Comparison of a Two-Dimensional Wave Prediction Model with Synoptic Measurements in Lake Michigan[1]

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ABSTRACT

We compare results from a simple parametric, dynamical, deep-water wave prediction model with two sets of measured wave height maps of Lake Michigan. The measurements were made with an airborne laser altimeter under two distinctly different wind fields during November 1977. The results show that the model predicted almost all of the synoptic features. Both the magnitude and the general pattern of the predicted wave-height contours compared well with the measurements. The model also predicts the direction of wave propagation in conjunction with the wave height map, which is useful for practical ship routing and can be significantly different from the prevailing wind direction.

1. Introduction

Analogous to a synoptic weather map, a synoptic wave height map is a useful tool for climatological studies, as well as for ship seakeeping operations. Almost all currently available numerical wave prediction models provide two-dimensional wave characteristics, from which a synoptic wave height map can be produced. However, because synoptic measurements are rare, most models are verified with only single point measurements, if at all. The accuracy of the synoptic features of the wave predictions remains unexplored. To remedy this situation, one needs a practical model, along with reliable measurements and reasonable comparisons between model and measurements. Here we present a brief study that includes all three of these elements. The results are encouraging.

2. The model

The model we used in this study was developed by Donelan (1977) and revised by Schwab *et al.* (1984). The model differs from most other numerical wave prediction models in that the basic equation is the momentum balance equation rather than the energy balance equation. The monentum balance equation is

$$\frac{\partial \mathbf{M}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{M} = \tau_w, \tag{1}$$

where $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$, **M** and **v** are the total momentum vector and the corresponding group velocity vector, and τ_w is that part of the momentum input from the wind that produces net wave momentum growth. Assuming equipartition of potential and kinetic energy in the wave field, the momentum vector can be expressed as

$$\mathbf{M} = \begin{pmatrix} M_x \\ M_y \end{pmatrix} = \rho_w g \int\int \frac{F(f,\theta)}{c(f)} \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} d\theta df, \tag{2}$$

where ρ_w is the water density, $c(f)$ the phase speed and $F(f, \theta)$ the two-dimensional frequency spectrum of wave energy as a function of frequency f and direction θ. Assuming further that the wave energy has a cosine-squared angular dependence about the mean angle independent of frequency and that there is no energy for $|\theta - \theta_0| > \pi/2$, we use

$$F(f,\theta) = \frac{2}{\pi} E(f) \cos^2(\theta - \theta_0), \tag{3}$$

where $E(f)$ is the one-dimensional spectral energy density. Since we are concerned with total momentum, we define the mean-square surface elevation:

$$\sigma^2 = \int\int F(f,\theta) df d\theta = \frac{|\mathbf{M}| C_p}{\rho_w g}, \tag{4}$$

where $|\mathbf{M}| = (M_x^2 + M_y^2)^{1/2}$ and C_p is the deep-water phase speed of the peak of the spectrum, $C_p = g/(2\pi f_p)$. Applying deep-water linear theory, Schwab *et al.* (1984) have shown that

$$\begin{pmatrix} v_x M_x \\ v_x M_y,\ v_y M_x \\ v_y M_y \end{pmatrix} = \frac{|\mathbf{M}| C_p}{4} \begin{pmatrix} \cos^2\theta_0 + \frac{1}{2} \\ \cos\theta_0 \sin\theta_0 \\ \sin^2\theta_0 + \frac{1}{2} \end{pmatrix} \tag{5}$$

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Thus the left-hand side of (1) is parameterized. We formulate the right-hand side as

$$\tau_w = 0.028\rho_a D_f|\mathrm{U} - 0.83C_p|(\mathrm{U} - 0.83C_p), \quad (6)$$

where ρ_a is the air density, U the 10 m wind speed, D_f the form drag coefficient given by $D_f = [0.4/\ln(50/\sigma)]^2$ with σ in meters. The factor 0.028 is the empirical fraction of the wind stress that is retained by the waves.

