Crown profile equations for stand-grown western hemlock trees in northwestern Oregon

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Abstract: Crown profile equations were developed for stand-grown western hemlock (*Tsuga heterophylla* (Raf.) Sarg.) in northwest Oregon. The profile model uses a segmented approach, dividing the crown into an upper and lower portion at the point of the largest crown width (LCW). The model explains about 86% of the variation in crown width when LCW is known but only 66% when LCW is predicted using a model developed from a larger data set collected in the same area as the data for developing the crown profile models were collected. The model can be adjusted using measurements or predictions of LCW for western hemlock in other populations. Comparisons are made with the crown form of Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco).

Résumé : Des équations qui décrivent le profil de la cime ont été développées pour la pruche de l'Ouest (*Tsuga heterophylla* (Raf.) Sarg.) en peuplement dans le Nord-Ouest de l'Oregon. Le modèle utilise une approche par segments en divisant la cime en portions supérieure et inférieure au point où la largeur de la cime est maximale. Le modèle explique environ 86 % de la variation de la largeur de la cime lorsque la largeur maximale est connue mais seulement 66 % lorsque la largeur maximale est dérivée d'un modèle développé pour un ensemble plus important de données collectées dans la même région que les données utilisées pour développer le modèle qui utilise l'approche par segments. Le modèle peut être ajusté pour la pruche de l'Ouest dans d'autres populations en utilisant des mesures ou des extrapolations de la largeur maximale de la cime. Des comparaisons ont été effectuées avec la forme de la cime du douglas (*Pseudotsuga menziesii* (Mirb.) Franco).

[Traduit par la Rédaction]

Introduction

Information that describes tree crown structure is becoming increasingly important to resource managers. This information is used to estimate crown cover, crown volume, the fractal dimension of the crown (e.g., Corona 1991; Zeide and Gresham 1991; Zeide and Pfeifer 1991; Zeide 1998), and the distribution of foliage and interception of solar radiation (e.g., Dubrasich et al. 1997; Grace 1990; Van Pelt and Franklin 2000). Crown width is used in calculating competition indices based on crown overlap (Daniels et al. 1986; Biging and Dobbertin 1992). Estimates of crown closure at various heights within a stand are also used as a competition measure in predicting various components of stand development in individual tree growth models (Biging and Dobbertin 1995), such as CACTOS (Wensel et al. 1986) and ORGANON (Hann et al. 1997). More recently, realistic crown shapes have been incorporated into forest visualization programs (e.g., Hanus and Hann 1997).

Much of the work to describe tree crowns in the past has focused on predicting the greatest width of either open-grown or stand-grown trees. Equations for predicting the widths of open-grown trees, or maximum crown width (MCW), have been developed for many tree species because of its use in computing crown competition factor (Krajicek et al. 1961). Some work has also been done to develop predictions of the largest crown width (LCW) of stand-grown trees (e.g., Smith 1966; Honer 1971; Hann 1997).

Methodologies to describe the entire tree crown shape or profiles have taken two general approaches. Indirect methods begin by predicting the attributes of branches along the tree's stem (e.g., branch length and branch angle) and then indirectly computing the crown width using trigonometric relationships between the lengths and angles of branches. Examples of this approach include the crown profile equations of Roeh and Maguire (1997), Deleuze et al. (1996), and Cluzeau et al. (1994). An alternative approach is to develop either deterministic or stochastic models for directly predicting crown width (radius or area) from tree attributes. Some deterministic examples of this approach are Doruska and Mays (1998), Baldwin and Peterson (1997), Nepal et al. (1996), Biging and Wensel (1990), Pretzsch (1992), and Mitchell (1975). Biging and Gill (1997) proposed directly predicting crown profiles with a stochastic approach to account for the rugged nature of crowns (Gill and Biging 2002*a*, 2002*b*).

Hann (1999) proposed using a direct, deterministic approach to develop a relative crown profile model for

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Table 1. Abbreviations and variables used.

