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OPTIMAL CURRENCY BASKET IN A WORLD OF GENERALIZED FLOATING: AN APPLICATION TO THE NORDIC COUNTRIES

by

Hali J. Edison and Erling Vardal

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Hali J. Edison*

and

Erling Vardal**

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* Economist, Division of International Finance, Board of Governors of the Federal Reserve System, Washington, D.C. 20551.

** Visiting Scholar, Division of International Finance, Board of Governors of the Federal Reserve System, Washington, D.C. 20551 and Associate Professor, Institute of Economics, University of Bergen, 19 Jonas Reinsgate, N5000 Bergen, Norway.

Abstract

The purpose of this paper is to derive optimal weights for a currency basket taking into consideration the objective of the policymaker. We carefully distinguish between the two terms: effective exchange rate index and currency basket, which are often used interchangeably in the literature. In general, our analysis is an extension of the work of Branson-Katseli and Lipschitz-Sundararajan and then applied to the Nordic countries. We use the policy objective of minimizing fluctuation in export production and illustrate our results using Norway, Sweden and Finland. The weights we derive create optimal currency baskets which are different from the ones used in the countries.

I. Introduction

Since the collapse of the fixed exchange rate regime in the early 1970s, countries have been faced with the question whether to peg their currencies and, if so, how this should be accomplished. Several countries have chosen to tie their currencies to unilaterally designed baskets of other currencies. European examples include Austria, Finland, Greece, Norway, Portugal, Spain, and Sweden. This paper considers the objectives of the policymakers and attempts to derive the optimal weights used in a currency basket. Given the formulas derived in the paper, optimal currency baskets are considered for the Nordic countries: Norway, Sweden, and Finland.

By the end of 1978 each of these countries had chosen to define its own currency basket and to stabilize the value of its currency in reference to this basket. Prior to 1978, Norway and Sweden had participated in the European Community's currency arrangement often referred to as the "snake". Under this arrangement both countries experienced appreciation of its kroner value measured by a general exchange rate index which was not consistent with their other macroeconomic goals. Therefore, to gain more control of the exchange rates, their authorities opted for currency baskets. The use of currency baskets introduced new and different problems.

There are a number of decisions which the policymaker must make when using a currency basket as a method of stabilization. Some of these are: (i) the choice of weights to be assigned to the different currencies, (ii) the choice of a geometric or an arithmetic index, and (iii) the choice of time period to be used for the base. In this paper we limit the analysis to the first problem: the choice of weights.

Furthermore, we will assume that the basket of any one country is independent of any other country.[1/] The immediate solution that each of these countries has used has been different and they have each made at least one change since 1978. In this paper we consider the selection of weights when the objective is to stabilize the production of home exports.

The point of departure for our analysis is the model presented in Branson and Katseli (BK) (1981,1982). BK developed a trade model which they used to select weights for an effective exchange rate index (also known as elasticity weights). Following Lipschitz and Sundararajan (LS) (1980,1982) we consider the weights derived by BK as the first step in obtaining optimal weights for the currency basket. Only under special circumstances are the two sets of weights equivalent. As we shall describe later in this paper, this difference is observed because of different objective functions and assumptions used to derive the weights.

The distinction between and the properties of exchange rate indices and currency baskets are investigated in Section II. In particular, a geometric exchange rate index is assumed and a small country stabilization policy is considered. Section III casts this paper in light of other studies. Section IV contains the basic trade model which is used to derive the elasticity weights. Section V derives the optimal weights for a currency basket assuming the objective is to minimize fluctuations in the production of exports. Section VI presents the empirical results. Optimal weights are presented and some special cases are examined. Section VII contains concluding remarks.

II. Definition and General Properties of Exchange Rate Indices

Often in the literature on optimal currency baskets the two terms effective exchange rate index and currency basket are used interchangeably. In this paper we will distinguish between the two terms. An effective exchange rate index is simply the weighted average of various bilateral exchange rates. Typically the weights associated with an effective exchange rate index represent "elasticity weights" derived to maintain a trade balance goal of some sort. Similarly, a currency basket is the weighted average of bilateral exchange rates. However, the weights for a currency basket are derived for any type of macroeconomic objective. Therefore, these currency basket weights tend to be more general. Note that the differences are because different objectives are used. Basket weights often utilize effective exchange rate weights, but will not be, in general, equal to them because additional factors are used in their derivation. Only in special cases will the two weights be identical. It is necessary to keep this distinction in mind when interpreting our further results.

The rest of the discussion in this section concerns the general properties of exchange rate indices, whether they are "effective" or "basket". In this paper the nominal exchange rate index is denoted as $I^r(t,w)$ where I, r, t, w are the exchange rate index, the reference currency, time, and weights respectively. The home country's (denoted by h) bilateral exchange rate with respect to currency j at time t is written as $E_j^h(t)$. The effective exchange rate index for the home currency is defined as a geometric average.

$$(1) \qquad I^h(t,w) = \prod_{j=1}^{N} (E_j^h(t)/E_j^h(0))^{w_j}$$

In this basket, there are N currencies with weights, w_j, $j=1,\ldots,N$ and the sum of the weights equals one, $\Sigma\, w_j=1$. When $I^h(t,w)$ is greater than $I^h(0,w)$ this implies a depreciation of the home currency. In later sections of this paper logarithms are used so (1) is rewritten in log form where lower case letters denote logs of the variables (with the exception of the weights and parameters)

$$(2) \qquad i^h(t,w) = \sum_{j=1}^{N} w_j\,(e_j^h(t) - e_j^h(0))$$

Further note that in subsequent discussions we shall drop the time subscript t. It is assumed, henceforth, that everything is at time t unless otherwise indicated.

