

The Competitive Effects of Mergers Between Asymmetric Firms

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Abstract

The *1992 Horizontal Merger Guidelines* suggest that the merger of two relatively weak competitors may result in a strong competitor and may lead to lower prices, despite the resulting increase in concentration. This paper introduces incomplete information into a simple model of repeated competition among firms that are asymmetric in their likely degree of efficiency at each stage of competition. In such a setting there do exist profitable yet price-reducing mergers among weaker firms. This model reasonably describes mergers between asymmetric firms that participate in auction or procurement settings and strengthens insights from the literature on asymmetric auctions regarding postmerger incentives for aggressive pricing. Finally, this model illustrates that the efficiencies described in the typical modeling of mergers in the asymmetric auction literature have private but not social benefits, and thus should not be permitted as a justification for merger.

1 Introduction

The existence of asymmetries between firms provides a strong motive for horizontal merger. Mergers often have the potential to combine complementary assets in a way that makes the parties more efficient and more effective competitors, leading to increased competition in the market. For example, merging firms may provide complementary products or their distribution areas may be non-overlapping. A merger between geographically distinct distributors may lower the parties' cost structure and create a more aggressive competitor for the business of customers requiring broad geographic coverage. Weighing against this possible social benefit of a merger is the potential for reduced competition resulting from the removal of a possibly strong rival. Contemporary horizontal merger analysis attempts to balance these conflicting forces in order to allow competition-enhancing mergers and prevent competition-dampening ones. This paper's results can help one to assess the competitive effects of a merger between firms that participate in procurement settings, with explicit accounting made for incomplete information and for both initial asymmetries among firms and the asymmetries induced by the merger.

Analysis of firms that differ in their product offerings or production processes typically involves what the *1992 Horizontal Merger Guidelines* ("Guidelines") refer to as "unilateral effects." In the language of economists, unilateral effects refer to changes in the outcomes of static Nash equilibria. One issue considered in deciding whether unilateral effects from a merger pose a threat to competition is the extent to which the parties are each other's closest competitors. With respect to product offerings, one can investigate whether the parties' products are particularly good substitutes for each other and not for non-parties' products. Simulating the merger by estimating cross-price elasticities between the parties' products can help one to assess the potential for harm from the merger. With respect to production processes, one can investigate whether the parties' costs are such that they currently constrain one another's pricing more than do the non-parties.

This paper examines mergers in which the parties' relative cost positions are a greater concern than the substitutability of their product offerings. Even if they produce homogenous goods, firms frequently differ in the efficiency with which they provide a given product or service. A simple modeling approach to merger analysis when firms differ in this respect is to assume that each firm has a particular cost schedule and then try to determine whether one of the parties is the marginal firm whose cost schedule determines the market price. For example, suppose there are six firms each capable of producing one unit with marginal production cost $c_i = i$ for $i \in \{1, \dots, 6\}$. If the buyer only wishes to purchase one unit, then the only merger with competitive significance is the

one between firms 1 and 2. The third most efficient firm and those beyond have no effect on the market outcome.¹

The preceding model is instructive, but it probably is too simple to provide much insight into transactions in most markets. The model's weakness is that if the firms' production costs are known perfectly and never vary through time, then firms $\{2, \dots, 6\}$ should have no sales and zero market shares. However, such a fact pattern normally is not seen in practice. One can appeal to noise in the marketplace to explain less efficient market participants' positive shares, but a more appealing approach is to assume that firms' cost schedules vary over time, either because of product changes or customer differences, or because of unobserved shocks to each firm's production process.² Under this approach, some firms may develop reputations for consistently being a low cost provider or a high cost provider. This idea is embodied in language that calls the firms strong and weak competitors, respectively.

The FTC apparently considered such ideas in its decision not to enjoin the recently proposed merger between Boeing and McDonnell Douglas. In a recent speech,³ FTC Chairman Robert Pitofsky stated that the predicted testimony of about 40 purchasers strongly would have suggested that McDonnell Douglas had little prospect of future aircraft sales. Consequently, he explained that the commercial aircraft market already consisted of two rather than three competitively significant players, and so the proposed merger likely would not have affected competition adversely.

If firms with different reputations for efficiency (or competitive significance) merge, and if the merged firm is more likely to be efficient than either firm is singly, then the merger-specific efficiencies may be cognizable according to the Guidelines. The Guidelines state that mergers generate efficiencies by "...enabling the combined firm to achieve lower costs in producing a given quantity and quality than either firm could have achieved without the proposed transaction."⁴ If the efficiencies are cognizable, then the parties can use the efficiency gains as a justification for the merger, despite the resulting increase in market concentration. This reasoning is embodied in a rule of thumb suggested by the Guidelines⁵ and used in merger analyses: the merger of two relatively weak competitors may result in a stronger competitor and may lead to lower prices. However,

¹However, see Baker [1996] for an interesting example in which the buyer demands multiple units and a merger of two inframarginal sellers can increase the buyer's payment by creating a hold-up problem.

²A second approach is to consider upward sloping marginal cost schedules. However, the constant marginal cost assumption frequently is employed and often appears to be sensible.

³"Staples and Boeing: What They Say About Merger Enforcement at the FTC." Prepared remarks of Robert Pitofsky, September 23, 1997.

⁴Guidelines, Section 4.

⁵Guidelines, Section 4.

efficiencies are no panacea for antitrust concerns. Another rule of thumb is that efficiencies almost never justify a merger to monopoly.

To explore the existence and effect of reputations and whether the preceding rules of thumb are justifiable, this paper employs a simple model of repeated price competition among firms that are asymmetric in their likely degree of efficiency at each stage of competition and that are privately informed of their respective efficiency aspects. Not only does the model incorporate asymmetries in reputations regarding productive efficiency and satisfy the predictions of the Guidelines and one's own intuition, it provides simple tools to assist in the evaluation of mergers. Moreover, I show that the "closest competitor" argument sometimes employed in unilateral effects analyses has some measure of validity in situations in which firms' cost schedules vary over time. Essentially, firms are not always each other's closest competitors, but a merger's competitive harm may derive from eliminating competition at the times that they are.

The model employed has price-setting firms participating in auction markets, and its use to explore these policy issues is the paper's second contribution. Given the relevance of auction models in actual transactions, it is important to see if standard predictions about equilibrium behavior from more familiar models hold in the auction framework. Auction models have been used profitably to examine markets in which asymmetric information plays a key role. For example, the Federal Trade Commission recently has begun to exploit auction models in merger analyses.⁶ In addition, intermediate goods markets, the markets in which most mergers take place,⁷ have many features similar to theoretical auction models. For example, if the intermediate good is a small element of the final product's total cost, then the input's derived demand likely is quite inelastic. One important feature of auction models is that all firms affect the price-setting process, and so intuitively all mergers have an impact on market outcomes. I show that by allowing firms privately to receive cost draws at each of several stages of competition and by ranking firms based on their likely degree of efficiency, one can model mergers in a reasonable manner.⁸

Auction models typically vary by whether the participating firms have private information about their own production costs or about the state of demand. These two variants are known in the auction literature as private value and common value frameworks, respectively. When firms are

⁶For example, auction results were used in evaluating the recent merger between Rite-Aid and Revco. See Baker [1996]. See also Section 2.21 of the Guidelines.

⁷This point is made in Scheffman [1993].

