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GREAT LAKES ICE THICKNESS PREDICTION

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Abstract. Weekly ice thickness data, collected from 24 bay, harbor, and river sites on the Great Lakes, were correlated with freezing degree-day accumulations to develop regression equations between ice thickness and freezing degree-days. The data base at ice measurement sites was 3 to 8 winters in length. Ths standard error of estimate varied for individual regression equations and averaged between 7 and 8 cm for five forms of regression equations. Because the regression equations are empirical, the range of input data used to predict ice thickness should be limited to the range of values used in the derivation.

INTRODUCTION

As part of a study of the characteristics of the Great Lakes ice cover, data on ice thickness in bays, harbors, and river sites on the perimeter of the lakes have been collected in accordance with written instructions[1], using standard report forms. When possible, sites were chosen to be representative of undisturbed natural ice growth, but they were also located in areas critical to navigation. Site selection was limited by the availability of observers living in the vicinity of each site. Using an ice auger and measurement rule, observers measured ice thickness weekly, starting from the time the ice was first considered safe to walk on and continuing until March in most locations.

The ice thickness data collected serves two functions: 1) it provides a climatology of ice thickness in various Great Lakes bays and harbors and 2) it provides the data base needed to develop forecast techniques of ice thickness. The latter function is considered in this paper. Two possible approaches to the solution of this forecast problem are: 1) heat budget analysis, as in studies made by Dutton and Bryson (1960), Scott and Ragotzkie (1961), and Bilello (1967), in which detailed observations of meteorological, hydrological, and ice parameters are used to solve the mass and energy equations necessary to predict ice formation and growth and 2) simple statistical analysis of available information, as illustrated by Williams' (1963) work on ice thickness prediction based on ice

[1] Provided by the National Oceanic and Atmospheric Administration, Lake Survey Center, Detroit, Michigan.

thickness data alone and Richards' (1964) work based on correlation of air temperature and percent ice cover to develop regression equations for the prediction of percent ice cover on the Great Lakes. The purpose of this paper is to present the results of a study to develop an elementary statistical technique for ice thickness prediction. The technique is based on regression analysis of ice thicknesses and freezing degree day (FDD) accumulations for individual ice measurement sites. Four forms of regression equations were derived. The utility of the resultant equations was evaluated in terms of standard error of estimate (SE) and empirical constraints. In addition, a fifth equation using both FDD's and thawing degree-days (TDD's) was also derived. As in Richards (1964), TDD's were considered to be an index of antecedent heat storage in the water.

DEVELOPMENT OF REGRESSION EQUATIONS

Parameters

As indicated in a study by Marshall (1965), ice sheets can be composed of a complex of lake ice, snow ice, slush, water, and snow layers. In this study only that part of the ice sheet composed of lake ice and snow ice directly attached to the lake ice layer was considered in what is termed ice thickness: additional slush, snow ice, water, and snow layers were excluded from consideration. Ice thicknesses up to the maximum reported thickness each winter at each site were used in the regression equations. From data bases of 3 to 8 winters in length, ice thickness data collected each winter at 24 sites were correlated with accumulated FDD's (ΣFDD's) and maximum accumulated TDD's (ΣTDD's). The ΣFDD's were calculated from the expression $\Sigma Fdd's = \Sigma(32 - \bar{T}_{24})$, where $\bar{T}$ is the mean daily air temperature, and ΣTDD's were calculated from the expression $\Sigma TDD's = \Sigma(\bar{T}_{24} - 32)$. Mean daily air temperature was calculated from daily maximum and minimum temperatures. Depending on site location, running ΣFDD's were started either the first day in November or the first day in December when $\bar{T}_{24} \leq 32^{o}F$ ($0^{o}C$). Sites in the northern Great Lakes (Duluth, Minn.; Alpena, Mich.; Sault Ste. Marie, Mich; Escanaba, Mich.; Houghton, Mich; Marquette, Mich.; and Green Bay, Wis.) normally have lower temperatures earlier in the winter season and so ΣFDD's at these locations are started in November although sites on the southern part of the Great Lakes (Muskegon, Mich.; Detroit, Mich.; Toledo, Ohio; Erie, Pa.; Rochester, N.Y.; and Oswego, N.Y.) normally do not have temperatures consistently below freezing until December and so ΣFDD's at these locations are started in December. Similarly, ΣTDD's are started the first day in March when $\bar{T}_{24} \leq 32^{o}F$ and continued through the end of December. The locations of ice and temperature recording sites used in this study are shown in Fig. 1.