To solve (1) we still need a relationship between wave momentum and wave height. We use the following empirical relation derived from JONSWAP relations (Hasselmann *et al.*, 1973) and linking σ^2 with peak energy frequency f_p and mean wind speed U:

$$\sigma^2 = 6.23 \times 10^{-6} \left(\frac{f_p U}{g}\right)^{-10/3} \frac{U^4}{g^2}. \quad (7)$$

The model is thus semi-empirical and parametric. A simple numerical integration scheme can then be applied to Eq. (1). Forward time differences are used to calculate the momentum components at the center of the elementary grid squares, and a combination of upwind and centered differences are used to evaluate the momentum advection terms at the edges of the grid squares. Model output at each grid point consists of significant wave height (defined by $H_{1/3} = 4\sigma$), peak-energy wave period and average wave direction. Schwab *et al.* (1984) have presented a detailed discussion of the assumptions and limitations used in the model. They tested the model against 2-month time series of wind, wave height, period and direction measured in Lake Erie during 1981. They showed that at two different points in the lake, the model predicted wave height and wave direction very well and wave period satisfactorily.

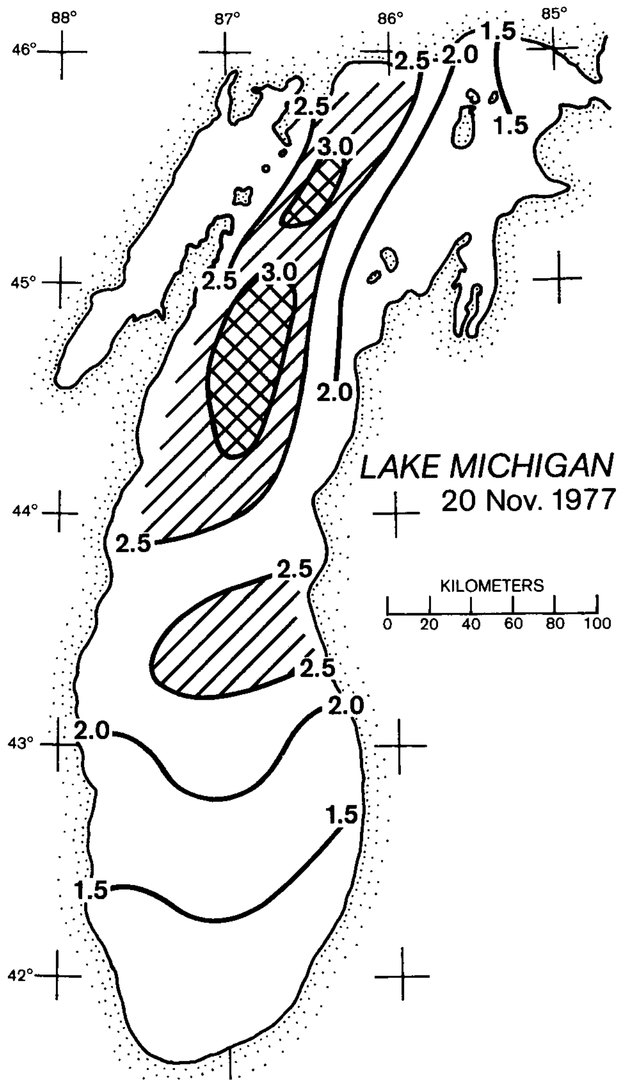

FIG. 1. Synoptic significant-wave-height contours for Lake Michigan during south wind, 20 November 1977 (from Liu and Ross, 1980).