⁸One might take another approach by letting firms' costs be constant over time but be unknown by their rivals. One then must account for attempts to acquire information about rivals. See Thomas [1998] for examples of this behavior.

privately informed about their costs, the most commonly used auction framework has all firms drawing their cost realizations from the same probability distribution. In these symmetric auction models, the expected payment by a buyer falls with the number of participating firms. Consequently, evaluating mergers without considering the asymmetry induced by the merger suggests that any merger will raise the price paid by buyers. One problem with the symmetry assumption is that firms typically are not symmetric, and welfare predictions may be sensitive to such a specification. A second problem with the symmetry assumption, and particularly the result relating the expected price to the number of firms, is that with symmetry the merger confers no advantage on the merging parties. That is, the merged firm is analyzed as if it were identical to the non-merged firms.⁹ Of course, real-world mergers often are motivated precisely because there is some efficiency gain to be had by combining assets. One area in which this problem has been addressed is in the recently emerging literature on asymmetric auctions.¹⁰ The basic premise of this literature is that different sellers receive their marginal production costs from different cost distributions. Assumptions of stochastic dominance between these distributions conform to beliefs that some sellers are more likely to be efficient than are others.

One interesting conclusion from this literature concerns changes in sellers' cost distributions in first-price auction settings.¹¹ If one seller's cost distribution changes to make the seller more likely to have low production costs, then the other sellers subsequently are worse off. Moreover, the seller that is more likely efficient may even be worse off. An "efficiency effect" and a "competitive effect" interact to generate these results. First, the seller with the changed cost distribution now is more likely to have low production costs, which, all else equal, increases the changed seller's expected profit. Second, the unchanged sellers must set their prices more aggressively, as they now face an opponent with a likely lower cost. The changed seller rationally anticipates his rivals' more aggressive price-setting and responds appropriately. As the unchanged sellers experience only a competitive effect, they unambiguously are harmed. In addition, the changes on net may harm the changed seller. Though he now faces the same opponents as before the change in his distribution, those opponents now are setting their prices more aggressively. This competitive effect may swamp the efficiency effect for the changed seller.

Using the sensible idea that a merged firm is more likely to be efficient than either one of its

⁹This is precisely the problem pointed out in the Cournot model in Salant, Switzer, and Reynolds [1983].

¹⁰See Maskin and Riley [1995], Lebrun [1995ab,1996, 1997], and Waehrer [1997].

¹¹See Lebrun [1997] and Thomas [1997].

component firms, it is tempting to use the above result to conclude that mergers can decrease buyers' expected payments. However, that conclusion ignores the reduction in the number of firms induced by the merger, which can be considered a "concentration effect." Though the firms set prices more aggressively in response to the increased likelihood of one firm's being efficient, they set prices less aggressively in response to the elimination of a competitor.

Conversely, for reasons of simplicity it is tempting to use the symmetric model and assume that the merged firm receives no benefit from the merger in terms of improving its likely degree of efficiency. Dalkir, Logan, and Masson [1998] illustrate that such a strategy may severely overstate the price effect from a merger. Moreover, one can show in both first-price and second-price settings that the merger will not be profitable, in the same spirit as the results in the Cournot model in Salant, Switzer, and Reynolds [1983].

Correctly evaluating the effect of mergers requires reconciling the efficiency, concentration, and competitive effects just described, and doing so reveals that the essential insights about the effects of mergers in auction settings are similar to those of mergers in non-auction settings. This is comforting because it implies that one need not necessarily employ a new methodology to make initial inferences about a merger's competitive effect. This is useful given the severe time constraints under which mergers usually are evaluated.

The results generally can be understood as follows. Assuming no efficiencies are generated by the merger, then the non-parties face the same competitive environment as before the merger. In other words, the non-parties' best response functionals, which for each cost realization yield optimal prices based on the price-setting strategies and the expected lowest price of all other firms, do not change following the merger. Consequently, there is no "competitive effect." However, when the merged firm determines its optimal price, it explicitly accounts for the removal of a competitor (its merger partner) by setting its price less aggressively. This is the "concentration effect," and an informal revealed preference argument suggests that this reduction in the parties' aggressiveness increases the non-parties' expected profits. Moreover, the consummation of the merger suggests that the parties find the merger profitable.

In unit demand auction models there does not exist allocative inefficiency, so social welfare increases if and only if productive efficiency increases. Accordingly, if a merger does not generate productive efficiencies in this model, then social welfare is unchanged. Consequently, the increase in all sellers' expected profits just described implies that the buyer is harmed by the merger, much in the same manner as in Farrell and Shapiro [1990]. Under a consumer welfare standard, such a

merger is deemed anticompetitive.

If the merger generates productive efficiencies, then the qualitative analysis is less clear. Even if the parties use the same price-setting rules as they did premerger, each non-party realizes that the expected low price of its rivals has decreased. Thus, the probability of that firm's setting the low price has fallen, which upsets its price-setting calculus and causes a "competitive effect." Each non-party may be less profitable postmerger than premerger, while again the parties presumably are more profitable. It is not clear whether the buyer will pay more or less, but social welfare will increase.

Sophisticated models of asymmetric auctions unfortunately are of little help in analytically assessing the sellers' competing incentives following a merger, stemming from the inability to solve explicitly for equilibrium price-setting functions. Dalkir, Logan, and Masson [1998], Waehrer [1997] and Froeb, Tschantz, and Croke [1996] do make in-roads toward understanding the effects of mergers in auction settings. Waehrer [1997] examines per-member expected profits for different sized coalitions in both first-price and second-price auction settings. While his analysis illustrates the critical incentives for entry following a merger, it suffers from the analytic limitations regarding the actual effect of the merger on a coalitions' expected profit, a buyer's expected payment, and social welfare. Froeb, Tschantz, and Croke [1996] assumes bidder types are drawn from an Extreme Value distribution and solves for equilibrium in open outcry auction settings, which are outcome-equivalent to second-price auction settings. In asymmetric settings, first-price and second-price auctions likely are not revenue equivalent,¹² so their results apply only to a (possibly large) subset of the market institutions seen in practice. As previously noted, Dalkir, Logan and Masson [1998] illustrates that naively using symmetric auction models to simulate mergers may severely overstate the acquisition's price effect. They simulate mergers among firms whose costs are drawn from different uniform distributions.

In contrast to the work just described, this paper uses a simple price-setting game, with asymmetric information about costs, that makes possible explicit calculation of sellers' expected profits and buyers' expected payments. The model also allows simple evaluation of proposed mergers using expected-efficiency rankings of the merging parties, which can be calculated using historical market share data. Moreover, the model's results show that the efficiencies typically described when modeling mergers using asymmetric auction models do not increase social welfare in any manner. This result suggests that they should not be considered relevant in merger analysis. Finally, socially

¹²See Maskin and Riley [1995].

valuable efficiencies are modeled, and it is shown that privately profitable mergers among weaker firms can benefit buyers.

The following are the paper's main results. First, a merger between the most efficient firm and any other viable competitor always is privately profitable and always is harmful to buyers, regardless of any efficiencies. Second, if a proposed merger does not generate efficiencies, then the buyer's expected payment increases. Therefore, the merged firm does not steal business from non-parties, and in fact all sellers gain at the buyer's expense. Finally, a privately profitable merger between two relatively weak competitors that generates efficiencies can reduce the buyer's expected payment despite the increase in concentration. Thus, the predictions of the Guidelines are borne out with respect to such consolidations. Finally, I show that entry does not always prevent mergers that lead to increased prices, and I show that complete information about costs leads to mergers that always are profitable and that may be harmful to the buyer.

Section 2 presents the basic model and explains its key features. Section 3 shows the effect of mergers that do not generate productive efficiencies, and it argues that this common representation of merger efficiencies in asymmetric auctions is inadequate in terms of merger policy. Section 4 introduces merger-specific productive efficiencies, and illustrates how the predictions of a merger's welfare consequences change because of this modeling improvement. Section 5 examines entry costs, complete information, and the relationship between firms' shares and efficiency rankings. Section 6 briefly concludes.