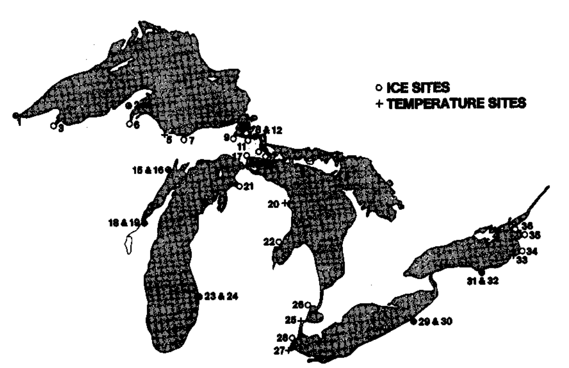

FIG. 1. Ice and temperature recording site location map.

TABLE 1. Temperature and associated ice-recording sites used in regression analysis. Maximum and minimum ΣFDD and ΣTDD values to be used in regression equations and mean value of ΣFDD's and standard deviation (SD) from mean for maximum ice thickness are also given below.

		ΣFDD				ΣTDD	
		Operating range		Maximum ice			
		Max	Min	Mean	SD	Max	Min
1.	Duluth Minn.	2610	190	2069	124	4758	3976
2.	Houghton, Mich.					4650	4211
3.	Ashland Harbor	2892	139	1864	705		
4.	Keweenaw Waterway	2206	181	1758	288		
5.	Marquette, Mich.					5020	4043
6.	Keweenaw Bay	2197	682	1755	312		
7.	Munising Bay	2162	405	1897	239		
8.	Sault Ste. Marie, Mich.					4801	4258
9.	Tahquamenon Bay	2065	282	1812	347		
10.	Gros Cap Light	2035	615	1453	529		
11.	Point Iroquois	1815	615	1502	132		
12.	Mosquito Bay	1951	590	1550	526		
13.	Lake Munuscong	2053	145	1670	271		
14.	Raber Bay	2075	10	1720	372		
15.	Escanaba, Mich.					5234	4502
16.	Escanaba	1543	261	1110	340		
17.	Straits-St. Martin Bay	1385	60	1178	200		
18.	Green Bay, Wis.					6043	5339
19.	Green Bay	1792	28	1271	352		
20.	Alpena, Mich.					5438	4445
21.	Little Traverse Bay	1277	732	1200	69		
22.	Saginaw	1190	30	907	348		
23.	Muskegon, Mich.					6528	5892
24.	Muskegon Lake	755	9	497	148		
25.	Detroit, Mich.					7561	6466
26.	Anchor Bay	737	35	351	229		
27.	Toledo, Ohio					7043	6221
28.	Brest Bay	584	87	388	157		
29.	Erie, Pa.					6753	5461
30.	Eire, Pa.-Marine Lake	597	30	324	150		
31.	Rochester, NY					6748	6088
32.	Rochester-Irondequoit Bay	883	86	628	217		
33.	Oswego, NY					6548	5616
34.	North Pond	681	14	441	141		
35.	Henderson Harbor	830	33	587	210		
36.	Wilson Bay	834	158	674	143		

Temperature sites underlined

Table 1 lists the temperature station locations and, directly under each location, the ice measurement sites using temperatures at that location for development of regression equations for that ice measurement site. The ΣFDD's and ΣTDD's used in this analysis were abstracted from ΣFDD's and ΣTDD's calculated by the Great Lakes Environmental Research Laboratory for a number of locations on the perimeter of the Great Lakes. The ΣFDD location most representative of area temperatures in the proximity of each ice measurement site was determined by visual examination of isothermal contour maps as given by Kopec (1965). This was done so that only those temperature stations having the greatest probability of producing strong correlations with ice thickness records at each ice site would be used.

