Innovation, Market Structure and the Holdup Problem:
Investment Incentives and Coordination

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Abstract: I analyze the innovation incentives under monopoly and duopoly provision of horizontally differentiated products purchased via bilateral negotiations, integrating the market structure and innovation literature with the holdup literature. I show that competition can improve local incentives for non-contractible investment. Because innovation levels are generally strategic substitutes, however, there can be multiple duopoly equilibria. In some circumstances, monopoly can provide a coordination device that can lead to greater expected welfare despite inferior local innovation incentives. The conditions for this to be the case, however, are quite restrictive.

I. Introduction

The question of what type of market structure provides the best innovation incentives dates back to Kenneth Arrow (1962). While many papers have analyzed this issue in a variety of settings (Arrow, 1962; Partha Dasgupta and Joseph Stiglitz, 1980; Jeremy Bulow, 1982; and many others), there has been no work on the effect of market structure on non-contractible investment (such as innovation) incentives in markets where trade occurs in bilateral contracts rather than in a spot market. In addition, most prior papers dealing with bilateral contracts and non-contractible investments have assumed the existence of bilateral monopoly (Oliver Williamson, 1985; Jean Tirole, 1986; Sanford Grossman and Oliver Hart, 1986; Oliver Hart and John Moore, 1988 among others), bypassing the question of how market structure affects investment incentives. Bilateral monopoly, however, is not the only situation where trade occurs via individually negotiated contracts rather than in spot markets. When customers are not final consumers, but rather firms purchasing inputs, these firms will often negotiate with multiple suppliers. [1] Even in cases where there is a bilateral monopoly, that monopoly will be often created by a choice between alternative suppliers in a prior period. In fact, this paper grows out of analysis of a proposed merger between the two dominant suppliers of accounting software for large law firms where the issues analyzed in this paper had direct policy relevance.

In markets where trade is governed by bilateral contracts, the issue of output distortion does not arise because trade is negotiated individually. [2] Thus, the effect of market structure on welfare will only be through its effect on non-contractible investments (as is standard in the holdup literature). Moreover, the effect of market structure on product innovation incentives is

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[1] Consider the market for various types of business planning software. Companies such as Oracle or PeopleSoft offer large, customizable software packages for business that perform a myriad of essential tasks such as billing and accounting, human resources management, supply chain management and many others. Large companies do not buy these products "off the shelf." Rather, they send out a request for proposal to several firms and negotiate the best deal with the company they prefer.

[2] Of course, there can be bargaining failures. In the model below, I assume that bargaining always results in the efficient transaction taking place. Even when bargaining failures can occur, however, market

substantially different when there are no set prices. For example, the "replacement effect" and the "product inertia effect" that greatly influence innovation incentives in standard models (Shane Greenstein and Garey Ramey, 1998 is one such example) are not present in this paper. I show that competition does alleviate the holdup problem by providing superior incentives for non-contractible investment. Since only the sellers are making non-contractible investments, if they had all the bargaining power, investment incentives would be optimal under duopoly and monopoly. Because the split of the surplus is determined by a bargaining game, however, I show that a seller only gets the entire marginal surplus from product improvements (though not the entire total surplus) when the buyer has a binding outside option (an option that gives the buyer at least half the total surplus available from trade with its preferred seller). Otherwise, the seller only receives half the marginal surplus from product improvements. Competition alleviates the holdup problem, then, because competition increases the options available to buyers, making it more likely that a buyer will have a binding outside option when negotiating with its preferred supplier. When more buyers have binding outside options, the marginal return (to the seller) from product improvement is closer to the social optimum.

Competition does not always increase total welfare, however, because it can create a coordination problem. There can be multiple equilibria because the two firm's innovation levels are (in most circumstances) strategic substitutes. This is due to a market share effect; the more my rival innovates the larger is her market share and the smaller is mine, reducing my incentive to innovate.

When there are two competing suppliers there is always one duopoly equilibrium that generates at least as much welfare as the monopoly outcome. When there are multiple duopoly equilibria, however, there will sometimes be one or more equilibria that generate less welfare than the monopoly outcome. This is more likely when non-contractible investment (innovation)

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structure still won't affect the degree of output distortion unless the probability of a bargaining failure is correlated with market structure.

costs are small, but not too small. Since it turns out that more asymmetric duopoly equilibria generate more welfare, when multiple equilibria exist, small innovation costs induce the monopolist to develop the two products asymmetrically. If innovation costs are too small, however, then the only possible duopoly equilibrium will be asymmetric as well. Of course, for monopoly to be superior even in these cases, buyers' value from trading outside the market must also not be too small or the inferior development incentives will overwhelm any potential coordination advantage. In addition, the best duopoly equilibrium (from a social welfare standpoint) must not occur with probability one when there are multiple equilibria.

This is not the first paper to study the relationship between competition and non-contractible investment incentives. Leonardo Felli and Kevin Roberts (2000) have also shown that competition can alleviate the holdup problem when only sellers invest. Their model, however, assumes that sellers can make take it or leave it offers to buyers. This guarantees that sellers get the entire marginal surplus from the transaction (whether there is competition or not). Thus, it does not address the central question of this paper: what is the effect of competition on ex ante investment incentives when trade is negotiated? Like Felli and Roberts (2000), Harold Cole et. al. (2001) model the effect of competition in a model with match specific investments (not product innovation investments as in this paper). An additional difference between their paper and this one is that their analysis of competition considers the effect of adding closer substitutes, whereas in this paper, the number of available products is fixed, but there is more competition when the products are under separate ownership. Neither of the above papers considers the effect of ownership on the holdup problem. Che and Ian Gale (2000) also show that competition in the form of contests can help improve sellers' incentives to make non-contractible cooperative investments. Because they focus on contests, however, their paper also does not consider the effect of competition when price is negotiated.

The plan of the rest of the paper is as follows. Section II develops the duopoly model, while Section III develops the monopoly model. Section IV discusses the welfare comparisons between the two. Section V concludes. All proofs are in the appendix.

II. The Duopoly Model

There are two producers, A and B, each producing their own, differentiated, product, at zero marginal cost (I will also refer to the products as A and B). Analyzing differentiated products is natural in this setting since negotiated trade is far less likely in a market for homogenous products. I consider horizontal differentiation, since vertical differentiation does not make sense in the context of bilaterally negotiated trade.

Consider a linear city model where product A is located at point zero and B is at point one. There is a unit mass of customers who are uniformly distributed over the interval from zero to one. Customers only have use for one unit of one of the two products (a business only needs one accounting software program or one supply chain management software program, for example).

There are two periods in the model. In period 0, products A and B start with an equal general value to customers, V. The products are differentiated, however, by their location in product space. A customer at point e between 0 and 1 gets a value from product A in period 0 of $V - ke$, where k is the parameter measuring the cost of purchasing a product whose specifications are one unit away from one's optimal specifications. Similarly, this customer's value from product B is $V - k(1-e)$. I assume that each customer's value of ε is common knowledge to the customer and both suppliers.[3] During period 0, the suppliers of product A and B choose an

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[3] This assumption allows me to use a bargaining solution without uncertainty to determine the price that each customer pays for the product and eliminates the possibility of bargaining failures. While this will not exactly reflect reality, for many markets it is not that far off. In the business planning software market, for example, the customers send out detailed requests for proposals, host demonstrations where they inquire about the capabilities that they are most interested in, and ask for specific customization of the software to meet their needs. All of this provides the supplier with detailed information about how important different capabilities are to that customer, giving it a very good idea of how the customer will value its product relative to its competitors.

amount of, non-stochastic, product development innovation. If supplier i develops its product by an amount d_i then it increases the value of its product by that amount. This costs the supplier $C(d_i) = \frac{c}{2} d_i^2$. Customers do not maker purchase decisions until period 1. At this time, a customer that purchases product A receives a value of $V+d_A-k\boldsymbol{e}$, whereas, if it purchases product B it receives a value of $V+d_B-k(1-\boldsymbol{e})$.

Thus, I am following Yeon-Koo Che and Donald Hausch (1999) in considering the case of cooperative investments rather than selfish investments, i.e., investments that benefit one's trading partner rather than oneself. In the markets I have in mind (such as software markets), the dominant form of innovation is product improvement innovation, which is fully cooperative.