Two attributes of an effective exchange rate are considered which are particularly relevant for a country with a small open economy. The first attribute applies when the home country can only alter one bilateral exchange rate. For the sake of convenience, we use the home-numeraire country exchange rate denoted as e_1^h. If the monetary authority in the home country attempts to fix the exchange rate index, that is $i=0$,[2/] then from (2) it can be shown that the sum of the weighted bilateral exchange rates must be zero.[3/]

$$(3) \qquad \sum_{j=1}^{N} w_j\, e_j^h = 0.$$

Further, if it is assumed that triangular arbitrage between exchange rates hold (that is, that $e_j^h = e_i^h + e_j^i$) then from (3) it can be shown that the authorities have the ability to fix the exchange rate index by using the home-numeraire rate to offset the movement of the sum of the weighted numeraire-bilateral exchange rates[4/]

$$(4) \qquad e_1^h = - \sum_{j=2}^{N} w_j \, e_j^1$$

Equation (4) can be interpreted as a policy rule. The home-numeraire exchange rate is manipulated to maintain a constant exchange rate index. This property may be true of either an exchange rate index or a currency basket. The only difference between the two is w_j.

The second attribute relates to the home country's exchange rate for any country j relative to the numeraire. This relationship can be obtained by manipulating equation (4)

$$(5) \qquad e_q^h = e_q^1 - \sum_{2}^{n} w_j \, e_j^1$$

If currency q appreciates by 1% against the numeraire ($\Delta e_q^1 = 0.01$, $\Delta e_j^1 = 0$ $j \neq q$), then the home currency will depreciate against currency q by $(1 - w_q)$% and will appreciate by w_q% against all the other currencies (that is assuming i=0).

III. Previous Studies

In a recent article Williamson (1982) surveyed many of the issues on currency baskets. Most of the existing literature on currency baskets have been applied to developing countries, in particular Latin America. This does not imply that it is not relevant for developed countries, because it is, especially for those countries with small open economies. This section highlights some of the important aspects of the literature and casts our research into the existing framework.

In his seminal article, Black (1976) suggests that exchange rate index weights should be selected so as to reflect bilateral trade using direction of trade measures or to reflect elasticities derived from a multilateral trade model. According to our use of terminology, Black describes appropriate weights for an exchange rate index, not a currency basket which policymakers use for their exchange rate goals. Black in this work only describes these weights and does not analytically derive them.

Various elasticity weights in several articles have been suggested as "optimal" in the sense that if policymakers adopt them when creating their exchange rate index they may be able to achieve their exchange rate target. Most notably, Flanders and Helpman (1981) (henceforth FH) derive such weights within a Keynesian model framework using a small open economy. One of the results FH derive is as follows: The weights used should reflect demand elasticity for traded goods because this allows changes in prices to have a small impact on the balance of trade which is considered a target in their analysis.

Another example in the literature of elasticity weights is BK (1981). This approach differs from FH because they consider both the demand and supply aspects of the market of traded goods. BK show that the weights are functions of trade shares and elasticities, which are much like the weights described by Black. These weights are also very similar, if not identical, to those used in an effective exchange rate index, for example, the Federal Reserve's trade weighted dollar exchange rate.

Lipschitz and Sundararajan (1980,1984) expand the calculations for basket weights by incorporating additional factors. LS's contribution allows for correlation between relative prices, domestic prices relative to foreign prices, and exchange rates while also including trade elasticities. In their latter paper LS synthesize the BK-FH approach with their own. Our work is an extension of LS.

This overview of the optimal currency basket literature would not be complete if we did not mention another strand which has been more recently considered. Turnovsky (1982) analyzes the choice of optimal currency basket using a general equilibrium macro model of a small open economy with perfect capital mobility. The optimality criterion he considers is the stabilization of domestic real income. Furthermore, he assigns a central role to international capital flows and exchange rates. Bhandari (1985) has reported on experiments using the Turnovsky model as his basic framework. The aim of the Bhandari paper is to analyze the sensitivity of the weights to the various assumptions rather than to estimate the weights relevant for a particular country. Note that the approach we have adopted, a partial equilibruim approach, should yield weights that can be derived as special cases of Turnovsky. It is also much easier to estimate optimal weights from our formation. Perhaps in a later paper we will consider this more general alternative.

IV. The Trade Model

A conventional trade model along the lines of Branson-Katseli (1981) serves as our basic framework for deriving and evaluating the optimal weights in a currency basket. Because our objective is to minimize fluctuations in the production of exports we report only the export side of the BK model. Note that the home country's export good is demanded by N foreign countries.

Table 1 reports the basic N+1 country trade model, and table 2 provides a summary of the definition of the variables used in this model. Equation (6) gives the home country's supply of the export good, x^s, as a function of the relative home currency prices. Here p_x^h is the home country's price index of exportables and p^h is the index of home domestic prices. Equation (7) presents the total foreign demand of home country good x^d. p_x^j is the price of the export in country j currency and p^j is an index of domestic prices in each j country. For simplicity it is assumed that the price elasticity of export demand, d_x, does not differ between countries.[5/] Equation (8) embodies the assumption of the "law of one price", that is that the home price of exports is equal to the foreign price of exports converted into home currency at the current exchange rate. Equations (9)-(10) are two reduced form equations, for the domestic price of exports (p_x^h) and for the quantity of exports (x), and are derived using the market equilibrium condition.

As pointed out in Branson and Katseli, k in (9)-(10) is an index of market power. In the small country case, k tends to unity as d_x tends to minus infinity. Observe that when this is the case equation (10) simplifies to

(10') $$p_x^h = \sum_{j=1}^{N} \left(\frac{X_j}{X}\right) (e_j^h + p^j)$$

This result suggests that changes in the home country price, p^h, does not have any effect on the price of exports.

From (9) it also follows that a 1% devaluation of currency j will, ceteris paribus, reduce the production of the export by $s_x\ k(X_j/X)$%. This result follows since a devaluation of currency j means a lower export-demand for country j, and consequently the equilibrium price of export decreases. This lower price leads to lower exports supplied. The larger country j's share of the export demand, the larger is the effect.