2 The Model

Consider a market with N firms that meet repeatedly in discrete periods. Each period the firms compete for the unit demand of a buyer. The firms produce homogenous goods within a period, but the goods and the buyer may differ across periods. Each period each firm privately learns its marginal production cost for the good demanded that period. A firm's marginal cost is drawn independently each period according to the firm's commonly known cost distribution, and is either c_L or c_H , with $0 = c_L < c_H$.¹³ Firms with marginal cost c_L are labeled "efficient" while those with marginal cost c_H are labeled "inefficient."¹⁴ Firm i is efficient with probability $\alpha_i \in (0, 1]$, with $\alpha = (\alpha_1, \dots, \alpha_N)$, and the firms are labeled such that $0 < \alpha_N \leq \alpha_{N-1} \leq \dots \leq \alpha_1 \leq 1$. Firm 1

¹³Assuming $c_L = 0$ does not affect the qualitative nature of the results, and it prevents distracting notational complexity. However, for purposes of calibrating the model to real world data, such as profit margins, one must deduce the appropriate value for c_L .

¹⁴It could be that the goods and their marginal costs differ across periods. However, there exists a normalization of the costs that permits applicability of the simpler model.

is the most likely to be efficient in any given stage of competition, while firm N is the least likely. The restriction $0 < \alpha_N$ is without loss of generality, as firms that always are inefficient have no effect on an efficient firm's behavior. I refer to the "best" firm as a firm i such that $\alpha_i \geq \alpha_j$ for all $j \in \{1, \dots, N\}$. Note that there may be several "best" firms.

The buyer organizes the competition by soliciting secret price offers from the firms, then buying from the low price firm at the price it offered. In the auction literature this is known as a first-price, sealed bid auction in an independent, private values environment.

It should be noted that this model formally is equivalent to the model of advertising previously and independently developed in McAfee [1994]. In that paper the α_i 's represent the coverage rate of a firm's advertising, but that is equivalent to the probability of a firm's having a cost low enough to serve the market profitably. Using the model, McAfee considers cartelization and merger formation, but does not evaluate social welfare and does not consider the effects of entry, efficiencies, or complete information.¹⁵ Thus, there is value in considering the model as it applies specifically to mergers in auction markets.

Because there are only two possible cost levels, the only equilibrium involves mixed strategies. Let $F_i(p)$ denote the equilibrium mixed strategy distribution of an efficient player i . I assume any given buyer can produce the item in-house for cost c_H , so buyers credibly can commit to a reserve price of c_H . With this convention, I also can make the simplifying assumption that inefficient players do not submit price offers. This assumption still yields a Nash equilibrium outcome, yet it avoids an uninteresting equilibrium existence question that arises due to the possibility of ties at the reservation price. If all firms are inefficient, then no firm submits an offer and each firm is awarded the contract with probability $\frac{1}{N}$ at price c_H .

Let $\pi_i(p|c_L)$ denote an efficient player i 's expected profit from setting price p . $\pi_i(p|c)$ is termed the firm's *interim expected profit*, as it is the firm's expected profit after receiving its cost draw but before competition takes place. In contrast, the firm's *ex ante expected profit* is the firm's expected profit before it receives its cost draw. Firm i wins whenever all other sellers set a strictly higher price, so

$$\pi_i(p|c_L) = \left(\prod_{k \neq i} [1 - \alpha_k F_k(p)] \right) p.$$

Because efficient players use mixed strategies, each price in the support of an efficient firm i 's

¹⁵Complete information has no intuitive role in his model, given the interpretation of advertising coverage rather than cost realizations.

mixed strategy distribution yields the same interim expected profit, π_i . Because of the reserve price, $\pi_i(p|c_H) = 0$. Denote firm i 's ex ante expected profit by $E\pi_i \equiv \alpha_i\pi_i$. Proposition 1 proves that all efficient players have the same equilibrium interim expected profit, $\pi(\alpha)$. In addition, it calculates a buyer's expected payment, $P(\alpha)$, and expected social welfare, $W(\alpha)$, for a given vector of efficiency probabilities, α . $W(\alpha)$ is the sum of consumers' and producers' surplus, so

$$W(\alpha) = [c_H - P(\alpha)] + \sum_{i=1}^N \alpha_i \pi(\alpha).$$

Proposition 1 *For a given vector of efficiency probabilities, α , the following are true:*

- (1) $\pi_i = \pi_j = \pi(\alpha) = \left(\prod_{k=2}^N (1 - \alpha_k)\right) c_H$
- (2) $P(\alpha) = (1 + \alpha_2 + \dots + \alpha_N)\pi(\alpha)$
- (3) $W(\alpha) = \left[1 - \left(\prod_{k=1}^N (1 - \alpha_k)\right)\right] c_H$

To sketch the proof of Proposition 1, which is completely presented in the Appendix, first note that the union of the supports of all firms' mixed strategy distributions must include c_H . If it did not, then at least one firm setting price near the maximum price $\bar{p} < c_H$ would prefer setting price c_H , which would not lower the firm's probability of winning by much, yet would raise its revenues when it wins.

One also can show that the supports of all firms' mixed strategy distributions have the same minimum. Moreover, this minimum is determined by the best firm, which faces weaker competition than any other firm. To see this, note that if the best firm sets price c_H , then its expected profit is $\left(\prod_{k=2}^N (1 - \alpha_k)\right) c_H$. As the best firm always can achieve this expected profit, it never sets price below $\left(\prod_{k=2}^N (1 - \alpha_k)\right) c_H$. As all firms must have the same minimum price in the support of their mixed strategy distributions, $\underline{p} = \left(\prod_{k=2}^N (1 - \alpha_k)\right) c_H$. This lowest price, which wins with probability one, also is an efficient firm's interim expected profit. One consequence of the best firm's determining the efficient firms' interim expected profits is that if α_1 increases, then all other firms' ex ante and interim expected profits are unchanged.¹⁶

Trade always occurs in this model, so there is no allocative inefficiency in the market. Social welfare is determined strictly by firms' productive efficiency, because the low cost firm always provides the product. Any merger that does not change the firms' overall productive efficiency therefore has no effect on social welfare, and any changes in outcomes are strictly transfers between

¹⁶Another consequence, in the spirit of Thomas [1997], is that the best firm always will implement a technology with a higher α . The same is not necessarily true for the remaining firms.

sellers and buyers.¹⁷ This can be seen by noting that

$$\frac{W(\tilde{\alpha})}{W(\alpha)} = 1,$$

where $\tilde{\alpha}$ represents a postmerger vector of efficiency parameters that lead to the same expected low cost.¹⁸

The proof of Proposition 1 does not require explicit calculation of the firms' price-setting strategies, though that may be done if desired. Instead, one moves directly to calculating the sellers' expected profits and the buyers' expected payments. Examination of the buyer's expected payment and an efficient player's interim expected profit provides an understanding of the forces driving the model, and it makes more transparent the effect of changes in α , such as would be caused by the mergers considered in the following sections.

3 Mergers without Productive Efficiencies

Using the results in Proposition 1, it is straightforward to evaluate the impact of any proposed merger in this model. Though with a large set of firms there are many potential mergers to be analyzed, for the purpose of determining the private and social gain or loss from merger, all potential mergers may be partitioned into three classes: (1) a merger between the best firm and any other firm; (2) a merger between two firms (neither initially the best) that does not result in a new best firm; and (3) a merger between two firms (neither initially the best) that does result in a new best firm.