Notice that I do not allow contracts in period 0 between customers and firms. There are several reasons for this. First, Che and Hausch (1999) prove, for cooperative investments in a bilateral setting, that there is no ex ante, renegotiation proof, contract that can improve upon ex post negotiation. To the extent this result is different when there are two suppliers, this only strengthens the result that competition often alleviates the holdup problem. Second, firms may not know the identity of all their potential customers in period 0, making ex ante contracts infeasible. Third, the type of contract suggested by W. Bentley MacLeod and James M. Malcomson (1993) for inducing efficient investments in a similar situation requires ensuring that a customer's outside option always binds. This necessitates payments from seller to buyer if the buyer does not trade with the seller. Unless the seller gets the entire surplus from their interaction, up front payments from the buyer to the seller will be strictly less than this payment. Thus, potential buyers have a strong incentive to misrepresent their desire to purchase this product so as to enter into this type of contract that gives them positive surplus even if they do not intend to trade. These losses could easily exceed the seller's share of the added surplus from more efficient development incentives. Finally, in many cases, such as the market for accounting

software for law firms that motivated the paper, we do not observe such contracts, thus we need a theory to explain the effect of market structure on welfare in these situations.

Since ε is common knowledge, and I assume that purchase decisions are made by bargaining without transactions costs, each customer will choose the product that gives it the greatest value. Thus, a customer will choose product A if and only if its $e < \dfrac{d_A - d_B + k}{2k} \equiv e*$.

The price each consumer pays is determined by the outcome of a bargaining game between the consumer and the two potential suppliers.[4] I posit an alternating offer bargaining game of the type used in Patrick Bolton and Michael D. Whinston (1993). The only difference between that bargaining game and this one is that in their game there are two buyers and one seller rather than two sellers and one buyer. Thus, the equilibrium of this bargaining game is a straightforward application of their equilibrium. They show that the equilibrium of the three player bargaining game is identical to the equilibrium of an outside option bargaining game between the two parties with the largest joint surplus where the party with the alternative trading partner has an outside option of trading with its less preferred partner and obtaining the entire surplus from that trade. It is well known that the solution to this outside option bargaining game (in the current application) gives the buyer the larger of half the surplus from the transaction and the surplus it could get from its outside option (getting the other product for free) (Ariel Rubenstein, 1982; Avner Shaked and John Sutton, 1984). As a result, a buyer located at $e \le e*$ will pay the following price for product A:

(1a) $\qquad p_A(e) = Min\{\dfrac{1}{2}(V + d_A - ke), (V + d_A - ke) - (V + d_B - (1-k)e)\}.$

Similarly, a buyer located at $e > e*$ will pay the following price for product B:

(1b) $\qquad p_B(e) = Min\{\dfrac{1}{2}(V + d_B - (1-k)e), (V + d_B - (1-k)e) - (V + d_A - ke)\}.$

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[4] Bargaining over price is common for intermediate goods. It is certainly the dominant method of trade when large firms purchase major software systems.

Given these prices, A and B in period 0 choose d_A and d_B respectively to maximize their profits. The profit functions are given below, where f is the probability density function for the uniform distribution ($f(e) = 1$ iff $e \in [0,1]$, 0 otherwise).

(2a)
$$P_A = \int_0^{e^*} p_A(e) f(e) de - C(d_A)$$

(2b)
$$P_B = \int_{e^*}^1 p_B(e) f(e) de - C(d_B) .$$

Differentiating (2), with these pricing functions, gives the following first order conditions:

(3a)
$$cd_A = F(\frac{d_A - d_B + k}{2k}) - \frac{1}{2} F(\frac{d_A - 2d_B - V + 2k}{3k}) \text{ and}$$

(3b)
$$cd_B = 1 - F(\frac{d_A - d_B + k}{2k}) - \frac{1}{2}(1 - F(\frac{2d_A - d_B + V + k}{3k})) .$$

Notice that the producer obtains the full marginal increase in value from product development from its customers with binding outside options and half the increase in value from its customers whose outside options are not binding.

Customers that prefer A have B as their outside option. For those A customers with smaller relative preferences for A over B, $\frac{d_A - 2d_B - V + k}{3k} < e \le \frac{d_A - d_B + k}{2k}$, the outside option of B will be binding in their bargaining game with A. These customers will get the surplus that they would receive if they bought B for free, and any increase in the value of A from more development accrues entirely to the producer of A. However, for those customers with larger relative preferences for A, for whom B is not a binding outside option, increases in the value of the product from added development are split evenly between the customer and the producer. Similarly, customers who prefer product B, but not by too much

$(\frac{d_A - d_B + k}{2k} < e < \frac{d_A - 2d_B - V + k}{3k})$, have a binding outside option of purchasing product A.

But for those customers with very strong preferences for B over A, purchasing product A is not a binding outside option.

While in models of innovation with non-negotiated prices, innovative investments are strategic substitutes, this is not always the case here. Typically, innovative investments are strategic substitutes because increased innovation by the rival decreases one's market share, reducing one's incentive to invest. With negotiated prices, however, there is an additional effect. When one's rival innovates this improves the outside option for one's customers. This increases the fraction of one's customers whose outside option is binding. And for those customers, the firm gets all, rather than just half, the added surplus from innovation. Because I assume that customers are uniformly distributed between zero and one, the market share effect will always dominate (innovative investments will be strategic substitutes) so long as at least some consumers buy each product. If, however, no consumers buy one product, then increased investment in that product will only affect the outside option for the other firm's customers, but will not reduce its market share, making investment by the non-selling firm a strategic complement for investment by the selling firm. If the density of consumers were greater near the middle of the unit interval than at the extremes, then investment by the firm selling to a positive (but very small) fraction of consumers could be a strategic complement for investment by the firm selling to the vast majority of the customers.

Differentiating the marginal benefit functions (the right hand sides of (3a) and (3b)) with respect to rival's development gives the precise conditions for when innovation is a strategic substitute or complement. For d_A, d_B will be a strategic substitute for d_A if and only if:

(4a)
$$f(\frac{d_A - d_B + k}{2k}) > \frac{2}{3} f(\frac{d_A - 2d_B - V + 2k}{3k})$$

Similarly, d_A will be a strategic substitute for d_B if and only if:

(4b)
$$f(\frac{d_A - d_B + k}{2k}) > \frac{2}{3} f(\frac{2d_A - d_B + V + k}{3k})$$

Figure 1 depicts the reaction functions for d_A and d_B for one particular set of parameter values.

Figure 1
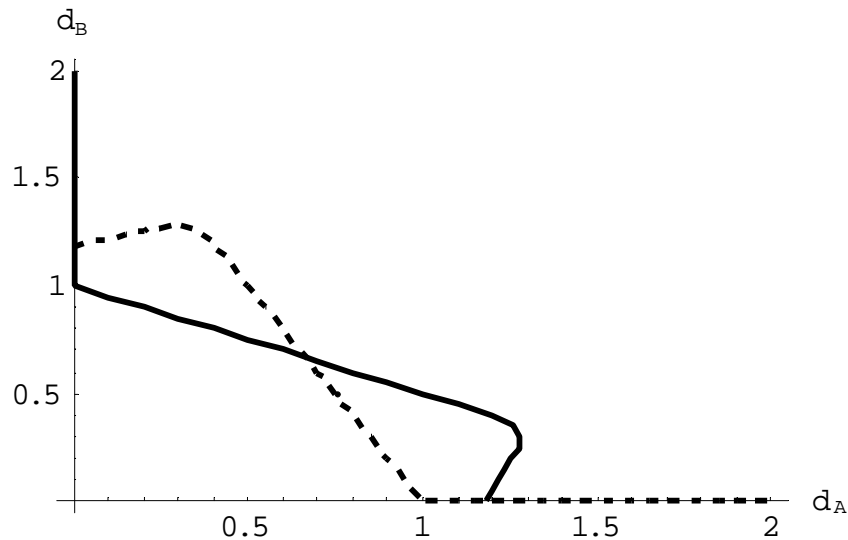
$$V = 2.5, c = .75, k = 1$$

The dashed line is B's reaction function. The solid line is A's reaction function.

Notice that there are three possible equilibria in the figure (the reaction functions intersect three times). There is a symmetric equilibrium where both products are developed equally and two equilibria where only one product is developed and sold. Of course, the number of equilibria is dependent on the values of the parameters. If product development is very costly (c is large), then there will be a symmetric equilibrium. In Figure 1, however, product development is cheap enough that if one firm (say A) is not going to develop its product then B's level of development will be large enough that no customers will buy product A, giving A no incentive to develop its product.

