ijt}(b)\}$$

set $P_{jt}(b) = P_{jt}(b-1)$ *and go to* (4).

(3) *Set*

$$P_{j't}(b) = \max\left[\max_i \{M_{ij't}(q_{it}(b) + e_{j'})\} - \tau, 0\right] + P_{j't}(b-1)$$

and allocate the free type j' *aircraft to the airline* i *with*

$M_{ij't}(q_{it}(b) + e_{j'}) = \max_i \{M_{ij't}(q_{it}(b) + e_{j'})\}$ *or to the scrapper if*

$\max_i \{M(q_{it}(b) + e_{j'})\} < 0.$ *Go to step* (4)

(4) *If* $j < J_t$, *set* $j = j+1,\ b = b+1,$ *and return to* (1). *If* $j = J_t$ *and*

$P_t(b) \neq P_t(b - J_t)$, *set* $j = 1,\ b = b+1,$ *and return to* (1). *If* $j = J_t$ *and*

$P_t(b) = P_t(b - J_t)$ *the algorithm has converged and* $q_t(b)$ *and* $P_t(b)$ *are an equilibrium.*

C Simulation

C.1 Simulation Algorithm

This subsection details the algorithm used to simulate Q_{it}. The simulator uses the law of total probability and strategically orders the errors of the model to maximize the amount of time the algorithm spends in the important regions of the errors given the parameters of the model, and therefore reduces the number of simulation draws that are necessary to simulate Q_{it} accurately.

Ordering aircraft in some way $((1), ..., (j), ..., (J))$ the probability

$$Q_{it}(\eta_t) = \Pr\left[\underline{b}_{ijt}(q_{it}^*, \eta_t) < \varepsilon_{ijt} < \overline{b}_{ijt}(q_{it}^*, \eta_t) \qquad \forall j\right]$$

can be rewritten

$$Q_{it}(\eta_t) = \prod_{j=1} \Pr\left[\underline{b}_{i(j)t}^* < \varepsilon_{i(j)t} < \overline{b}_{i(j)t}^* | \varepsilon_{i(1)t}, ..., \varepsilon_{i(j-1)t}\right], \tag{29}$$

where

$$\begin{aligned} \overline{b}_{i(j)t}^* &= \min\Big(\Delta_{i(j)t}^+, \varepsilon_{i(1)t} + \Delta_{i(j)(1)t}, ..., \varepsilon_{i(j-1)t} + \Delta_{i(j)(j-1)t}, \\ &\qquad \Delta_{i(j+1)t}^+ + \Delta_{i(j)(j+1)t}, ..., \Delta_{i(J)t}^+ + \Delta_{i(j)(J)t}\Big) \text{ and} \\ \underline{b}_{i(j)t}^* &= \min\Big(\Delta_{i(j)t}^-, \varepsilon_{i(1)t} - \Delta_{i(1)(j)t}, ..., \varepsilon_{i(j-1)t} - \Delta_{i(j-1)(j)t}, \\ &\qquad \Delta_{i(j+1)t}^- - \Delta_{ij+1jt}, ..., \Delta_{i(J)t}^- - \Delta_{iJjt}\Big). \end{aligned} \tag{30}$$

The simulation algorithm described below constructs the probability $Q_{it}(\eta_t)$ as suggested by equation (29) by sequentially drawing errors $\varepsilon_{i(j)t}$, where the draw $\varepsilon_{i(j)t}^r$ is conditional on the draws $\varepsilon_{i(1)t}^r, ..., \varepsilon_{i(j-1)t}^r$.

(1) *Order aircraft* $\{j : \overline{q}_{jt} > 0\}$ *according to some criterion (the criterion used in practice is described below). Let* (j) *be the* j*th element of the ordered set. Let* $J^* = \#\{j : \overline{q}_{jt} > 0\}$.

(2) *Initialize* $P_i^r = 1$.

(3) *For each* $(j) \leq J^*$,

(a) *Let*

$$\overline{b}_{i(j)t}^* = \min\begin{pmatrix} \Delta_{i(j)t}^+, \varepsilon_{i(1)t}^r + \Delta_{i(j)(1)t}, ..., \varepsilon_{i(j-1)t}^r + \Delta_{i(j)(j-1)t}, \\ \overline{\varepsilon}_{i(j+1)t}\left(\varepsilon_{i(1)t}^r, ..., \varepsilon_{i(j-1)t}^r\right) + \Delta_{i(j)(j+1)t}, ..., \\ \overline{\varepsilon}_{i(J)t}\left(\varepsilon_{i(1)t}^r, ..., \varepsilon_{i(j-1)t}^r\right) + \Delta_{i(j)(J)t} \end{pmatrix}, \tag{31}$$

be a upper bound for $\varepsilon_{i(j)t}$, *where*

$$\overline{\varepsilon}_{i(j+1)t}\left(\varepsilon_{i(1)t}^r, ..., \varepsilon_{i(j-1)t}^r\right) = \min\left(\Delta_{i(j+1)t}^+, \varepsilon_{i(1)t}^r + \Delta_{i(j+1)(1)t}, ..., \varepsilon_{i(j-1)t}^r + \Delta_{i(j+1)(j-1)t}\right).$$

Also let

$$\underline{b}^{*}_{i(j)t} = \max \begin{pmatrix} \Delta^{-}_{i(j)t}, \varepsilon^{r}_{i(1)t} - \Delta_{i(1)(j)t}, ..., \varepsilon^{r}_{i(j-1)t} - \Delta_{i(j-1)(j)t}, \\ \underline{\varepsilon}_{i(j+1)t}\left(\varepsilon^{r}_{i(1)t}, ..., \varepsilon^{r}_{i(j-1)t}\right) - \Delta_{ij+1jt}, ..., \\ \underline{\varepsilon}_{i(J)t}\left(\varepsilon^{r}_{i(1)t}, ..., \varepsilon^{r}_{i(j-1)t}\right) - \Delta_{iJjt} \end{pmatrix}$$

be a lower bound, where

$$\underline{\varepsilon}_{i(j+1)t}\left(\varepsilon^{r}_{i(1)t}, ..., \varepsilon^{r}_{i(j-1)t}\right) = \max\left(\Delta^{-}_{i(j+1)t}, \varepsilon^{r}_{i(1)t} - \Delta_{i(1)(j+1)t}, ..., \varepsilon^{r}_{i(j-1)t} - \Delta_{i(j-1)(j+1)t}\right).$$