3. The measurements

While there are numerous wave measurements at single-point locations reported in the literature, synoptic wave measurements are very rare. The synoptic wave maps shown in Figs. 1 and 2 are from Liu and Ross (1980). The maps, which show Lake Michigan wave heights under south and west winds, resulted from a joint program combining airborne laser profilometer and *in situ* Waverider buoy measurements in Lake Michigan. The measurements were made in November 1977 during passage of an intense cold front. The dominant wind directions over Lake Michigan before and after the frontal passage were from the south and west, respectively. The direction shift occurred in the early morning hours of 21 November. Measurements were made in the afternoons of 20 and 21 November from a NOAA C-130 aircraft flying at a speed of 106 m s^{-1} and an altitude of 150 m above the lake surface in a direction generally parallel to the wind direction. The airborne wave measurements, made continuously along the flight tracks from a Spectra Physics Geodolite laser altimeter, were generally consistent with those measured from the Waveriders. By contouring all the point measurements of significant wave heights made during the flight, synoptic wave height maps were obtained. The duration of each flight was about 5 h, so the wave height measurements are really only approximately synoptic. However, because wind speed and direction during the flights remained remarkably steady, the results are in fact quite realistic. Figures 1 and 2, showing the Lake Michigan wave height maps under southerly and westerly wind fields, respectively, are the subjects of our model application.

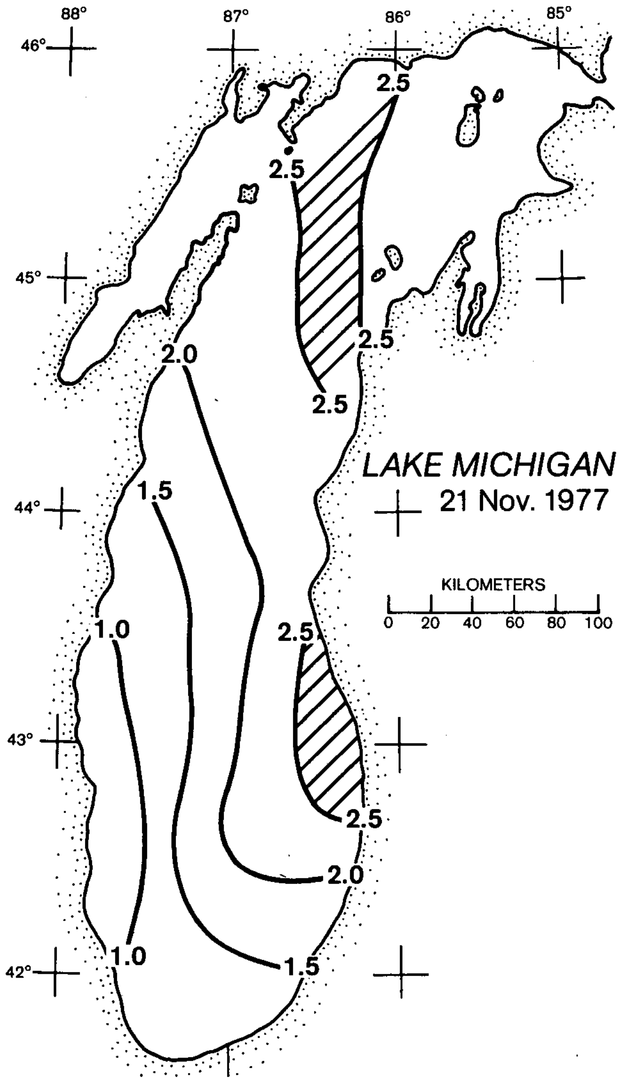

FIG. 2. As in Fig. 1 except for west wind, 21 November 1977 (from Liu and Ross, 1980).