Abbreviation	Units	Definition
BD	m	Diameter of the branch, outside bark, at about one branch diameter from the bole
BL_f	m	Length of the branch measured when felled
BL_s	m	Length of the branch measured when standing
BP		Relative branch position within the crown
CL	m	Live crown length
CR		Crown ratio (CR = CL/HT)
CW_h	m	Crown width at height h above the ground
CWA_h	m	Crown width at height h above the ground for the portion of the crown above LCW
CWB_h	m	Crown width at height h above the ground for the portion of the crown below LCW
DACB	m	Distance above the crown base to LCW
DBH	cm	Diameter at breast height
DOB	cm	Diameter outside bark at height h
h	m	Height above the ground to the crown width of interest (CW_h)
HB	m	Height above the ground to the base of the branch
HCB	m	Height above the ground to the crown base
HLCW	m	Height above the ground to LCW
HT	m	Total height
LCW	m	Largest crown width of a stand-grown tree
MCW	m	Maximum crown width of an open-grown tree
RPA_h		Relative position of CW_h within the live crown and above LCW
RPB_h		Relative position of CW_h within the live crown and below LCW
VA_f	degrees	Angle between the bole and the branch when felled
VA_s	degrees	Angle between the bole and the branch when standing

Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco) that is scaled by an estimate or measurement of LCW. Once LCW (which itself is scaled from MCW) is defined, the crown width at any height within the crown (CW_h) is predicted by the relationship

$$[1] \qquad CW_h = LCW \times RP_h^k$$

where RP_h is the relative position in the crown (i.e., $RP_h = (HT - h)/CL$), h is the distance from the ground, HT is the total tree height, and CL is the crown length. The variables used in this paper and their units of measure are given in Table 1, which are consistent with Hann (1999). The value of k in eq. 1 characterizes the shape of the crown for a range of geometric solids (i.e., cylinder, cone, parabola, or neiloid). Because MCW and LCW can vary across the geographic range of a species (Hann 1997), this methodology potentially allows for easier calibration of crown profile by rescaling, or measuring, values of MCW and LCW for a particular area. This paper uses the methods of Hann (1999) to develop crown profile prediction equations for western hemlock (*Tsuga heterophylla* (Raf.) Sarg.) in northwestern Oregon. In application, the minimal inputs to the resulting model are direct measurements of tree diameter at breast height (DBH), HT, and height to crown base (HCB) and predictions of LCW and its distance above crown base (DACB) from these direct measurements. However, direct measurements of LCW and DACB can also be used to localize predictions. The effects of measuring or predicting LCW and DACB are tested for the modeling data set and an independent validation data set.

Materials and methods

Data

Modeling data set

Data for this study were collected as part of a larger project to develop a tree-level growth model for western hemlock in northwestern Oregon. The study area was in coastal northwest Oregon from Tillamook (about 45°30′N) to Astoria (about 46°10′N) and inland from the Pacific Ocean to the crest of the coast range (about 123°30′W). Stands in the study area that were predominantly western hemlock (greater than or equal to 60% of the basal area) were stratified into three age-classes (15–30, 30–45, and 45–60 years), three site classes (high, medium, and low), and three density classes (high, medium, and low) based on inventory data. A total of 43 stands were measured. These were selected randomly from the available stands in each age–site–density class to cover the entire matrix. Stands generally had not received a thinning within the last 15 years, except for some young stands that had been precommercially thinned. No stands were selected if thinning had occurred within the previous 5 years.

A cluster of five measurement plots was established in each stand selected for measurements. When possible, four

Table 2. Summary of data set for modeling LCW of stand-grown trees from measurements of 2293 trees in 43 stands.

	No. of observations	Mean	Minimum	Maximum
DBH (cm)	2293	34.04	0.25	121.41
HT (m)	2293	25.18	1.49	46.60
CR	2293	0.54	0.01	1.00
DBH (cm)/HT (m)	2293	1.34	0.13	3.24
LCW (m)	2293	6.10	0.49	14.91
BA (m^2/ha)	43	60.82	15.29	111.03
Site (m)	43	34.41	29.84	41.85