One can also see in Figure 1 that when d_B, for example, is very small, it becomes a strategic complement for d_A. A similar effect occurs for d_B when d_A is very small (the reaction functions have a positive slope in these regions).

Figure 2 depicts the reaction functions when product development is relatively more important (its is cheaper and the initial value of the product is smaller). In this case, there five, rather than three, equilibria. In addition to the symmetric equilibria and the equilibria at the corners, there are two interior asymmetric equilibria.

Figure 2

$$V = 1, c = .6, k = 1$$
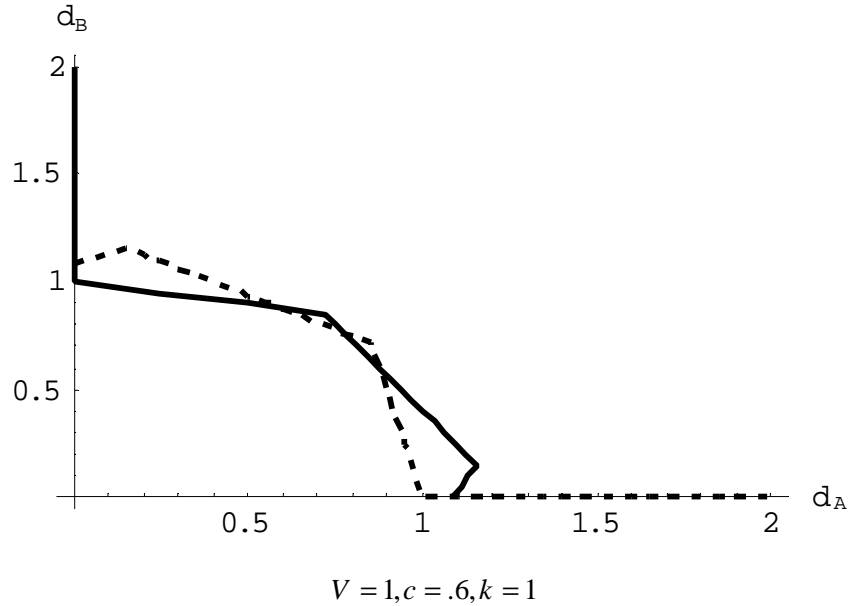
The interior asymmetric equilibria occur when one product (say B) is developed enough more than A that the outside option of purchasing product B is binding for all customers that purchase product A. The reverse, however, is not true. Thus, even though A has a smaller market share, its development incentive is only slightly smaller than B's since A receives the entire surplus from added development from all its customers. This ensures that its market share is large enough to justify its development level.

Because I assume that customers are uniformly distributed, explicitly solving the duopoly first order conditions (3) requires considering six possible cases. There is the fully interior case, where $0 < \dfrac{d_A - 2d_B - V + 2k}{3k} < \dfrac{d_A - d_B + k}{2k} < \dfrac{2d_A - d_B + V + k}{3k} < 1$. In this case both A and B have positive sales, and, moreover, both A and B sell to some customers with binding outside

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options and to some customers without binding outside options. I call this case (2I) since two firms sell and the outside option cutoff points are interior. There is the case where all of one firm's customers have a binding outside option, but the other firm has customers of both types and both firms make sales. I call this case (2B); two firms sell and one has all its customers with a binding outside option. This case requires that the following condition hold:

$$\frac{d_A - 2d_B - V + 2k}{3k} < 0 < \frac{d_A - d_B + k}{2k} < \frac{2d_A - d_B + V + k}{3k} < 1.$$ Notice that this condition

indicates that it is firm A whose customers all have a binding outside option. Of course, there is an analogous equilibrium where it is firm B whose customers have the binding outside option. Since these two equilibria are identical, I consider them as one equilibrium type. The third case where both firms make positive sales is where all the customers of both firms have binding outside options. I call this case (2BB) (for two binding outside options). It requires that:

$$\frac{d_A - 2d_B - V + 2k}{3k} < 0 < \frac{d_A - d_B + k}{2k} < 1 < \frac{2d_A - d_B + V + k}{3k}.$$ Notice, that when both firms

make sales, it is not possible that either firm will have all its customers have a non-binding outside option since the marginal customer is necessarily indifferent between the two goods.

The three remaining cases all involve only one firm making sales. For each case, there is an equilibrium where the firm that does not make sales is A and where that firm is B. I will describe the case where A makes no sales. When A makes no sales, its value can be such that for some customers purchasing A for free gives them a binding outside option while for some customers it does not. I call this case (1I) (one firm sells, the outside option cutoff is interior).

This requires that: $\frac{d_A - d_B + k}{2k} < 0 < \frac{2d_A - d_B + V + k}{3k} < 1$. Alternatively, the value of A could

be such that all consumers have a binding outside option:

$$\frac{d_A - d_B + k}{2k} < 0 < 1 < \frac{2d_A - d_B + V + k}{3k}.$$ This is case (1B). There is also case (1NB), where A

is not a binding outside option for any customers. This requires that:

$$\frac{d_A - d_B + k}{2k} < \frac{2d_A - d_B + V + k}{3k} < 0 < 1.$$

While there are six distinct types of equilibria that can obtain, only some of subset of these equilibria will be feasible for any given set of parameter values. To determine when any of these equilibria are feasible, one must solve the first order conditions for d_A and d_B under the assumption that a given equilibrium obtains and then determine for what parameter values the development levels are consistent with the conditions for that equilibrium.

For example, if equilibrium (2I) exists then the first order conditions are:

(5a) $$cd_A = \frac{d_A - d_B + k}{2k} - \frac{1}{2}\frac{d_A - 2d_B - V + 2k}{3k} \quad \text{and}$$

(5b) $$cd_B = 1 - (\frac{d_A - d_B + k}{2k}) - \frac{1}{2}(1 - (\frac{2d_A - d_B + V + k}{3k})).$$

Solving these two equations for equilibrium development levels gives:

(6) $$d_A{}^{2I} = d_B{}^{2I} = \frac{V + k}{6ck - 1}.$$

It is easy to see that the second order conditions in this case require that $ck>1/3$, and that these development levels are consistent with the conditions for (2I) if and only if

$\frac{2ck - 1}{4c} \le V \le \frac{4ck - 1}{2c}$. If V is larger than this upper bound then every customer's outside option binds, while if V is too small the customer at ½ will not get positive value from either product (when development levels are given by (6)).

Table 1 gives the development levels and the conditions for each of the six possible equilibria. The entries in the table are obtained exactly as they were obtained for (2I) in the above paragraph.

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Table 1

Equilibria	Development Levels	Conditions on ck	Conditions on V
(2I)	$d_A{}^{2I} = d_B{}^{2I} = \dfrac{V+k}{6ck-1}$	$ck \geq \dfrac{1}{3}$	$\dfrac{2ck-1}{4c} \leq V \leq \dfrac{4ck-1}{2c}$
(2B)	$d_A{}^{2B} = \dfrac{V+3k(1-2ck)}{-1+10ck-12(ck)^2}$; $d_B{}^{2B} = \dfrac{V(1-2ck)+2k(1-ck)}{-1+10ck-12(ck)^2}$	$\dfrac{1}{2} \leq ck \leq \dfrac{5+\sqrt{13}}{12}$ or $\dfrac{5+\sqrt{13}}{12} < ck \leq 1$	$3k(2ck-1) \leq V \leq \dfrac{4ck-1}{2c}$ or $\dfrac{4ck-1}{2c} \leq V \leq 3k(2ck-1)$
(2BB)	$d_A{}^{2BB} = d_B{}^{2BB} = \dfrac{1}{2c}$	$ck \geq \dfrac{1}{2}$	$V \geq \dfrac{4ck-1}{2c}$
(1I)	$d_A{}^{1I} = 0$; $d_B{}^{1I} = \dfrac{V+4k}{6ck+1}$	$0 \leq ck \leq \dfrac{1}{2}$ or $\dfrac{1}{2} < ck \leq 1$	$\dfrac{1-2ck}{2c} \leq V \leq \dfrac{2ck+1}{c}$ or $3k(2ck-1) \leq V \leq \dfrac{2ck+1}{c}$
(1B)	$d_A{}^{1B} = 0$; $d_B{}^{1B} = \dfrac{1}{c}$	$0 \leq ck \leq 1$	$V \geq \dfrac{2ck+1}{c}$
(1NB)	$d_A{}^{1I} = 0$; $d_B{}^{1NB} = \dfrac{1}{2c}$	$0 \leq ck \leq \dfrac{1}{2}$	$V \leq \dfrac{1-2ck}{2c}$

Note that for equilibria (2B) and (1I) there are two different conditions for ck and for V. The first condition for V are applicable only when the first condition for ck holds and similarly for the second conditions. Thus, in (2B), for example, the equilibria will exist if and only if

$\dfrac{1}{2} \leq ck \leq \dfrac{5+\sqrt{13}}{12}$ and $3k(2ck-1) \leq V \leq \dfrac{4ck-1}{2c}$ or $\dfrac{5+\sqrt{13}}{12} < ck \leq 1$ and

$\dfrac{4ck-1}{2c} \leq V \leq 3k(2ck-1)$. Also, note that the lower bounds on ck in the first three equilibria

ensure that the second order conditions are satisfied. The second order conditions for the last

three equilibria hold for any $ck>0$.

