Estimating water temperatures and time of ice formation on the Saint Lawrence River

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Abstract

Monthly mean heat losses from the surface of the St. Lawrence River during the fall–winter cooling period were determined by an empirical heat budget which incorporated the processes of radiation, conduction, convection, and precipitation. Calculations indicate that the heat loss can be reasonably represented by a simple linear relation with air–water temperature differential. It is suggested however, that the coefficient of proportionality changes with variations in the ratio of radiation to evaporation. An equation was evaluated which relates surface heat loss to temperature decline along the international section of the river. Within the limits of accuracy of the heat loss calculations, the equation provides adequate estimates of water temperature changes for the period of study. The water temperature decline equation was used as the basis for developing a prediction technique which enables river freeze-up estimates to be made as early as 1 October. When observed freeze-up dates were used, predictions for a 6-year period (1965–1970) yielded standard deviations of 4.7, 3.3, and 3.5 days for predictions starting at the beginning of October, November, and December. Observed freeze-up occurred within 2 days of the predicted date in 4 of the 6 years examined. Experimental predictions for two additional years yielded similar results.

Commercial shipping activity on and into the Great Lakes normally ceases for 3 to 4 months each winter because of the extensive ice cover on the interlake connecting channels and the St. Lawrence River. Historically, lake shipping ends in mid-December, before ice formation and reopens in spring, when ice no longer poses a problem. The closing date has been dictated not by the capability of vessels to navigate ice fields, but rather by the problems inherent to lock and hydroelectric plant operations when quantities of floating ice are present. Temporary ice stabilization booms are often placed across high velocity sections of river channels in early winter to reduce surface velocity and induce sheet ice formation above the booms.

In establishing the navigation closing date, inadequate attention has been given to the annual, seasonal variations in climate governing the time of ice formation in a specific year. Those variations can cause differences in time of freeze-up of as much as 3 weeks from year to year. Reliable predictions of the time of ice formation would obviously be of great use.

This report describes a technique for estimating the time of river ice formation 2 to 3 months in advance. As energy exchange at the air–water interface is one of the more important factors governing the temperature of a water body, an analytical method for evaluating the heat flux from a water surface during the fall–winter cooling period is described in detail. The technique was developed specifically for the international section of the St. Lawrence River (Fig. 1) but is sufficiently general for possible application to the St. Marys and St. Clair Rivers, which also have as their sources lakes with large heat storage capacities.

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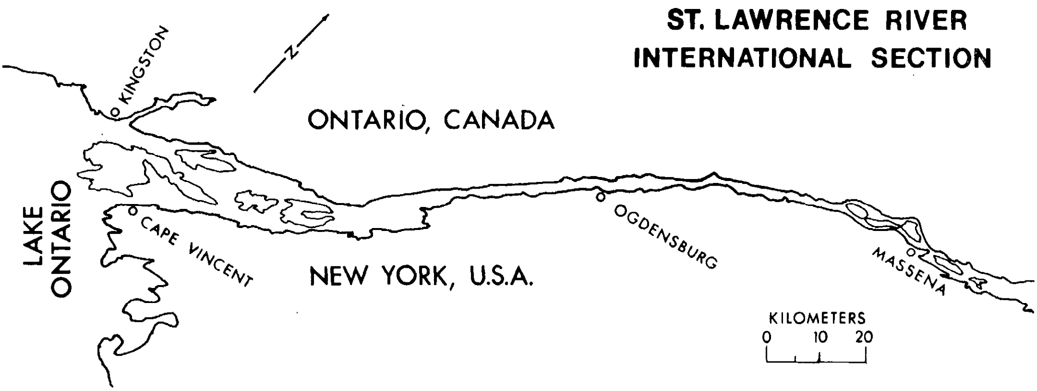

Fig. 1. Plan of the international section of the St. Lawrence River.

Navigation Season Extension Demonstration Program.