(*b*) *Update*

$$P_i^r = P_i^r\left[\Phi\left(\frac{\overline{b}_{i(j)t}}{\sigma_\varepsilon}\right) - \Phi\left(\frac{\underline{b}_{i(j)t}}{\sigma_\varepsilon}\right)\right]$$

(*c*) *Draw* $\varepsilon^{r}_{i(j)t}$ *conditional on* $\underline{b}_{i(j)t} \leq \varepsilon^{r}_{i(j)t} \leq \overline{b}_{i(j)t}$ *as*

$$\varepsilon^{r}_{i(j)t} = \sigma_\varepsilon \Phi^{-1}\left\{\left[\Phi\left(\frac{\overline{b}_{i(j)t}}{\sigma_\varepsilon}\right) - \Phi\left(\frac{\underline{b}_{i(j)t}}{\sigma_\varepsilon}\right)\right]u^r + \Phi\left(\frac{\underline{b}_{i(j)t}}{\sigma_\varepsilon}\right)\right\},$$

where $u^r \sim U(0,1)$.[27]

(4) *if* $(j) = J^*$ P_i^r *is the simulator, otherwise set* $(j) = (j) + 1$ *and return to* (3).

C.2 Properties of the Simulator

Next I discuss some properties of the above simulator of Q_{it}, namely that it satisfies the criterion of a importance sampling simulator. Let

$$h(\varepsilon_{it}) = \prod_{(j)} 1\left(\underline{b}_{i(j)t} < \varepsilon_{i(j)t} < \overline{b}_{i(j)t}\right)$$

be a function that is equal to one if the vector $\varepsilon_{it} = \left(\varepsilon_{i(1)t}, ..., \varepsilon_{i(J)t}\right)$ is consistent with equilibrium and zero otherwise. The likelihood contribution of each airline is given by

$$Q_{it} = \int \cdots \int h(\varepsilon_{it}) f(\varepsilon_{it}) d\varepsilon_{it}, \tag{32}$$

where

$$f(\varepsilon_{it}) = \prod_{(j)} \frac{1}{\sigma_\varepsilon^2}\phi\left(\frac{\varepsilon_{i(j)t}}{\sigma_\varepsilon}\right)$$

is the joint density of the $\varepsilon_{i(j)t}$. Note that since the integrand is zero when the errors are outside of the bounds defined by equilibrium. Therefore, the value of the right hand side of equation (32)

[27] For the first couple of guesses of the parameters of the model, both $\overline{b}_{i(j)t}$ and $\underline{b}_{i(j)t}$ may be in the extreme tail of the distribution. In cases where both $\Phi\left(\frac{\overline{b}_{i(j)t}}{\sigma_\varepsilon}\right)$ and $\Phi\left(\frac{\underline{b}_{i(j)t}}{\sigma_\varepsilon}\right)$ are very close to zero or one the estimation algorithm may crash. In such cases, I use L'Hopital's rule to derive the alternative simulator $\varepsilon^{r}_{i(j)t} = \phi^{-1}\left\{\left[\phi\left(\frac{\overline{b}_{i(j)t}}{\sigma_\varepsilon}\right) - \phi\left(\frac{\underline{b}_{i(j)t}}{\sigma_\varepsilon}\right)\right]u^r + \phi\left(\frac{\underline{b}_{i(j)t}}{\sigma_\varepsilon}\right)\right\}$.

does not change if we integrate only over the portion of the support of $f(.)$ that is consistent with equilibrium. That is,

$$\int \cdots \int h(\varepsilon_{it}) f(\varepsilon_{it}) d\varepsilon_{it} = \int \cdots \int_{\underline{b}_{i(j)t} < \varepsilon_{i(j)t} < \overline{b}_{i(j)t}} h(\varepsilon_{it}) f(\varepsilon_{it}) d\varepsilon_{it}.$$

Now, rewrite equation (32) as

$$\int \cdots \int_{\underline{b}_{i(j)t} < \varepsilon_{i(j)t} < \overline{b}_{i(j)t}} \frac{h(\varepsilon_{it}) f(\varepsilon_{it})}{g(\varepsilon_{it})} g(\varepsilon_{it}) d\varepsilon_{it},$$

where

$$g(\varepsilon_{it}) = \prod_{(j)} \frac{\frac{1}{\sigma_\varepsilon^2} \phi\left(\frac{\varepsilon_{i(j)t}}{\sigma_\varepsilon}\right)}{\Phi\left(\frac{\overline{b}_{i(j)t}}{\sigma_\varepsilon}\right) - \Phi\left(\frac{\underline{b}_{i(j)t}}{\sigma_\varepsilon}\right)}, \tag{33}$$

is the joint *pdf* of independent truncated normal random variables with truncation points $\left(\overline{b}_{i(j)t}, \underline{b}_{i(j)t}\right)$. Note that $g(\varepsilon_{it})$ can be written as

$$g(\varepsilon_{it}) = \prod_{(j)} \frac{1\left(\underline{b}_{i(j)t} < \varepsilon_{i(j)t} < \overline{b}_{i(j)t}\right) \frac{1}{\sigma_\varepsilon^2} \phi\left(\frac{\varepsilon_{i(j)t}}{\sigma_\varepsilon}\right)}{\Phi\left(\frac{\overline{b}_{i(j)t}}{\sigma_\varepsilon}\right) - \Phi\left(\frac{\underline{b}_{i(j)t}}{\sigma_\varepsilon}\right)}.$$

$g(\varepsilon_{it})$ satisfies the four features of a good importance sampling distribution: (1) $g(.)$ is easy to draw from, (2) has the same support as $f(\varepsilon_{it})$, (3) $\frac{h(\varepsilon_{it}) f(\varepsilon_{it})}{g(\varepsilon_{it})}$

$$\frac{h(\varepsilon_{it}) f(\varepsilon_{it})}{g(\varepsilon_{it})} = \prod_{(j)} \left[\Phi\left(\frac{\overline{b}_{i(j)t}}{\sigma_\varepsilon}\right) - \Phi\left(\frac{\underline{b}_{i(j)t}}{\sigma_\varepsilon}\right)\right],$$

is easy to evaluate, and (4) $\frac{h(\varepsilon_{it}) f(\varepsilon_{it})}{g(\varepsilon_{it})}$ is bounded between 0 and 1, and is smooth in the parameters of the model. Thus, the algorithm described above is an importance simulator that simulates $E\frac{h(\varepsilon_{it}) f(\varepsilon_{it})}{g(\varepsilon_{it})}$, where $\varepsilon_{it} \sim g$. The fact that I can write the simulator as an importance sampling simulator means that it is unbiased.