According to equation (9), the size of the production of the export varies with

(11) $$\Sigma \left(\frac{X_j}{X}\right)(e_j^h + p^j - p^h),$$

which is a weighted average of foreign prices measured in the home currency, $e_j^h + p^j$, relative to the home domestic price, p^h. If the measure in (11) increases, this then leads to an increase in the production of export. Equation (11) may be used as a measure of the overall competitiveness of the home country's export industry. Moreover, the relative domestic price between the home country and country j, $e_j^h + p^j - p^h$, may be taken as a measure of the competitiveness between these two countries.

Using the triangular arbitrage condition, $e_j^h = e_1^h + e_j^1$, (9) can be rewritten as

$$(12) \qquad x = s_x \cdot k\left[\sum_1^N \tau_j(e_j^1 + p^j - p^h) + e_1^h\right]$$

where $\tau_j = X_j/X$, that is the trade (export) share of country j.

If the home country is small, it can not influence e_j^1 and p^j. Under this circumstance the home country can affect the production of the export either by manipulating the home-numeraire exchange rate or the domestic price level. This holds even if the home country is so small that it exerts no market power, that is when k = 1.

The variable τ_j in (12) is a trade weight, and can be used to define an exchange rate index and under certain circumstances optimal currency basket weights. In the next section we shall define a policy objective function which allows us to derive weights that incorporate other factors.

V. The Optimal Weighting Scheme

In this section we focus on the type of exchange rate policy targets used in the Nordic countries. Typically, in these countries, exchange rate policy is aimed at promoting and maintaining the competitiveness of the industries which produce traded goods. The background is that the level of employment is viewed as the most important overall target in the economy and a substantial fraction of the labor force is employed in the traded-goods sector. Since production is directly linked to employment, the following exposition will utilize the production of export goods as the target of exchange rate policy.[6/]

The previous choice needs some additional comments. First, in the Nordic countries a substantial part of the traded goods produced is import-competing and these should therefore be included in the target. However, inclusion of these types of goods do not alter the subsequent conclusions we arrive at and for the sake of simplicity are therefore excluded. Second, we may use a relative unit cost of labor measure as an alternative target for the exchange rate policy. This requires that wage rates are brought explicitly into the analysis. Again this only adds complexity to the model without altering the main results.

In accordance with the previous discussion, it is assumed that the policymaker is concerned with maintaining the production of exports at some target x^*, and that the weighting scheme in the currency basket is a policy instrument. The policy problem is to find the weights at a particular point in time which minimizes the cost of deviations from the target. To this end the policymaker is assumed to possess the following quadratic objective function.

$$(13) \qquad L = E(x - x^*)^2$$

To simplify matters it is further assumed that the target is expected to be achieved, that is $x^* = E(x)$. According to equation (4), rewritten below as equation (14), basket weights are shown to effect the home-numeraire exchange rate. It has already been shown in equation (12) this will in turn affect the level of export production.

(14) $$e_1^h = - \sum_2^N w_j e_j^1,$$

A direct expression for this dependency is obtained by substituting (14) into (12):

(15) $$x(w) = s_x \cdot k[\sum_1^N \tau_j (p^j - p^h) + \sum_2^N (\tau_j - w_j) e_j^1]$$

Equation (15) gives the expression x(w) which is used in (13). For our empirical work we separate out the numeraire-home relative price from the numeraire-country j prices by adding and subtracting p^1 from the right hand side of (15). This yields

(16) $$x(w) = s_x \cdot k[(p^1 - p^h) + \sum_1^N \tau_j (p^j - p^1) + \sum_2^N (\tau_j - w_j) e_j^1]$$

Note that x(w) is a function of both τ_j's and w_j's, the export shares and the optimal basket weights, respectively. It will be shown that in only special cases will the basket weights equal the export shares. The problem of choosing weights can now be seen as minimizing the objective function with respect to the weights

(17) $$\text{Min } L = E(x(w) - E(x(w)))^2$$

w.r.t.(w)

where $x(w)$ is given in (16), and the weights satisfying this problem $w^o = (w_1^o,\ldots,w_N^o)$, are defined as optimal.

The first-order conditions after minimizing (17) are as follows:

$$(18) \qquad \sum_{j=2}^{N} (w_j - \tau_j)\mathrm{Cov}(e_j^1,e_q^1) - \sum_{2}^{N} \tau_j \,\mathrm{Cov}(p^j - p^1, e_q^1)$$

$$\mathrm{Cov}(p^1 - p^h, e_q^1) = 0 \text{ for } q = 2,\ldots,N.$$

To simplify the expression in (18) we switch our notation to matrices and vectors. The N-1 equations in (18) are rewritten as:

$$(19) \qquad \Omega(w - \tau) - \Pi\,(\tau) - \Gamma = 0$$

where $\Omega = \lfloor \mathrm{Cov}(e_j^1,e_q^1)\rfloor$ = an (N-1) by (N-1) variance covariance matrix of exchange rates.

$\Pi = \lfloor \mathrm{Cov}(p^j-p^1,e_q^1)\rfloor$ = a (N-1) by (N-1) covariance matrix between the numeraire-country q exchange rates and the relative price of the (N-1) foreign countries and the numeraire country.

$\Gamma = \lfloor \mathrm{Cov}(p^1-p^h,e_q^1)\rfloor$ = a (N-1) column vector of the covariance between the numeraire-country q exchange rate and the relative price between the numeraire and home country.

From (19), the solution with respect to the optimal weights, w^o, can be derived as:

(20) $$w^o = \tau + \Omega^{-1} \Pi \cdot \tau + \Omega^{-1} \Gamma .$$

By utilizing the BK-model, LS(1982) derive optimal basket weights when the assumed target is to minimize the variance of the home country's <u>terms of trade</u>. Surprisingly, the optimal weights LS derive are similar to those in (20). In fact, the optimal weights are identical except that the trade weight, τ, in (20) are export shares while in LS, τ reflects total shares.7/

As LS note, terms of trade is not a meaningful exchange rate policy target when the home country is small (k=1). The basket weights derived by LS therefore only apply for economies that have some power in the markets for traded goods (k<1). On the other hand, the basket weights we derive in (20) is based on minimizing the variance of the production of exports. This target is relevant (or particularly relevant) for economies with no power in the market for export goods. Our result is striking because we derive the same weights as LS except using a different policy objective function.