Energy exchange at the air–water interface

A natural body of water gains heat primarily by the absorption of short-wave solar radiation and long-wave atmospheric radiation. Heat gain by biochemical processes, conduction through the bottom and transformation of kinetic into thermal energy is of small magnitude and generally is neglected in studies of lakes, reservoirs, and rivers (Rodgers and Sato 1971; Velz 1970). Heat is lost primarily by long-wave back radiation, evaporation, and conduction and convection. During the period before freezing, the water can be substantially cooled with snow falling on or blowing into the water.

The sum of the significant energy exchange processes can be represented by the equation

$$Qt = Qs - Qr + Qa - Qar - Qbs - Qe - Qh - Qp, \quad (1)$$

where Qt = heat storage; Qs = global solar radiation; Qr = reflected solar radiation; Qa = downward atmospheric radiation; Qar = reflected atmospheric radiation; Qbs = upward terrestrial surface radiation; Qe = latent heat transfer (evaporation); Qh = sensible heat transfer (conduction and convection); Qp = heat transfer by precipitation.

Comprehensive discussion of the individual terms of Eq. 1 is given by Jacobs (1951) and others. It has been generally concluded that estimation of the terms must be based on formulae which utilize readily measurable physical parameters. Several empirical formulae are available from which to choose. The type of available meteorological data and the physical setting of the study area were used as criteria for selecting a representative formula for each term. The results have been expressed in units of ly day^{-1}. Some of the equations were developed from data collected over relatively short periods and estimation errors might therefore be expected when averaged data are used in those equations. Any high estimates should be nearly balanced by corresponding low estimates, however, because the distribution of sample means approximates a normal distribution (Mack 1967). The result should be a value which is within the error range, estimated at 10–15%, of the heat budget computation technique.

An equation for indirectly estimating incoming solar energy was derived by Fritz and MacDonald (1949) and states that

$$Qs/Qo = a + bs, \quad (2)$$

where Qs = global solar radiation; Qo = radiation of a perfectly transparent atmosphere; a = constant; b = constant; s = bright sunshine expressed as a fraction of the total possible.

The Fresnel formula can be used for determining the reflectivity of a plane water surface for unpolarized light. Although the computed reflectivity values are valid only for a plane undisturbed water surface, observed reflection from a disturbed surface should deviate only slightly from the computed values (Angstrom 1925) and further, those deviations should be completely masked by the averaging process.

The value for reflected solar radiation thus becomes

$$Qr = Qs \times R, \qquad (3)$$

where R = reflectivity from the Fresnel formula.

Anderson and Baker (1967) developed an equation for calculating downward atmospheric radiation which can be stated as

$$Qa = \sigma Ta^4 - [228.0 + 11.16(es^{1/2} - ea^{1/2}) - A][Qs/Qsc]^2, \qquad (4)$$

where σ—Stefan-Boltzmann constant = 11.71×10^{-8}; Ta = surface air temperature —°K; es = saturated vapor pressure at Ta—*mb*; ea = vapor pressure of the air—*mb*; A = station adjustment term; Qs = global solar radiation; Qsc = clear sky global radiation.

A reflectance factor of 0.03 for water (Koberg 1958) gives the reflected atmospheric radiation:

$$Qar = Qa \times 0.03. \qquad (5)$$

With the use of the Stefan-Boltzmann law and an emissivity factor for water of 0.97 (Anderson 1952), the upward terrestrial surface radiation becomes

$$Qbs = 0.97\sigma Tw^4, \qquad (6)$$

where Tw = water surface temperature—°K; $\sigma = 11.71 \times 10^{-8}$.

The relationship developed from the Lake Hefner studies (Anderson 1952) is used to estimate heat loss due to evaporation:

$$Qe = 5.0 \times 10^{-3} W(ew - ea)L, \qquad (7)$$

where ew = saturation vapor pressure at the water surface—*mb*; ea = vapor pressure of the air at ambient air temperature—*mb*; W = wind speed at 11 m above the water surface—knots; L = latent heat of evaporation.