Finally, I discuss the ordering criterion used in practice in step (1) of the simulation algorithm. Since I am using MSL it is very important that I reduce the variance of my simulator as much as possible. One way to improve the variance of the simulator is to strategically order $\{j : q_{jt} > 0\}$ in Step 1 of the above algorithm to reduce the variance of the simulator. Consider the case where $J^* = 2$. In Figure 4, the area within the six-sided figure represents the values of ε_1 and ε_2 that are consistent with equilibrium, and the dotted circles represent the level sets of the joint distribution of ε_1 and ε_2. The GHK algorithm first computes the probability that $\underline{b}_{(1)} \leq \varepsilon_{(1)} \leq \overline{b}_{(1)}$. Then it simulates a value of $\varepsilon_{(1)}$ conditional on $\underline{b}_{(1)} \leq \varepsilon_{(1)} \leq \overline{b}_{(1)}$. Finally, it computes the probability that $\varepsilon_{(2)}$ is such that the simulated values of $\varepsilon_{(1)}$ and $\varepsilon_{(2)}$ are within the area consistent with equilibrium.

The simulated probability is a product of two probabilities, and the variance of the simulator is proportional to the variance of the second probability, which is a function of the simulated value of $\varepsilon_{(1)}$. In order to reduce the variance of the simulator as much as possible, I would arrange $\{j : q_{jt} > 0\}$ in descending order of the variance of the contribution of the simulator with respect to those elements of ε that precede it. However, such a rule is too costly to construct. Alternatively, I develop a simpler rule that is similar to the above rule that is less costly to implement. Specifically, I order $\{j : q_{jt} > 0\}$ in increasing order of

$$\left| \frac{\phi\left(\frac{\overline{b}^*_{(j)}}{\sigma_\varepsilon}\right) - \phi\left(\frac{\underline{b}^*_{(j)}}{\sigma_\varepsilon}\right)}{\Phi\left(\frac{\overline{b}^*_{(j)}}{\sigma_\varepsilon}\right) - \Phi\left(\frac{\underline{b}^*_{(j)}}{\sigma_\varepsilon}\right)} \right|,$$

where

$$\begin{aligned}
\overline{b}^*_{(j)} &= \min \begin{pmatrix} \Delta^+_{(j)}, \Delta^+_{(1)} + \Delta_{(j)(1)}, ..., \Delta^+_{(j-1)} + \Delta_{(j)(j-1)}, \\ \Delta^+_{(j+1)} + \Delta_{(j)(j+1)}, ..., \Delta^+_{(J)} + \Delta_{(j)(J)} \end{pmatrix} \\
\underline{b}^*_{(j)} &= \max \begin{pmatrix} \Delta^-_{(j)}, \Delta^-_{(1)} - \Delta_{(1)(j)}, ..., \Delta^-_{(j-1)} - \Delta_{(j-1)(j)}, \\ \Delta^-_{(j+1)} - \Delta_{(j+1)(j)}, ..., \Delta^-_{(J)} - \Delta_{(J)(j)} \end{pmatrix}.
\end{aligned}$$

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Table 1

Aircraft

Model	***N***
Boeing 747	1078
Boeing 767	690
Boeing 777	104
Airbus 300/310	682
Airbus 330/340	191
Douglas DC-10	371
Douglas MD-11	174
Lockheed L-1011	236
Total	3526

Table 2

Observed Aircraft Attributes

Attribute	Mean	Standard Deviation	Minimum	Maximum
Length (meters)	59.44	8.33	46.7	70.7
Seating Capacity	410.93	98.98	253	550
Fuel Capacity (1000 gallons)	128.05	57.65	44.01	219.62
Payload Range (1000 miles)	5.87	2.01	1.04	9.21

Table 3
Structural Estimates:
Industry and Cost

ϕ: Tax Reform	-0.0013
	(0.0159)
c: Adjustment cost	0.06495
	(0.0299)
σ_ε: Error standard deviation	3.0368
	(0.0677)
λ_1: Boeing transaction cost	0.9650
	(0.0628)
λ_2: Airbus transaction cost	0.9566
	(0.0284)
λ_3: McDonnell-Douglas transaction cost	0.9747
	(0.1946)
λ_4: Lockheed-Martin transaction cost	0.9445
	(0.0506)

Standard Errors are in parentheses.

Table 4
Structural Estimates:
Airline Preferences

Large U.S. passenger airline:		**Small U.S. passenger airline:**		**Other airline:**	
Trend (α):	0.2663 (3.37e-04)	Trend (α):	0.0728 (7.29e-04)	Trend (α):	0.0951 (2.80e-04)
Rate of decreasing returns (δ):	0.00976 (3.92e-05)	Rate of decreasing returns (δ):	0.05223 (0.0013)	Rate of decreasing returns (δ):	9.58e-05 (9.29e-05)
Aircraft manufactured 1978-84 (γ_1):	0.1702 (0.0676)	Aircraft manufactured 1978-84 (γ_1):	0.0047 (0.0029)	Aircraft manufactured 1978-84 (γ_1):	0.7898 (0.0096)
Aircraft manufactured 1985-91 (γ_2):	0.2101 (0.0708)	Aircraft manufactured 1985-91 (γ_2):	0.0099 (0.1797)	Aircraft manufactured 1985-91 (γ_2):	0.9474 (0.0232)
Aircraft manufactured 1992-97 (γ_3):	0.2593 (0.0191)	Aircraft manufactured 1992-97 (γ_3):	0.0196 (0.0721)	Aircraft manufactured 1992-97 (γ_3):	2.1857 (0.2239)
Boeing 747 (γ_4):	-0.9780 (0.0912)	Boeing 747 (γ_4):	4.0334 (0.085)	Boeing 747 (γ_4):	2.5071 (0.0215)
Boeing 767 (γ_5):	-0.3374 (0.2002)	Boeing 767 (γ_5):	1.4712 (0.2018)	Boeing 767 (γ_5):	1.7003 (0.0278)
Airbus 300/310 (γ_6):	-1.7071 (0.0789)	Airbus 300/310 (γ_6):	2.9528 (0.0604)	Airbus 300/310 (γ_6):	4.2430 (0.2986)
Douglas DC-10 (γ_7):	0.7132 (0.0515)	Douglas DC-10 (γ_7):	4.1726 (0.2326)	Douglas DC-10 (γ_7):	0.9427 (0.0857)
Douglas MD-11 (γ_8):	-2.2373 (0.1601)	Douglas MD-11 (γ_8):	1.9768 (0.1732)	Douglas MD-11 (γ_8):	1.4847 (0.2267)
Lockheed L-1011 (γ_9):	-3.2784 (0.0111)	Lockheed L-1011 (γ_9):	4.1864 (0.5171)	Lockheed L-1011 (γ_9):	1.0731 (0.0299)

Standard errors are in parentheses.

