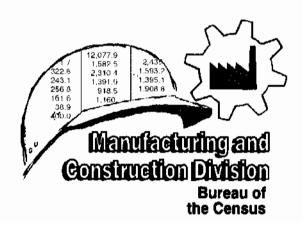
Working Papers

Using Planned Spending From The Plant and Equipment Expenditures Survey As A Forecast

> John H. Gates May 1991

Industrial and noitental authoritation Statistics



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The data were compiled and the statistical tabulations were performed by Steven Rudolph, Kali Kong, and Dale Lehmann. Helpful comments were received from several people including Gaylord Worden, John Berry, and Easley Hoy.

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Using Planned Spending From The Plant and Equipment Expenditures Survey As A Forecast

Executive Summary

This paper addresses the question, "How accurate is planned spending from the Plant and Equipment Expenditures Survey as a forecast of future investment spending?" We begin by explaining that planned spending is not a forecast. Several reasons for distinguishing between planned spending and forecasted spending are presented. However, an adjustment is made to the estimates of planned spending from the Plant and Equipment Expenditures Survey that creates a hybrid plan/forecast. This hybrid is published in the P&E Survey press releases. Since many analysts use this hybrid estimate directly as a forecast, a comparison of the published estimates with actual spending is made.

Forecasts based on two simple models of future investment are also compared to actual spending. The exercise of making these comparisons is used as an opportunity to alert the reader to various pitfalls that should be avoided when working with data from the Plant and Equipment Expenditures Survey. Spending on plant and equipment is briefly compared to nonresidential fixed investment for the benefit of analysts who are primarily interested in the latter as a measure of investment spending.

The paper concludes that while the hybrid plan/forecast of spending can be used as a forecast it is more effective to use it as one of several inputs into a forecast. Two basis for this conclusion are presented. First, we explain in the introduction that company plans are one factor affecting demand while actual spending is the result of the interplay of both demand and supply. Second, in the comparison of the accuracy of plans with the accuracy of simple models, we show that plans can be used to improve a model so that the model becomes more accurate than the plans themselves.

Using the plans to improve a model is demonstrated by first comparing the accuracy of planned spending as a forecast to the accuracy of a very simple model. The two are found to be closely matched when forecasting quarterly change. Then the planned spending is combined with the simple model, to produce a new model. This new model is more accurate than either the plans or the simple model.

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Using Planned Spending From The Plant and Equipment Expenditures Survey As A Forecast

Introduction

Each quarter, the Bureau of the Census conducts the Plant and Equipment Expenditures (P&E) Survey. The survey is the basis for estimates of actual and planned spending by business on plant and equipment in the domestic U.S. Planned spending is not a forecast of future spending. Planned spending is an important factor in determining the demand for investment goods and services but it is not the sole determinant of that demand. A forecast of future spending is an estimate of what the future level of spending will be when the interplay of demand and supply has been completed. A forecast can take into account additional factors that affect market demand, and can consider factors that affect market supply.

There are at least two reasons why future market demand for investment goods and services will not be simply the sum of individual companies' plans. First, and foremost, circumstances change and so do plans. Revenue, interest rates, prices of investment goods, and other factors that affect a company's plans are not totally controlled by the company. As these factors change, companies adapt by modifying and updating their plans. The actual spending finally observed is the result of a dynamic process of change and adaptation to change. Second, plans are particularly susceptible to observation induced change. An observation induced change occurs when a company changes its plans as a result of the publication of estimates of planned spending. For example, during a downturn in business activity a company may plan to reduce costs by cutting back on investment. However, when the company sees that its competitors are planning to increase investment, it may change its plans to avoid having less production capacity than its competitors when business activity picks up. Thus, future market demand will not be just the sum of individual plans but will result from a complex interplay of planning by companies and unexpected developments that affect that planning. In addition, the process of surveying companies' plans and publishing the results may itself lead to changes in the plans1.

^{1.} For more discussion of the role of plans in economic analysis and for more analysis of plans see Part IV of The Quality and Economic Significance of Anticipations Data, Conference Proceedings, Princeton University Press for the National Bureau of Economic Research, Princeton, New Jersey, 1960, pp. 351-403.

Even if future market demand was the simple sum of individual companies' plans, a forecast would need to consider the effects of future market supply. Plans are closely guarded secrets of individual companies and, as such, are subject to implementation bottlenecks. That is, since there is no coordination among companies of planned purchases, bottlenecks may develop when all companies attempt to implement their plans. For example, if many companies plan to purchase the same type of equipment, shortages of that equipment may develop when the companies begin to make their purchases. A forecast should at least attempt to capture such possibilities. Despite this distinction between planned and forecasted investment, analysts frequently use the estimates of planned spending as forecasts.