plots were located 45.72 m in cardinal directions from a central plot. The sample plots were a nested design of two fixed plots for smaller trees and a variable plot for larger trees following the methodologies of Hann (1992). Trees with diameters of 0.0–10.16 cm were sampled on a 0.0018-ha fixed plot. A 0.0071-ha fixed plot was used to sample for trees with diameters of 10.41–20.32 cm. A 4.59 m^2/ha basal area factor prism was used to select sample trees that were 20.57 cm and larger in diameter. All live trees greater than 1.37 m on the plots were identified by species and measured for DBH, HT, and HCB and any damage noted. In addition, all live trees greater than 1.37 m were measured for LCW in 32 of the 43 stands. In the 11 remaining stands, trees were subsampled for LCW throughout each plot's range of DBH. LCW was computed as the geometric mean of two crown widths based on measurements of four crown radii representing two azimuths. The first azimuth was defined by the direction from the subject tree towards the center of the measurement plot and the second crown width was in a perpendicular direction. The crown radii were measured as the horizontal distance from the center of the tree bole to the greatest extent of the crown from the bole in each quadrant by vertically sighting with a clinometer to locate the branch tip. This resulted in measuring 2293 western hemlock trees for developing LCW equations, which are summarized in Table 2.

After all plots were measured, 61 trees were selected to represent the range in size (DBH and HT) and crown ratio (CR) for felling. These trees were spread out across the study area as much as possible. After felling, the live crown was divided into 10 equal intervals along the bole, the largest branch was selected at each of these locations and at the base of the live crown, and the following measurements were taken at each of the selected branches: (*i*) the height of the branch base (HB) from the ground, (*ii*) the length of the branch (BL_f), (*iii*) the angle between the bole at the base of the branch and the branch tip (VA_f), and (*iv*) the branch diameter outside bark at approximately one branch diameter from the bole (BD). Measurements began at the whorl nearest to the crown base with the first undamaged branch. Three trees were dropped from further analysis because of excessive damage to the branches during felling. A further 22 trees were dropped because they did not exhibit a clear LCW, owing primarily to broken branches in the lower part of the crown. This provided 322 undamaged branches from 36 trees in 18 stands for modeling crown profile and the data are summarized in Table 3.

Table 3. Summary of crown width modeling data set from measurements made on 36 felled trees in 18 stands.

	No. of observations	Mean	Minimum	Maximum
DBH (cm)	36	36.83	13.97	84.33
HT (m)	36	26.18	12.86	41.61
CR	36	0.50	0.26	0.93
LCW (m)	36	6.95	2.77	11.74
Site (m)	18	34.80	30.82	41.85
BA (m^2/ha)	18	33.43	28.72	76.36
QMD (cm)	18	31.01	14.33	42.42
DACB (m)	36	4.43	1.16	8.72
CWB_h (m)	73	5.27	1.16	10.49
CWA_h (m)	213	3.54	0.34	10.03

Since the branch angle of prone, felled trees can differ from the branch angle of standing trees because of the weight of the branch, the tree sections with measured branches were stood upright and a standing branch length (BL_s) and angle (VA_s) were also measured whenever possible. The standing branch length and angle for the branches that had only felled measurements were estimated using the following models from Hann (1999) fit to data for the branches with both standing and felled measurement (n = 349 branches with both standing and felled measurements):

$$[2]\qquad BL_s = BL_f \times \exp(-0.184\,220\,3 \times BL_f^{-0.307\,769})$$

$$[3]\qquad VA_s = 60.0 - [27.160\,832 - 8.780\,759 \times (BL_f/BD)^2 - 0.677\,032 \times VA_f \times (1.0 - BP)^{0.5}]$$

where BP is the relative position of the branch within the crown computed as (HT – BH)/(HT – HCB). The adjusted R^2 for eq. 2 was 0.9633 and for eq. 3 was 0.2397.

The crown width (CW_h) of each measured branch was computed as the vertical projection of the standing branch length (BL_s) using the standing branch vertical angle (VA_s) and taking into consideration the diameter outside bark (DOB) of the tree bole using

$$CW_h = 2[\sin(VA_s) \times BL_s] + DOB$$

Because DOB at the height of the branch tip was not measured in the field, it was predicted using the taper equations of Flewelling (1994) and Flewelling and Raynes (1993) with the location specified as coastal Oregon. The height of the projected crown width above the ground (h) was computed from the height to the base of the branch (HB) and the vertical distance from the base of the branch to the branch tip as follows:

$$h = \mathrm{HB} + \cos(\mathrm{VA_s}) \times \mathrm{BL_s}$$

Validation data set

In addition to the modeling data set, 35 western hemlock trees were made available as a validation data set (Kershaw 1993; Kershaw and Maguire 1995). These trees were from two installations located in the same drainage in the Cascade Mountains of western Washington. At each location, trees were sampled from two 0.041-ha plots that had received an initial thinning to 471 trees/ha. In addition, one of the plots at each location received a urea nitrogen fertilizer application of 223 kg/ha. Both sites naturally regenerated after clear-cutting and slash burning. At the time that the measurements were taken, the two stands were 18 and 33 years of age, at breast height, and had been treated 10 years earlier.