Table 5
Chi-Square Tests for the Total Quantity of Aircraft Owned

	Small U.S. Passenger Airlines		Large U.S. Passenger Airlines		"Other" Airlines		
Period	**Predicted**	**Observed**	**Predicted**	**Observed**	**Predicted**	**Observed**	χ^2 **-Statistic**
1978	225.2	189	102.4	119	489	486	8.53
1979	292	218	131.8	130	591.4	594	18.79
1980	283.8	268	151	140	681.6	710	2.86*
1981	272.8	292	185.4	150	777.4	819	9.13
1982	265.8	292	210.2	173	838.6	885	11.73
1983	259.2	298	216	202	968.4	968	6.72
1984	267.2	296	211.8	201	1000.2	1058	7.00
1985	255	325	223.2	205	1181	1126	23.26
1986	257.2	301	243	243	1228.6	1216	7.59
1987	239.2	309	242.2	270	1341.8	1283	26.14
1988	267.4	311	281.2	302	1383.2	1372	8.74
1989	256.4	295	286	312	1523.8	1512	8.27
1990	269	304	330.6	327	1606.8	1648	5.65*
1991	298.2	220	392.2	395	1740	1800	22.60
1992	283.4	211	419.4	446	1947.2	1950	20.19
1993	280.6	220	459.4	477	2083.8	2071	13.84
1994	267.2	229	444.2	465	2191.4	2172	6.61
1995	256.2	221	440.4	449	2297.8	2273	5.27*
1996	243.6	247	472	447	2364.2	2340	1.62*
1997	240	242	476.6	437	2446.8	2427	3.47*

The test statistics are distributed $\chi^2(2)$.
The * indicates that the model fits the data at the 5% level of significance.

Table 6

Chi-Square Tests for the Quantity of Boeing 747 Aircraft Owned

	Small U.S. Passenger Airlines		Large U.S. Passenger Airlines		"Other" Airlines		
Period	**Predicted**	**Observed**	**Predicted**	**Observed**	**Predicted**	**Observed**	χ^2 **-Statistic**
1978	93.4	85	20.2	40	288.8	261	22.84
1979	139.6	102	40.8	40	316.8	315	10.15
1980	129.6	117	33.2	41	346.6	367	4.26*
1981	115.6	113	38.4	44	388.4	412	2.31*
1982	104.2	113	44.4	44	410.4	438	2.60*
1983	94.6	116	35.8	43	448.6	452	6.31
1984	102.4	132	35	31	446.6	461	9.48
1985	92.4	144	25.2	25	508.8	475	30.62
1986	88	125	36.6	38	527.4	508	15.61
1987	101	132	18.8	44	553.2	521	43.37
1988	108.2	130	38.4	45	549	542	5.75*
1989	90.4	117	51.8	48	611	599	12.66
1990	135.6	119	58	55	634	657	5.65*
1991	133	86	70.2	64	686.2	712	26.75
1992	135	84	68.8	69	753	772	41.24
1993	118.8	90	92.8	70	799.6	810	20.87
1994	108	101	89.8	73	837.4	831	6.77
1995	100.8	100	79.6	69	878.2	850	2.33*
1996	98.4	112	78.8	71	906	865	4.51*
1997	97.4	112	74.4	73	934.8	897	3.74*

The test statistics are distributed $\chi^2(2)$.

The * indicates that the model fits the data at the 5% level of significance.

Table 7

Predicted Quantities of Aircraft Owned by Airlines 1998-2002 with and without a 10% Investment Tax Credit

		Year of Manufacture					
	Year	**1969-1977**	**1978-1984**	**1985-1991**	**1992-1997**	**1998-2002**	**Total**
Small U.S.	1998	77.7	62.4	31.4	62	19.4	252.9
Airlines:	1999	77.3	50	17.6	39.1	21.3	205.3
No Policy	2000	46.3	63.7	26.3	48	16.4	200.7
	2001	45.3	90.6	25.3	35.4	14.4	211
	2002	58	82	18.9	31.9	12.3	201.3
	1998	73.6	64.7	32.3	64	19.3	253.9
10% Tax	1999	74.7	55.3	16.4	38.7	22	207.1
Credit	2000	43	67.3	27.6	43.6	12.7	194.2
	2001	41	85.1	29.1	36.6	14.7	206.5
	2002	51.9	81.3	23.3	29.4	13	198.9
Large U.S.	1998	98.3	98.3	111.3	241	41.7	590.6
Airlines:	1999	95.9	94.4	144.9	241.7	18.1	595
No Policy	2000	105.9	93.7	146.3	208	20.6	574.5
	2001	127	85.7	129.7	230	24.7	597.1
	2002	111.7	87.3	132.7	223.6	35.4	590.7
	1998	92	96	110.3	241.3	43.4	583
10% Tax	1999	90	88.3	148	245.7	23	595
Credit	2000	97.6	87.7	150.7	212.6	16.4	565
	2001	125	81.9	129.7	228.3	29	593.9
	2002	106.1	80.3	138.6	221.1	39.1	586.2
"Other"	1998	507.1	493.6	445.9	498.1	393.9	2338.6
Airlines:	1999	503.1	507.7	424.4	517.3	414	2367
No Policy	2000	523.2	489.8	411.1	540.6	423	2387.9
	2001	491.7	470	424.4	529	427.7	2342.8
	2002	492	475.6	421	537.4	440.7	2367.7
	1998	518	493.6	446	498	393.4	2349
10% Tax	1999	512	508.7	422.4	517.6	409.7	2370.4
Credit	2000	535.3	494.1	404.7	544.7	412.7	2391.5
	2001	498	479.7	420.3	535.3	418.3	2351.6
	2002	505.9	484	411.7	547.3	432.4	2381.3