This model represents a merger by letting the merged firm take two draws, one from each firm's distribution of marginal costs, and select the minimum draw as its production cost. This is the common representation of mergers in the asymmetric auction literature.¹⁹ Using this representation, if firms i and j merge, then the probability that the merged firm is efficient is $\tilde{\alpha}_{ij} = 1 - (1 - \alpha_i)(1 - \alpha_j) = \alpha_i + \alpha_j - \alpha_i\alpha_j$. Another obvious alternative is to use a new cost distribution, $G(c)$ say, for the new firm. While probably more realistic from the perspective of applications, this alternative certainly complicates comparisons of premerger and postmerger scenarios,

¹⁷The unit demand structure assumed here makes easier the determination of social welfare, in contrast to the more general demand structure assumed in McAfee [1994].

¹⁸Note that this outcome differs from the outcome in a situation with more cost levels. Generally speaking, the buyer's expected payment is the expected price, while the seller's expected profit is a function of $p - c$, taking account of the probabilities of the various cost levels and their associated optimal prices. In the present model, the only cost that matters has been normalized to zero. Thus, in a more general model, if expected costs change but the expected price does not, then welfare changes.

¹⁹For example, see Waehrer [1997] or Dalkir, Logan, and Masson [1998].

and operationally is likely to lead to assumptions with no formal basis.

It is evident that $\tilde{\alpha}_{ij}$ exceeds both α_i and α_j , so the merged firm is more likely to be efficient than either of the firms is singly. Therefore, one might conclude from a strict reading of the Guidelines that this “efficiency” is cognizable when analyzing a merger. However, this representation is missing an important element of the potential efficiency gains from the merger. After all, the probability that the merged firm is not efficient is the same as the probability premerger that neither firm is efficient. Put another way, the probability that none of the firms in the market is efficient is the same premerger as it is postmerger.²⁰ Thus, this merger offers only what I call pseudo-efficiencies: The parties have become more efficient from their point of view, but not from society’s. Section 4 examines mergers that actually generate productive efficiencies. Analyzing the two cases separately highlights the role real efficiencies play in determining the welfare implications of mergers in auction markets, giving them the same importance they play in merger analyses in general.

I evaluate the three classes of merger in turn.²¹ The vector of premerger efficiency probabilities is denoted α , while the vector of efficiency probabilities following a merger between firms i and j is denoted $\tilde{\alpha} = (\alpha_1, \dots, \tilde{\alpha}_{ij}, \dots, \alpha_N)$.

A merger between firms i and j is privately profitable if and only if $E\pi_{ij} > E\pi_i + E\pi_j$. If this condition holds, then there exists a mutually agreeable split of the merger-generated surplus. Breaking this expression into its component pieces, one sees that

$$\frac{E\pi_{ij}}{E\pi_i + E\pi_j} > 1 \Leftrightarrow \left(\frac{\tilde{\alpha}_{ij}}{\alpha_i + \alpha_j} \right) \left(\frac{\pi(\tilde{\alpha})}{\pi(\alpha)} \right) > 1. \quad (1)$$

A merger between firms i and j is harmful to the buyer if and only if $P(\tilde{\alpha}) > P(\alpha)$. Under a consumer welfare standard, a merger that increases the buyer’s expected payment is deemed anti-competitive. One can show for mergers not involving the best firm either premerger or postmerger that

$$\frac{P(\tilde{\alpha})}{P(\alpha)} > 1 \Leftrightarrow \left(\frac{1 + \dots + \tilde{\alpha}_{ij} + \dots + \alpha_N}{1 + \dots + \alpha_i + \dots + \alpha_j + \dots + \alpha_N} \right) \left(\frac{\pi(\tilde{\alpha})}{\pi(\alpha)} \right) > 1. \quad (2)$$

Similar expressions arise following mergers either involving the best firm premerger or creating a

²⁰Dalkir, Logan, and Masson [1998] does this neatly with two-dimensional cost attributes, such as for the provision of two services by a hospital. However, it is not clear from the model why unbundling of services is possible by the merged firm but not by unmerged firms. That is, it appears that premerger the buyer would demand separate contracts from providers.

²¹I do not impose the same structure on cartel or merger formation as does McAfee [1994]. I simply examine exogenously imposed mergers that are privately profitable. This allows for more arbitrary cartel and merger formation, perhaps arising due to costs of coordinating a cartel or merger.

new best firm. Noting that $\tilde{\alpha}_{ij} \leq \alpha_i + \alpha_j$, examination of the two previous expressions illustrates that a necessary condition for the merger to be profitable or to harm the buyer is that an efficient firm's interim expected profit must increase. However, as this condition is not sufficient for either of the two conclusions, there may exist privately profitable mergers that do not harm the buyer.

Suppose firm 1 and firm $j \neq 1$ merge. The merged firm's new efficiency probability is $\tilde{\alpha}_{1j} = 1 - (1 - \alpha_1)(1 - \alpha_j)$. The merged firm necessarily is still the most likely to be efficient.

Proposition 2 *A merger between the best firm and any other firm is privately profitable and is harmful to the buyer.*

Recall from its definition that $\pi(\alpha)$ is determined by the expected cost of all firms but firm 1. By merging with firm j , $\pi(\tilde{\alpha})$ is determined by the expected cost of all firms but firms 1 and j , which necessarily is higher than $\pi(\alpha)$. Because the efficiency probability $\tilde{\alpha}_{1j}$ does not enter $\pi(\tilde{\alpha})$ or $P(\tilde{\alpha})$, the merger's only effect is to reduce competition by removing a competitor. Consequently, the buyer is harmed. Also note that the remaining competitors benefit from the merger.²² One consequence of Proposition 1 is that in an initially symmetric setting, mergers always harm the buyer.

On a more intuitive level, the best firm now expects greater profits from setting price c_H , because it faces one less rival. Due to the mixed strategy requirement that all prices yield the same expected profit to the firm, the minimum price also rises. The increase in the minimum price shifts the non-parties' prices to higher levels. Thus, price-setting becomes less aggressive after the merger.

Suppose firms i and j ($i, j \neq 1$) merge, and the merged firm is not the most likely to be efficient. The merged firm's new efficiency probability is $\tilde{\alpha}_{ij} = 1 - (1 - \alpha_i)(1 - \alpha_j) \leq \alpha_1$.

Proposition 3 *A merger between two firms that are not the best and that does not result in a new best firm is not privately profitable and benefits the buyer.*

Given that $\pi(\tilde{\alpha}) = \pi(\alpha)$ and that the merged firm's probability of being efficient is less than the sum of the firms' probabilities of being efficient ($\tilde{\alpha}_{ij} < \alpha_i + \alpha_j$), the firms do better apart than they do merged. Because the merger is not privately profitable, it will not be proposed.

²²It would be useful to know if the gain from not merging exceeds the gain from merging. Of course, a formal examination of why particular mergers are proposed and others are not is outside the scope of this paper.

The mergers described in Proposition 3 illustrate the lack of a “competitive effect” from the merger. As the best firm experiences no change in its competitive environment, it has no incentive directly from the merger to change its behavior. The parties have an incentive to change their behavior due to the concentration effect, but in this mixed strategy environment the pricing reactions by rivals keep an efficient firm’s interim expected profit unchanged. As a consequence, the parties do not find the merger profitable.

Suppose firms i and j ($i, j \neq 1$) merge, and the merged firm is the most likely to be efficient. The merged firm’s new efficiency probability is $\tilde{\alpha}_{ij} = 1 - (1 - \alpha_i)(1 - \alpha_j) > \alpha_1$.

