

Power-law residence time distribution in the hyporheic zone of a 2nd-order mountain stream

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Received 16 January 2002; revised 14 March 2002; accepted 28 March 2002; published 10 July 2002.

[1] We measured the hyporheic residence time distribution in a 2nd-order mountain stream at the H. J. Andrews Experimental Forest, Oregon, and found it to be a power-law over at least 1.5 orders of magnitude in time (1.5 hr to 3.5 d). The residence time distribution has a very long tail which scales as $t^{-1.28}$, and is poorly characterized by an exponential model. Because of the small power-law exponent, efforts to characterize the mean hyporheic residence time (t_s) in this system result in estimates that are scale invariant, increasing with the characteristic advection time within the stream channel (t_{ad}). The distribution implies the hyporheic zone has a very large range of exchange timescales, with significant quantities of water and solutes stored over timescales very much longer than t_{ad} . The hyporheic zone in such streams may contribute to short-time fractal scaling in time series of solute concentrations observed in small-watershed studies. **INDEX TERMS:** 1871 Hydrology: Surface water quality; 1824 Hydrology: Geomorphology (1625); 3250 Mathematical Geophysics: Fractals and multifractals; 1832 Hydrology: Groundwater transport

1. Introduction

[2] The hyporheic zone is the region of near-stream aquifers containing exchange flows of stream water—water that originates from, and is returned to the stream channel. The hyporheic zone may extend 10s of meters horizontally from the channel and a meter or more beneath it [Wondzell and Swanson, 1996, 1999], and is an important location for biologically-mediated reactions that modify the chemistry of stream water [Findlay, 1995; Triska et al., 1989]. Exchange flows bring stream water into close contact with biofilms on sediment surfaces where reactions tend to occur. However, the extent to which transformations occur is dependent, in part, on the residence time of stream water in the subsurface. Further, because the hyporheic zone may have a large volume with long residence times, it can retain solutes present in stream water and later release them back to the stream channel, thus delaying or attenuating chemical signals produced by precipitation and watershed processes. Differences in residence time distributions, then, will determine the relative importance of biogeochemical transforma-

tions and the transport of contaminants and other solutes in streams and rivers.

[3] Conventional studies of hyporheic exchange at reach scales use a 1-dimensional advection, dispersion and transient storage model [Bencala and Walters, 1983; Runkel, 1988] that assumes residence times are distributed exponentially [Harvey et al., 1996; Wörman et al., 2002]. Harvey et al. [1996] showed that nominal travel times to wells located in distinct areas of the hyporheic zone display exponential distributions, but with shorter mean residence times for wells located in near-stream gravel bars than those located in deeper alluvium further from the stream. Those observations suggest that the combined behavior of all flow paths in the hyporheic zone is not exponential, but instead reflects a broader distribution of timescales. Conventional model simulations with the transient storage model are sensitive only to short-time hyporheic storage, ignoring the longer timescales of exchange [Harvey et al., 1996]. Choi et al. [2000] suggested that simultaneously modeling several hyporheic zones with different mean residence times could broaden the timescale of hyporheic storage that is considered by models. Wörman et al. [2002] show that the residence time distribution of the hyporheic zone of Säva Brook, Sweden is characterized by a lognormal distribution. Examining results of conventional studies of hyporheic exchange conducted in many stream reaches shows that, in general, reaches with short (or long) channel residence times yield correspondingly short (or long) estimates of hyporheic residence times [Harvey and Wagner, 2000] (Figure 1). We propose that this scale-invariant behavior is associated with the underlying hyporheic residence time distribution following a power-law.