Table 8

Predicted Quantities of Aircraft Owned by Airlines 1998-2002 – Mandated Modernization for Aircraft Over 20 Years Old

		Year of Manufacture					
	Year	**1969-1977**	**1978-1984**	**1985-1991**	**1992-1997**	**1998-2002**	**Total**
Small U.S.	1998	77.7	62.4	31.4	62	19.4	252.9
Airlines:	1999	77.3	50	17.6	39.1	21.3	205.3
No Policy	2000	46.3	63.7	26.3	48	16.4	200.7
	2001	45.3	90.6	25.3	35.4	14.4	211
	2002	58	82	18.9	31.9	12.3	201.3
	1998	73	64.4	33	63.3	19.3	253
Modernization	1999	70.6	53.3	17.9	38.6	23.1	203.5
Mandate	2000	43	64	26.3	48	17	198.3
	2001	40.7	90.4	27.7	35.4	13.7	207.9
	2002	55.6	87	19.4	36.6	8	206.6
Large U.S.	1998	98.3	98.3	111.3	241	41.7	590.6
Airlines:	1999	95.9	94.4	144.9	241.7	18.1	595
No Policy	2000	105.9	93.7	146.3	208	20.6	574.5
	2001	127	85.7	129.7	230	24.7	597.1
	2002	111.7	87.3	132.7	223.6	35.4	590.7
	1998	90.7	96.3	106	241.4	42.6	577
Modernization	1999	91.9	92.4	141	246.7	18.3	590.3
Mandate	2000	100.4	91.3	143.1	213	20.9	568.7
	2001	126.3	82	123.2	237	25.7	594.2
	2002	107.6	83.4	135.4	224.7	32.3	583.4
"Other"	1998	507.1	493.6	445.9	498.1	393.9	2338.6
Airlines:	1999	503.1	507.7	424.4	517.3	414	2367
No Policy	2000	523.2	489.8	411.1	540.6	423	2387.9
	2001	491.7	470	424.4	529	427.7	2342.8
	2002	492	475.6	421	537.4	440.7	2367.7
	1998	519.3	493.6	449.3	498	392.7	2352.9
Modernization	1999	514	507	428.4	514.3	408.6	2372.3
Mandate	2000	533	492.8	414.4	536.7	411	2387.9
	2001	497.7	474.3	428.4	523.1	415.7	2339.2
	2002	501	475.3	421.1	532.4	427.7	2357.5