Proposition 4 *If a merger between two firms that are not the best and that does result in a new best firm is privately profitable, then it harms the buyer.*

Notice that a merger creating a new strongest firm is not always privately profitable. Though an efficient firm’s interim expected profit increases after such a merger, the parties may not gain enough to find a mutually agreeable split of the resulting profits.

The following theorem collects the previous three results.

Theorem 1 *If efficiencies are not generated by a merger in this auction setting, then all privately profitable mergers harm the buyer.*

Mergers in this environment do not change social welfare, and they result only in wealth transfers between the sellers and the buyer. Because $\tilde{\alpha}_{ij} < \alpha_i + \alpha_j$, equation 1 shows that the only way a merger can be profitable for the merging parties is if the interim expected profits increase. If this is the case, then the non-parties’ ex ante expected profits must increase. Because welfare is unchanged, the only way both the parties and the non-parties can have their expected profits increase is if the buyer is harmed.

This model of efficiency-enhancing mergers clearly lacks some important features. First, it generates no real efficiencies in a social sense, in that the probability that none of the firms in the market is efficient is unchanged by the merger, when it is natural to presume that it will fall following a merger that generates efficiencies. This standard method of modeling mergers in the asymmetric auction literature is suspect not only with the discrete cost distribution I use here, but also with the continuous distributions used in more sophisticated analyses. Second, in this model no firms merge unless the merger makes them the new best firm. This is uncharacteristic of many real-world mergers. The next section introduces real efficiencies generated by merger and shows

how such a modeling change produces a greater number of profitable mergers, including mergers that both increase social welfare and reduce buyers' expected payments.

4 Mergers with Productive Efficiencies

To add the possibility of real efficiencies from a merger, I employ an efficiency factor, $(1 - \phi) \geq 0$, such that the probability that the merged firm is efficient is $\tilde{\alpha}_{ij} = 1 - (1 - \phi)(1 - \alpha_i)(1 - \alpha_j)$. ϕ can be considered a correlation coefficient between the firms' draws that is relevant only when they are merged. As ϕ grows, a merger confers a greater efficiency advantage.²³ Note that ϕ describes the degree of negative correlation between the firms' costs, and therefore is precisely the sort of complementarity that one likely would view as giving a procompetitive reason for merging.²⁴ If $\phi = 0$, then there is no complementarity between the firms and the analysis of the preceding section holds. If $\phi = 1$, then the firms are perfectly complementary and always will be efficient. The three cases analyzed in Section 3 simply are special cases of the three examined here.

Suppose firm 1 and firm $j \neq 1$ merge. The merged firm's new efficiency probability is $\tilde{\alpha}_{1j} = 1 - (1 - \phi)(1 - \alpha_1)(1 - \alpha_j)$. The merged firm necessarily is still the most likely to be efficient.

Proposition 5 *For any merger specific efficiency $\phi > 0$, a merger between the best firm and any other firm is privately profitable and is harmful to the buyer.*

Following the merger, the non-parties' ex ante and interim expected profits increase. The merged firm sees an identical effect with respect to its interim expected profit, but sees a greater increase in its ex ante expected profit due to the merger-specific efficiency factor. The merger-specific efficiency parameter affects the merged firm's ex ante expected profit but not the buyer's expected payment. Thus, none of the merger-specific efficiencies are passed on to consumers. Consequently, if one adopts a consumer welfare standard, then, under the model's assumptions, a merger between the best firm and one of its rivals always should be enjoined, regardless of any alleged efficiency gains.

Of tangential interest is that a firm can illustrate statistically that its price falls when its costs are lower, and thus may argue that it passes through a large fraction of realized cost savings. Such arguments were offered as one justification for the proposed merger of Staples and Office Depot.²⁵

²³Though ϕ does not vary across mergers, which seemingly indicates that all mergers are equally efficient, mergers between firms that are relatively inefficient actually generate a greater percentage increase in the expected degree of efficiency. Moreover, one could make ϕ vary by merger with no difficulty other than a notational one.

²⁴The cost difference also can be considered to be location-based. That is, imagine customers are in randomly assigned geographic locations, firms are located in different geographic areas, find transport of the product costly, and thus wish to align themselves with firms in non-overlapping areas.

²⁵Federal Trade Commission v. Staples, Inc., 970 F. Supp. 1066 (D.D.C. June 30, 1997) (Hogan, J.).

However, as illustrated here, savings following a merger are not passed through. This apparent contradiction is resolved in this model as follows. Absent structural changes, lower costs imply lower prices. However, lower costs more often, via merger, combined with decreased postmerger competition imply higher prices. Thus, past experience should not necessarily be a guide, because it ignores the change in industry structure. Consequently, pass-through estimates should be considered to overstate pass-through, unless the degree of competition explicitly is accounted for.

Suppose firms i and j merge ($i, j \neq 1$), and the merged firm is not the most likely to be efficient. The merged firm's new efficiency probability is $\tilde{\alpha}_{ij} = 1 - (1 - \phi)(1 - \alpha_i)(1 - \alpha_j) \leq \alpha_1$.

Proposition 6 *There exist privately profitable mergers, between two firms that are not the best either premerger or postmerger, that benefit the buyer. Formally, suppose a merger between firms i and j ($i, j \neq 1$) does not result in a new best firm. For some such mergers, there exist $\phi > 0$ such that the merger is privately profitable and benefits the buyer.*

When there are efficiencies generated by a merger, then the merger's private profitability no longer requires that an efficient firm's interim expected profit increases. Instead, the interim expected profit can decrease, but the parties can more than make up for this loss through their increased likelihood of being efficient. One important consequence of this result is that the merging firms need not become the best firm in order to find the merger profitable. Therefore, two relatively weak competitors can merge and become a stronger competitor, with consumer welfare increasing despite the increase in concentration.

Suppose firms i and j merge ($i, j \neq 1$), and the merged firm is the most likely to be efficient. The merged firm's new efficiency probability is $\tilde{\alpha}_{ij} = 1 - (1 - \phi)(1 - \alpha_i)(1 - \alpha_j) > \alpha_1$.

Proposition 7 *There exist privately profitable mergers, between two firms that are not the best premerger but that are the best postmerger, that benefit the buyer. Formally, suppose a merger between firms i and j ($i, j \neq 1$) results in a new best firm. For some such mergers, there exist $\phi > 0$ such that the merger is privately profitable and benefits the buyer.*

The intuition for the preceding result is much the same as that for the result of Proposition 6. The parties can benefit from the merger even though the non-parties might be harmed. If the sum of the parties' gain and the gain to social welfare exceeds the non-parties' losses, then the buyer benefits.

The following theorem collects the previous three results.

Theorem 2 *If a privately profitable merger in this auction setting generates real efficiencies, then the buyer may benefit.*

This result illustrates that a merger in this model must generate efficiencies to benefit consumers. More importantly, it confirms the idea that the merger of two relatively weak competitors can create a stronger competitor and can be procompetitive, despite the resulting increase in concentration. Moreover, the addition of efficiencies to the model makes its predictions more credible and creates a larger number of profitable mergers. The following two examples show that the buyer may or may not be harmed by a privately profitable merger that generates real efficiencies.

Example 1: Suppose $\phi = \frac{1}{2}$ and that there are twelve firms in the market, with $\alpha_i = \alpha$ for all i . Thus, the firms are symmetric and according to Proposition 5 a merger between two of the firms will be profitable and will harm the buyer. According to Proposition 1,

$$\frac{P(\tilde{\alpha})}{P(\alpha)} = \frac{(1 + 10\alpha)}{(1 + 11\alpha)(1 - \alpha)} > 1.$$

Thus, the price effects of the merger are identical regardless of ϕ . This is the case because the merger-specific efficiencies are not passed along to the buyer.