2. Methods

2.1. Mathematics

[4] The effect of the hyporheic zone on a non-reactive or linearly sorbing solute can be expressed within the mass balance equation as a convolution of the hyporheic memory function $g^*(t)$ with solute concentration in the stream channel,

$$\frac{\partial c}{\partial t} + \frac{R_h V_h}{V_c} \int_0^t \frac{\partial c(t-\tau)}{\partial t} g^*(\tau) d\tau = D_L \frac{\partial^2 c}{\partial x^2} - v \frac{\partial c}{\partial x} + \frac{\partial c_{inj}}{\partial t} \quad (1)$$

where c is solute concentration in the channel, R_h is the retardation factor in the hyporheic zone due to linear

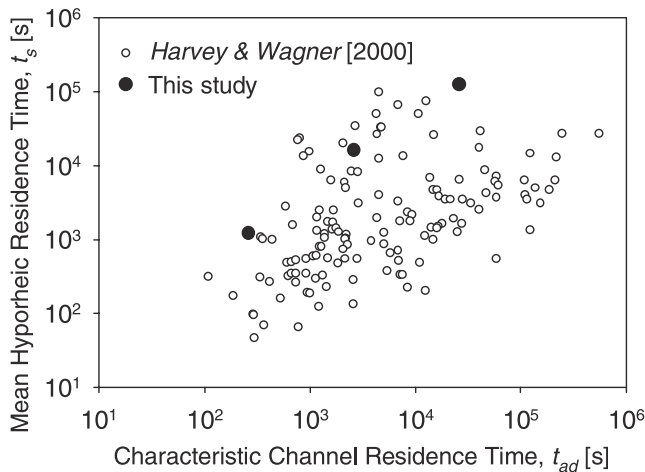


Figure 1. Mean residence time in hyporheic zone versus mean residence time in surface channel from *Harvey and Wagner [2000]* (with permission). Middle value from this study is estimated from experiment, where an exponential residence time distribution was assumed. Other two values from this study show estimates that would be obtained from shorter and longer reaches, given the same underlying residence time distribution in those reaches (see Figure 4 and associated discussion).

sorption, V_h the reach volume of the hyporheic zone, V_c is the reach volume of the channel, τ is a lag time, D_L is the dispersion coefficient in the channel, v is the mean velocity in the channel, and c_{inj} is a concentration injected into the channel. Equation (1) is equivalent to equations (1) and (8) of *Haggerty et al. [2000]*; the mathematics of the convolution approach have been expounded in several papers [*Hornung, 1997; Hornung and Showalter, 1990; Peszyńska, 1996*]. Equation (1) assumes that prior to injection of tracer, the stream and hyporheic zone have the same concentration (could be 0). In the case of a pulse injection, $c_{inj} = M\delta(t)/Q$ at $x = 0$ and equals 0 at $x > 0$, $t = 0$, where M is the tracer mass injected, $\delta(t)$ is a Dirac function, and Q is channel discharge.

[5] The memory function $g^*(t)$ can be thought of as the probability density that a tracer molecule entering the hyporheic zone at $t = 0$ will still be in the hyporheic zone at t . The residence time distribution of the hyporheic zone (the probability that a tracer molecule entering the hyporheic zone at $t = 0$ will leave the hyporheic zone at t) is proportional to the derivative of the memory function:

$$RTD(t) = -t_h \frac{\partial g^*(t)}{\partial t}, \quad t > 0 \quad (2)$$

where t_h is the harmonic mean residence time in the hyporheic zone.

[6] Given a pulse injection of tracer into the channel, the breakthrough curve $c(t)$ is proportional to the residence time distribution of water in the stream, $f(t)$. At late time (significantly later than the peak breakthrough), the relationship between the residence time distributions is straightforward [*Haggerty et al., 2000*]:

$$f(t) = \frac{c(t)Q(t)}{M} = -t_{ad} \frac{R_h V_h}{V_c} \frac{\partial g^*(t)}{\partial t}, \quad t \gg t_{ad} \quad (3)$$

where t_{ad} is the characteristic channel residence time (equal to L/v where L is the distance from injection to measurement) and $Q(t)$ indicates that the discharge varies in time. Consequently, the hyporheic residence time distribution may be directly measured by injecting a pulse of tracer and measuring late-time concentrations.

2.2. Tracer Test

[7] We injected a pulse (11.0 g) of rhodamine WT (RWT, Formulabs, Piqua, Ohio) into a 2nd-order mountain stream. A section of the reach is shown in Figure 2. Stream discharge and concentrations of RWT in stream water were measured at a flume 306.4 m downstream. The data and further experimental details are given in Figure 3. Sediment samples taken from the stream were used to measure the RWT sorption isotherm using standard techniques [*Shiau et al., 1993*]; the isotherm was found to be linear with a distribution coefficient of $15.9 \text{ cm}^3/\text{g}$, yielding a retardation factor R_h of approximately 100.