For trees selected for branch measurements, all live branches were numbered consecutively from the lowest live branch upward. Prior to felling the tree, all live branches up to 8.99 m high above the ground were measured with a height pole for the height to the branch tip and the height to where the branch was attached. After measurement, the live branches were cut off and saved. The tree was then felled and the section above 8.99 m set upright and reoriented and the remaining branches measured. The removed branches were measured for total length (Kershaw 1993).

A secondary sampling process was then performed on the validation data set to mimic the procedures used to collect the modeling data set. In this process, sampling points were determined for each tree by dividing the live crown into 10 equal intervals and then selecting the whorl closest to each location and the lowest whorl in the crown. All branches at each selected whorl were then examined and the branch with the largest crown radius was chosen for inclusion in the validation data set to be consistent with the modeling data set. The validation data set is summarized in Table 4.

Table 4. Summary of crown width validation data set from 35 western hemlock (*Tsuga heterophylla*) trees from Kershaw and Maguire (1995).

	No. of observations	Mean	Minimum	Maximum
DBH (cm)	35	20.12	12.70	27.69
HT (m)	35	15.93	13.69	18.68
CR	35	0.76	0.60	0.85
LCW (m)	35	6.49	4.69	8.69
$\mathrm{CWB_h}$ (m)	75	5.34	2.53	8.38
$\mathrm{CWA_h}$ (m)	240	3.44	0.52	8.66

Analysis methods

The crown profile model of Hann (1999) uses a two-segment approach to predict tree crown profiles for Douglas-fir. The two segments are divided within the crown at the location of LCW. This method begins by predicting LCW as a scaled function of MCW. Ritchie and Hann (1985), Dubrasich et al. (1997), and Hann (1997) found that the following equation predicted LCW well and has the desirable property that LCW = MCW when CR = 1.0:

$$[4] \qquad \mathrm{LCW} = \mathrm{MCW} \times \mathrm{CR}^{\theta_0 + \theta_1 \times \mathrm{CL} + \theta_2 (\mathrm{DBH/HT})}$$

The MCW has been shown to be strongly related to DBH in numerous studies. Data from open-grown trees were not collected as part of this study, but three published MCW models for western hemlock were available. Farr et al. (1989) developed a MCW equation from 102 trees (0.3–56 cm) from southeast Alaska:

$$[5a] \qquad \mathrm{MCW} = 0.98 + 0.443 \times \mathrm{DBH}^{0.7340}$$

Smith (1966) developed a MCW equation from data in British Columbia (the number and size range of trees was not reported):

$$[5b] \qquad \mathrm{MCW} = 1.280 + 0.170 \times \mathrm{DBH}$$

Paine and Hann (1982) developed an equation using 62 trees (1.5–60.5 cm) from southwest Oregon:

$$[5c] \qquad \mathrm{MCW} = 1.388\,84 + 0.169\,77 \times \mathrm{DBH}$$

These models represent the range of western hemlock (Packee 1990). Models 5*b* and 5*c* are both linear and nearly identical and model 5*a* gives similar results through about 65 cm where it begins flattening out from the curved nature of the model form.

Fits of LCW eq. 4 with the MCW values computed using Paine and Hann's (1982) eq. 5*c* showed residuals that exhibited trends in larger trees indicating that MCW might be overpredicted in trees greater than 60 cm DBH. This suggested that the MCW relationship over the extended range of diameters was curvilinear, as found by Farr et al. (1989). We therefore used the western hemlock data of Paine and Hann (1982) to develop an estimator of MCW:

$$[5d] \qquad \mathrm{MCW} = 1.328\,52 + 0.188\,94 \times \mathrm{DBH} - 0.000\,484\,971 \times \mathrm{DBH}^2$$

The parameters were estimated using weighted regression and the same weight ($1/\mathrm{DBH}^{1.244}$) used by Paine and Hann (1982). The MCW estimates are similar for the all of the previous estimates up to about 40 cm where the curvature of the Farr (1989) model and eq. 5*d* becomes evident. Above 50 cm, eq. 5*d* predicts MCW values that are about 1 m greater than the Farr (1989) estimates.