Example 2: Suppose there are twelve firms in the market, with $\alpha_1 = \dots = \alpha_{10} = \frac{5}{8}$ and $\alpha_{11} = \alpha_{12} = \frac{1}{8}$. If $\phi = \frac{17}{49}$, then a merger between firms 11 and 12 does not result in a new best firm. Using the results of Proposition 1, one can show that

$$\frac{E\pi_{11,12}}{E\pi_{11} + E\pi_{12}} = 1.31 > 1$$

and that

$$\frac{P(\tilde{\alpha})}{P(\alpha)} = 0.677 < 1.$$

Thus, the merger is profitable and the buyer's expected payment falls.

5 Extensions

This section considers three extensions of the preceding model: entry, complete information, and calculating efficiency probabilities. The first two affect equilibrium behavior and predictions about the welfare consequences following mergers, and they illustrate that institutional details surrounding a particular transaction must be known in order to implement the most reasonable form of the model. The third extension explains how one may determine the efficiency probabilities

from historical market share data, thus providing a simple and potentially useful way of estimating firms' competitive significance when they are asymmetric in their likely degree of efficiency.

The Role of Entry: Sections 3 and 4 implicitly assume that entry is blockaded and that there exists an exogenously determined number of firms participating in the market. However, an important element of horizontal merger analysis is determining whether entry or the threat of entry mitigates any competitive harm from the acquisition being examined. To that end, this section incorporates entry costs that endogenously determine the number of market participants, and it shows how the previous analysis is affected by the presence of potential competitors. In this manner it complements the analysis in Waehrer [1997] that shows that the profitability of entry following a merger may increase in first-price auction environments and does not change in second-price auction environments.

I limit my discussion in the following ways. First, firms must pay an entry cost, F , before receiving their cost draw. Second, there potentially exist a large number of entry equilibria. I assume that entry occurs in order of firms' efficiency rankings.²⁶ Third, it is reasonable only to look at mergers that are privately profitable when there is no entry threat. Other mergers will not be proposed and so require no further consideration.

If interim expected profits fall ($\pi(\alpha) \geq \pi(\tilde{\alpha})$), then entry will not occur following the merger. Recall that $\pi(\alpha)$ can exceed $\pi(\tilde{\alpha})$ following a privately profitable merger only if the merger generates real efficiencies. Thus, if there are efficiency gains that decrease efficient firms' interim expected profit, then entry will not be induced. The next firm that would enter, $N + 1$, did not enter pre-merger, while postmerger its ex ante expected profit will be even lower. While this result appears innocuous given the presence of merger-specific efficiencies, recall from Propositions 6 and 7 that prices may rise even if a merger generates efficiencies.

If interim expected profits rise ($\pi(\alpha) < \pi(\tilde{\alpha})$), then entry may occur following the merger. However, if profitable entry reduces efficient firms' interim expected profit by a sufficient amount, then the merger will not be profitable for the parties and will not be proposed. Thus, entry can prevent anticompetitive mergers if entry costs are not too high. In fact, entry will prevent all mergers in a symmetric setting without efficiencies. In this case, the analyst need not even know F , as entry will bring $\pi(\tilde{\alpha})$ back down to $\pi(\alpha)$.

Another interesting consequence of considering entry decisions is that efficiencies may promote

²⁶With more than two cost levels, expected efficiency likely should be used as the entry criterion.

price-increasing mergers that are profitable only if they induce exit. A reduction in interim expected profits may induce exit by non-parties. This exit raises the interim expected profits to a new level, which may be sufficient to make profitable an otherwise unprofitable merger. The following example illustrates this possibility.

Example 3: Suppose there are five firms in the market, with $\alpha_1 = \frac{7}{8}$, $\alpha_2 = \alpha_3 = \frac{1}{2}$, $\alpha_4 = \alpha_5 = \frac{3}{8}$, and $\phi = \frac{1}{8}$. Consider a merger between firms 2 and 3. Using the results of Proposition 1, one can show that

$$E\tilde{\pi}_5 = 0.032c_H < 0.037c_H = E\pi_5.$$

If the fixed cost to participate in the market each period is such that $E\tilde{\pi}_5 < F < E\pi_5$, then firm 5 will exit following the merger. Following the exit, denote firm profit levels as $\hat{\pi}$ rather than $\tilde{\pi}$. One can show that

$$E\tilde{\pi}_{2,3} < E\pi_2 + E\pi_3 < E\hat{\pi}_{2,3}.$$

Thus, the merger is profitable only if firm 5 exits. Moreover, one can show that the buyer's expected payment rises following the merger and the exit, which condemns the merger under a consumer welfare standard. However, social welfare increases in this example, strengthening claims by the parties that the merger simply drives out inefficient firms and improves competition in the market.

The Role of Complete Information: Sections 3 and 4 explicitly assume that firms' cost information is private at the price-setting stage and that the winning firm is paid the price that it offered, in the *first-price* manner of auction theory. However, there are situations in which it is reasonable to assume that firms know their rivals' costs before prices are set. Also, buyers may find it preferable to use *second-price* rules, in which the winning firm is paid the price offered by the second lowest priced firm. Fortunately, in the unit demand auction model used here, both the presence of complete cost information and the use of second-price rules are outcome equivalent.

Suppose firms learn their rivals' costs before prices are set. An efficient firm i earns profit c_H if its rivals all are inefficient, and earns zero otherwise. Thus, an efficient firm i 's interim expected profit is

$$\pi(\cdot|c_L) \equiv \pi_i = \left(\prod_{k \neq i} (1 - \alpha_k) \right) c_H$$

and firm i 's ex ante expected profit is $E\pi_i \equiv \alpha_i \pi_i$. Social welfare is unchanged by the change in

the price-setting format, so

$$P(\boldsymbol{\alpha}) = \left(\sum_{k=1}^N \alpha_k \pi_k \right) + \left(\prod_{k=1}^N (1 - \alpha_k) \right) c_H.$$

Proposition 8 *With perfect information about costs, all mergers are profitable. If there are no efficiencies from the merger, then the non-parties are unaffected and the buyer is harmed.*

In this setting, all firms' pricing strategies are unaffected by the merger. For the parties, the merger simply reduces the number of competitors and increases the frequency with which the merged firm earns a positive profit. In fact, those additional times that the parties earn a positive profit are precisely those times that both of the parties are efficient and all other firms are inefficient. In essence, the merger can be harmful because there are instances premerger in which the parties are each other's closest competitors. Such competition is eliminated by the merger. Although the merger decreases each party's share of the resulting profits, each party also is more likely to be efficient and it is less likely that there is an efficient rival than before the merger. Real efficiencies only make the merger more appealing, as they further increase the frequency with which the parties are efficient and are able to exploit any lack of efficient rivals.

In this setting, any merger without real efficiencies is profitable and harms the buyer. The non-parties' profits are unaffected by the merger. Consequently, an entrant that must pay a fixed cost has no greater incentive to enter postmerger than it did premerger. Thus, entry is not induced following a merger that raises the price paid by the buyer, consistent with the analysis in Waehrer [1997].

Calculating Efficiency Rankings: Sections 3 and 4 assume the analyst knows the vector of expected efficiency parameters, $\boldsymbol{\alpha}$. For the proposed methodology to be of practical value there must be a means of determining the expected efficiency parameters. With a great enough number of bidding observations, each firm's market share should approach its probability of winning a given contract. Thus, a firm's market share over a long time frame, assuming the firm's characteristics largely are unchanged, reasonably approximates the probability of the firm's winning a given auction. From this probability estimate, one can estimate the vector of efficiency probabilities, $\boldsymbol{\alpha}$. With an estimate of $\boldsymbol{\alpha}$, one can evaluate the competitive effects of a proposed merger under the model's assumptions.