3. Analysis

[8] Figure 3 shows the channel residence time distribution, which at late times scales as $f(t) \sim t^{-1.28}$. We know from equations (2) and (3) that the hyporheic residence time must scale in the same way, with $RTD(t) \sim t^{-1.28}$, and the memory function $g^*(t) \sim t^{-0.28}$ over time periods ranging from 5×10^3 to 3×10^5 s (1.5 hr to 3.5 d). The exponential residence time distribution assumed in conventional models is $g^*(t) = \exp(-t/t_s)/t_s$, where t_s is the mean hyporheic residence time. A best fit of the conventional model results in $t_s = 1.63 \times 10^4$ s and is inadequate for this stream, since all data except the peak are poorly represented (Figure 3).

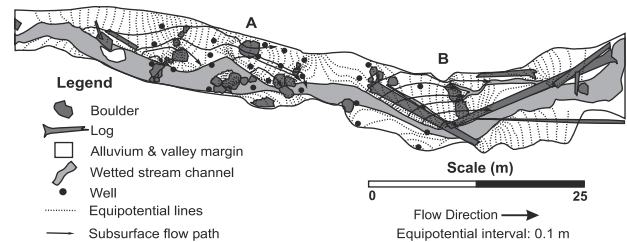


Figure 2. Water table elevation along a portion of WS03, H. J. Andrews Experimental Forest, Oregon. Conditions are for summer baseflow from a groundwater flow model [*Kasahara, 2000*]. The calibrated model has average error of 0.10 m between predicted water table elevations and those observed at wells. Some subsurface flow paths in uppermost model layer (surface 0.5 m of alluvium) are illustrated. Region “A” shows nested flow paths, with short, arcuate flows around abrupt boulder-steps in main channel, and longer flow paths extending to valley margin and following the overall down-valley gradient. Region “B” shows development of relatively long flow paths through deep accumulations of sediment trapped behind two large logs that create 1.5 m to 2.0 m high steps in main channel. Much longer flow paths are present, especially in region “B”, flowing through deeper alluvium, but these cannot be shown in this 2-dimensional illustration. The shortest flow paths occur at scales smaller than finest resolution of the model (0.3 m horizontally; 0.5 m depth).

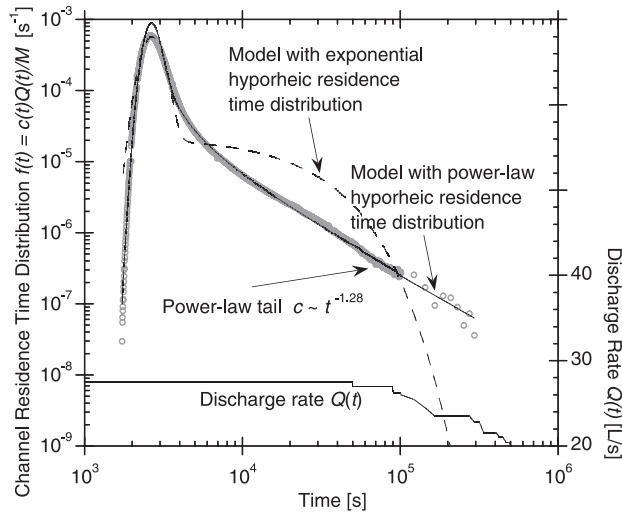


Figure 3. Stream residence time distribution and models of distribution. Dense data (grey circles prior to 10^5 s) were collected in real time using a field fluorometer with a flow-through cell (Turner Designs, Sunnyvale, California); later sparse data were taken from discrete water samples obtained with an automated sample collector. Calibration was performed with stream water prior to test. Discrete samples were taken throughout the experiment and measurements (not shown) overlie the flow-through data. Stream residence time distribution is obtained by scaling concentration by the discharge rate at time of measurement $Q(t)$ and dividing by the mass of tracer injected M . Stream discharge was steady at 27.46 L/s for the first 4.99×10^4 s (13.9 hrs) of the test. Peak concentration was $237 \mu\text{g/L}$ at 2.63×10^3 s (0.731 hr). Total mass recovered (assuming Q measured perfectly) at end of data was 0.791 of injected mass. Missing mass results in model peak higher than the data.