REGRESSION EQUATION CHARACTERISTICS

A number of regression curves were fit to the data with the objective of minimizing the standard error of estimate (SE). To achieve this objective, two approaches were investigated. The first was to develop regression equations for each winter of the data base at one site. The characteristics of ice growth vary from winter to winter so that, if the same curve is used, its coefficients must be parameterized to reflect changes in growth rates, which are a function of weather conditions and ice sheet characteristics. In studies on ice growth, Shaw (1965), Deriugin (1972), and Bydin (1972) have indicated that ice thickness, snow cover, snow ice, and slush layers all effect the growth of ice. As these parameters vary from year to year, so should the coefficients of the regression curve. This is clearly illustrated in Fig. 2, which shows linear regression equations of ΣFDD's for

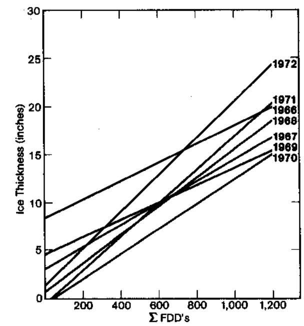

FIG. 2. Raber Bay regression equations for individual winters 1966-67 through 1972-73. Equation is of the form $Y = A_0 + A_1$ (ΣFDD's).

Raber Bay for individual winters. The average SE obtained by using these equations was 1.7 in. (4.3 cm). Attempts to parameterize the coefficients of the regression equation by using TDD's as an index of antecedent summer heating and ΣFDD's during the early part of the winter season did not prove successful. The SE obtained from the parameterized regression coefficients was 4.7 in. (11.9 cm). As a result, a second approach taken in this study was to develop a regression equation based on all years of record at a given site for the development of a single regression equation. With this approach, the coefficients of the regression equation would be representative of the climatic norm of ice sheet characteristics for the period of its data base. With the objective of minimizing the SE, five types or forms of regression equations were developed at each of the 24 ice sites. These equations and the average SE values for the 24 ice sites are summarized in Table 2. From this Table it is seen that Y_2 and Y_3, that is, the quadratic form of the equation

TABLE 2. Summary of regression equation forms, coefficients, and standard errors of estimate ($X_1 = \Sigma FDD$, $X_2 = \Sigma TDD_{max}$, A's = coefficients, Y = ice thickness in inches.)