Figure 1
Proportion of Aircraft Sold in Secondary Markets

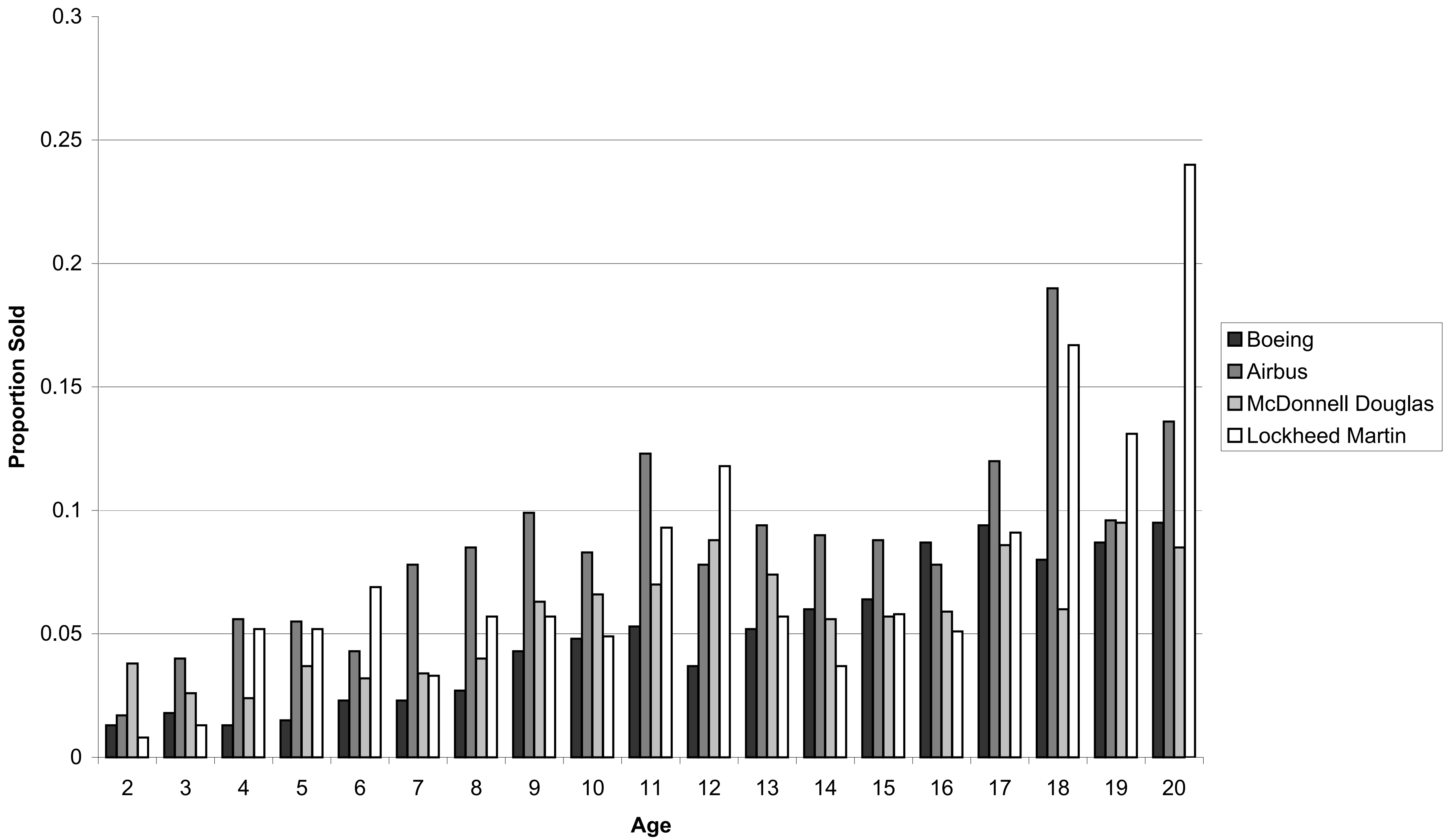


Figure 2
Proportion of Aircraft Sold to Passenger Airlines

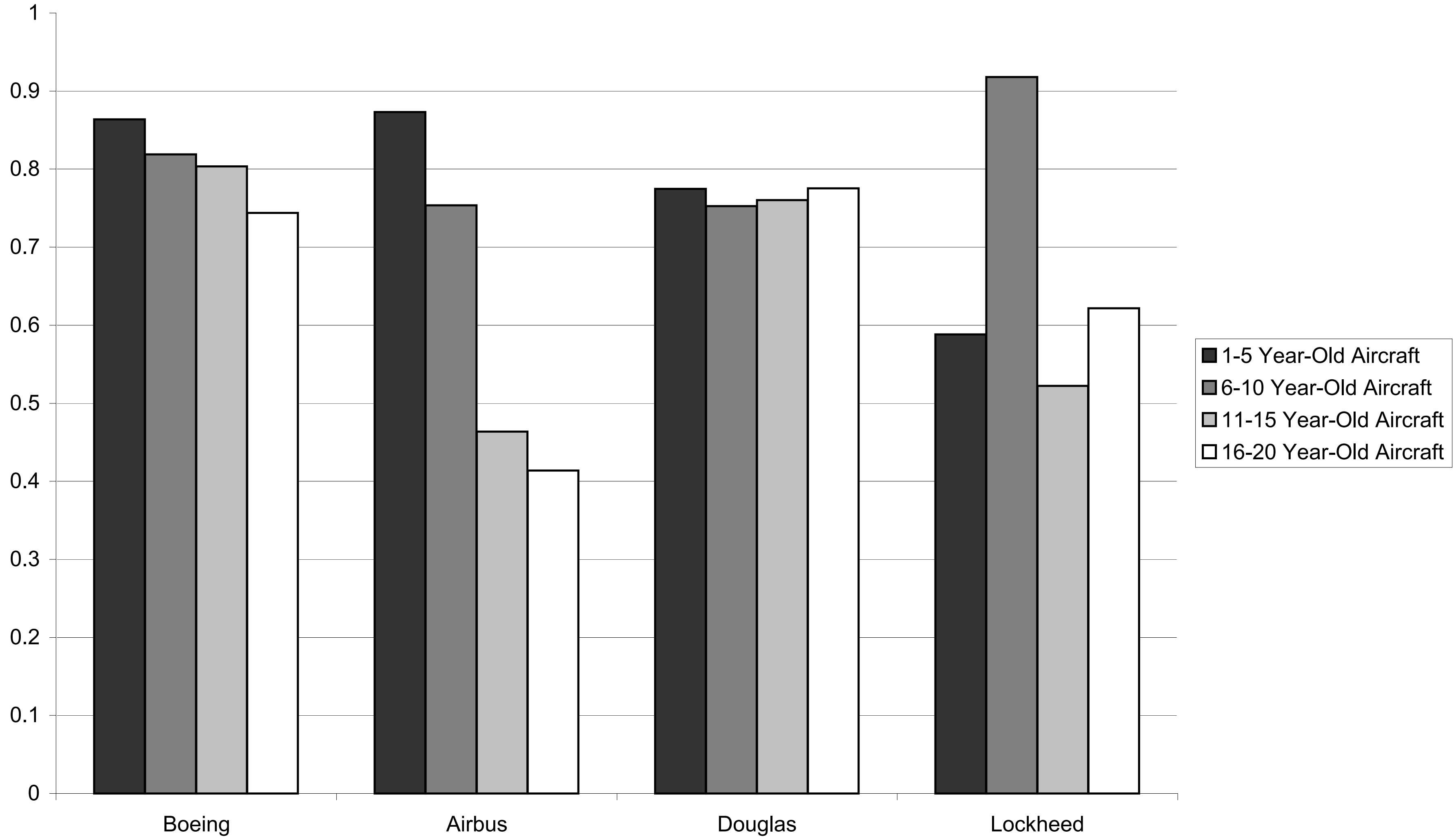