Three special cases for the weights derived in equation (20) are considered below:

<u>Case 1</u>

If relative domestic prices, $p^j - p^1$, are uncorrelated with exchange rates, that is,

$$(21) \qquad \mathrm{Cov}(p^j - p^1, e^1_q) = 0 \qquad j=h,2,\dots,N \text{ and } q=2,\dots,N$$

and it is assumed that $\Gamma=0$ then from (20) the optimal weights are equivalent to the trade weights:8/

$$w^o_q = \tau_q \qquad \text{for all } q.$$

It also follows from (18) that if $\mathrm{Cov}(p^j - p^1, e^1_q) \neq 0$ for at least one j and q, that is if we have a relationship between exchange rates and relative domestic prices, then the trade weights are not the optimal basket weights. The importance of these results are that they set a benchmark: Only when all $\mathrm{Cov}(p^j - p^1, e^1_q) = 0$ are the trade weights the optimal basket weights.9/

Let us consider an extreme case of a relationship between exchange rates and the relative domestic price:

Case 2

If we have "purchasing-power-parity" (PPP) between the home country and all other countries in the basket, that is

$$(22) \qquad p^j + e^h_j = p^h \qquad j=1,\dots,N,$$

there does not exist a unique optimal weighting scheme.

This result is easily explained by substituting (22) into (9). We obtain $x(w) = 0$, that is the production of the export equals zero no matter what are the values of the chosen weights.

Another interesting case is the following:

Case 3

If PPP holds between the home country and another country, say country N,

$$(23) \qquad p^h - p^N = e^h_N$$

and if there is no correlation between relative domestic prices, $p^j - p^h$, and exchange rates, e^h_j, for the additional N-1 countries,

$$(24) \qquad \mathrm{Cov}(p^k - p^j, e^h_q) = 0 \qquad\qquad j,q=1,..,N-1,$$

then the optimal weights simplify to[10/]

$$(25) \qquad w^o_N = 0 \quad w^o_j = \tau_j/(1-\tau_N) > \tau_j \qquad\qquad j=1,...,N-1.$$

In other words, (23)-(24) are sufficient conditions for the optimal basket weight of one country, N, to be zero.

An intuitive explanation for the result in case 3 is as follows: According to our discussion related to equation (11), country N affects the production of the export through the relative domestic price $(p^N + e^h_N) - p^h$. If this relative price rises so will the production of the export. If (23) holds, the relative price, $(p^N + e^h_N) - p^h$, is always zero, and no distortion will be transmitted from country N to the export market of the home country. Consequently, the home country needs no protection against country N, that is, it is optimal to set this country's basket weight equal to zero.

Following the same line of argument as above, it can be shown that it is optimal to peg to a single currency L, if PPP holds between the home country and all it's trading partners, except L.[11/] The explanation for this result is that in this case the export market is only disturbed from country L, and it is therefore optimal to give a maximal weight (= 1) to this country.[12/]

The results in Cases 1-3 suggest that two conditions are of importance when the basket weight for a country, say country N, is chosen: (i) the share (or the elasticty of trade) between the home country and country N (τ_N); and (ii) the relationship between the relative price, $p^N - p^h$, and the exchange rate e_N^h. The stronger this relationship is, the smaller is the optimal weight.

VI. Numerical Illustrations

It is possible to describe one or two special cases, but to demonstrate more fully the implications of these formulas, we proceed to actual calculations. It should be noted that our method of calculating the weights differ somewhat from that of LS or BK (for elasticity weights). They use regression analysis whereas we manipulate the matrices of equation (20) directly.

Tables 3 to 5 illustrate the computations of the optimal currency basket weights for each of the countries. The first column, labelled trading partners, represents those countries present in the existing baskets calculated by the monetary authorities. These lists may exclude important countries, but we decided to limit our investigation to those countries now used in each currency basket. The column labelled actual basket weights presents the τ_j's used in each country. Note that the U.S. dollar is used as the numeraire currency and its weight is considered as the residual (that is, $1 - \sum_2^N w_j$) and is therefore reported below the total of the other weights. The columns labelled 1-3 refer to calculations using equation (20) and actual basket weights. Following these columns is another column of τ_j weights which represent actual export shares used to evaluate equation (20). The columns labelled 4-6 correspond to those in columns 1-3 but export share weights (τ_j's) are used.

The evaluation of the optimal weights is performed using data over the time period January 1973 to March 1982. Columns labelled 1 and 4 report calculations using matrix manipulation which correspond directly to equation (20). In order to ascertain the impact of third-party effects we recalculate the optimal weights by imposing two restrictions: (i) the Ω matrix is forced to be diagonal, that is the covariance between-exchange rate is zero -- $Cov(e_j^1, e_q^1) = 0$ for $j \neq q$ (see columns labelled 2 and 5), (ii) and in addition the column vector $\Gamma = 0$ - the covariance between numeraire exchange rate and relative price between numeraire and home country is zero (see columns labelled 3 and 6). This latter assumption is used in LS. In fact, LS argue that there is no reason to expect $\Gamma\Omega^{-1}$ to be stable and therefore set it to zero.

It is quite difficult to generalize, but some observations can be gleaned from tables 3-5 and other experiments we have tried:

a) The historical export share indices for each country tend to be higher (weaker home currency) than the actual basket.

b) In most cases the calculated optimal weights for the United States tend to be higher and those weights for United Kingdom and Germany lower.

c) There is a complicated interaction of the effects of covariances of exchange rates which yields a rather different pattern to the weights making them quite difficult to interpret.

d) The interpretation of negative weight for currency basket is important. In the tables a zero is assigned to the weight, if the calculated weight optimal is negative. However, for Finland and Sweden we were unable to interpret our results for the weights when no restrictions were imposed on equation (20). Both Turnovsky and BK argue that negative weights have an economic interpretation. Negative weights imply that the index will move in the opposite direction as the currency movement and at least one other country currency will have a larger weight (perhaps greater than unity) to compensate for this feature. We have not interpreted our results in this light.