4. The wind field

Although wind forecasts are generally an important part of a comprehensive wave prediction system, it is not our intent to pursue it in this study. However, in order to implement the wave prediction model given in Section 2 and obtain synoptic wave fields corresponding to Figs. 1 and 2, we need continuous wind field data as input. We estimated the wind field from observations of wind speed and direction and air temperature recorded every 2 hours by the nine U.S. Coast Guard stations around Lake Michigan (Fig. 3). Since the stations are located sufficiently close to the water, we did not correct the reported wind when the wind was blowing off the lake at that station. When the wind was blowing from land, we applied an over-land/over-lake correction to the wind speed and the wind direction. The corrections are

$$U_W = U_L\left(1.2 + \frac{1.85}{U_L}\right)\left[1 - \frac{\Delta T}{|\Delta T|}\left(\frac{|\Delta T|}{1920}\right)^{1/3}\right], \quad (8)$$

$$\Delta\theta = (12.5 - 1.5\Delta T) - (0.38 - 0.03\Delta T)U_W, \quad (9)$$

where U_W and U_L are over-lake and over-land wind speeds (m s^{-1}), respectively; ΔT is air–water temperature difference (°C), with water temperature estimated from local climatology and assumed constant throughout the lake; and $\Delta\theta$ is the clockwise angle between over-land and over-lake winds (deg). These formulas, developed by Schwab (1978) based on graphs given by Resio and Vincent (1977), have been successfully applied to modeling storm surge and current fluctuations in the Great Lakes (Schwab, 1978; 1983). With this scheme, we thus obtained a set of nine nearshore over-lake wind data every 2 hours. The two components of the wind vector were then assumed to be linear functions of the spatial coordinates, with the coefficients determined by least-squares fitting to the nine stations to obtain a continuous, two-dimensional wind field. Figure 4 shows a comparison of the wind speeds obtained with this scheme, designated "Model Wind Speed," with wind speeds measured at the wave observation points from Liu and Ross (1980). With a root-mean-square dif-

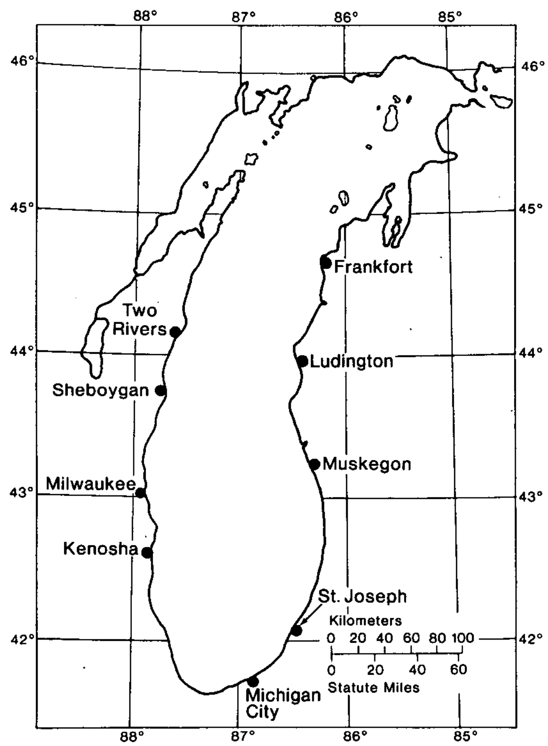

FIG. 3. Locations of Coast Guard stations that report meteorological data around Lake Michigan.

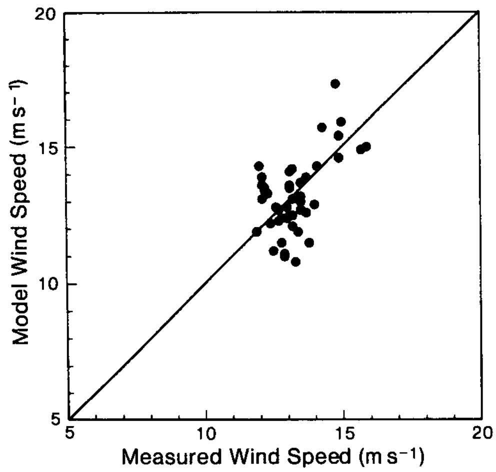

FIG. 4. Direct comparison of model and measured wind speeds in Lake Michigan during 20–21 November 1977.

ference of 1.2 m s^{-1} between the model and the measured data, this result is excellent.