Figure 3
Deliveries and Sales per Period

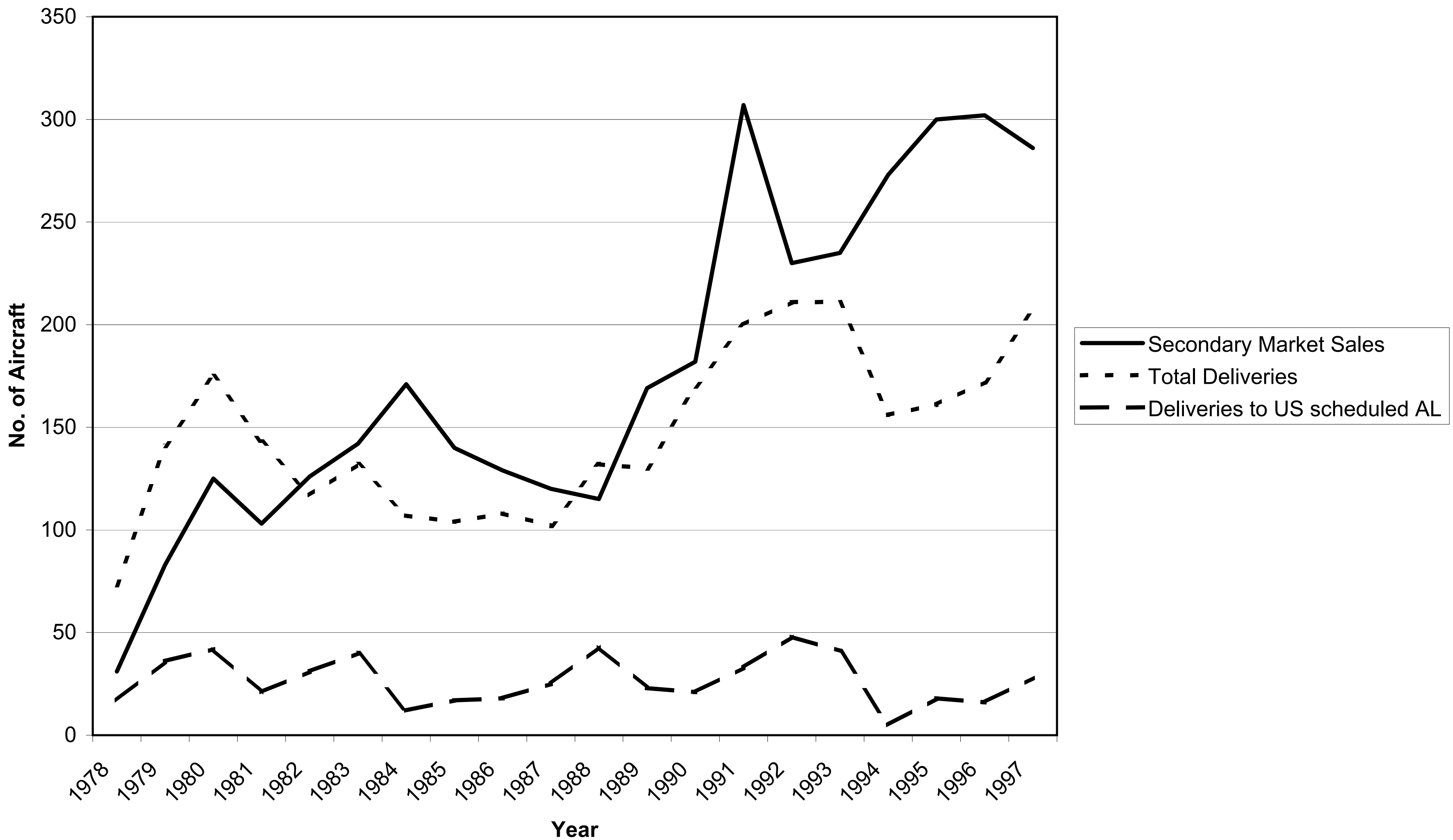
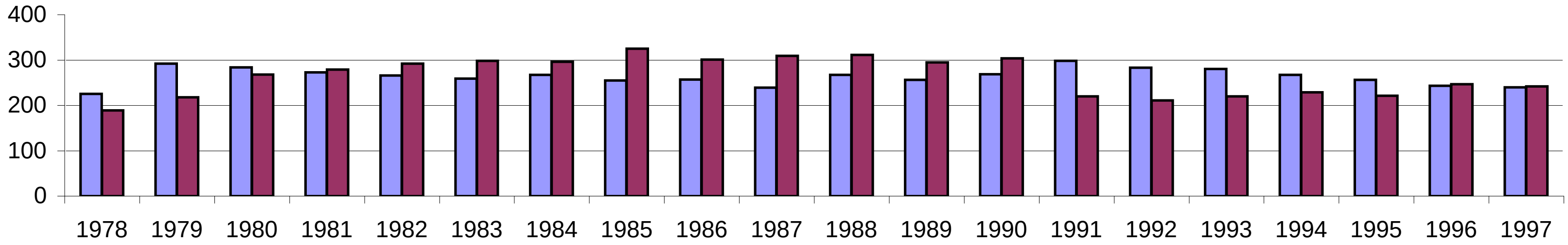
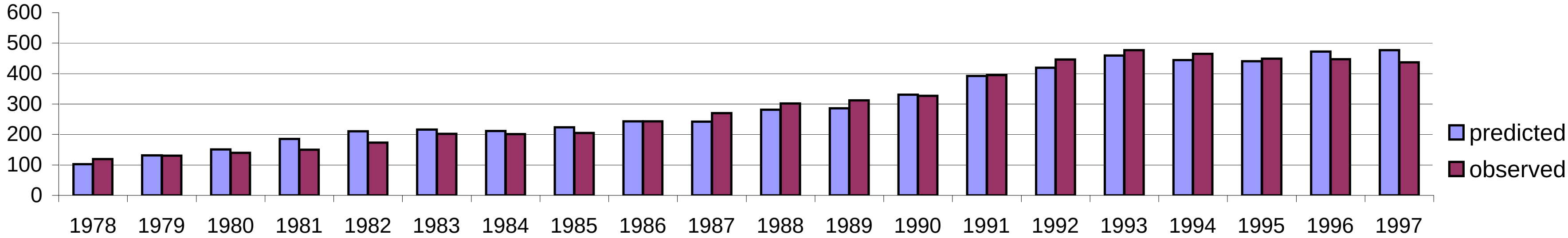