Proposition 9 *Market shares are given by*

$$S_N = \frac{1 - (1 - \alpha_N)^N}{N} + \left(\frac{1}{N}\right) \prod_{k=1}^N (1 - \alpha_k)$$

and

$$S_i = S_{i+1} + \left(\frac{1}{i}\right) \left[(1 - \alpha_{i+1})^i - (1 - \alpha_i)^i \right] \prod_{k=i+1}^N (1 - \alpha_k)$$

for $i \in \{1, \dots, N - 1\}$.

The Guidelines suggest that $\frac{1}{N}$ is the correct measure of shares if firms are in auction markets and are equally likely to win contracts on a forward looking basis, despite firms' past histories of success. However, if the parties are asymmetric, then the Guidelines offer no methodology for determining whether current shares reasonably describe firms' competitive significance. Proposition 9, by permitting the calculation of firms' expected efficiency rankings from market share data, offers a measure of firms' competitive significance that should provide additional predictive power.

Though the formulae for the shares are complicated and do not necessarily imply unique values for the expected efficiency parameters, one can solve for the expected efficiency parameters either through direct calculation or simulation. Calibration to prevailing prices and margins should permit one to select a narrow set of expected efficiency parameters on which to focus. One then can predict the merger's competitive effects within the context of the present model as a complement to the traditional analysis examining concentration measures, entry, and efficiencies.

6 Conclusion

This paper provides a rough guide for policy makers analyzing mergers between firms that participate in auction markets. Of particular importance is that the model employed explicitly accounts for the possibility of premerger asymmetries among firms in the industry as well as the asymmetries likely induced by the merger. The model's predictions are in accord with the analysis implicitly used in the Merger Guidelines, which provides some comfort regarding the Guideline's predictions. In particular, the privately profitable merger of two relatively weak competitors can create a strong competitor and can lead to lower expected payments by buyers, despite the resulting increase in market concentration. Under a consumer welfare standard, such a merger is procompetitive. Also, the merger of the largest or strongest firm and any other leads to increased expected payments, despite any efficiency gain. Under a consumer welfare standard, such a merger is anticompetitive.

More importantly than the model, which should not necessarily be taken as a close approximation to reality in all procurement settings, this paper lays out the issues that must be examined when a merger is proposed by firms that participate in auction settings. Despite the additional complications arising from the incomplete information aspect of the problem, the analysis of these mergers follows much the same approach as currently is used. One must account for the decreased number of competitors, the possibility of efficiencies realized by the parties, and the likely response of non-parties to the merger.

The paper also shows that entry may not prevent anticompetitive effects from merger. If an efficient firm's interim expected profit falls, as may happen when the merger generates real efficiencies, then entry will not occur. However, it is possible in this situation that the buyer's expected payment increases.

Finally, the paper shows that firms' expected efficiency rankings may be estimated using historical market share data. This method thus provides a meaningful way to evaluate the competitive significance of different firms and to account for asymmetries among firms. It also provides a useful rebuttal to standard arguments that likely would be offered by the merging parties if beneficial to their case.

Appendix

Proof of Proposition 1: I first show there are no pure strategy equilibria. Suppose each efficient firm i sets price p_i with probability one. No firm sets a price less than $\left(\prod_{k \neq i} (1 - \alpha_k)\right) c_H$, as firm i can guarantee itself that expected profit by setting price arbitrarily close to c_H . Thus, the minimum price set is strictly greater than zero. At least one efficient firm sets the minimum price. If no other firm sets the minimum price, then the firm setting the minimum price has an incentive to raise its price to be arbitrarily close to the next highest price of the other firms. If at least one other firm sets the minimum price, then a firm setting the minimum price has an incentive to undercut that price slightly. Thus, there can be no pure strategy equilibria. For the same reasons, there be no mass points on any price strictly less than c_H , and at most one firm can set price c_H with positive probability.

Given that efficient firms must use a mixed strategy in equilibrium, I now show that the supports of all efficient firms' mixed strategy distributions have the same minimum. Suppose firms i and j have different lower supports for their mixed strategy distributions, with firm i setting the minimum price, so that $\underline{p} = \underline{p}_i < \underline{p}_j$. Price \underline{p}_i wins with probability one, while price \underline{p}_j wins with probability less than one.

Now $\pi_i(\underline{p}_i|c_L) = \pi_j(\underline{p}_i|c_L)$. Also,

$$\pi_i(\underline{p}_j|c_L) = \prod_{k \neq i} [1 - \alpha_k F_k(\underline{p}_j)] \underline{p}_j$$

and

$$\pi_j(\underline{p}_j|c_L) = \prod_{k \neq j} [1 - \alpha_k F_k(\underline{p}_j)] \underline{p}_j.$$

Comparing the two, it is evident that

$$\pi_i(\underline{p}_j|c_L) > \pi_j(\underline{p}_j|c_L).$$

Now

$$\pi_i(\underline{p}_i|c_L) \geq \pi_i(\underline{p}_j|c_L) > \pi_j(\underline{p}_j|c_L).$$

However, $\pi_i(\underline{p}_i|c_L) = \pi_j(\underline{p}_i|c_L)$, which implies $\pi_j(\underline{p}_i|c_L) > \pi_j(\underline{p}_j|c_L)$, which violates the equilibrium requirement $\pi_j(\underline{p}_j|c_L) \geq \pi_j(\underline{p}_i|c_L)$.

Then solve for the minimum price, which yields each efficient firm's expected profit.

The buyer's expected payment is the sum of the expected profits of all the firms and the expected price the buyer must pay if all firms are inefficient. For each combination of efficient and inefficient firms, calculate the number of efficient firms and the sum of their interim expected profits. This calculation is quite messy, but the following induction argument proves

$$P(\alpha_1, \dots, \alpha_N) = (1 + \alpha_2 + \dots + \alpha_N)\pi(\alpha_1, \dots, \alpha_N).$$

Suppose

$$P(\alpha_1, \dots, \alpha_N) = (1 + \alpha_2 + \dots + \alpha_N)\pi(\alpha_1, \dots, \alpha_N).$$

$P(\alpha_1, \dots, \alpha_{N+1})$ is determined by

$$\begin{aligned} P(\alpha_1, \dots, \alpha_{N+1}) &= (1 - \alpha_{N+1}) [(1 + \alpha_2 + \dots + \alpha_N)\pi(\alpha_1, \dots, \alpha_{N+1})] + \\ &\quad \alpha_{N+1} [(1 + \alpha_2 + \dots + \alpha_N)\pi(\alpha_1, \dots, \alpha_{N+1}) + \pi(\alpha_1, \dots, \alpha_{N+1})] \\ &= (1 + \alpha_2 + \dots + \alpha_N + \alpha_{N+1})\pi(\alpha_1, \dots, \alpha_{N+1}). \end{aligned}$$

Now

$$\begin{aligned}
P(\alpha_1, \alpha_2) &= \alpha_1 \alpha_2 [2\pi(\alpha_1, \alpha_2)] + \alpha_1(1 - \alpha_2) [\pi(\alpha_1, \alpha_2)] + \\
&\quad (1 - \alpha_1)\alpha_2 [\pi(\alpha_1, \alpha_2)] + (1 - \alpha_1)(1 - \alpha_2) [c_H] \\
&= (1 + \alpha_2)\pi(\alpha_1, \alpha_2),
\end{aligned}$$

so $P(\alpha_1, \dots, \alpha_N)$ is correctly defined for $N = 2$, and thus for all $N > 2$.