Because the nature of the eddy coefficient is poorly understood, recourse can be made to the Bowen ratio (Bowen 1926) and the sensible heat flux term becomes

$$Qh = 3.05 \times 10^{-3} WL(Tw - Ta). \qquad (8)$$

Laevastu (1960) developed a formula for the heat loss due to snow falling on or blowing into the water whereby

$$Qp = 7.97Ps + 0.1Ps(Tw - Tp), \qquad (9)$$

where Qp = heat loss by snowfall; Ps = snowfall per day in mm water equivalent; Tw = water temperature (°C); Tp = snow (air) temperature (°C).

Water temperature decline

St. Lawrence River surface heat losses—In northern latitudes beginning in late September or early October, the net flux of heat is from water to air, and water temperature starts to decline toward the freezing point. In the Great Lakes region an ice cover normally develops on rivers and along the lake shores during the period from mid-December to late January. The months of October through January are thus the most important in terms of water surface heat loss and temperature decline.

Monthly mean surface heat losses from the international section of the St. Lawrence River were calculated for the 6-year period 1965–1970 (Table 1) together with long term mean heat losses for the period 1931–1960 for the months of October through January. Meteorological parameters necessary for the solution of Eq. 2 through 9 were extracted from annual meteorological summaries published by Canada and the United States. Water temperature data from the St. Lawrence River at Massena, N.Y., were provided by the Power Authority of the State of New York (PASNY). The Public Utilities Commission of the City of Kingston, Ontario, furnished water temperatures for that location. At Cape Vincent and Ogdensburg, N.Y., data were made available by the Limnology Division, Lake Survey Center, NOAA. The

Table 1. Monthly mean values of St. Lawrence River heat budget terms (ly day^{-1}).

Year		Oct	Nov	Dec	Jan
	Qs	205	123	91	118
	Qr	16	17	17	19
	Qa	665	608	587	506
	Qar	20	18	18	15
1965–66	Qbs	749	703	666	642
	Qh	59	89	76	164
	Qe	156	132	106	105
	Qp	0	17	6	28
	Qt	-130	-245	-211	-349
	Qs	246	128	100	112
	Qr	20	18	19	18
	Qa	643	644	558	565
	Qar	19	19	17	17
1966–67	Qbs	752	718	662	643
	Qh	54	55	93	85
	Qe	175	115	123	114
	Qp	0	2	11	11
	Qt	-131	-155	-267	-211
	Qs	206	138	103	121
	Qr	17	20	20	20
	Qa	676	583	558	483
	Qar	20	18	17	14
1967–68	Qbs	766	710	666	641
	Qh	57	117	110	209
	Qe	159	143	118	118
	Qp	0	4	4	14
	Qt	-137	-291	-274	-412
	Qs	238	112	92	123
	Qr	19	16	18	20
	Qa	669	613	535	514
	Qar	20	18	16	15
1968–69	Qbs	784	718	658	637
	Qh	2	127	169	141
	Qe	193	175	135	102
	Qp	0	9	20	9
	Qt	-111	-338	-389	-287
	Qs	237	117	102	130
	Qr	19	17	20	21
	Qa	649	628	519	459
	Qar	20	19	16	14
1969–70	Qbs	766	718	661	638
	Qh	62	86	169	212
	Qe	162	128	124	109
	Qp	0	6	21	12
	Qt	-143	-229	-390	-417
	Qs	220	119	106	117
	Qr	18	17	20	19
	Qa	683	639	503	490
	Qar	20	19	15	15
1970–71	Qbs	786	733	661	640
	Qh	52	83	191	199
	Qe	143	134	126	117
	Qp	0	4	28	12
	Qt	-116	-232	-432	-395