Inspection of Table 1 reveals the above figures do not represent special cases; for many parameter values there are multiple equilibria. In Section IV, I will discuss how the equilibria compare in terms of social welfare.

III. The Monopoly Model

Because trade is negotiated individually under perfect information, market structure does not affect the purchase decisions of any customers (though, it will affect the price). When one firm owns both A and B, it doesn't allow them to compete against each other for any customer, eliminating this outside option for every customer. I assume that there exists an inferior third product that provides an exogenously given value v to all customers regardless of location. This product would not affect the equilibrium in the duopoly model because I assume that $v \leq V - k$: even if A or B did not develop their product at all one of these products would be at least as good an outside option for all customers as the third product. Now that customers are only offered A or B, but not both, however, this third product serves as every customer's outside option. As a result, a customer at $e \leq e*$ now pays the following price for product A:

(7a)
$$p_A(e) = Min\{\frac{1}{2}(V + d_A - ke), (V + d_A - ke) - v\}.$$

Similarly, a buyer located at $e > e*$ will pay the following price for product B:

(7b)
$$p_B(e) = Min\{\frac{1}{2}(V + d_B - (1-k)e), (V + d_B - (1-k)e) - v\}.$$

With these pricing functions, the monopolist's profit function is the following:

(8)
$$P_M = \int_0^{e*} p_A(e)f(e)de - C(d_A) + \int_{e*}^{1} p_B(e)f(e)de - C(d_B)$$

Differentiating (8) gives the following first order conditions in the monopoly case:

(9a)
$$cd_A = F(\frac{d_A - d_B + k}{2k}) - \frac{1}{2}F(\frac{d_A - 2v + V}{k}) \text{ and}$$

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(9b) $$cd_B = 1 - F(\frac{d_A - d_B + k}{2k}) - \frac{1}{2}(1 - F(\frac{d_B - k - 2v + V}{k})) \,.$$

Because $v \leq V - k$, the right hand sides of (9) are (at least weakly) smaller than the right hand sides of (3), the first order conditions in the duopoly case. The reason the marginal benefit to innovating is smaller with monopoly than duopoly is that, since the value of every consumer's outside option smaller, a smaller fraction of consumers will have a binding outside option. Thus, the seller will more often split the gross social surplus from the transaction rather than get the residual surplus over the consumer's outside option. So, even though the total price the producer receives for the product is (weakly) greater for every consumer, the marginal effect of innovation on price is (weakly) smaller with monopoly than with duopoly.

Since there can be multiple solutions to the monopolist's first order conditions, (9), to accurately compare the monopoly case to the duopoly one, I must determine which solution maximizes profits. To do so, I first explicitly solve the first order conditions to determine the candidate profit maximizing development levels for any given set of parameter values. This is done in the same way as I determined the possible solutions to the duopoly first order conditions above. Of course, since every customer's outside option is now to purchase the third product and obtain surplus v, the conditions for when each possible solution is valid are different. (I give each of these possible solutions the same abbreviated label as in the duopoly case except that I add the letter M in front in the monopoly case.) In addition, there are also two other possible solutions when both products are sold: the outside option can now be not binding for all customers of either one or both products. I call these two cases (M-2NB) and (M-2NBNB) respectively. (There is also the possibility of a case where one outside option is always binding and the other is not, however, one can show that there is no solution to the monopolist's first order conditions that is consistent with this case. It also turns out there is no solution in case (M-2B) either.)

For both products to have positive sales and an interior outside option cutoff, (M-2I)

requires that $0 < \dfrac{d_A - 2v + V}{k} < \dfrac{d_A - d_B + k}{2k} < \dfrac{d_B - 2v - k + V}{k} < 1$. Notice that this is identical

to the duopoly conditions for (2I) except that $\dfrac{d_A - 2v + V}{k}$ replaces $\dfrac{d_A - 2d_B - V + 2k}{3k}$ and

$\dfrac{d_B - k - 2v + V}{k}$ replaces $\dfrac{2d_A - d_B + V + k}{3k}$. The conditions for the other cases are also identical

to the duopoly conditions with the same replacements. The conditions for the two new cases are:

(10) $0 < \dfrac{d_A - d_B + k}{2k} < \dfrac{d_A - 2v + V}{k}$ and $\dfrac{d_A - d_B + k}{2k} < \dfrac{d_B - 2v - k + V}{k} < 1$ (M-2NB) and

(11) $0, \dfrac{d_B - 2v - k + V}{k} < \dfrac{d_A - d_B + k}{2k} < \dfrac{d_A - 2v + V}{k}, 1$ (M-2NBNB).

After obtaining the solutions to the monopolist's first order conditions (and the conditions

for when those solutions are valid) for every case, I compare the profit among solutions that exist

for overlapping parameter values. For example, if equilibrium (M-2I) exists then the first order

conditions are:

(12a) $$cd_A = \dfrac{d_A - d_B + k}{2k} - \dfrac{1}{2}\dfrac{d_A - 2v + V}{k} \quad \text{and}$$

(12b) $$cd_B = 1 - (\dfrac{d_A - d_B + k}{2k}) - \dfrac{1}{2}(1 - (\dfrac{2d_A - d_B + V + k}{k})) .$$

Solving these two equations for equilibrium development levels gives:

(13) $$d_A{}^{M-2I} = d_B{}^{M-2I} = \dfrac{2v + k - V}{2ck + 1} .$$

It is easy to show that these development levels are consistent with the conditions for (M-2I) if

and only if $\dfrac{2ck - 1}{4c} \geq V - 2v \geq -\dfrac{1}{2c}$. If $V\text{-}2v$ is larger than this upper bound then the outside

options are never binding, while if $V\text{-}2v$ is too small they are always binding when development

levels are given by (13).

When equilibrium (M-1I) exists, on the other hand, the first order conditions are:

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(14a) $$cd_A = 0 \text{ and}$$

(14b) $$cd_B = 1 - \frac{1}{2}(1 - (\frac{2d_A - d_B + V + k}{k})) .$$

Solving these two equations for equilibrium development levels gives:

(15) $$d_A{}^{M-1I} = 0; d_B{}^{M-1I} = \frac{2v + 2k - V}{2ck + 1} .$$

One can show that these development levels are consistent with the conditions for (M-1I) if and only if $\frac{2ck-1}{2c} \geq V - 2v \geq -\frac{1}{c}$ and $ck<1/2$ or $k(1-2ck) \geq V - 2v \geq -\frac{1}{c}$ and $1/2 \leq ck \leq 1$. If *V-2v* is larger than this upper bound then the outside options are never binding, while if *V-2v* is too small they are always binding.

In comparing (M-2I) and (M-1I), one can see that both solutions are valid when

$\frac{2ck-1}{2c} \geq V - 2v \geq -\frac{1}{2c}$ and *ck<1/2* or when $\frac{2ck-1}{4c} \geq V - 2v \geq -\frac{1}{2c}$ and $1/2 \leq ck \leq 1$. Since the monopolist will never choose the one that yields lower profit, I must compare the profit in the two solutions under these conditions. Using the monopolist's profit function, (8), I can write the profit difference between (M-1I) and (M-2I) as follows:

(16) $$\frac{k(1 - 2ck) + 2c(V - 2v)^2}{4(1 + 2ck)}$$

When *ck<1/2*, this is obviously negative, so the monopolist will never choose (M-2I). If $1/2 \leq ck \leq 1$, then the monopolist will only choose (M-2I) if:

(17) $$V - 2v \in [-\sqrt{\frac{k(2ck-1)}{2c}}, \sqrt{\frac{k(2ck-1)}{2c}}] .$$

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Since $\sqrt{\dfrac{k(2ck-1)}{2c}} > k(1-2ck)$ whenever $ck<1$, (M-2I) will be more profitable than (M-1I) if

and only if $V - 2v > -\sqrt{\dfrac{k(2ck-1)}{2c}}$. This condition only limits the possible parameters for when

(M-2I) or (M-1I) can be chosen if $ck < \dfrac{1+\sqrt{5}}{4}$, since otherwise $-\dfrac{1}{2c} > -\sqrt{\dfrac{k(2ck-1)}{2c}}$.