The location of LCW within the crown is predicted from the DACB. Hann (1999) found that this was proportional to live crown length (CL):

$$[6] \qquad \mathrm{DACB} = c_1 \times \mathrm{CL}$$

In addition to CL, DACB was also plotted against CR, DBH, HT, HT/DBH, and LCW, but no trends were found.

Knowing the position of LCW (eq. 6) and the size of LCW (eq. 4), crown widths are predicted above LCW (i.e., CWA_h) using the model:

$$[7] \qquad \mathrm{CWA}_h = \mathrm{LCW} \times \mathrm{RPA}_h^{a_0 + a_1 \mathrm{RPA}^{1/2} + a_2 (\mathrm{HT/DBH})}$$

where RPA is the relative position of CWA_h in the crown above LCW and is computed as

$$\mathrm{RPA}_h = \frac{\mathrm{HT} - h}{\mathrm{HT} - \mathrm{HLCW}}$$

where HLCW is the height above the ground to LCW (HLCW = HCB + DACB). Crown widths below the LCW (i.e., CWB_h) are predicted from

$$[8] \qquad \mathrm{CWB}_h = b_1 \times \mathrm{LCW}$$

The values of CWB_h were also plotted against DBH, HT, CL, CR, HT/DBH, LCW, and RPB_h, where RPB_h is a relative position below LCW and is calculated as

$$\mathrm{RPB}_h = \frac{\mathrm{HT} - \mathrm{HCB}}{\mathrm{DACB}}$$

No trends were found across any of these other variables.

The parameters, and their asymptotic standard errors, for all equations were estimated using linear or nonlinear least squares. Weighted regression was used if the variance of residuals was not homogeneous. Final fits were then summarized by calculating the root mean squared error (RMSE) and the adjusted coefficient of determination (R_a^2) for each equation using unweighted residuals. The R_a^2 was computed using the method recommended by Kvålseth (1985). Unweighted residuals were used because these are the values of most interest by users of the equations.

Given values of LCW and DACB, the two-component crown profile equations (eqs. 7 and 8) can be combined into the following estimator of CW_h (Hann 1999):

$$[9] \qquad \mathrm{CW}_h = I \times \mathrm{CWA}_h + (1.0 - I) \times \mathrm{CWB}_h$$

where $I = 1.0$ if $h \geq \mathrm{HLCW}$ and $I = 0.0$ if $h < \mathrm{HLCW}$. Equation 9 was evaluated by examination of the residuals (predicted minus observed) of the modeling and validation data sets. The residuals were summarized by the mean, variance, MSE (to take into account bias), and R_a^2 (variation explained by the equations). The evaluation was repeated to compare differences of predicting versus measuring DACB and LCW for both the modeling and the validation data sets.

Results

The parameter estimates and standard errors of LCW eq. 4 fit using MCW eq. 5*d* and a weight of $1/\mathrm{MCW}^2$ are given in Table 5. The unweighted RMSE is 1.1 m and the unweighted R_a^2 is 0.743. The parameter estimate for the DBH/HT term was not significant, so this term was not included. The residuals for the fitted model are shown in Fig. 1. Figure 2 illustrates the expected trends of increasing LCW with larger crown lengths for a range of tree sizes.

The parameter estimate and its standard error of DACB eq. 6 fit using a weight of $1.0/\mathrm{CL}^2$ is given in Table 5. The resulting unweighted RMSE is 1.4 m and the unweighted R_a^2 is 0.371.

Table 5. Parameter estimates and associated asymptotic standard errors for eqs. 4, 6, 7, and 8.

Equation	Parameter	Parameter estimate	Asymptotic standard error
4	θ_0	0.105 590	0.012 824
	θ_1	0.011 699	0.001 099
6	c_1	0.355 270	0.021 51
7	a_0	0.461 782	0.071 2
	a_1	0.552 011	0.129 1
8	b_1	0.809 414	0.015 98

Fig. 1. Unweighted residuals for eq. 4 plotted against predicted LCW (Loess line plotted through residuals).