values of *Qo* and *Qsc* in Eq. 2 and 4 were published by Bolsenga (1964). The constants *a* and *b* in Eq. 2 were calculated by regression for the Great Lakes area by Bolsenga (unpublished) who reported a correlation coefficient of 0.85. For 1965–1970, mean monthly values of air temperature and snowfall were derived by averaging data from Kingston, Ontario, and Alexandria Bay, Ogdensburg, and Massena, N.Y. Water temperature was taken as the mean of Kingston, Ogdensburg, and Massena data. The requirement that the temperature represent that of the surface water necessitated an assumption of isothermy since the water temperatures available were from subsurface sensors; justification for this assumption is discussed below. Vapor pressure, wind speed, and solar radiation values represent the mean of data from Kingston and Massena. The long term mean values were based on averaged data from Kingston and Montreal. The monthly values for the 6-year period are thus representative of only the international section of the river, whereas, the long term means represent the river between Lake Ontario and Montreal.

Table 2. Monthly average heat loss from the St. Lawrence River.

		Oct	Nov	Dec	Jan
Lake Survey Center	1965–1971	138	250	327	345
	1931–1960	201	300	353	351
Wardlaw	1954	---	---	374	488
Witherspoon & Poulin	1970	197	239	332	353

A summary of the heat loss calculations together with comparison values from studies by Wardlaw (1954) and Witherspoon and Poulin (1970) are presented in Table 2. Although the 6-year and long term mean calculations were based on different geographical areas and significantly different time periods, the December and January values were similar. This suggested that monthly mean heat loss from the river between Lake Ontario and Montreal was relatively uniform in space and time during the months of December and January. The lower values for the 1965–1970 data reflect, to some extent, the modifying influence of Lake Ontario on the thermal regime of the river nearest the lake, i.e. the international

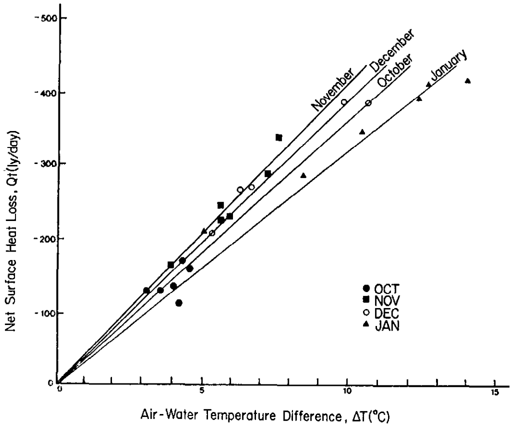

Fig. 2. Variation of net surface heat loss with air–water temperature difference for the months October–January on the St. Lawrence River.

section, the effects of which were particularly evident during the first half of the cooling season and, to a lesser extent, during the second half. Secular climatic changes and the disparity in the lengths of the two time periods over which the calculations were made may also have contributed to the differences between the two sets of values.

Wardlaw (1954) used 55 years of meteorological data from Montreal to calculate monthly heat losses from the St. Lawrence River (Table 2). A comparison with the long term means from this study showed relatively good agreement in December, although both the December and January values were less than those reported by Wardlaw. The differences might have also been related to the diminishing influence on climate of Lake Ontario at increasing distances from the lake.