	$Y_1 = A_{11} + A_{12}X_1$		$Y_2 = A_{21} + A_{22}X_1 + A_{23}X_1^2$		
	A_{11}	A_{12}	A_{21}	A_{22}	A_{23}
Duluth Harbor	3.4028240	1.2017006 10^{-2}	-1.7208171 10^{-1}	1.8980196 10^{-2}	-2.6859027 10^{-6}
Ashland Harbor	6.8240477	7.6296082 10^{-3}	-3.4831938	2.5429245 10^{-2}	-5.8614496 10^{-6}
Keweenaw Waterway	2.9488674	9.0272436 10^{-3}	-1.1036787	1.7928885 10^{-2}	-3.8016715 10^{-6}
Keweenaw Bay	-2.2021258 10^{-1}	9.9218409 10^{-3}	-4.6730061	1.6748374 10^{-2}	-2.4023639 10^{-6}
Munising Bay	-4.0849520	1.2102158 10^{-2}	-5.5369687	1.4592228 10^{-2}	-9.4325378 10^{-7}
Tahquamenon Bay	3.3315191 10^{-1}	1.2004298 10^{-2}	8.6637497 10^{-1}	1.0824584 10^{-2}	5.1530105 10^{-7}
Gros Cap Light	-5.6725706	1.3743082 10^{-2}	-1.3057198 10^{1}	2.5345165 10^{-2}	-4.2345691 10^{-6}
Point Iroquois	-5.1283188	1.4371503 10^{-2}	-8.3006954	2.0064171 10^{-2}	-2.3819175 10^{-6}
Mosquito Bay	1.8914255	6.7687948 10^{-3}	3.6683617	3.6237706 10^{-3}	1.2527198 10^{-6}
Lake Munuscong	2.1219510 10^{-1}	1.5032481 10^{-2}	-3.4393291	2.3091810 10^{-2}	-3.6360098 10^{-6}
Raber Bay	3.5393900	1.1766615 10^{-2}	1.1969509	1.7650667 10^{-2}	-2.7120427 10^{-6}
Escanaba	1.4505962	2.0636584 10^{-2}	-2.1652718	3.0844921 10^{-2}	-6.1189085 10^{-6}
Straits-St. Martin Bay	-6.9819737 10^{-1}	1.8008394 10^{-2}	-2.2141027 10^{-1}	1.6462231 10^{-2}	1.0104868 10^{-6}
Green Bay	1.2064214 10^{1}	3.1906188 10^{-3}	8.8698702	1.1776232 10^{-2}	-4.6541074 10^{-6}
Little Traverse Bay	2.4122000 10^{-2}	1.5372550 10^{-2}	-1.4641357 10^{1}	4.5855165 10^{-2}	-1.5325204 10^{-5}
Saginaw Bay	2.0463734	1.6139926 10^{-2}	-2.6199436	3.6112973 10^{-2}	-1.6421488 10^{-5}
Muskegon Lake	4.1214804	1.4888599 10^{-2}	2.7078401	2.5322751 10^{-2}	-1.3640471 10^{-5}
Anchor Bay	5.3662081	1.7541704 10^{-2}	4.1741304	2.7366015 10^{-2}	-1.3297794 10^{-5}
Brest Bay	1.7058731	2.2838643 10^{-2}	1.4140168	2.6490654 10^{-2}	-6.9277239 10^{-6}
Erie, Pa.-Marine Lake	6.1852887	-1.1974013 10^{-4}	2.5271755	2.8095185 10^{-2}	-1.8544553 10^{-5}
Rochester-Irondequoi	2.9199783	1.3868954 10^{-2}	3.3975837	1.1340433 10^{-2}	2.6842336 10^{-6}
North Pond	5.1403924	2.5683588 10^{-2}	3.9811918	3.8547887 10^{-2}	-2.2876346 10^{-5}
Henderson Harbor	4.8643066	2.2303224 10^{-2}	2.6353613	3.8095173 10^{-2}	-1.8588689 10^{-5}
Wilson Bay	-1.1998849	3.1917809 10^{-2}	-1.2807941	3.2308890 10^{-2}	-3.9752740 10^{-7}