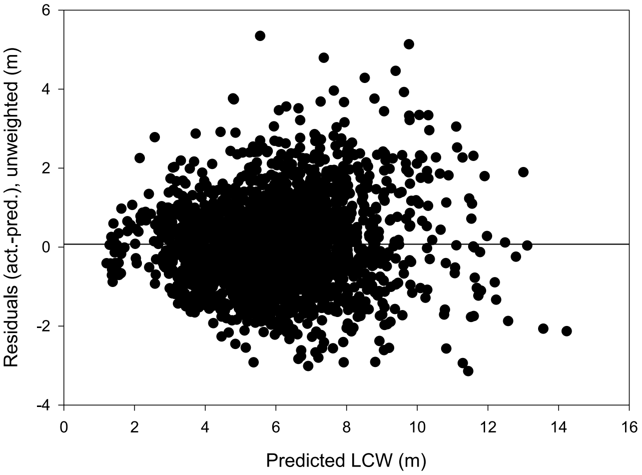
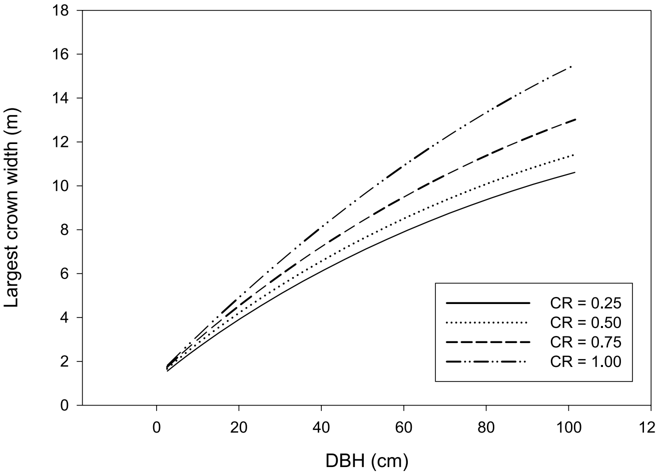
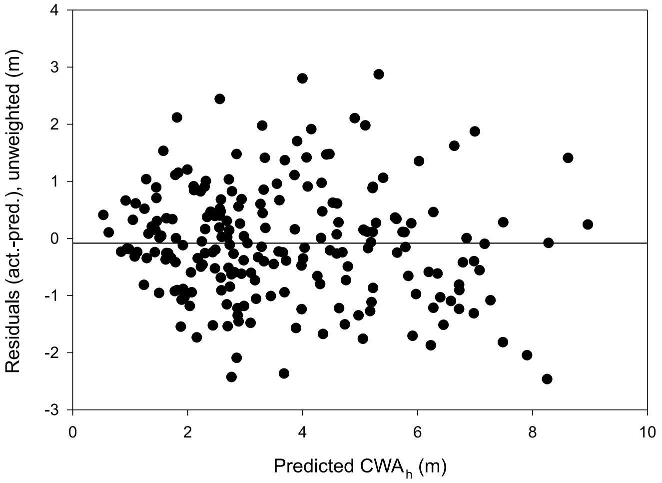


Fig. 2. LCW predicted from eq. 4 for CRs of 0.25, 0.5, 0.75, and 1.00. MCW is based on eq. 5*d*, and HTs used to compute CL from CR were predicted using the western hemlock (*Tsuga heterophylla*) height–diameter equation from Hanus et al. (1999): HT = 1.37 + exp(5.367 257 327 − 7.216 169 91 × DBH − 0.364 550 8).



The final parameter values and their standard errors for CWA_h eq. 7 fit with a weight of $1/\mathrm{LCW}^2$ are given in Table 5 and the residuals are shown in Fig. 3. The HT/DBH term was not significant and was therefore not included in the final model. The resulting unweighted RMSE is 1.1 m and the unweighted R_a^2 is 0.627. The presence of serial correlation between residuals within each tree's crown profile was evaluated by making individual fits to each tree's

Fig. 3. Unweighted residuals for eq. 7 plotted across predicted CWA_h (Loess line plotted through residuals).



CWA_h data using weighted nonlinear regression and computing the Durbin–Watson statistic (Kmenta 1986). Only three of the 36 trees in the modeling data set had Durbin–Watson statistics that were significant (p = 0.10). This is consistent with Hann (1999) who found 10 out of 101 and Raulier et al. (1996) who found three tests in 32 trees in their data set with significant tests. We therefore concluded that serial correlation was not a serious problem and could be ignored.