Based on observations of the cooling rate of the St. Lawrence River between Lake Ontario and Montreal, Witherspoon and Poulin (1970), utilizing a measurement procedure, derived a constant heat exchange coefficient for the river which relates net surface heat loss to the difference between water and air temperatures ($Qt = K\Delta T$). To examine the applicability of the relationship for determining heat losses from the St. Lawrence River, I compared the net heat loss, Qt, with air–water temperature differences for the period 1965–1970 (Fig. 2) from which the slope of the curve represents the cooling coefficient, k. With the possible exception of October, the monthly values exhibited good linear correlation. The major feature of Fig. 2 is the marked similarity between the value of k for November and December and, to a lesser extent, the January value. This suggests that reasonably accurate surface heat loss values for the St. Lawrence River would be obtainable with the above relationship, utilizing a constant cooling coefficient, for the period in question. Williams (1963) pointed out, however, that because neither solar radiation (Eq. 2) nor evaporation (Eq. 7) is a function of air–water temperature difference, the value of k should vary in proportion to the ratio of the two terms. The variability of k for the St. Lawrence River is indicated by the wide range of values presented by the Joint Board of Engineers (1926) and the difference between their average value and that determined by Witherspoon and Poulin (1970). Using the data presented in Table 2 and average air–water temperature differences for the same time periods, I computed average values of k; these values ranged from 42 to 53 ly day^{-1} °C^{-1} within a given month and from 33 to 53 ly day^{-1} °C^{-1} throughout the cooling period. It is interesting that the maximum monthly difference occurred during November, the month in which local meteorological conditions might be expected to be most variable. Although a constant cooling coefficient appears valid over brief times such as the 1965–1970 period, extrapolation to much longer periods appears tenuous. Random variations in the ratio of solar radiation to evaporation both temporally and spatially could, therefore, lead to large errors in the estimation of net surface heat loss. Except for October, loss of heat by evaporation generally exceeds heat gained by solar radiation. Thus there would be a tendency to underestimate heat losses.

Estimating water temperatures—The temperature change of a completely mixed water column of unit surface area responding only to heat exchange at the surface can be represented as

$$T_1 = T_0 - (1/\rho Cph)Q_t t, \qquad (10)$$

where T_0 = initial water temperature; ρ = water density; Cp = specific heat of water; h = water depth; Qt = surface heat loss rate; T_1 = water temperature at time t.

Equation 10 has further application to water flowing in a confined channel with a uniform surface heat flux. In such a case, temperature T_1, at the downriver end of any channel section, can be calculated if the temperature, T_0, at the upriver end of the section, the average surface heat flux, Qt, along the section, and the travel time of water, t, through the section are known. Ince and Ashe (1964) and Poulin et al. (1971) used similar approaches in their studies of temperature variation along the St. Lawrence River.

It must be assumed that convection and turbulent processes result in isothermal water conditions during the cooling period and that water temperatures can be adequately represented by a series of point measurements. Unpublished bathythermograph profiles of the St. Lawrence River taken by personnel from the Lake Survey Center in November 1972 indicated that vertical and lateral temperature variations did not exceed 0.1–0.2°C. Temperature profiles immediately before and after ice formation also indicated that the water was nearly isothermal (R. O. Ramseier personal communication). A further assumption is that river discharge is steady and uniform and can be accurately represented by the monthly mean flow value from which travel time is derived. This is generally true when Lake Ontario levels are near or below normal and relatively low, steady flows are maintained during the winter. During periods of high Lake Ontario levels, such as at present, the low flows necessary to facilitate ice formation and the higher flows for maintaining an acceptable lake level must be properly balanced. That frequently requires rapid decreases of flow of up to 15% of normal over relatively short periods.

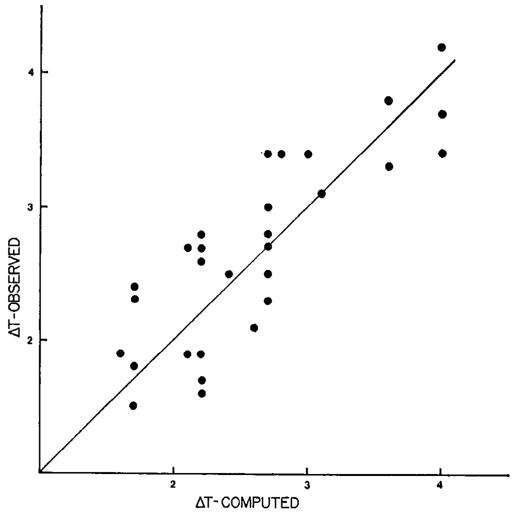

Fig. 3. Calculated versus observed temperature declines for the St. Lawrence River between Lake Ontario and Massena, New York.