By conducting comparisons of this type between all possible solutions of the monopolist's first order conditions, one can determine the profit maximizing development levels for all possible parameter values. The results are summarized in Table 2. (I do not need to worry about second order conditions in Table 2 since the development levels have been compared to all other possible local maxima.)

Table 2

Case	Development Levels	Conditions on ck	Conditions on V
(2I)	$d_A^{M-2I} = d_B^{M-2I}$ $= \dfrac{2v+k-V}{2ck+1}$	$\dfrac{1}{2} \le ck < \dfrac{1+\sqrt{5}}{4}$ or $\dfrac{1+\sqrt{5}}{4} \le ck$	$-\sqrt{\dfrac{k(2ck-1)}{2c}} \le V-2v \le \dfrac{2ck-1}{4c}$ or $-\dfrac{1}{2c} \le V-2v \le \dfrac{2ck-1}{4c}$
(2BB)	$d_A^{M-2BB} = \dfrac{1}{2c}$ $d_B^{M-2BB} = \dfrac{1}{2c}$	$\dfrac{1+\sqrt{5}}{4} \le ck < 1$ or $ck \ge 1$	$\dfrac{-2+\sqrt{2(1+ck-2(ck)^2}}{2c} \le V-2v \le -\dfrac{1}{2c}$ or $V-2v \le -\dfrac{1}{2c}$
(2NB)	DNE	DNE	DNE
(2NBNB)	$d_A^{M-2NBNB} = \dfrac{1}{4c}$ $d_B^{M-2NBNB} = \dfrac{1}{4c}$	$ck>1/2$	$\dfrac{2ck-1}{4c} \le V-2v$
(1I)	$d_A^{M-1I} = 0$; $d_B^{M-1I} = \dfrac{2k+2v-V}{2ck+1}$	$0 \le ck \le \dfrac{1}{2}$ or $\dfrac{1}{2} \le ck < \dfrac{1+\sqrt{5}}{4}$ or $\dfrac{1+\sqrt{5}}{4} \le ck < 1$	$-\dfrac{1}{c} \le V-2v \le \dfrac{2ck-1}{2c}$ or $-\dfrac{1}{c} \le V-2v \le -\dfrac{\sqrt{ck(2ck-1)}}{\sqrt{2}c}$ or $-\dfrac{1}{c} \le V-2v \le \dfrac{-2+\sqrt{2(1+ck-2(ck)^2)}}{2c}$
(1B)	$d_A^{M-1B} = 0$; $d_B^{M-1B} = \dfrac{1}{c}$	$ck \le 1$	$V-2v < -\dfrac{1}{c}$
(1NB)	$d_A^{M-1NB} = 0$; $d_B^{M-1NB} = \dfrac{1}{2c}$	$ck \le 1/2$	$\dfrac{2ck-1}{2c} < V-2v$

IV. Welfare Comparisons

I define social welfare as total surplus, the total value purchased by customers less the costs of innovation. Given that the social planner will want to assign each customer to the product it values most, the social welfare function can be written as follows:

$$(18) \qquad SW = \int_0^{e^*} (V + d_A - ke) f(e) de + \int_{e^*}^1 (V + d_B - (1-k)e) f(e) de - C(d_A) - C(d_B).$$

The first order conditions for social welfare maximization are the following:

$$(19a) \qquad\qquad cd_A = F(\frac{d_A - d_B + k}{2k}) \text{ and}$$

$$(19b) \qquad\qquad cd_B = (1 - F(\frac{d_A - d_B + k}{2k})).$$

By comparing these first order conditions with those of the duopoly and monopoly models ((3) and (9) respectively) one can see that the social planner wants larger local innovation incentives than exist in either duopoly or monopoly, but that local innovation incentives are closer to the social optimum under duopoly than monopoly.

Of course, socially inferior local innovation incentives do not necessarily imply socially inferior innovation levels. To do precise welfare comparisons, one must compare all of the possible equilibria in the duopoly model for various parameter values to the outcome for those parameter values in the monopoly case. When there are multiple equilibria with duopoly, welfare comparisons may also depend on the probability of any of particular duopoly equilibria obtaining. This is particularly true because, as the first proposition demonstrates, welfare varies among duopoly equilibria in a particularly straightforward fashion.

Proposition 1. Whenever there are multiple duopoly equilibria, social welfare is always largest when development levels are most asymmetric.

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The advantage of asymmetric equilibria is that they have less duplicative investment and more customers are taking advantage of the product that invests more. The disadvantage is that customers are, on the average, purchasing products that are farther away from their ideal location in product space. It turns out, however, that when this effect dominates for a particular equilibrium, more asymmetric equilibria do not exist.

Because, when multiple equilibria are possible, more asymmetric equilibria generate more social welfare than the less asymmetric ones, the level of social welfare in a duopoly is uncertain. It will depend on the probability that any given equilibria, among those that are possible, will obtain. When one firm owns both products, however, this uncertainty is absent. While there still will be several different solutions to the monopolist's first order conditions, (9), Table 2 identifies exactly which solution maximizes profits for any particular set of parameter values. If the relative profit levels are closely enough aligned with the relative welfare levels among these solutions, then having a monopoly could provide a socially beneficial coordination device. Of course, whether or not the (potential) coordination benefits are ever sufficient to overcome the inferior local development incentives in monopoly will depend on just how inferior the development incentives are. This will depend on v, the value of the third product that provides the outside option for buyers once A and B are under common ownership. The larger v is the larger the fraction of customers that will have a binding outside option for any given set of parameter values, thus reducing the difference in product development incentives between monopoly and duopoly. The second proposition compares the total welfare in duopoly and monopoly market structures for the largest v the model allows (the best case for monopoly).

Proposition 2. Assume $v=V-k$.

(a) If $\dfrac{1}{3} \le ck < \dfrac{1}{2}$ and $V \le \dfrac{1-2ck}{2c}$ or $\dfrac{1}{2} \le ck < 1$ and $\dfrac{2ck+1}{c} \le V$, then expected

welfare is strictly greater with monopoly whenever there is a positive probability

of all feasible duopoly equilibria occurring.

(b) If $\dfrac{1}{3} \le ck < \dfrac{1}{2}$ and $\dfrac{1-2ck}{2c} \le V \le \dfrac{4ck+1}{2c}$ or $\dfrac{1}{2} \le ck < \dfrac{1+\sqrt{5}}{4}$ and

$\dfrac{\sqrt{ck(2ck-1)}}{\sqrt{2}c} + 2k \le V < \dfrac{2ck+1}{c}$ or $\dfrac{1+\sqrt{5}}{4} \le ck < 1$ and

$\dfrac{2 - \sqrt{2(1+ck-2(ck)^2)}}{2c} + 2k \le V < \dfrac{2ck+1}{c}$, then there exists some probability

distribution for what duopoly equilibria will obtain over feasible duopoly

equilibria where monopoly generates greater expected welfare than duopoly.

(c) For any other parameter values, duopoly generates as much or more welfare

than monopoly for any probability distribution over feasible duopoly equilibria.

What Proposition 2 says is that while it is possible for the coordination benefits of
monopoly to outweigh the superior local innovation incentives of duopoly, in most cases welfare
will be greater with duopoly. For welfare to be greater with monopoly, there must be multiple
duopoly equilibria. Whenever there is just one duopoly equilibrium, the monopolist will
distribute its product development effort across the two products in the same way as occurs in the
duopoly equilibrium. Since there is more development in duopoly (and there is too little
development in both cases), duopoly is necessarily superior.

When there is at least one duopoly equilibrium where both products are developed and
one where only one product is developed, however, then sometimes, but not always, the
monopolist will only develop one product. This is when there is a coordination advantage in
monopoly. Welfare is almost always greater when a monopolist develops only one product than
in a duopoly equilibrium where both products are developed (the exception is that welfare is

greater in (2B) than in (M-1I)). Because I assume a finite distribution of consumer types, so there are cases where local development incentives are identical in monopoly and duopoly, it is possible for the monopoly outcome to be identical to the duopoly equilibrium where only one product is developed. When this happens, this is part (a) of the proposition, duopoly can never generate more welfare than monopoly. If the monopoly outcome where only one product is developed has some, but not all, consumers with binding outside options, however, then there will always be a duopoly equilibrium where only one product is developed that is superior to the monopoly outcome (because local development incentives differ). Nonetheless, if there are other possible duopoly equilibria where both products are developed, part (b) says that monopoly could still generate more welfare in expectation provided the probability of these equilibria occurring is large enough.