The plots showed that a simple linear relationship with LCW through the origin was adequate to describe CWB_h. Parameter estimates and their standard errors for CWB_h eq. 8 fit with a weight of $1.0/LCW^2$ are given in Table 5. The resulting unweighted RMSE is 2.2 m and the unweighted R_a^2 is 0.861.

Discussion and conclusions

For western hemlock, Hann (1997) found that MCW = LCW based on 32 trees from his study area in southwest Oregon. The larger data set used in this study found that while LCW was similar to MCW for long-crown trees, as would be expected, LCW became less than MCW in smaller crowned trees.

Figure 4 shows the relative crown width (CWA_h/LCW) above LCW for the position of the branch in the crown. In comparison with the same model developed for Douglas-fir (Hann 1999), the crown profiles above LCW are wider near the tip of the tree and a little narrower near LCW for western hemlock. These results indicate that western hemlock crowns are parabolic near the tree tip and become more conic in shape in the lower crown. Douglas-fir, on the other hand, have generally conic crowns that become slightly parabolic towards LCW. In Douglas-fir, Hann (1999) found that crown shape differs by the social position of the tree as measured by DBH/HT. The lack of a DBH/HT variable in western hemlock indicates that shape does not differ by social position. We have found that social position does affect CR and CL, which would impact LCW, causing different, but proportional, crown widths.

Fig. 4. Relative crown width (CWA_h/LCW) plotted against relative position within the crown (RPA_h) for eq. 4 and for a dominant Douglas-fir (*Pseudotsuga menziesii*) (HT/DBH = 0.48) and a suppressed Douglas-fir (HT/DBH = 1.92) from Hann (1999) for comparison.

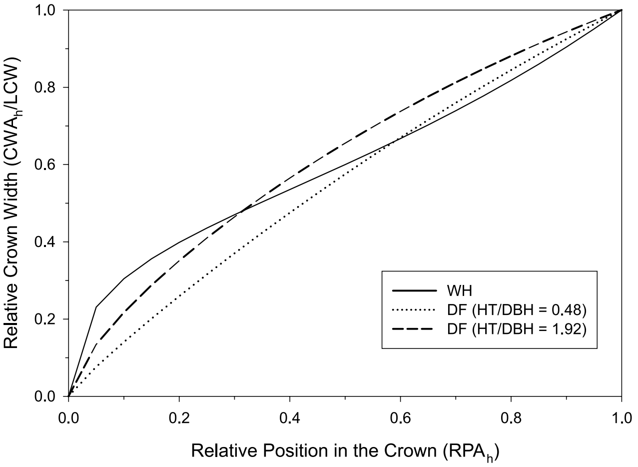
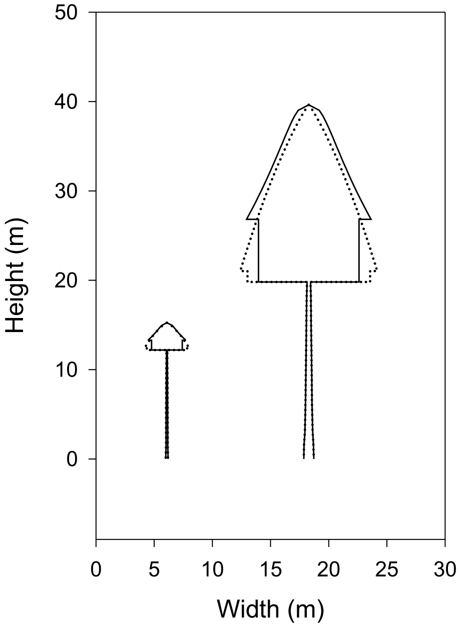


Fig. 5. Predicted crown profiles for a dominant tree with a DBH of 81.3 cm, an HT of 39.6 m, and a CR of 0.50 and a suppressed tree with a DBH of 15.2 inches, an HT of 15.2 feet, and a CR of 0.20. Solid lines are for a western hemlock (*Tsuga heterophylla*) and the dotted lines are the same trees using the Douglas-fir (*Pseudotsuga menziesii*) equations of Hann (1999).