[9] A model with power-law scaling was fit to the data. The memory function employed was a truncated power-law [Haggerty *et al.*, 2000]:

$$g^*(t) = \int_{\alpha_{\min}^{-1}}^{\alpha_{\max}^{-1}} \frac{(k-2)\alpha^{k-2}e^{-\alpha t}}{\alpha_{\max}^{k-2} - \alpha_{\min}^{k-2}} d\alpha. \quad (4)$$

This is a summation of exponential distributions (i.e., the conventional model) with a power-law weighting function. The function scales as $g^*(t) \sim t^{1-k}$ between the times of α_{\max}^{-1} and α_{\min}^{-1} , and $RTD(t) \sim t^{-k}$ over the same times. We will refer to α_{\max}^{-1} as t_{\min} and α_{\min}^{-1} as t_{\max} , the limits of power-law behavior. Equation (4) with $k = 1.28$ is substituted into Equation (1), solved in the Laplace domain, and inverted numerically [Haggerty *et al.*, 2000]. The values of t_{\min} and t_{\max} were set to 0 and 10^6 s, respectively. The results are shown in Figure 3.

4. Discussion and Conclusions

[10] The power-law residence time distribution ($k < 3$) implies a very wide range of timescales over which hyporheic exchange flow occurs. Figure 2 shows the water table elevation along the middle portion of the studied reach under summer baseflow conditions. Exchange flows between the stream and the hyporheic zone result from

channel morphologic features. For example, stream water flows into the streambed upstream of sharp steps in the channel's longitudinal gradient. Steps created by small logs and boulders may only be 10s of cm high and only drive localized hyporheic exchange whereas large log jams can create steps 1–2 m high and drive hyporheic exchange through the streambed for 10s of m upstream. Exchange flows occur at even larger scales, driven by channel sinuosity and variations in width and depth of alluvium stored in the stream valley. The longest hyporheic flow paths may exceed 100 m in length, within which shorter flow paths are inset, down to the scale of only a few centimetres because of the effect of multiple morphologic features. Further, the residence time of water flowing along any given flow path will depend on the conductivity of the substrate. Thus, a complex flow net is present in the alluvium of the valley floor through which water flows at vastly different time scales. Any single feature may create flow paths in which residence times are distributed exponentially, but the summation of all flow paths within the stream reach is the hyporheic residence time distribution, which scales as a power-law in this stream.

[11] Two important characteristics of hyporheic exchange flow cannot be determined or become scale invariant when the residence time distribution is a power-law and k is small (< 3). First, the mean hyporheic residence time is strongly dependent upon the maximum timescale of hyporheic exchange flow (t_{\max}). As long as the tail of the tracer breakthrough follows a power-law, t_{\max} must be beyond

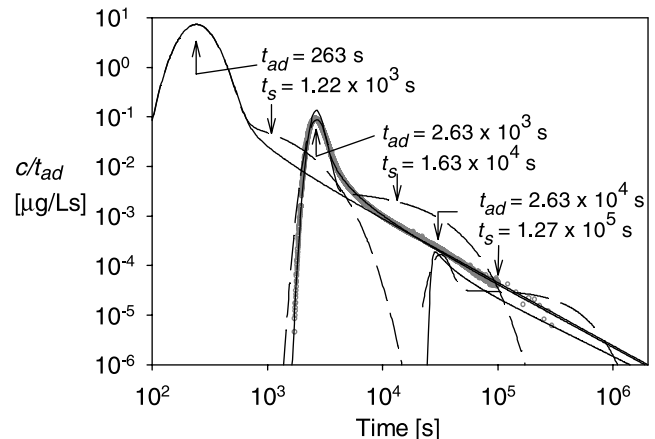


Figure 4. Breakthrough curves and models for hypothetical reach lengths 1/10 and 10 times as long as the experimental reach length. Concentration is normalized by advection time ($t_{ad} = L/v$), showing scale-invariant behaviour of the tracer pulse over different reach lengths. Assuming the same stream velocity, dispersion, and hyporheic RTD as measured in the tracer test, a numerical model was run over two other reach lengths, the first 1/10 as long as the test reach, and the second 10 times as long as the test reach. Power-law models of data and shorter/longer reaches are shown by solid lines. Next, models with an exponential residence time distribution (shown with dashed lines) were fitted to each of the power-law models. Exponential models provide estimates of the mean residence time in the hyporheic zone, t_s , which is scale invariant with t_{ad} . Estimated t_s are plotted in Figure 1.