	$Y_3 = A_{31} + X_1A_{32} + X_2A_{33}$			$Y_4 = 10^{(A_{41} + A_{42}\text{Log } X_1)}$	
	A_{31}	A_{32}	A_{33}	A_{41}	A_{42}
Duluth Harbor	2.0248889 10^{1}	1.1837887 10^{-2}	-3.6981295 10^{-3}	-1.6127218	9.3017736 10^{-1}
Ashland Harbor	5.7920276 10^{1}	7.5703878 10^{-3}	-1.1291549 10^{-2}	-1.5314181	8.8488972 10^{-1}
Keweenaw Waterway	1.4407030 10^{1}	9.0268398 10^{-3}	-2.5535390 10^{-3}	-1.5967426	8.8933599 10^{-1}
Keweenaw Bay	1.1266769 10^{1}	1.0305761 10^{-2}	-2.7480362 10^{-3}	-2.4246826	1.1274021
Munising Bay	3.0782579 10^{1}	1.3075438 10^{-2}	-8.2185176 10^{-3}	-4.6130958	1.8088461
Tahquamenon Bay	-4.7599380	1.1961102 10^{-2}	1.1481742 10^{-3}	-2.3851107	1.1513470
Gros Cap Light	1.6752886 10^{1}	1.4279045 10^{-2}	-5.1509675 10^{-3}	-5.4615045	2.0829005
Point Iroquois	4.0751622 10^{1}	1.8011248 10^{-2}	-1.1135898 10^{-2}	-5.3430591	2.0735706
Mosquito Bay	2.2386462 10^{1}	7.6131305 10^{-3}	-4.7254385 10^{-3}	-1.9430761	9.4875681 10^{-1}
Lake Munuscong	1.6773200	1.5048871 10^{-2}	-3.3258781 10^{-4}	-2.1665243	1.1115717
Raber Bay	1.9748460 10^{1}	1.1961527 10^{-2}	-3.6695495 10^{-3}	-3.6968023 10^{-1}	5.1965843 10^{-1}
Escanaba	-3.3276020	2.0551708 10^{-2}	9.9278739 10^{-4}	-1.9693885	1.1094463
Straits-St. Martin Bay	-2.9450308 10^{1}	1.9831541 10^{-2}	5.6140832 10^{-3}	-1.2627167	8.1698087 10^{-1}
Green Bay	-3.4022070 10^{1}	5.6329344 10^{-3}	7.7250200 10^{-3}	3.5531318 10^{-1}	2.7604413 10^{-1}
Little Traverse Bay	2.1959434 10^{1}	1.2046729 10^{-2}	-3.5863044 10^{-3}	-2.4985209	1.2234750
Saginaw Bay	-6.2259393	1.5679414 10^{-2}	1.6960373 10^{-3}	1.1664068	7.9933353 10^{-1}
Muskegon Lake	-1.9473033 10^{1}	1.5133620 10^{-2}	3.7304656 10^{-3}	-4.7290713 10^{-1}	5.7299951 10^{-1}
Anchor Bay	1.6706935 10^{1}	1.6884497 10^{-2}	-1.6027222 10^{-3}	3.4218845	5.5523789 10^{-1}
Brest Bay	1.9188125	2.3250936 10^{-2}	-5.2639270 10^{-5}	-1.3889037	9.2870471 10^{-1}
Erie, Pa.-Marine Lake	2.5924859	2.1774076 10^{-2}	-2.5778863 10^{-5}	-1.0851179	8.3308000 10^{-1}
Rochester-Irondequoi	1.4807235 10^{1}	1.4166753 10^{-2}	-1.8713945 10^{-3}	-6.4587617 10^{-1}	6.0420144 10^{-1}
North Pond	6.7767504	2.5853105 10^{-2}	-2.7223056 10^{-4}	-2.6097238 10^{-2}	4.5996548 10^{-1}
Henderson Harbor	-3.5826466	2.1664343 10^{-2}	1.4090431 10^{-3}	-3.7859353 10^{-1}	5.8680254 10^{-1}
Wilson Bay	1.7865682 10^{1}	3.4844063 10^{-2}	-3.3355376 10^{-3}	-2.6423205	1.4054223