When the period over which a rapid change occurs corresponds to the travel time based on mean flow, the assumption of steady state fails to describe the flow regime adequately. The use, at those times, of a constant travel time value for periods of up to a month could result in errors in estimating water temperatures.

Critical locations on the international section of the river in terms of navigation constraints are ice boom and lock locations. Since the purpose of the study was to develop a technique for estimating time of freeze-up at critical locations, the temperature decline of the river between Lake Ontario and Massena, near which the U.S. locks are located, was examined. Adequate water temperature data were available from the two locations to establish the general validity of Eq. 10 as it applied to the international section of the river.

Average cross-sectional area and mean water depth for the international section were determined from Lake Survey Center charts of the river. River flow data were used in conjunction with cross-sectional areas to compute water velocity through the section.

Calculations of temperature decline be-

Table 3. Values of constants in Eq. 11.

Constant	Oct	Nov	Dec
a	0.00935	0.01750	0.05180
b	0.0492	0.0920	0.2175
c	1.09886	1.09886	1.09886

tween Lake Ontario and Massena were made for periods corresponding to water travel time between those two points, using computed heat flux values and starting temperatures from Kingston, Ontario. The travel times ranged from about 8 to 12 days and surface heat flux rates ranged from about 160 to 430 ly day^{-1}. A plot of the observed versus predicted temperature declines is shown as Fig. 3. On an average, the equation predicted water temperature decline between Lake Ontario and Massena within about 0.4°C. The average error was less than 15% of the average temperature drop during a corresponding period of time, which is consistent with the estimated error in computing surface heat flux. The mean difference between observed and predicted values was 0.1°C with a standard deviation of 0.4°C. Values nearest the perfect fit line were obtained from those data that bracketed the middle of a month and may be explained by the manner in which the heat loss values were calculated, i.e. on a monthly basis. The Kingston starting temperatures were approximated only to ±0.3°C. That together with river flow variations could account for much of the deviation between observed and calculated temperature values. Equation 10 thus provided sufficiently accurate estimates of water temperature at Massena for use in estimating the time of ice formation.

Freeze-up prediction technique

The temperature decline equation discussed above was used as the basis for development of a long-range freeze-up prediction technique for the international section of the river. Freeze-up is defined as the development of continuous ice cover across the entire channel width. The technique permits an estimate of the time of ice formation at Massena, New York, beginning as early as 1 October and for any subsequent time before ice formation. Input to the technique includes initial temperature and surface heat flux, both of which are based on observed water temperature at Kingston, Ontario, on the prediction start date and water travel time which can be estimated from published river regulation plans. Since freeze-up at Massena normally occurs between 15 December and 15 January, initial temperature values and surface heat flux rates are required only during that time period to solve the prediction problem.

Table 4. Differences between computed and observed temperatures at Kingston, Ontario, on 15 December.

Starting date	Temp difference (°C) 1 Oct	1 Nov	1 Dec
1965	-0.1	+0.2	-0.2
1966	-1.7	-1.7	-2.0
1967	+0.5	+0.5	+0.3
1968	+0.2	+0.3	+0.3
1969	-0.2	0.0	-0.2
1970	-0.3	-0.3	-0.2
SD	0.7	0.7	0.7

The St. Lawrence River drains Lake Ontario, which has a relatively small surface area to volume ratio, comparable to that of the much larger Lakes Superior, Michigan, and Huron. A lake with a small area to volume ratio might be expected to be relatively unresponsive to short term meteorological extremes and to gain or lose heat at a relatively uniform and quasi-predictable rate from year to year. Examination of Kingston water temperature data for the period 1965–1971 indicated that decline of water temperature during the cooling period was relatively uniform and could be estimated from the equation

$$\Delta T = (aT_0 - b)t^c, \tag{11}$$

where ΔT = temperature change; T_0 = Kingston water temperature on the starting date; t = time in days from the starting date; $a,b,c,$ = constants.