Figure 4

Total Quantity of Aircraft Owned by Small U.S. Airlines



Total Quantity of Aircraft Owned by Large U.S. Airlines



Total Quantity of Aircraft Owned by "Other" Airlines

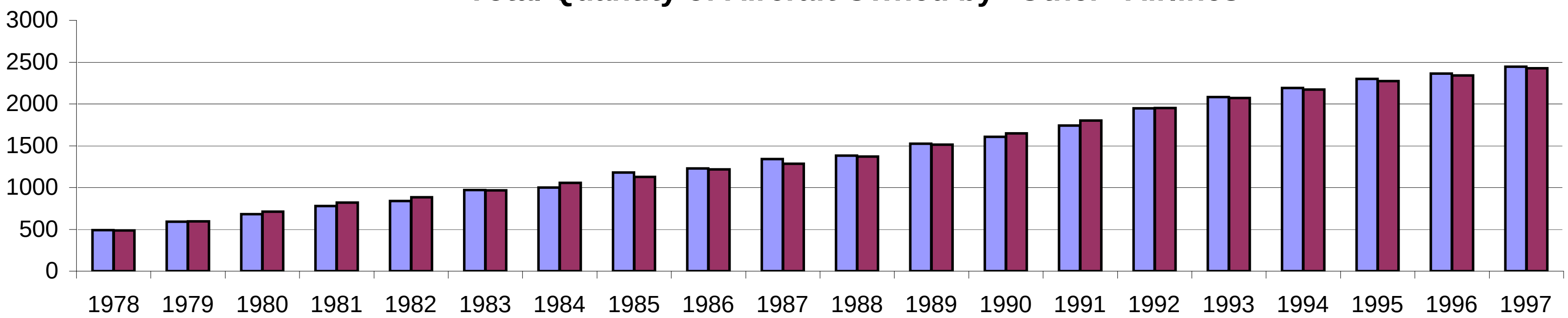


Figure 5
Boeing 747 Aircraft Owned by Small U.S. Airlines

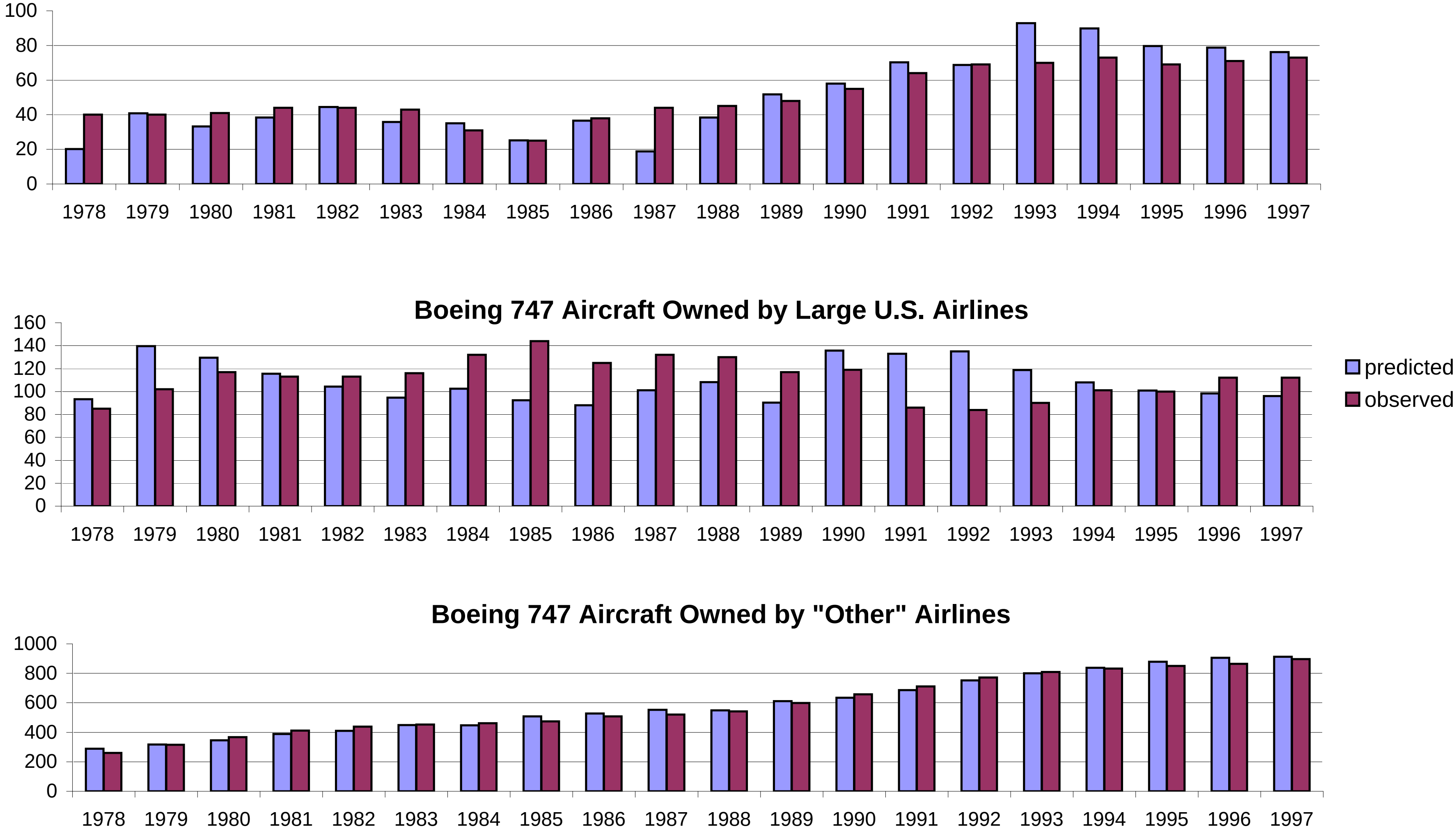
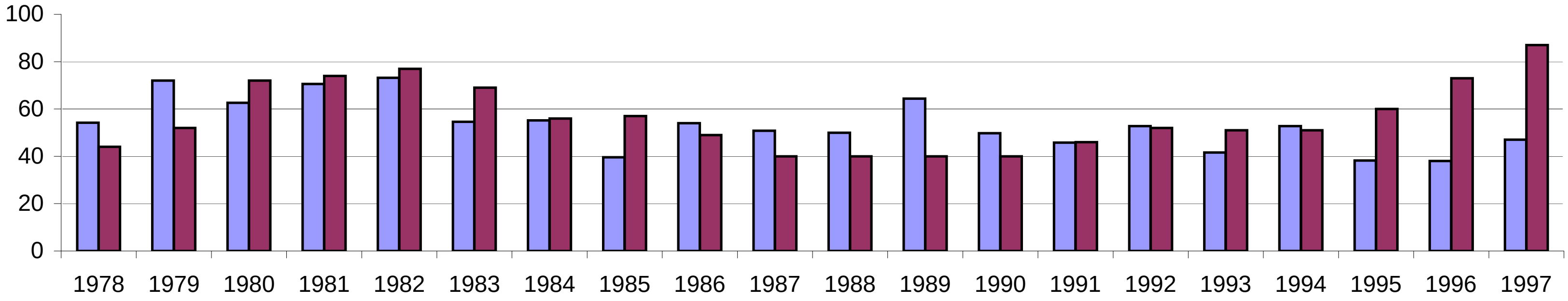
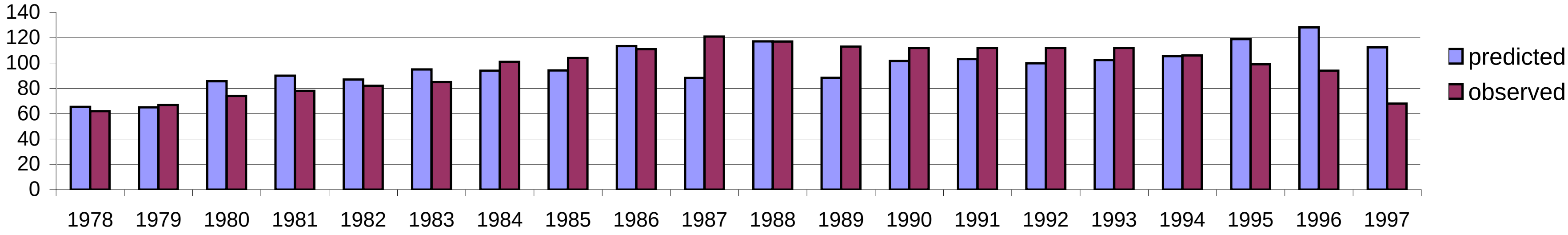


Figure 6

DC-10 Aircraft Owned by Small U.S. Airlines



DC-10 Aircraft Owned by Large U.S. Airlines



DC-10 Aircraft Owned by "Other" Airlines