Charts 1-3 show the time paths of the actual currency basket indices and two 'optimal' indices using the optimal weights from columns (2) and (3) of the corresponding country tables 3-5. The indices have been arbitrarily calculated, in the sense that we do not replicate the existing baskets, by using 1980 as the base year. It is clear from the charts that during most of the time period the optimal baskets are above the actual baskets which suggest weaker home currencies. The patterns of the country indices themselves are quite different reflecting differences in home currency historical experiences.

VII. Conclusion

The purpose of this paper is to derive optimal weights for a currency basket taking into consideration the objective of the policy makers in the Nordic countries. Our analysis is based on the work of Branson-Katseli and Lipschitz-Sundararajan. The existing literature in this area often interchange the terms effective exchange rate index and currency basket. We carefully distinguish between the two terms. Both are defined as weighted average of bilateral exchange rates. However, the weights used in their calculations are different. Generally, the weights in an effective index are some form of trade weights, while the weights in a basket are more broadly defined.

In this paper we derive both export share weights by using a simple multi-country model, and basket weight by assuming that the objective of the policymakers is to minimize fluctuations in the production of exports. We show that only under special circumstances are the two weights the same. The basket weights are a function of these export share weights but also other factors such as the covariances of relative prices and exchange rates. Using the formulas we have derived, various calculations are made for Norway, Finland and Sweden.

Table 1: Simple N+1 Country Trade Model

Export-supply:

(6) $x^s = s_x(p_x^h - p^h)$ $\qquad s_x > 0$

Export-demand:

(7) $x^d = d_x \sum_{j=1}^{N} \frac{X_j}{X} (p_x^j - p^j)$ $\qquad d_x < 0$

Export-prices:

(8) $p_x^h = e_j^h + p_x^j$ $\qquad j=1,\ldots,N$

Reduced forms:

(9) $x = s_x\, k \sum_{j=1}^{N} (\frac{X_j}{X})(e_j^h + p^j - p^h)$

(10) $p_x^h = (1-k)p^h + k \sum_{j=1}^{N} \frac{X_j}{X} (e_j^h + p^j)$

Table 2: Definitions of Variables

Lower case letters indicate that the variable is in logarithm

e_j^h	- Exchange rate (home currency in term of a unit of currency j)
p_x^h	- Domestic price of exports
p^h	- Domestic price index
p^j	- Foreign country j index of prices
x (x^s,x^d,x_j)	- Exports (supply, demand, exports for country j)
s_x	- Export elasticity of supply with respect to relative price term
d_x	- Export elasticity of demand with respect to relative price term
k	- Ratio $\left(\frac{d_x}{d_x - s_x}\right)$

Table 3: Optimal Basket Calculations for Norway

Trading Partners	Basket Weights τ_j	(1)	(2)	(3)	Export Share Weights τ_j	(4)	(5)	(6)
Austria	.015	.085	.013	.014	.009	.041	.010	.009
Belgium	.024	.240	.022	.023	.023	.31	.024	.022
Canada	.036	-.041	-.064	.030	.009	-.003	-.087	.007
Switzerland	.012	.046	.013	.009	.018	.051	.029	.014
Denmark	.068	-.037	.066	.069	.104	-.069	.106	.105
United Kingdom	.147	.160	.147	.116	.176	.177	.165	.139
France	.092	.030	.089	.094	.047	.013	.049	.048
Finland	.03	.022	.027	.031	.035	.028	.037	.036
Germany	.177	.120	.175	.143	.164	.139	.172	.132
Holland	.046	.037	.045	.041	.063	.050	.069	.056
Italy	.033	.030	.032	.034	.032	.019	.032	.032
Japan	.06	-.068	.059	.059	.021	-.103	.021	.020
Sweden	.15	.190	.147	.152	.216	.266	.218	.218
Total	.89	1.00	.835	.815	.917	.684	.932	.838
United States	.11	0	.165	.185	.-83	.316	.068	.162

Notes to Table: Trading partners listed above are those currently used in Norway's basket. The column labelled basket weights represents weights used in the calculation of the Norwegian basket while the column labelled export shares are derived from direction of trade statistics. Columns (1) and (4) represent weights derived using equation (20). Columns (2), (3), (5), (6) are special cases of equation (20).

Table 4: Optimal Basket Calculations for Finland

Trading Partners	Basket Weights τ_j	(1)	(2)	(3)	Export Share Weights τ_j	(4)	(5)	(6)
Belgium	.027	-.06	.024	.026	.023	-.027	.024	.022
Switzerland	.025	-.06	.019	.018	.02	-.048	.032	.015
Denmark	.05	.305	.045	.051	.064	.20	.066	.064
United Kingdom	.147	.35	.153	.116	.186	.38	.174	.147
France	.061	-.09	.055	.062	.061	-.07	.063	.062
Germany	.189	.58	.18	.153	.149	.54	.157	.121
Holland	.052	-.35	.043	.046	.056	-.33	.063	.050
Italy	.039	.189	.037	.04	.033	.17	.033	.033
Japan	.05	.08	.048	.049	.02	.01	.021	.019
Norway	.056	-.163	.051	.054	.07	-.16	.072	.068
Sweden	.20	.152	.194	.202	.191	.18	.193	.193
Total	.896	*	.849	.817	.873	*	.898	.794
United States	.104	*	.151	.183	.127	*	.102	.206

Note to Table: Trading partners listed above are those currently used in Finland's basket. The column labelled basket weights represent weights used in the calculation of Finnish basket while the column labelled export share are derived from direction of trade statistics. Columns labelled (1) and (4) represent weights derived using equation (20), the other columns are special cases of equation (20).