Equation form and Coefficients	$Y_5 = A_{51} + A_{52} \text{Log } X_1$		Standard error* of estimate									
	A_{51}	A_{52}	Y_1		Y_2		Y_3		Y_4		Y_5	
			in	cm	in	cm	in	cm	in	cm	in	cm
Duluth Harbor	$-6.7447299\ 10^1$	$2.8318618\ 10^1$	2.76	(7.01)	2.54	(6.45)	2.54	(6.45)	2.93	(7.43)	2.71	(6.87)
Ashland Harbor	$-4.7829256\ 10^1$	$2.1275120\ 10^1$	4.88	(12.39)	3.45	(8.76)	4.22	(10.73)	3.45	(8.76)	3.96	(10.05)
Keweenaw Waterway	$-4.3999217\ 10^1$	$1.9148110\ 10^1$	2.66	(6.76)	2.41	(6.11)	2.60	(6.60)	2.47	(6.28)	2.54	(6.44)
Keweenaw Bay	$-8.1379326\ 10^1$	$3.0403152\ 10^1$	3.00	(7.62)	3.00	(7.62)	2.91	(7.40)	2.96	(7.51)	3.00	(7.62)
Munising Bay	$-8.6790934\ 10^1$	$3.1901840\ 10^1$	3.50	(8.88)	3.50	(8.88)	2.68	(6.80)	3.37	(8.57)	3.61	(9.18)
Tahquamenon Bay	$-5.9628768\ 10^1$	$2.4504737\ 10^1$	3.35	(8.50)	3.35	(8.50)	3.35	(8.50)	3.57	(9.07)	3.71	(9.42)
Gros Cap Light	$-1.1405596\ 10^2$	$4.0780670\ 10^1$	3.05	(7.76)	3.00	(7.61)	2.82	(7.16)	3.11	(7.90)	3.00	(7.61)
Point Iroquois	$-1.0317160\ 10^2$	$3.7636724\ 10^1$	2.73	(6.93)	2.73	(6.93)	1.39	(3.53)	2.83	(7.18)	2.63	(6.67)
Mosquito Bay	$-4.4584400\ 10^1$	$1.7877812\ 10^1$	3.16	(8.02)	3.16	(8.02)	3.04	(7.73)	3.10	(7.88)	3.19	(8.09)
Lake Munuscong	$-6.9977175\ 10^1$	$2.9055736\ 10^1$	2.52	(6.41)	2.41	(6.11)	2.52	(6.41)	2.15	(5.47)	3.04	(7.73)
Raber Bay	$-2.5846230\ 10^1$	$1.4369567\ 10^1$	3.22	(8.17)	3.13	(7.95)	3.13	(7.95)	4.11	(10.44)	4.78	(12.15)
Escanaba	$-7.9622005\ 10^1$	$3.4057352\ 10^1$	3.09	(7.84)	3.00	(7.62)	3.09	(7.84)	3.09	(7.84)	3.00	(7.62)
Straits-St. Martin Bay	$-4.0618158\ 10^1$	$1.9030352\ 10^1$	3.09	(7.84)	2.78	(7.06)	2.33	(5.93)	3.93	(9.98)	4.10	(10.41)
Green Bay	-1.6118956	5.7456274	4.24	(10.76)	4.12	(10.46)	3.72	(9.44)	3.66	(9.30)	4.02	(10.21)
Little Traverse Bay	$-8.8397354\ 10^1$	$3.4681492\ 10^1$	3.66	(9.30)	3.63	(9.23)	3.57	(9.07)	3.69	(9.38)	3.66	(9.30)
Saginaw Bay	$-2.8073674\ 10^1$	$1.4765895\ 10^1$	3.67	(9.33)	3.36	(8.54)	3.62	(9.20)	3.08	(7.82)	3.86	(9.82)
Muskegon Lake	-8.8297839	7.6236953	2.27	(5.78)	2.21	(5.61)	2.17	(5.52)	1.39	(3.52)	2.21	(5.61)
Anchor Bay	$-1.4351305\ 10^1$	$1.0507311\ 10^1$	2.74	(6.96)	2.70	(6.87)	2.70	(6.87)	3.01	(7.66)	2.88	(7.32)
Brest Bay	$-2.4347346\ 10^1$	$1.3594342\ 10^1$	2.12	(5.38)	2.12	(5.38)	2.12	(5.38)	3.00	(7.62)	2.50	(6.35)
Erie, Pa.-Marine Lake	$-1.3927830\ 10^1$	9.5017074	2.28	(5.79)	2.18	(5.54)	2.25	(5.71)	2.25	(5.71)	1.89	(4.80)
Rochester-Irondequoit	$-1.8458497\ 10^1$	$1.0643651\ 10^1$	2.01	(5.10)	2.01	(5.10)	1.97	(5.02)	2.46	(6.24)	2.33	(5.91)
North Pond	$-1.1395185\ 10^1$	$1.0063628\ 10^1$	2.31	(5.86)	2.21	(5.61)	2.31	(5.86)	2.54	(6.45)	2.62	(6.66)
Henderson Harbor	$-2.2261724\ 10^1$	$1.4443080\ 10^1$	3.02	(7.66)	2.82	(7.17)	2.95	(7.50)	3.20	(8.13)	3.20	(8.13)
Wilson Bay	$-6.5771524\ 10^1$	$3.0284198\ 10^1$	2.23	(5.67)	2.23	(5.67)	2.02	(5.13)	3.30	(6.39)	2.52	(6.39)
Overall average standard error for each equation form			2.98	7.57	2.84	7.20	2.75	6.99	3.03	7.69	3.12	7.93

