

## A Simple Index of Stand Density for Douglas-fir

Robert O. Curtis

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ABSTRACT. The expression  $RD = G/(Dg^{1/2})$ , where  $G$  is basal area and  $Dg$  is quadratic mean stand diameter, provides a simple and convenient scale of relative stand density for Douglas-fir, equivalent to other generally accepted diameter-based stand density measures. FOREST SCI.28:92-94.

ADDITIONAL KEY WORDS. *Pseudotsuga menziesii*, competition.

MANY DIAMETER-BASED MEASURES OF STAND DENSITY have a common interpretation as expressions of area available to the average tree, compared to either the open-grown condition or the normal stand (Curtis 1970, 1971). In closed stands, these measures are expressions of average crown development and competition level relative to the standard condition and are approximately equivalent; they differ mainly in details of algebraic form and method of estimation of the constants.

The diameter-based expressions of relative stand density include such measures as stand density index (Reineke 1933), ratio of basal area to normal basal area for the given diameter, crown competition factor (Krajicek and others 1961), and tree-area ratio (Chisman and Schumacher 1940). These are useful in analysis of growth relationships and in stand modelling as predictor variables not intrinsically correlated with age and site and for description of competition levels and thinning regimes. They provide numerical scales convenient for description of such concepts as maximum density, zone of occurrence of suppression mortality, and acceptable limits of stand density in thinning practice; and are directly related to average crown development and potential response to release.

This note presents a slightly different form of one of the generally accepted measures, which we have found simple and convenient to use for coastal Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco var. *menziesii*).

In either the normal or maximum density stand, number of stems ( $N_L$ ) per unit area can be expressed (Reineke 1933) as a power function of quadratic mean diameter ( $Dg$ ):

$$N_L = a_0(Dg^c) \quad (1)$$

and

$$\ln N_L = \ln a_0 + c \ln Dg.$$

Since average area per tree is  $1/N_L$ ,

$$\begin{aligned} (1/N_L)/(1/N_{obs}) &= (\text{area per tree})_L/(\text{area per tree})_{obs} \\ &= \frac{N_{obs}}{a_0(Dg^c)} \end{aligned} \quad (2)$$

expresses stand density as a ratio of average area per tree in the normal stand to average area per tree in the observed stand, given that both stands have the same  $Dg$ .

Equivalently, since  $G = k(N)(Dg^2)$ , where  $G$  is basal area and  $k$  is the basal area constant, then

$$G_L = a_1(Dg^b), \text{ in which } b = c + 2 \quad (3)$$

and

$$\ln G_L = \ln a_1 + b \ln Dg.$$

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The author is Principal Mensurationist, Pacific Northwest Forest and Range Experiment Station, USDA Forest Service, Olympia, WA 98502. Manuscript received 3 November 1980.



Since  $[I/N_L]/[I/N_{obs}] = N_{obs}/N_L = [N_{obs} Dg^2]/[N_L Dg^2] = G_{obs}/G_L$ , an expression of density equivalent to (2) is the ratio of observed basal area ( $G_{obs}$ ) to normal basal area ( $G_L$ ) for the given  $Dg$ :

$$G_{obs}/G_L = \frac{G_{obs}}{a_1(Dg^b)} \quad (4)$$

and, since  $a_1$  is merely a constant multiplier, relative ranking of stands is unchanged if we replace  $a_1$  by 1.0 and express relative stand density on a scale of

$$\text{Relative density} = \text{RD} = G_{obs}/(Dg^b). \quad (5)$$

In this form, density is on a continuous scale ranging from zero for zero basal area through some biological maximum for the species. There is now only a single parameter (the exponent of  $Dg$ ) to be determined. Although estimation of this exponent requires data which represents an average density (level of competition) which is proportional to the biological maximum over a range of  $Dg$ , any uncertainty in defining the level of density to be regarded as “normal” does not affect the numerical value of the index or its use for density measurement in managed stands.

Estimates of the exponent  $b$  can be obtained by fitting either equations (1) or (2) to normal stand data; or, alternatively, by the tree area ratio technique using a power function in place of the original polynomial (Curtis 1971). Since available information suggests that the curve of crown area over diameter in the open-grown condition is proportional to that of tree area over diameter in the normal stand, it may also be possible to estimate  $b$  from measurements on open-grown trees. Here, the equation to be fitted would be

$$(CW)^2 = a_2(Dg^{2-b}) \quad (6)$$

or

$$\ln (CW)^2 = \ln a_2 + (2 - b)\ln Dg,$$

where  $CW$  is crown width.

For coastal Douglas-fir,  $b$  is in the range 0.45 to 0.50, as shown by equation 1-0 in Bruce and others (1977), Curtis (1971), and table 25 in McArdle and others (1961).<sup>1</sup> Rounding to 0.5, the relative density expression becomes

$$\text{RD} = G_{obs}^{0.5}/(Dg^{0.5}). \quad (7)$$

which (for  $G$  in  $\text{m}^2/\text{ha}$  and  $Dg$  in  $\text{cm}$ ) has a range from zero for zero basal area, through a value of about 9.5 for the “normal” of McArdle and others (1961), to some biological maximum which appears to be about 14 for small plot observations. (Equivalent values for  $G$  in  $\text{ft}^2/\text{acre}$  and  $Dg$  in inches are 65 and 100.) This simple expression is directly proportional to a normality calculated from the widely used table 25 in McArdle and others (1961). We have found it useful as a predictor variable and in defining thinning regimes in recent simulation work (Curtis and others 1981).

An equivalent expression is

$$\text{RD} = a_3 N / (Dg^{3/2}) \quad (8)$$

where  $N$  is number of stems per unit area, and

$$\begin{aligned} a_3 &= 0.00007854 \text{ for } G \text{ in } \text{m}^2/\text{ha} \text{ and } Dg \text{ in } \text{cm}, \\ &= 0.00545415 \text{ for } G \text{ in } \text{ft}^2/\text{acre} \text{ and } Dg \text{ in } \text{inches}. \end{aligned}$$

From this,

$$N = \frac{R D}{a_3(Dg^{3/2})} \quad (9)$$

which is useful in controlling thinning or planting by number of stems. For example, one might specify that  $\text{RD}$  should not exceed 7 (~50 in English units) at the time of first commercial thinning. (This is a species-specific value; the value 7 will—for Douglas-fir—

<sup>1</sup> Patrick H. Cochran, Pacific Northwest Forest and Range Experiment Station, obtained a similar estimate of  $b = 0.49$  for interior Douglas-fir in eastern Oregon and Washington (personal communication, 1980).

avoid suppression mortality and excessive restriction of crown development.) Substitution in (9) then gives, for any desired diameter  $Dg$  at the time of first commercial thinning, the number of trees  $N$  per unit area to be left after precommercial thinning, plus a small mortality allowance. This simple formula gives estimates very close to those of Reukema (1975).

The same procedure should be applicable to even-aged stands of other species. Reineke (1933) suggested a single slope coefficient of -1.605 for the form

$$\ln N = a + (b - 2)\ln Dg$$

equivalent to a value of 0.40 for the exponent  $b$  in

$$RD = G_{\text{obs}}/(Dg^b)$$

This suggests that 0.4 would be a usable value in the absence of better information for a species. More recent literature and experience suggest a possible range among species of perhaps 0.3 to 0.5. After rounding consistent with the precision of such estimates, recognition of a few species groups should suffice.

Maximum attainable value will of course differ among species, as will coefficients relating increment and mortality to the density measure.

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