Table 5: Optimal Basket Calculations for Sweden

Trading Partners	Actual Basket Weights τ_j	(1)	(2)	(3)	Export Share Weights τ_j	(4)	(5)	(6)
Austria	.015	.31	.012	.014	.014	.321	.012	.013
Belgium	.038	-.22	.035	.037	.052	-.201	.05	.050
Canada	.011	-.014	-.09	.009	.014	-.022	-106	.012
Switzerland	.022	.038	.017	.016	.021	-.044	.021	.016
Denmark	.082	.37	.077	.083	.107	.40	.104	.108
United Kingdom	.132	.12	.137	.104	.137	.119	.137	.108
France	.055	.05	.049	.056	.063	.059	.059	.064
Finland	.072	.05	.065	.073	.074	.047	.069	.075
Germany	.162	-.17	.154	.131	.144	-.218	.141	.116
Holland	.051	.074	.043	.045	.06	.093	.056	.053
Italy	.036	-.02	.034	.037	.041	-.022	.040	.042
Japan	.027	-.03	.025	.027	.018	.041	.017	.017
Norway	.094	.192	.089	.092	.128	.241	.125	.125
Spain	.012	.10	.01	.012	.015	.093	.014	.015
Total	.809	*	.747	.736	.888	*	.845	.84
United States	.191	*	.253	.264	.112	*	.155	.186

Note to Table: Trading partners listed above are those currently used in Swedish basket. The columns labelled basket weights represent weights used in the calculation of Swedish basket while the column labelled export share are derived from direction of trade statistics. Columns labelled (1) and (4) represent weights derived using equation (20), the other columns are special cases of equation (20).