using ΣFDD's and maximum ΣTDD's, produced, on the average, the lowest SE values. As Y_2 would require less work (it would not be necessary to compile ΣTDD's), it is generally preferred over Y_3. The SE values and regression equation coefficients for each type of equation are also given in Table 2. To minimize the SE value at individual ice sites, one can use the regression equation form that gives the lowest SE.

EVALUATION OF REGRESSION EQUATIONS

Attempts to normalize the regression equations by making the Y intercept zero resulted in significant increases in the SE value of Equation Y_1. The primary reason for this is the fact that, as noted earlier, ice growth is partly a function of ice sheet characteristics which vary from winter to winter and even during a given winter. As a result, ice growth and ice thickness are variable for a given ΣFDD or ΣTDD value. Therefore, the equation that gives the best fit of the data will not necessarily be one with a Y intercept equal to zero even though it would be more satisfying from a causative view. This points out the fact that the range of ice thickness and ΣFDD and ΣTDD values over which each regression equation is considered a valid predictor is limited to the values from which the equation was derived, as these equations are empirical and no cause and effect relationship is claimed for them. (Maximum and minimum ΣFDD and ΣTDD values for each ice location are given in Table 1 and should be used as guidelines when making forecasts.)

The regression equations predict ice thicknesses up to maximum ice thickness; however, there is no method for identifying occurrence of maximum ice thickness. It cannot be assumed that maximum ice thickness is attained at maximum ΣFDD's because there is evidence to the contrary. Sydor (1973), in a study of ice growth in Duluth Harbor, found that maximum ice thickness occurred before maximum ΣFDD's. As an estimate of when to expect maximum ice thickness, an average value of ΣFDD's at the time of maximum reported ice thickness was calculated for each ice measurement site from its data base. These ΣFDD values and their standard deviations are given in Table 1.

Ice thickness predicted by regression equations is representative of on-site conditions, but may not be representative of area conditions. Ice thickness measurements are point source data and may or may not be representative of thicknesses over an entire bay or harbor as parameters affecting ice thickness over a given area may vary considerably. The use of the regression equations is also limited by virtue of their dependence on accurate ΣFDD forecasts, which in turn are a function of accurate long-range temperature forecasts. At the present time, perhaps the best application of these equations is for short-range forecasts or for estimating current ice conditions during the winter. This can be done very easily as only a running ΣFDD and maximum ΣTDD value (for Y_3) at the appropriate location is needed. Then the ice thickness can be calculated or looked up from tabulations. In using this technique, it must be remembered that the same conventions used by the author in calculating ΣFDD's and ΣTDD's (noted earlier) must be used or erroneous ice thickness values will be generated.

SUMMARY AND CONCLUSIONS

A method to predict ice thickness from ΣFDD and ΣTDD regression equations was derived for 24 bay and harbor sites on the Great Lakes. On the average, SE values ranged between 7 to 8 cm for various regression equation types. The equations are limited by the lack of accurate long-range temperature forecasts, the range of ice thicknesses and ΣFDD and ΣTDD values of the data base from which they were derived, and the fact that they were derived for the ice growth period only and thus can no longer be used with any validity once maximum ice thickness is attained.

Presently the primary use of the equations is for predicting short-range or current ice thickness. Clearly, if potential users of these equations need significantly greater accuracy, heat budget analysis of on-site conditions should be considered. However, the question of area representativeness of the predictions should be addressed as this is potentially a great source of error and a limiting factor in application of the present and future forecast techniques.

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