To illustrate the complete tree profile, eq. 9 was used to predict the profiles for dominant and suppressed western hemlock trees shown in Fig. 5. These are compared with Douglas-fir trees of the same DBH, HT, and CR with the crown profile computed using the equations of Hann (1999). Again, we see that the crowns of western hemlock are a little wider above HCLW, while the crowns of Douglas-fir are a little wider below HLCW. The location of LCW is higher in the crown profile for western hemlock.

Table 6. Unweighted residual statistics for eq. 9 using the actual measurements on the felled trees (f) and predicted values (p) of LCW and DACB for trees in the modeling data set.

	Unweighted residual			
CW_h predicted using	Mean	Variance	MSE	R_a^2
LCW_f and $DACB_f$	–0.0388	0.783 712	0.845 466	0.859
LCW_f and $DACB_p$	–0.0635	0.821 645	0.857 061	0.849
LCW_p and $DACB_p$	0.5069	1.617 175	1.898 167	0.656

Table 7. Unweighted residual statistics for eq. 9 using the actual measurements on the felled trees (f) and predicted values (p) of LCW and DACB for trees in the validation data set.

	Unweighted residual			
CW_h predicted using	Mean	Variance	MSE	R_a^2
LCW_f and $DACB_f$	0.0447	0.841 044	0.845 466	0.746
LCW_f and $DACB_p$	0.0716	0.845 661	0.857 061	0.748
LCW_p and $DACB_p$	1.1640	1.070 734	2.458 172	0.278

The use of the simplified CWB model shows that the crown widths decrease below LCW but does not give a smooth tapered transition between LCW and CWB. For most applications involving calculation of crown widths, this should not be a problem; however, a more complex function could be used to smooth the crown widths below LCW to the base of the live crown if necessary (e.g., in visualization applications). Future efforts may want to sample more intensely in this region of the crown to better determine the profile.

Equation 9 was used to predict the entire crown profile for the trees of the modeling data set. Residuals were calculated and are summarized in Table 6 for actual and predicted measures of LCW and DACB. If LCW and DACB are known, nearly 86% of the variation in unweighted predictions of CW_h can be explained with a small average underprediction bias of about 0.04 m. There is little difference between using the actual values of DACB and predicting DACB for the modeling data set. By predicting DACB, the average underprediction bias increases to about 0.06 m and the variance increases 4.8% while the model still predicts nearly 85% of the variation in the modeling data set. However, predicting LCW nearly doubles the variation in the residuals while dropping the predictability to about 66% with an average overprediction bias of about 0.51 m. The magnitudes of prediction biases are similar to those found by Hann (1999) in his Douglas-fir model (after converting his estimators from metres to feet). The variances for western hemlock predictions are about 50%, 35%, and 14% higher than for Douglas-fir when LCW and DACB are known, LCW is known, and DACB is predicted and when LCW and DACB are both predicted, respectively. This suggests that the western hemlock crowns tended to be rougher than Douglas-fir crowns.

For the validation data set, the average residuals of the 18 unfertilized and 17 fertilized trees were 0.25 and –0.18 m, respectively, but were not significantly different from each other ($p = 0.16$). The variances of the residuals were 0.728 and 0.870 m^2, respectively, and were also not significantly different from each other. Therefore, the unfertilized and fertilized trees were combined for validation. The average error in the resulting residual analysis with measured DACB and LCW was of similar magnitude to that found in the modeling data set, but the variance of the residuals was higher (Table 7). The model explained about 75% of the variation in unweighted residuals of the validation data set compared with about 86% for the modeling data set. As with the modeling data set, predicting DACB had only a small impact on the predictions of crown width (increased average residual from 0.04 to 0.07 m), while also predicting LCW caused a large increase in average residual (from 0.07 to 1.16 m) and a 26% increase in the variation of residuals. This resulted in a large loss in the precision of predictions. The poor results using predicted LCW might be partially due to the narrow range in the validation data set (only two sites).

Most of the increase in MSE for the predictions using eq. 9 came with the use of predicted LCW. The agreement with Hann's (1999) findings for Douglas-fir suggests that one can greatly increase the precision and accuracy of the crown width predictions by using localized estimates of LCW. These could be obtained by directly measuring LCW or calibrating eq. 4. These results further suggest that broadly applicable tree crown profile equations can be developed with a small number of trees sampled for the detailed and costly branch measurements needed to characterize relative crown shape.

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