The situations where monopoly can be superior occur when development costs are not too large and the products are not too differentiated (ck small) so that a monopolist will want to focus all its product development effort on one product. On the other hand, if ck is too small, then there will also be no duopoly equilibrium where both products are developed. The conditions on the products' initial value, V, are important because they determine whether the point at which outside options are binding or not is interior in each case. (This is true in the monopoly case because this proposition examines the case where the value of the third product, v, is V-k, the largest it can be and still not affect anyone's decisions in the duopoly case.) Thus, when ck is between a third and a half, V must be small for a duopoly equilibrium where both products are sold to exist. For larger ck, a monopolist will choose to develop only one product only if enough consumers have a binding outside option, i.e., only if the value of the third product is high enough. Since this is bounded by the original value of the dominant products, this value must be large also. If I were not explicitly assuming that the value of the third product were at its maximum, V-k, there would be an added minimum on its value, v. Thus, the smaller the value of the third product the less likely monopoly is to generate more welfare than duopoly.

It is important to note that the existence of cases where monopoly can never be worse than duopoly is dependent on the assumption of a finite distribution of consumer types. If the support of consumer types extended across the real line, then there would never be any regions where local development incentives were identical in monopoly and duopoly. If this distribution had very small density in the tails, however, there would be cases where the local innovation incentives were almost identical. Thus, the probability that duopoly would result in both products being developed would not have be much greater than zero for monopoly to generate more expected welfare than duopoly. So, while the exact result in part (a) is sensitive to the finite support assumption, a very similar result would obtain for most plausible distributions with infinite support.

V. Conclusion

Analyzing the welfare effects of market structure in markets where bilateral contracting is the norm is very different from analyzing these effects in markets where spot market transactions are predominate. With efficient bilateral contracting, there will be no ex post allocation inefficiency; market structure will only affect welfare when it affects the hold up problem. Because of the holdup problem, firms will have insufficient incentives to improve the quality of their products. I show that competition among suppliers, however, will reduce this problem.

Competition, however, brings with it the problem of equilibrium selection. I show that in a duopoly there is the potential for multiple equilibria in development levels. When that is the case, the more asymmetric is the development of the two products the greater is welfare. When one firm owns both products, however, then there is no such problem; the monopolist chooses the outcome that maximizes its profits. If the duopoly market sometimes arrives at an equilibrium where both products are developed, even when there is also an equilibrium where only one product is developed, then it is possible that, for some parameter values, the coordination benefits of monopoly can generate greater welfare than duopoly, despite the inferior local development incentives. As Proposition 2 makes clear, however, there are many more parameter values for

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which the benefit of added competition in increasing the value of the customers' outside option will result in greater total welfare. Moreover, Proposition 2 considers the case where the value of the third option is as large as it can be and still have the original market be a duopoly. When the third option is of lower value, then the set of parameter values where monopoly is superior will be even more restrictive.

Of course, the paper assumes that a firm's ability to develop its product(s) (the development cost function) is independent of market structure. If innovation synergies occur when one firm owns both products, then this makes monopoly more advantageous than the model in this paper predicts. In fact, because (efficiently) negotiated trade eliminates the effect of market structure on allocation efficiency, it will be much easier for synergies to result in greater welfare with less competition than in markets governed by fixed price transactions.

Clearly, analysis of the holdup problem should not be restricted to bilateral monopoly. I extend the holdup model by showing how market structure impacts non-contractible investment incentives in the presence of holdup problems. Of course, this paper is just a first step in this direction. There is a great deal of room for further research. I assume that the buyer's valuation from the product was independent of the value other buyers received from the product they bought. If there were downstream competition among the buyers, this might not be the case. Analyzing how this would affect the results of the paper would be quite interesting.

Even more interesting would be to relax the assumption that ε, the relative preference parameter, is common knowledge. While the suppliers often know much more about their products than customers, and often learn a great deal about how the customer will use the product in the sales process, the customer will always know more about its valuation for each product than the suppliers. Because I assumed away this type of information asymmetry, there were no bargaining failures in the model. Since bargaining failures reduce welfare, the effect of market structure on the incidence of bargaining failures will be important. The direction of this effect is unclear. On the one hand, under monopoly the disagreement option is worse for the buyer, so it

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has a greater incentive to avoid a bargaining failure. On the other hand, the consequences of bargaining failure in this situation are more severe.

Bargaining failures could also impact the results to the extent they affect innovation incentives. The social planner will always weigh bargaining failures more severely than the monopolist since the monopolist only bears part of the cost of a failure. Asymmetric information could also affect innovation incentives if it creates a business stealing incentive in duopoly. This is unlikely to occur, however, since it can only happen if an indifferent customer must pay a positive price for either product. Since the most likely form of bargaining failure from asymmetric information is delay, there should be no business stealing incentive even with asymmetric information. That said, analyzing more precisely the effect of asymmetric information would be quite interesting.

APPENDIX

Proof of Proposition 1. To prove this Proposition, I compare welfare in each equilibria to welfare in every other equilibria that can exist for the same set of parameter values.

Lemma 1.1. Welfare is greater in (2B) than in (2I) when both exist.

Proof. Evaluating the social welfare function, (18), for each equilibria and subtracting the welfare in (2I) from the welfare in (2B) gives:

(A1)
$$\frac{k(-5 + 50ck - 108(ck)^2 + 72(ck)^3)(1 - 4ck + 2cV)^2}{4(1 - ck)^2(1 - 10ck + 12(ck)^2)^2}.$$

This has the sign of $(-5 + 50ck - 108(ck)^2 + 72(ck)^3)$. Since it is only possible to have both

equilibria (2I) and (2B) when $\frac{1}{2} \leq ck \leq \frac{5 + \sqrt{13}}{12}$, I only need to determine the sign of this

expression within this region. The expression is convex in ck in this region, so the minimum

value will be at the value of ck where the first derivative of $(-5 + 50ck - 108(ck)^2 + 72(ck)^3)$ is

zero. Taking this derivative, setting it equal to zero and solving for ck gives $ck = \frac{9 + \sqrt{6}}{18}$. At

this value for ck, $(-5 + 50ck - 108(ck)^2 + 72(ck)^3) = 2(27 - 2\sqrt{6})/27 > 0$. Thus, (A1) is always

positive, proving the lemma.

Lemma 1.2. When both exist, welfare is always greater in (1I) than (2I).

Proof. Evaluating the social welfare function, (18), for each equilibria and subtracting the welfare in (2I) from the welfare in (1I) gives:

(A2)
$$\frac{-k\{-19 + 8cV - 72(cV)^2 + 4ck(25 - 18(cV)^2) + 72(ck)^2(3 + 4cV) - 1584(ck)^3 + 1296(ck)^4\} + 2V(4 + cV)}{4(1 - 36(ck)^2)^2}.$$

The denominator is positive. The numerator is convex in V, so its minimum is again at the value

of V that solves its first order condition. This is $V = \frac{2(-1 + ck + 36(ck)^3)}{c(1 + ck + 36(ck)^2)}$. At this V, the

numerator is $\dfrac{(1-36(ck)^2)(-8+35ck-36(ck)^3)}{c(1+ck+36(ck)^2)}$. This has the sign of $-8+35ck-36(ck)^3$.

Since these two equilibria can both exist only when $\dfrac{1}{3} \le ck \le \dfrac{5+\sqrt{13}}{12}$, and $-8+35ck-36(ck)^3$ is

always positive in this interval, the numerator of (A2) is always positive, proving the lemma.

Lemma 1.3. Welfare is greater in (1NB) than in (2I) when both exist.

Proof. Evaluating the social welfare function, (18), for each equilibria and subtracting the

welfare in (2I) from the welfare in (1NB) gives:

(A3)
$$\frac{3-30ck+92(ck)^2-72(ck)^3+8cV(1-4ck+cV)}{8c(1-6ck)^2(1-10ck+12(ck)^2)^2}.$$

This has the sign of the numerator, which is decreasing in V whenever (2I) exists. So, the

smallest the numerator can be is when $V = \dfrac{4ck-1}{2c}$. At this V, the numerator is

$(1-6ck)^2(1-2ck)$, which is positive since (1NB) requires that *ck<1/2*. Thus, the numerator is

always positive, proving the lemma.