5. Results

While the measurements were made in 5-hour flights, the model results were produced in hourly

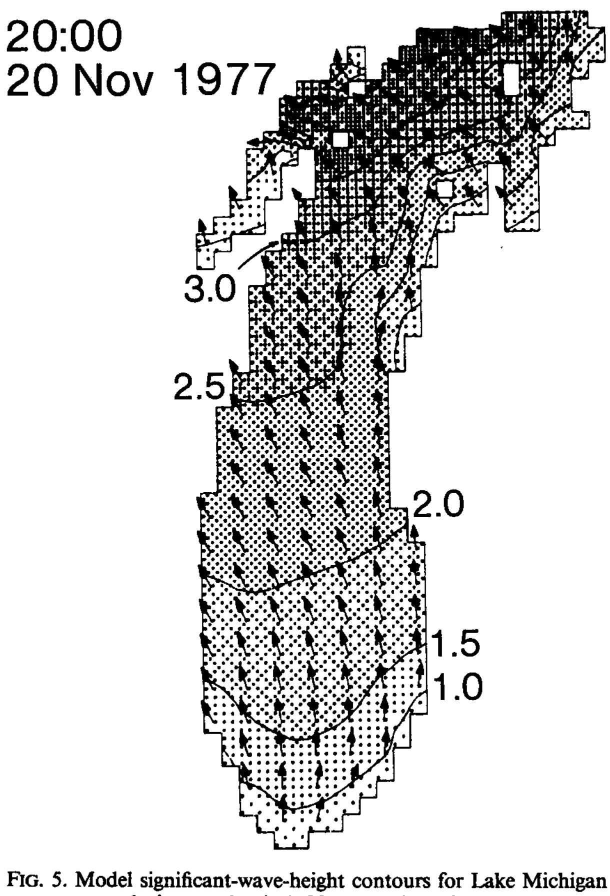

FIG. 5. Model significant-wave-height contours for Lake Michigan during south wind, 20 November 1977.

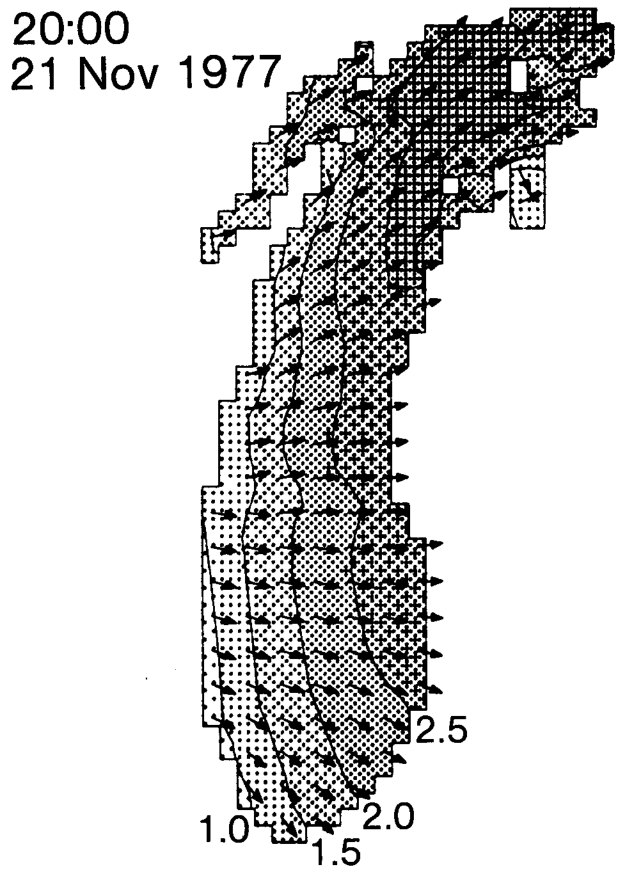

FIG. 6. As in Fig. 5 except for west wind, 21 November 1977.