Chart 1

CALCULATION OF NORWEGIAN BASKET

USING 1980 AS BASE YEAR

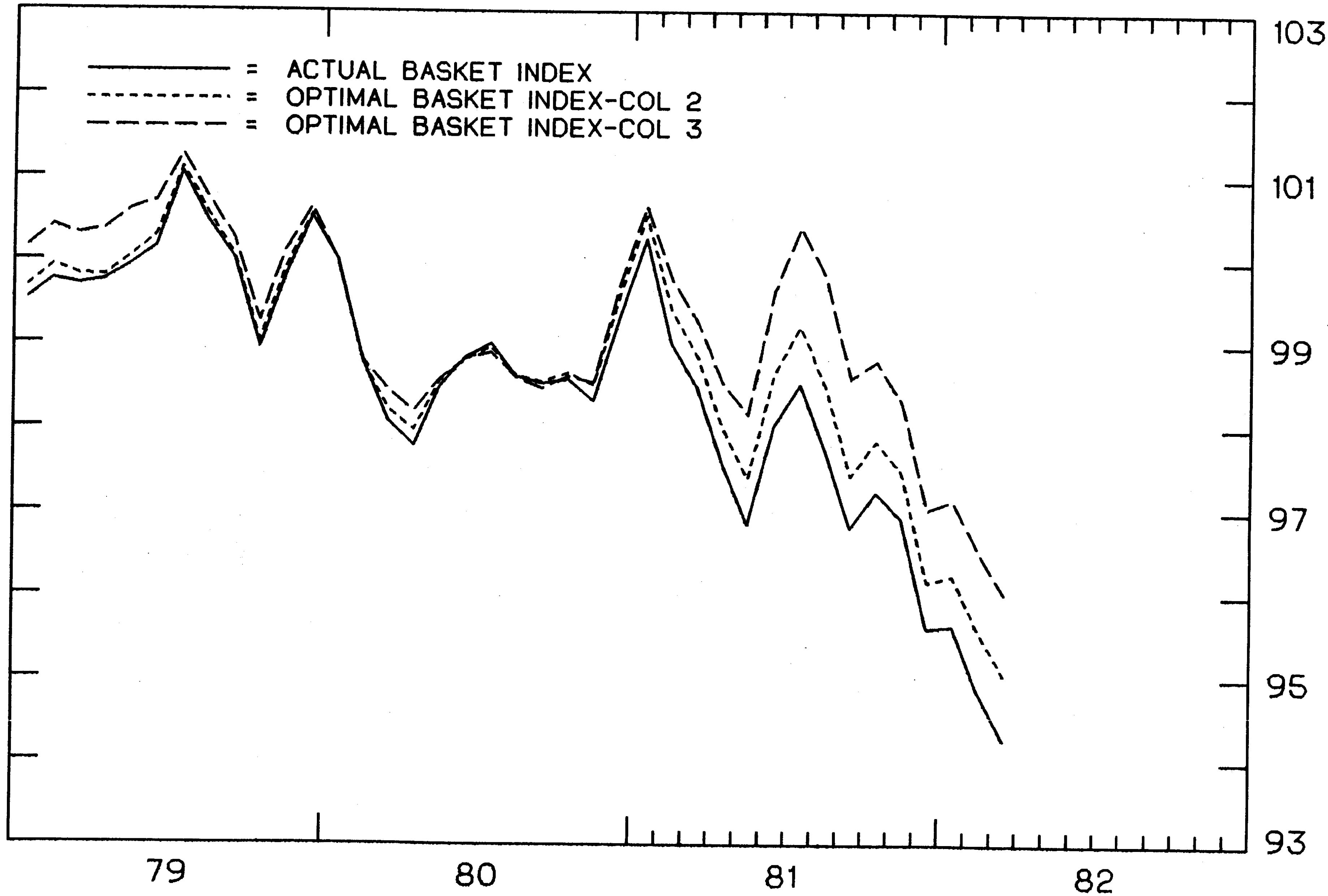


Chart 2

CALCULATION OF FINNISH BASKET INDEX
USING 1980 AS BASE YEAR

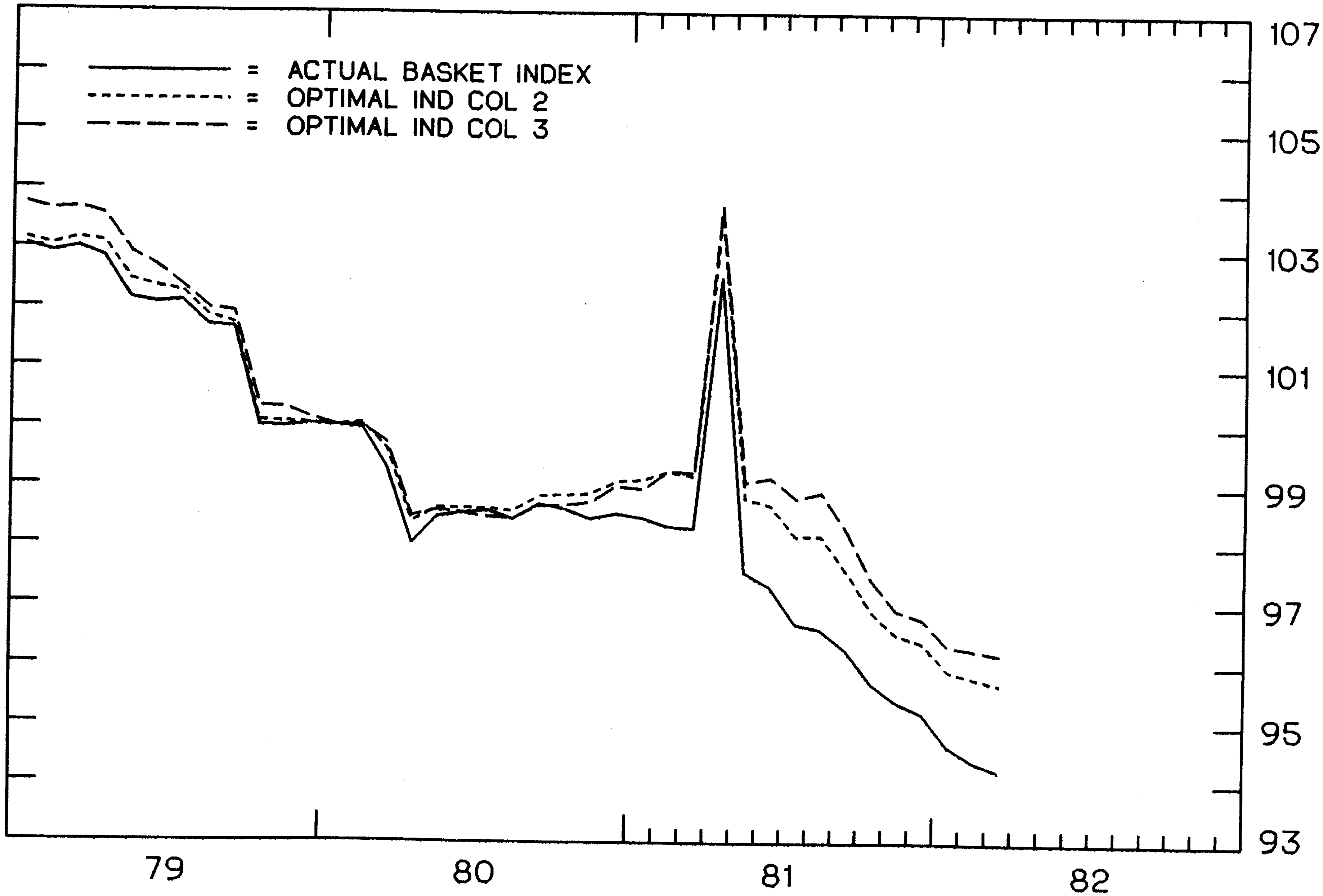
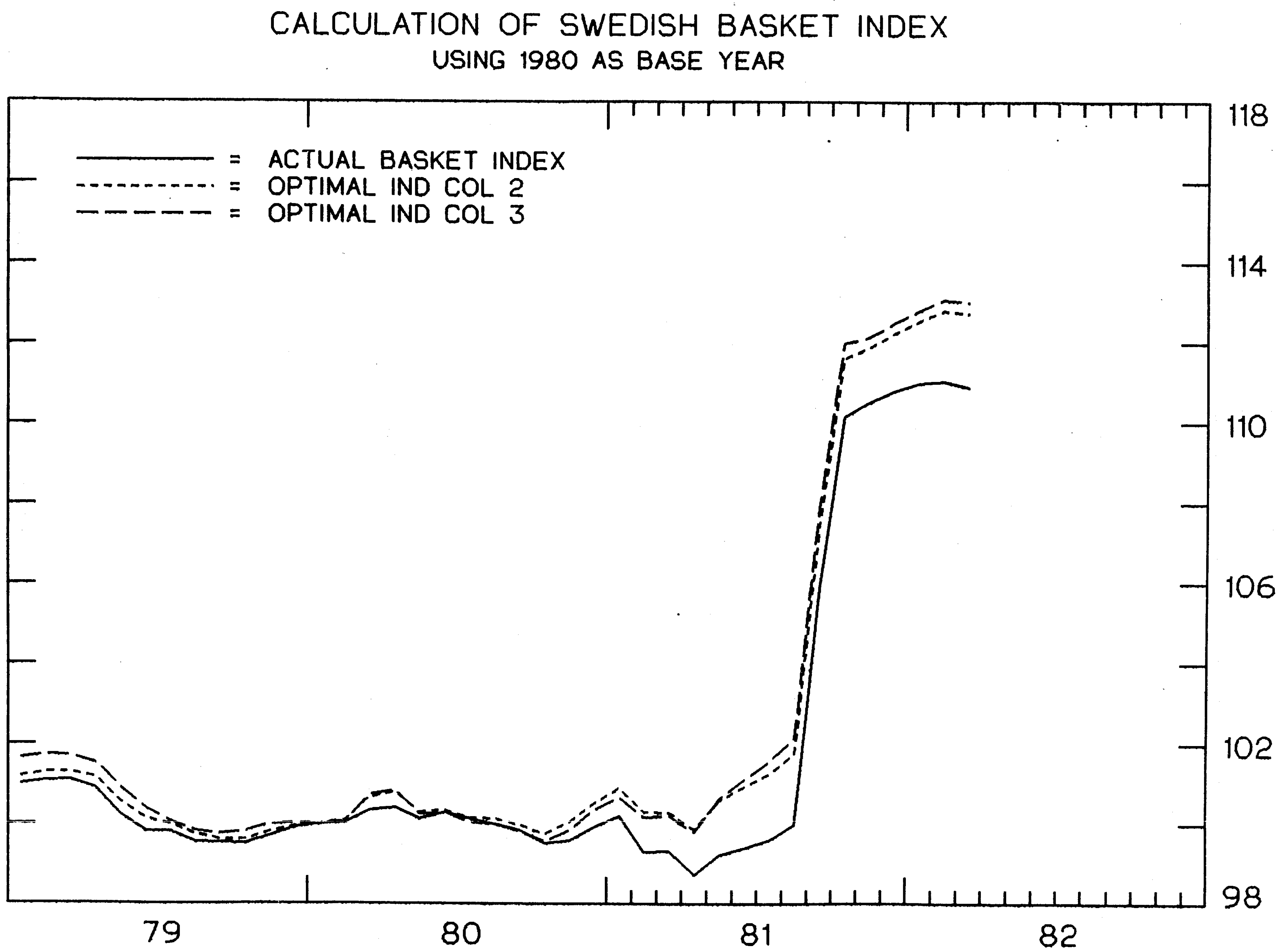