Lemma 1.4. Welfare is greater in (2B) than in (2BB) when both exist.

Proof. Evaluating the social welfare function, (18), for each equilibria and subtracting the

welfare in (2BB) from the welfare in (2B) gives:

(A4)
$$\frac{-(1-3ck+2(ck)^2)(1-4ck+cV)^2}{4c(1-10ck+12(ck)^2)^2}.$$

This has the sign of $-(1-3ck+2(ck)^2)$, which is always negative when $\dfrac{1}{2} < ck \le 1$, as must be

true when (1B) exists.

Lemma 1.5. Welfare is greater in (1I) than in (2B) when both exist.

Proof. Evaluating the social welfare function, (18), for each equilibria and subtracting the

welfare in (2B) from the welfare in (1I) gives:

(A5)
$$\frac{\begin{array}{c}(V+3k(1-2ck))\{4-29ck-8(ck)^2+420(ck)^3-792(ck)^4+432(ck)^5\\+8cV(1+38ck-120(ck)^2+72(ck)^3)\}\end{array}}{2(1+6ck)^2(1-10ck+12(ck)^2)^2}$$

This has the sign of the term in curly braces (when both (1I) and (2B) exist the first term in the numerator is positive). The term in curly braces is decreasing in V since

$1+38ck-120(ck)^2+72(ck)^3<0$ whenever $\frac{1}{2}\le ck\le\frac{5+\sqrt{13}}{12}$ (which must be rue when both

(1I) and (2B) exist). When (2B) exists and $\frac{1}{2}\le ck\le\frac{5+\sqrt{13}}{12}$, we know that $V\le\frac{4ck-1}{2c}$. Thus,

the curly braces term cannot be any smaller than it is at $V=\frac{4ck-1}{2c}$, where it is:

(A6)
$$(1-10ck+12(ck)^2)(7-22ck-48(ck)^2+72(ck)^3)/2$$

The first term is negative when $\frac{1}{2}\le ck\le\frac{5+\sqrt{13}}{12}$ and it is easy to verify that the second is also

negative in this region. Thus, (A5) is always positive, proving the lemma.

Lemma 1.6. Welfare is always greater in (1I) than (2BB) when both exist.

Proof. Evaluating the social welfare function, (18), for each equilibria and subtracting the welfare in (2BB) from the welfare in (1I) gives:

(A7)
$$\frac{(2ck-1)(-1+ck+18(ck)^2)+2cV(2+4ck-cV)}{8c(1+6ck)^2}.$$

This is positive since $\frac{1}{2}\le ck\le 1$ and $V\le\frac{2ck+1}{c}$ when these two equilibria both exist.

Lemma 1.7. Welfare is always greater in (1B) than (2BB) when both exist.

Proof. Evaluating the social welfare function, (18), for each equilibria and subtracting the welfare in (2BB) from the welfare in (1B) gives $(1-ck)/4c>0$ when (1B) exists.

Table 1 shows that these seven situations exhaust all the possibilities for multiple equilibria in the duopoly model. Q.E.D.

Proof of Proposition 2 . By combining the information in Tables 1 and 2, and setting $v=V-k$, one can get the following list of feasible duopoly outcomes and the monopoly outcome for various parameter values. The list is divided into eight cases based on the value of ck. The feasible duopoly equilibria are given immediately after the parameter restriction and the monopoly outcome then follows the semi-colon. Where there is a (***) this denotes parameter values that are covered by part (a) of the Proposition. The (**) denotes parameter values covered by part (b) of the Proposition. All other parameter values are covered by part (c).

1. If $ck < 1\ 3$ then:
$$V < \frac{1-2ck}{2c} : \quad \text{NB}$$
$$\frac{1-2ck}{2c} \pounds V < \frac{2ck+1}{2c} : \quad I$$
$$\frac{2ck+1}{2c} \pounds V < \frac{2ck+1}{} :$$
$$\frac{2ck+1}{c} \pounds V : \quad B$$

2. If $1\ 3 \pounds ck < 1\ 2$ then:
$$V < \frac{1-2ck}{2c} : \quad \text{NB}$$
$$\frac{1-2ck}{2c} \pounds V < \frac{4ck-1}{2c} : \quad I$$
$$\frac{4ck-1}{2c} \pounds V < \frac{2ck+1}{2c} : \quad I$$
$$\frac{2ck+1}{2c} \pounds V < \frac{2ck+1}{} :$$
$$\frac{2ck+1}{c} \pounds V : \quad B$$

3. If $1 \ 2 \pounds \ ck < \dfrac{1}{12}$ ... then

$$\dfrac{-1+2ck}{4c} \pounds \ V < 3k \quad 1+2ck \qquad \qquad 2 \ \text{BNB}$$

$$3k \quad 1+2ck \pounds V \dfrac{4ck-1}{2c} \qquad \qquad 2 \ \text{BNB}$$

$$\dfrac{4ck-1}{2c} \pounds \ V < \dfrac{6ck+1}{c} \qquad \text{I} \quad \text{BB} \quad 2 \ \text{BNB}$$

$$\dfrac{6ck+1}{c} \pounds \ V < \dfrac{\overline{ck} \quad 1+2ck}{\overline{2} \ c} + 2k : \ \text{I} \quad \text{BB} \quad 2$$

$$\dfrac{\overline{ck} \quad 1+2ck}{\overline{2} \ c} \qquad \pounds \ V \qquad \dfrac{2ck+1}{c} \qquad \text{BB} \quad 1$$

$$\dfrac{2ck+1}{c} \pounds \ V : \ \text{B} \quad \text{BB} \quad 1 \quad **$$

4. If $\dfrac{1}{12} \quad \overline{13} \pounds \ c \ V < \dfrac{1}{24} \quad 0$ ... then

$$\dfrac{-1+2ck}{4c} \pounds \ V < \dfrac{4ck-1}{2c} \qquad 2 \ \text{BN}$$

$$\dfrac{4ck-1}{2c} \pounds V \ 3k \quad 1+2ck \qquad \text{BB} \quad 2 \ \text{BNB}$$

$$3k \quad 1+2ck \pounds V \dfrac{6ck+1}{c} : \ \text{I} \quad \text{BB} \quad 2 \ \text{BNB}$$

$$\dfrac{6ck+1}{c} \pounds \ V < \dfrac{\overline{ck} \quad 1+2ck}{\overline{2} \ c} + 2k : \ \text{I} \quad \text{BB} \quad 2$$

$$\dfrac{\overline{ck} \quad 1+2ck}{\overline{2} \ c} \qquad \pounds \ V \qquad \dfrac{2ck+1}{c} \qquad \text{BB} \quad 1$$

$$\dfrac{2ck+1}{c} \pounds \ V : \ \text{B} \quad \text{BB} \quad 1 \quad **$$

5. If $\frac{1}{24}$ $+$ $\overline{105}$ $\text{\textsterling} c$ $< \frac{1}{4}$ $+$ 5 then:

$$\frac{-1+2ck}{4c} \text{\textsterling} V < \frac{4ck-1}{2c} : \quad \text{NB}$$

$$\frac{4ck-1}{2c} \text{\textsterling} V < \frac{6ck+1}{4c} : \quad \text{BN}$$

$$\frac{6ck+1}{4c} \text{\textsterling} V - 3k \quad \overline{ck} \quad \overline{1+2ck}$$

$$3k \quad \overline{1+2ck} \text{\textsterling} V < \frac{\overline{ck} \quad \overline{1+2ck}}{\overline{2} c} + 2k : \quad$$

$$\frac{\overline{ck} \quad \overline{1+2ck}}{\overline{2} c} - 2k \text{\textsterling} V < \frac{2ck+1}{c}$$

$$\frac{2ck+1}{c} \text{\textsterling} V : \quad$$

6. If $\frac{1}{4}$ $+$ $\overline{5}$ $\text{\textsterling} c$ < 968339 then:

$$\frac{-1+2ck}{4c} \text{\textsterling} V < \frac{4ck-1}{2c} : \quad \text{NB}$$

$$\frac{4ck-1}{2c} \text{\textsterling} V < \frac{6ck+1}{4c} : \quad 2 \quad \text{NB}$$