maps. As the wind fields were steady during each flight, the series of hourly maps corresponding to the flights showed very little change from hour to hour. Figures 5 and 6 represent the typical model wave height maps corresponding to the measured results shown in Figs. 1 and 2, respectively. The agreement between them is excellent. They show that the simple parametric model based on the momentum balance concept combined with wind fields derived from Coast Guard measurements has predicted almost all the measured synoptic features. Under the southerly wind field on 20 November (Figs. 1 and 5) the equal-wave-height contours in the southern part of the lake tend to be perpendicular to the wind direction, parallel to each other, and increase northward with the indication of a ridge in the central part of the lake. This pattern is evident in maps of both modeled and measured data with similar magnitudes. In the northern part of the lake, the measured results show the highest waves (height ~ 3 m) in the center of the northwest part of the lake. The model results show that the highest waves are located along the western shore of the northwest part of the lake. However, the location of the 2.5 m height contour is almost identical in both measured and model maps. Under the westerly wind field on 21 November (Figs. 2 and 6), the wave height contours are again perpendicular to the wind direction, parallel to each other, and now increasing toward west northwest. The highest waves are 2.5 m along the east shore. The modeled and measured results substantially agree both in pattern and in magnitude. The model also provides the direction of wave propagation plotted concurrently on the model maps. This does not always follow the prevailing wind. To make quantitative comparisons, the predicted significant wave heights at various points in the lake are compared to the corresponding measured heights given by Liu and Ross (1980), Table 1, as shown in Fig. 7. Although there is some indication of underestimation for higher wave heights and overestimation of lower wave heights by the model, the results, with a root-mean-square difference of 0.3 m, are very encouraging.

It should be mentioned here that contrary to many model studies that start with model results and then seek measurements for verification, this study started with specific measurements (Figs. 1 and 2) and then attempted the model applications. The successful results certainly reflect the usefulness of the model in steady winds. The model also produced maps during transition wind fields. However, we do not have actual measurements to substantiate their accuracy. With these understandings we feel the model has passed a primary test for providing realistic and verifiable synoptic wave height maps, at least for steady wind fields.

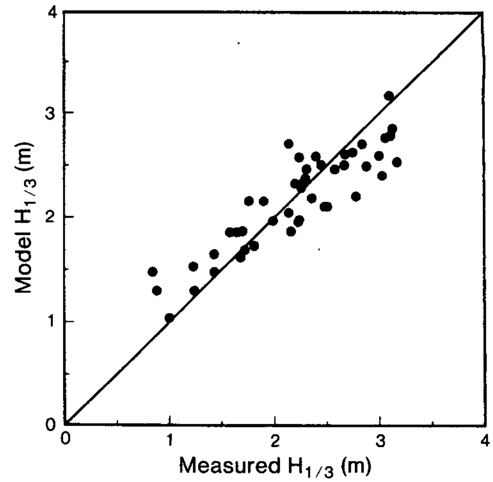

FIG. 7. Direct comparison of model and measured significant wave heights in Lake Michigan during 20–21 November 1977.

6. Concluding remarks

We have presented a simple, semi-empirical, parametric wave prediction model that successfully predicts synoptic wind-wave heights in Lake Michigan. The comparison between model results and measured two-dimensional synoptic wave height fields under two distinctly different wind conditions is excellent. Since accurate synoptic prediction is a major goal of wave prediction, this model shows promise.

The model is not without drawbacks. It is purely a wind-wave prediction model and has no provision for swell propagation at present. Hence it is more appropriate for Great Lakes waves than for ocean swell predictions. In addition, the model is for deep-water waves and the results may not be accurate for shallow-water waves.

One of the main advantages of this model is its simplicity; the empirical formulation of the momentum from the wind is simpler than most source functions used in energy transport models. Its simplicity, computational economy and insensitivity to grid size certainly make the model adaptable for many practical applications.

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