Chart 3


CALCULATION OF SWEDISH BASKET INDEX
USING 1980 AS BASE YEAR
= ACTUAL BASKET INDEX
= OPTIMAL IND COL 2
= OPTIMAL IND COL 3
118
114
110
106
102
98
79
80
81
82

Footnotes:

* The first author is an Economist in the International Finance Division of the Board of Governors of the Federal Reserve System and the second author is an Associate Professor at the Institute of Economics, University of Bergen, Bergen, Norway. Work was done on this project while the second author was a visiting scholar in the Division of International Finance of the Board of Governors. An earlier version of this paper was presented at the 5th International Symposium on Forecasting, Montreal, Canada, June 9-12, 1985. The views expressed herein are solely those of the authors and do not necessarily represent the views of the Federal Reserve System or any members of its staff.

1. A paper by Asheim (1984) examines the properties of a system of currency baskets.

2. According to (1), at time 0 the index equals 1. We assume that the monetary authority keeps the index constant. Therefore, $I^h(t,w) = 1$ and $i^h(t,w) = 0$.

3. Without loss of generality, we assume that the exchange rates at time 0, $E_j^h(0)$, are scaled such that they equal 1. Therefore, $e_j^h(0) = 0$, and (3) is immediately obtained from (2).

4. The result of equation (4) can be more clearly seen by noting

$$e_j^h = w_i^h + e_j^i$$

$$i = (1 - w_1 - w_2 \dots w_N)\ e_1^h + w_1(e_1^h + e_j^1) + w_2(e_2^h + e_j^2)$$

And since I = 0 then

$$e_1^h = - \sum_2^N w_j\ e_j^1$$

5. Without changing the results we derive in any significant way, this assumption simplifies the formulaes in the text.

6. In the subsequent table references are given to articles written by "authorities" on the exchange rate policy specific for the countries in question.

	Finland	Norway	Sweden
	Puro (1978)	Skanland (1983) Solheim and Sporastoyl (1983)	Lundström (1978) Kredit-och valuta-oversikt (1981)
General goal	Competitiveness of Finnish industry	Competitiveness of Norwegian industry	Sweden's ability to compete internationally
Specific measures	Stabilizing cost and price levels	Relative unit cost cost of labor, relative export prices, relative consumer prices	Unit labor cost, relative consumer prices, balance of trade

The target we consider in this paper, to minimize fluctuations in the production of exports, accords with the general goal suggested by the authorities listed in the table above.

7. For the sake of comparison, the matrix notation in (19) is adopted from LS(1982).

8. LS(1982), p. 130, show that if the relative domestic prices, $p^j - p^1$, are fixed, the optimal basket weights are equal to the trade weights. Clearly, this proposition is a special case of our result in case 1.

9. If (21) holds, we see from (15) that the variance of the production of the export can be separated into two terms: (i) the variance of the relative domestic prices and (ii) a variance term related to exchange rates. The first of these is not affected by the basket weights. Since the second term becomes zero if the trade weights are chosen as basket weights, the variance of the production of the export will also be minimized by the choice of these weights.

10. The proof of the proposition in case 3 is as follows:

By substituting (23) into (9), we obtain:

$$x(w) = (s_x \cdot k)\left[\sum_{1}^{N-1} \tau_j (p^j - p^h) + \sum_{1}^{N-1} \tau_j e_j^h\right].$$

Because of (24)

$$Var(x(w)) = (s_x \cdot k)^2 \lfloor Var(\sum_1^{N-1} \tau_j (p^j - p^h)) + Var(\sum_1^{N-1} \tau_j e_j^h) \rfloor.$$

Since $e_j^h = e_1^h + e_j^1$, by using (14)

$$\sum_1^{N-1} t_j e_j^h = \sum_2^N (\tau_j - w_j(1 - \tau_N))e_j^1 - (1 - \tau_N)w_N \, s_e^1 N$$

Consequently, $Var(\sum_1^{N-1} \tau_j e_j^h) = 0$ when the weighting scheme in (25) is chosen, and Var(x(w)) will be minimal.

11. An additional requirement is that $cov(p^h - p^L, e_L^h) = 0$.

12. LS(1980), p. 88, have a PPP-proposition that is different from cases9 2-3. They show that if PPP holds between the trading partners of the home country, i.e. $p^1 - p^j = e_j^1$, j=2,....N, and if in addition $cov(p^h - p^1, e_j^1) = 0$, j=2,....,N, then it is optimal for the home country to peg to the numeraire.

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