$$\frac{6ck+1}{4c} \text{\textsterling} V < \frac{4ck+1}{2c} :$$

$$\frac{4ck+1}{2c} \text{\textsterling} V - 3k \quad \overline{1+2ck}$$

$$3k \quad \overline{1+2ck} \quad \frac{4ck+2-\overline{2} \quad \overline{1+ck-2c^2k^2}}{2c} : \quad$$

$$\frac{4ck+2-\overline{2} \quad \overline{1+ck-2c^2k^2}}{2c} \text{\textsterling} V < \frac{2ck+1}{c} :$$

$$\frac{2ck+1}{c} \text{\textsterling} V : \quad$$

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7. If $.968339 \le ck < 1$ then:

$$\frac{-1+2ck}{4c} \le V < \frac{4ck-1}{2c} \quad : \quad 1\text{I} \quad , \quad 2\text{ }\text{NB}$$

$$\frac{4ck-1}{2c} \le V < \frac{6ck+1}{4c} \quad : \quad 2\text{B} \quad , \quad 2\text{ }\text{NB}$$

$$\frac{6ck+1}{4c} \le V < \frac{4ck+1}{2c} \quad : \quad 2\text{ }\text{B} \quad , \quad 2\text{I}$$

$$\frac{4ck+1}{2c} \le V < \frac{4ck+2-\overline{2}\sqrt{1+ck-2c^2k^2}}{2c} \quad : \quad 1\text{B} \quad , \quad 2\text{B}$$

$$\frac{4ck+2-\overline{2}\sqrt{1+ck-2c^2k^2}}{2c} \le V < 3k\sqrt{1+2k} \quad : \quad 1\text{B} \quad , \quad 11\text{ }^{**}$$

$$3k\sqrt{1+2k} \le V < \frac{2ck+1}{c} \quad : \quad 1\text{I} \quad , \quad 11\text{ }^{**}$$

$$\frac{2ck+1}{c} \le V \quad : \quad 1\text{B} \quad , \quad 11\text{ }^{**}$$

8. If $1 \le ck$ then:

$$\frac{-1+2ck}{4c} \le V < \frac{4ck-1}{2c} \quad : \quad 1\text{I} \quad , \quad 2\text{ }\text{BNB}$$

$$\frac{4ck-1}{2c} \le V < \frac{6ck+1}{4c} \quad : \quad 2\text{BB} \quad , \quad 2\text{ }\text{NB}$$

$$\frac{6ck+1}{4c} \le V < \frac{4ck+1}{2c} \quad : \quad 2\text{B} \quad , \quad 2\text{I}$$

$$\frac{4ck+1}{2c} \le V \quad : \quad 1\text{BB} \quad , \quad 2\text{B}$$

First, I prove part (c). Where there are no stars next to an outcome, the following observations establish that all of the possible duopoly equilibria provide at least as much welfare as the monopoly outcome. First, whenever a duopoly equilibria and a monopoly outcome involve either equal development of both products or only development of one product, then the duopoly outcome will provide equal welfare if the outside options are never binding or always binding and greater welfare when they bind for some customers and not for others (the interior cases). If development is distributed identically across the products in the two regimes, then, since in both regimes there is (weakly) too little development, the regime that provides more development generates more welfare. Development is identical when the outside options never bind or always

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bind but is strictly greater in duopoly when the cutoff point is interior. Second, when comparing monopoly outcomes where an equal number of products are developed, welfare is greater the greater the fraction of the customers with a binding outside option (this obviously follows from the first observation). Third, by Proposition 1, duopoly will always be (weakly) superior to monopoly when the least asymmetric equilibrium is (weakly) superior to the monopoly outcome. In every case above when there are no stars, the monopoly outcome develops the same number of products and has the same or fewer number of customers with binding outside options as the least asymmetric duopoly equilibrium. Thus, the three observations guarantee that duopoly provides (at least weakly) greater welfare than duopoly in all of these cases.

Now, I prove part (a). Monopoly and duopoly development levels are identical between (1B) and (M-1B) and between (1NB) and (M-1NB) (see Tables 1 and 2). Thus, welfare must be identical in these cases also. In every parameter set with (***), there are more than one duopoly equilibria, the best of which (by Proposition 1) is either (1B) or (1NB). The monopoly outcome in these cases is always (M-1B) (when the best duopoly outcome is (1B) or (M-1NB) (when the best duopoly outcome is (1NB)). This proves (a).

To prove part (b), I show that in every case with (**), the monopoly outcome provides strictly greater welfare than at least one possible duopoly equilibrium and strictly less welfare than one possible duopoly equilibrium. The observations for part (c) establish the second part of this contention in every case with (**) except the third to last case of 7, where I need to show that welfare is greater in (2B) than (M-1I).

Lemma 2.1. In Case 7 welfare is greater in (2B) than (M-1I).

Proof. Using the social welfare function, (18), and substituting in for development levels in each case using Tables 1 and 2, I can write the welfare difference between (2B) and (M-1I) as follows:

$$(A8) \quad \frac{k\left(4+3ck-26c k^2 - 64c^3 k^3 + 72c^4 k^4\right)}{2\left(-10ck + 12c^2 k^2\right)} \ldots$$

(A8)

Taking the second derivative of (A8) with respect to V gives the following:

$$2c\left(1 - 22ck + 136c^2 k^2 - 232c^3 k^3 + 128c^4 k^4\right)$$

(A9)

This is always positive when $.968339 \leq ck < 1$, so the first derivative of (A8) is increasing in ck, meaning it can be no larger than when $V = 3k(2ck - 1)$, the upper bound in (2B) in case 7. At this V, the first derivative of (A8) is:

$$4\left(1 + c\right)\left(-10ck + 12c^2 k^2\right)\left(Vk - 26c^2 k^2 + 32c^3 k^3\right)$$

(A10)

This is negative for $.968339 \leq ck < 1$, so the welfare difference, (A8), is smallest at this same, maximal value of V. Evaluating (A8) at this V gives the following:

$$8k\left(+4c\right)\left(-11ck + 22c^2 k^2 - 12c^3 k^3\right)$$

(A11)

This is clearly positive, proving the lemma.

Now I prove that wherever there is a (**) that the monopoly outcome generates strictly greater welfare than one duopoly equilibrium. Doing so requires proving the following lemmas.

Lemma 2.2. In cases 3 through 7 welfare is greater in (M-1I) than in (2BB).

Proof. Using the social welfare function, (18), and substituting in for development levels in each case using Tables 1 and 2, I can write the welfare difference between (2BB) and (M-1I) as follows:

$$\frac{1 + 4c^3 k^3 + c\left(k - \ldots\right)\left(2k + V\right)}{4c\left(+2c\right)}$$

(A12)

This has the sign of the numerator, which is increasing in V since its derivative with respect to V is:

(A13) $$4\,c\,(c\,(V\text{-}2\,k)\text{-}1)$$

This is positive whenever (2BB) exists. So the welfare difference, (A12), can be no larger than it

is when $V = \dfrac{2ck+1}{c}$, the maximum value for V when (M-1I) exists (at $v=V\text{-}k$). At this V, the

welfare difference is:

(A14) $$(1+2ck)^2(ck-1)$$

This is negative since $ck<1$ in these cases, proving the lemma.

Lemma 2.3. In case 2 welfare is greater in (M-1NB) than (2I).

Proof. Using the social welfare function, (18), and substituting in for development levels in each

case using Tables 1 and 2, I can write the welfare difference between (2I) and (M-1NB) as

follows:

(A15) $$\frac{-\,3+72c^3k^3+2c\ \blacksquare 5k\,-\ \blacksquare\blacksquare\blacksquare\blacksquare\ \blacksquare3k^2\,\text{-}\ \blacksquare kV+2V^2\ \blacksquare}{8c\ \blacksquare\,\text{-}\ 6c\blacksquare\blacksquare}.$$

This has the sign of the numerator. The derivative of the numerator with respect to V is:

(A16) $$8c(2c(2k-V)-1)$$

When (2I) exists, this is positive. So the numerator of (A15) is at its maximum at $V = \dfrac{4ck-1}{2c}$,

the maximum V for (2I). At this V, the numerator of (A15) is:

(A16) $$(1-6ck)^2(2ck-1)$$

This is negative in case 2 since $ck<1/2$, proving the lemma.

Lemmas 2.2 and 2.3 together establish that when the conditions of (b) hold, the

monopoly outcome provides more welfare than one duopoly equilibrium. This completes the

proof. Q.E.D.

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