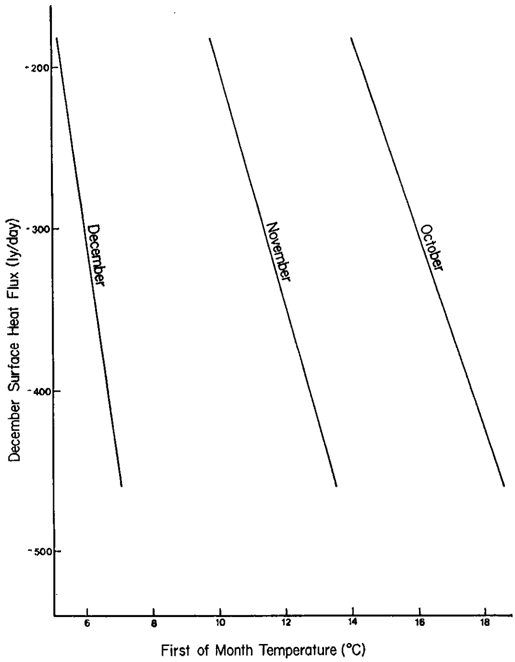

Fig. 4. Surface heat flux as a function of first of the month water temperatures at Kingston, Ontario.

Equation 11 is readily recognized as logarithmic with a variable intercept. The constants (Table 3) were evaluated by regression and found to be characteristic for a particular forecast start date.

The results obtained from Eq. 11 are summarized in Table 4, which indicates good correspondence between computed and observed values, over three different periods for 5 of the 6 years. The same order of accuracy was attained for the three prediction start dates. The error in the 1966 data may have resulted partially from lower than normal precipitation and above normal air temperatures during autumn 1966.

Evaluation of the various air–water energy exchange processes requires a knowledge of a number of meteorological variables. Because of current limitations in long-range weather predictions, those variables are not now available sufficiently far ahead to make use of them for predicting surface heat loss rates 2 to 3 months in advance. Recourse must therefore be made to an indirect method.

Table 5. Dates of observed and predicted freeze-up on the St. Lawrence River at Massena, New York.

	Observed freeze-up	Estimated date for prediction starting		
		1 Oct	1 Nov	1 Dec
1965–1966	9 Jan*	1 Jan	8 Jan	3 Jan
1966–1967	26 Dec*	3 Jan	1 Jan	24 Dec
1967–1968	2 Jan*	3 Jan	3 Jan	6 Jan
1968–1969	25 Dec†	25 Dec	26 Dec	28 Dec
1969–1970	28 Dec†	28 Dec	2 Jan	30 Dec
1970–1971	24 Dec†	26 Dec	25 Dec	26 Dec
Mean	29 Dec	30 Dec	1 Jan	30 Dec
SD (days)	5.7	4.7	3.3 5.3*	3.5 5.3*
1972–1973	30 Dec‡	29 Dec	3 Jan	31 Dec
1973–1974	30 Dec‡	29 Dec	30 Dec	29 Dec

*From Poulin et al. (1971).
†From PASNY records.
‡From Saint Lawrence Seaway Development Corporation records.

The relationship between December surface heat flux and first of the month temperature at Kingston for the months of October, November, and December was examined, with the results shown in Fig. 4. The linear correlations were similar in each case with correlation coefficients of 0.85, 0.86, and 0.88 for the October, November, and December data respectively. The curves thus provided December heat flux values based only on Kingston water temperatures on the prediction start date. The similarity of December and January average heat flux values (Table 2) permitted the use of December values during both months. Projected river flow rates can be obtained from the current river regulation plan.