the experimental timescale, and is therefore unknown. Consequently, attempts to describe the hyporheic zone with a mean residence time will produce estimates that scale with characteristic channel residence time ($t_{ad} = L/v$). We demonstrate this with a hypothetical example. We used equation 1 with the measured memory function $g^*(t) \sim t^{-0.28}$ to produce breakthrough curves for hypothetical stream reaches 1/10 and 10 times as long as the experimental reach length (solid lines, Figure 4). We assume that the power-law memory function and all other parameters are the same over the shorter and longer reaches. The fact that longer reaches are likely to encounter longer hyporheic flow paths is implicitly accounted for within the power-law model. Assumed values of t_{min} and t_{max} have no effect on the overall behavior of the breakthroughs as long as they are beyond the time limits of the test, which our values are. The resulting tracer breakthroughs in Figure 4 are self-similar, having the same overall appearance and regressing to a common late-time “tail”. This long tail can be matched, at least approximately over each stream reach, using the conventional model with an exponential residence time distribution (dashed lines, Figure 4). The estimates of the mean hyporheic residence time from the conventional model values scale linearly with mean channel residence time (Figure 1), having values of t_s/t_{ad} of 4.6 to 6.2. The data from other streams in Figure 1 are consistent with the existence of power-law distributions of hyporheic residence times. It may be that many tracer tests, having been modeled with an assumed exponential distribution, yield estimates of mean hyporheic residence time that scale with mean channel residence time. We suggest that whenever possible, late-time, low-concentration tracer data should be examined for the presence of a non-exponential tail. If data do not drop exponentially, then a non-conventional model of hyporheic exchange may be more appropriate.

[12] Second, the estimates of the volume of the hyporheic zone (V_h) will also scale with reach length. For example, the conventional model estimates the relative volume $R_h V_h/V_c$ as 0.315, 3.60, and 7.66 when fitted to the short-reach power-law model, the data, and the long-reach power-law model, respectively. These estimates grow with reach length because larger proportions of the hyporheic zone contribute to the development of the breakthrough curve as the sample reach is lengthened. Consequently, the conventional model may underestimate the volume of the hyporheic zone, particularly in experiments that are short relative to the time-scale over which most hyporheic exchange occurs. The power-law model of the hyporheic zone, which estimated $R_h V_h/V_c$ as 141, is also insufficient for estimating the volume of the hyporheic zone. Because the data exhibit power-law scaling between the peak arrival time and the time at which the lower RWT detection limit is reached, both t_{min} and t_{max} for hyporheic exchange are unknown. Therefore, the limits on the power-law scaling are also unknown. Without knowledge of these limits, neither the true mean hyporheic residence time nor the true volume of the hyporheic zone can be estimated from our data.

[13] Recent work indicates that water residence times in some catchments may exhibit fractal scaling, converting a pulse input into an output with a power-law tail [Kirchner *et al.*, 2000, 2001]. Our study suggests that the last step in transport out of the catchment, the stream, may contribute to

fractal scaling at shorter time scales (hours to days) due to a power-law residence time distribution in the hyporheic zone. Since the stream is the last filter in the system, all short-time information about upstream conditions - chemistry, hillslope residence times, and groundwater ages - may be significantly smoothed or altered by transport through the hyporheic zone.

[14] **Acknowledgments.** This work was supported by the National Science Foundation (EAR 99-09564), by the H. J. Andrews Experimental Forest, and by the U.S.D.A. Forest Service, Pacific Northwest Research Station. We thank P. J. Wigington and the U.S. EPA for loan of equipment, and Mike Gooseff, Jud Harvey and Anders Wörman for valuable discussions.

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