Proof of Proposition 2: Using the definition of $\tilde{\alpha}_{1j}$ and the results of Proposition 1,

$$\pi(\tilde{\alpha}) = \left(\prod_{k \neq 1, j} (1 - \alpha_k) \right) c_H = \frac{\pi(\alpha)}{(1 - \alpha_j)}.$$

I first show $E\pi_{1j} > E\pi_1 + E\pi_j$.

$$\begin{aligned}
\frac{E\pi_{1j}}{E\pi_1 + E\pi_j} &= \left(\frac{\tilde{\alpha}_{1j}}{\alpha_1 + \alpha_j} \right) \left(\frac{\pi(\tilde{\alpha})}{\pi(\alpha)} \right) \\
&= \frac{\tilde{\alpha}_{1j}}{(\alpha_1 + \alpha_j)(1 - \alpha_j)} \\
&= \frac{\alpha_1 + \alpha_j - \alpha_1 \alpha_j}{\alpha_1 + \alpha_j - \alpha_1 \alpha_j - \alpha_j^2} \\
&> 1.
\end{aligned}$$

A similar argument establishes

$$\frac{P(\tilde{\alpha})}{P(\alpha)} = \frac{1 + \sum_{k \neq 1, j} \alpha_k}{(1 - \alpha_j) \left(1 + \sum_{k \neq 1} \alpha_k \right)} > 1.$$

Thus, the merger increases the merged firm's expected profits from the sum of the parties' pre-merger expected profits, indicating that the firms can find a mutually agreeable split of the profits. Additionally, the buyer's expected payment increases. \square

Proof of Proposition 3: Using the definition of $\tilde{\alpha}_{ij}$ and the results of Proposition 1,

$$\pi(\tilde{\alpha}) = \left(\prod_{k=2}^N (1 - \alpha_k) \right) c_H = \pi(\alpha).$$

It is straightforward to show

$$\frac{E\pi_{ij}}{E\pi_i + E\pi_j} = \frac{\tilde{\alpha}_{ij}}{(\alpha_i + \alpha_j)} < 1$$

and

$$\frac{P(\tilde{\alpha})}{P(\alpha)} = \frac{1 - \alpha_i \alpha_j + \sum_{k=2}^N \alpha_k}{1 + \sum_{k=2}^N \alpha_k} < 1.$$

Thus, the merger decreases expected profits, indicating that the firms cannot find a mutually agreeable split of the profits. The buyer's expected payment decreases. \square

Proof of Proposition 4: Using the definition of $\tilde{\alpha}_{ij}$ and the results of Proposition 1,

$$\pi(\tilde{\alpha}) = \left(\prod_{k \neq i, j} (1 - \alpha_k) \right) c_H.$$

It is straightforward to show

$$\frac{W(\tilde{\alpha})}{W(\alpha)} = 1.$$

Because social welfare is unchanged,

$$[c_H - P(\tilde{\alpha})] + \left[\sum_{k \neq i, j} \alpha_k \pi(\tilde{\alpha}) \right] + \tilde{\alpha}_{ij} \pi(\tilde{\alpha}) = [c_H - P(\alpha)] + \left[\sum_{k \neq i, j} \alpha_k \pi(\alpha) \right] + (\alpha_i + \alpha_j) \pi(\alpha),$$

which simplifies to

$$[P(\alpha) - P(\tilde{\alpha})] + [\tilde{\alpha}_{ij} \pi(\tilde{\alpha}) - (\alpha_i + \alpha_j) \pi(\alpha)] = \sum_{k \neq i, j} \alpha_k [\pi(\alpha) - \pi(\tilde{\alpha})].$$

By assumption $(1 - \alpha_1) > (1 - \alpha_i)(1 - \alpha_j)$, so $\pi(\tilde{\alpha}) > \pi(\alpha)$ and the right hand side of the above expression is strictly negative. Thus, if the merger is privately profitable, then the buyer's expected payment must increase. \square

Proof of Proposition 5: Using the definition of $\tilde{\alpha}_{1j}$ and the results of Proposition 1,

$$\pi(\tilde{\alpha}) = \left(\prod_{k \neq 1, j} (1 - \alpha_k) \right) c_H = \frac{\pi(\alpha)}{(1 - \alpha_j)}.$$

It is straightforward to show

$$\frac{E\pi_{ij}}{E\pi_i + E\pi_j} = \frac{\tilde{\alpha}_{ij}}{(\alpha_i + \alpha_j)(1 - \alpha_j)} > 1$$

and

$$\frac{P(\tilde{\alpha})}{P(\alpha)} = \frac{1 + \sum_{k \neq 1, j}^N \alpha_k}{(1 - \alpha_j) \left(1 + \sum_{k \neq 1}^N \alpha_k\right)} > 1.$$

Thus, the merger increases the ex ante expected profits such that the firms can find a mutually agreeable split, while the buyer's expected payment increases. \square

Proof of Proposition 6: Using equations (1) and (2), a profitable merger that reduces the buyer's expected payment requires

$$\left(\frac{\tilde{\alpha}_{ij}}{\alpha_i + \alpha_j}\right) > \frac{1 + \left(\sum_{k \neq 1, i, j} \alpha_k\right) + \tilde{\alpha}_{ij}}{1 + \left(\sum_{k \neq 1, i, j} \alpha_k\right) + \alpha_i + \alpha_j}.$$

Straightforward algebra shows that the above requirement is equivalent to

$$\phi > \frac{\alpha_i \alpha_j}{(1 - \alpha_i)(1 - \alpha_j)}.$$

For $\alpha_i + \alpha_j < 1$, there exist $\phi < 1$ such that the above relationship holds. Therefore, for appropriate choice of ϕ , there exist profitable mergers that lower the buyer's expected payment. \square

Proof of Proposition 7: Using the definition of $\tilde{\alpha}_{ij}$ and the results of Proposition 1,

$$\pi(\tilde{\alpha}) = \left(\prod_{k \neq i, j} (1 - \alpha_k)\right) c_H$$

and

$$\frac{E\pi_{ij}}{E\pi_i + E\pi_j} = \left(\frac{\tilde{\alpha}_{ij}}{\alpha_i + \alpha_j}\right) \frac{1 - \alpha_1}{(1 - \alpha_i)(1 - \alpha_j)}.$$

For the merger to be profitable, it must be the case that the above expression exceeds 1. The condition for this to hold can be written

$$\frac{(1 - \alpha_1) - (\alpha_i + \alpha_j)(1 - \alpha_i)(1 - \alpha_j)}{(1 - \alpha_i)(1 - \alpha_j)} > 1 - \phi.$$

If the merged firm is the best firm, then

$$\frac{1 - \alpha_1}{(1 - \alpha_i)(1 - \alpha_j)} > 1 - \phi.$$

There is no apparent relationship between the left hand sides of the two preceding expressions. However, it is possible that both are satisfied, so that there exist privately profitable mergers that result in a new best firm. \square

Proof of Proposition 8: This proof follows from straightforward algebra. \square

Proof of Proposition 9: (Sketch) To calculate S_i one must calculate the probability θ_i that an efficient firm i wins a given auction. Firm i 's share will be $\alpha_i\theta_i + \frac{1}{N} \prod_{k=1}^N (1 - \alpha_k)$, where the second term is the probability that the firm wins when it and all other firms are inefficient. The probability that firm i wins depends on its price offer, p . To calculate θ_i , one must integrate $\theta_i(p)$ over all prices in firm i 's mixed strategy price distribution. \square

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