With inputs to Eq. 10 as determined above, predictions of times of freeze-up at Massena, New York, were made for the years 1965–1970. Massena temperature data and information from Poulin et al. (1971) were used to infer freeze-up at a water temperature of about 0.3°C. In 4 of the 6 years examined (Table 5), observed freeze-up occurred within 2 days of the date predicted on 1 October and 1 November. Somewhat more variation was noted for the 1 December prediction. Both the 1 November and 1 December predictions showed some statistical improvement over the 1 October estimate. It should be noted that the arguments used in the technique were derived from the 1965–1970 data and relatively good correspondence between actual and forecast freeze-up dates for those years might therefore be expected. To test the method further, a freeze-up hindcast was made for 1972, and in 1973 experimental forecasts were provided to all interested parties. The 1 October predictions agreed with the observations within 1 day in both years (Table 5).

The Canadian Department of Energy Mines and Resources (DEMR) previously had developed a technique whereby freeze-up dates at various locations along the St. Lawrence River can be inferred from probability forecasts of water surface temperatures (Poulin et al. 1971). The DEMR method differs from the method described herein in several respects. Included in those differences are the manner in which heat losses are calculated and forecast starting temperatures are obtained, both of which might be expected to influence ultimate forecast results. The nature and magnitude of that influence is suggested by the differences in the standard deviation values (Table 5). The DEMR method apparently cannot be used for forecasting before 1 November and this is perhaps the major and most significant difference between the two.

A major limitation of the prediction technique is its failure to account for certain meteorological phenomena during the late stages of cooling but before freeze-up. The effect of snow falling on a water surface is perhaps the prime example. A 15-cm snowfall, together with an air–water temperature difference of 9°C, neither uncommon along the St. Lawrence River in December or January, can lead to a short term precipitation heat loss which is about 35% of the total average daily loss during those months. In contrast, an average daily snowfall yields a value which is not more than 5% of the storage term. Since snow could provide centers of nucleation on which ice crystals could grow, a heavy snowfall might have a profound influence on surface heat loss and the inception of ice growth. The effect could be responsible for some of the discrepancy between observation and prediction. Limitations notwithstanding, the results indicate that the present technique permits a sufficiently accurate estimate of the time of freeze-up at Massena on and after 1 October to be of value for the long-range scheduling of shipping and other navigation and power related activities.

Conclusions

Energy exchange at the air–water interface is one of the more important factors controlling the temperature of a water body. The rate of energy exchange is governed by the physical processes of radiation, evaporation, conduction, convection, and precipitation. Empirical formulae, selected on the basis of available meteorological data and the physical setting of the water body were used to quantify the various processes.

In northern latitudes, the net flux of heat is from water to air during the months of October through January, and water temperature declines to the freezing point, at which time an ice cover develops. For the St. Lawrence River between Lake Ontario and Montreal, calculations showed heat losses were relatively uniform, both temporally and spatially, during the latter half of the cooling period, but varied directly with distance from Lake Ontario during the months of October and November. Except for the month of November, the calculated heat losses were in good agreement with the values determined by assuming a linear relationship between heat loss and air–water temperature difference. The difference in the November values was related to the increase in the ratio of evaporation to solar radiation with increasing air–water temperature differential.

With a uniform heat flux, isothermal water conditions, and a steady flow, the temperature decline of a flowing river can be approximated by a simple equation. When applied to the international section of the St. Lawrence River and over a range of flow conditions and surface heat flux rates, the equation estimated water temperatures to within about 0.5°C. The error was consistent with the purported accuracy of the heat loss calculations. Short period variations in flow and imprecise observations of temperature could also account for part of the discrepancy between observed and estimated temperatures.

The equation for decline of water temperatures can be used as early as 1 October to estimate the time of ice formation on the river, and at any later time before freeze-up. The method requires knowledge of only three variables: the temperature of Lake Ontario for the day on which the prediction is made, the surface heat flux rates during the latter half of the cooling period, and the projected river flow rates. The latter two variables must be evaluated indirectly. Test predictions for 1 October yielded a discrepancy between predicted and observed dates of freeze-up of not more than 2 days in 5 of 7 years tested. Predictions for dates after 1 October were slightly improved. The method does not account for sudden heat losses and the consequent temperature changes resulting from meteorological events such as a heavy snowfall during the latter stages of cooling.

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