In response to this use of the estimates as forecasts and to Statistical Policy Directive No. 3 issued by the Office of Management and Budget, an adjustment is made for consistent differences between planned and actual spending. This adjustment is made to the estimates of planned spending. Specifically, planned spending is divided by the median of the ratios of planned to actual spending for the same time horizon in

Anderson and Erceg recently evaluated the estimates of planned spending as forecasts of both actual spending on plant and equipment, and of nonresidential fixed investment. Credible are Capital Spending Surveys as Forecasts?" by Gerald H. Anderson and John J. Erceg, Economic Commentary, Federal Reserve Bank of Cleveland, December 1, 1990, ISSN 0428-This continues a tradition of more than 20 years. Examples are: "A Comparison of Alternative Econometric Models of Quarterly Investment Behavior," Dale W. Jorgenson, Jerald Hunter, and M. Ishag Nadiri, <u>Econometrica</u>, Vol. 38, Number 2, March 1970, pp. 187-212. "A Comparison of Anticipatory Surveys and Econometric Models in Forecasting U.S. Business Investment," J. Steven Landefeld and Eugene Seskin, Journal of Economic and Social Measurement, Volume 14, Number 1, 1986, pp. 77-86. "How Accurate Are Capital Spending Surveys?" Karen Bradley and Avril Euba, Federal Reserve Bank of New York Quarterly Review, Winter 1977-78, p. 10-15.

^{2.} Paragraph 6.d of Statistical Policy Directive No. 3 states, "If preliminary estimates show signs of a consistent bias (for example, if revisions are consistently in the same direction), the agency shall take steps to correct this bias. While a change from planned to actual spending is not the same sort of revision as a change from a preliminary to a final estimate of the same quantity, at some point in the past it was decided to adjust the estimates of planned spending.

the same quarter of each of the last eight years¹. By adjusting estimates of planned spending for consistent differences between planned and actual spending, we actually create a hybrid plan/forecast². The estimate is not strictly a forecast because no attempt is made to anticipate the status of factors thought to affect future market supply. However, the estimate is no longer strictly an estimate of planned spending either. It is a hybrid.

We should note that if we assume the interaction of market demand and market supply results in actual spending that deviates from planned spending in a stable manner, than the adjustment just cited transforms the estimate of planned spending into a forecast. The basis for such an assumption is not clear however, and we have not endorsed it. For simplicity, we will refer to our hybrid estimates as estimates of planned spending.

In this document, we discuss the accuracy of the estimates of planned spending as forecasts of future investment. We do this by comparing planned percentage changes to actual percentage changes, and by comparing the errors associated with planned percentage changes to the errors associated with two simple models. We find that combining the planned changes with a simple model produces a model that is more accurate, in at least one sense, than either of its inputs. The process of making the comparisons is used as an opportunity to warn readers of pitfalls that frequently "trip up" users of data from the P&E Survey.

^{1.} The adjustments are made at the industry level. That is estimated planned spending for an industry is adjusted. Economic explanations of the consistent differences between planned and actual spending have tended to focus on the behavior of individual companies. For example see, "The Realization of Investment Plans: A Microeconometric Approach," Michael J. McKelvey, Ph.D. dissertation, University of Pennsylvania, 1980. However, most empirical work on these same discrepancies has tended to utilize data at the industry level. For example, "The Realization of Plans Reported in the BEA Plant and Equipment Survey," Frank de Leeuw and Michael J. McKelvey, Survey of Current Business, October 1981, pp. 28-37.

^{2.} If we adjusted individual company data, we would be adjusting for unexpected events that affect the realization of the company's plans. When we adjust industry data, we take into account factors affecting individual companies and the overall effects of the interplay of companies in the marketplace. This results in smaller differences between estimated planned and actual spending for the industry but it further blurs the definition of what we are estimating.

We will not discuss other important uses of the survey results. For example, the P&E Survey produces timely estimates of investment spending by detailed industries. These estimates are important to analysts who need timely estimates of actual investment by industry instead of, or in addition to, estimates of planned future investment.

The paper is divided into an introduction, three discussion sections, a summary, and two appendices. The discussion sections are devoted to the technical aspects of using data from the P&E Survey and comparing forecasted and actual investment. The reader who wishes to read only results will want to skip to at least the third discussion section and possibly to the summary. In the first discussion section, we discuss the choice of data series for use in the evaluation. The second discussion section deals with two simple forecast models we have chosen for comparison to the plans, and with a description of the statistics we will present. In the third discussion section, we discuss the results of comparing the chosen estimates of planned spending and the model forecasts to actual spending. The results found here are updates of the results found by earlier researchers.

Data Series

In this section, we discuss the level of detail at which estimates are tabulated and/or evaluated, the measure of actual spending to be used, and the measures of planned spending to be used. These are technical matters that may not interest the more general reader but are very important to anyone who constructs forecasts of investment spending or needs to understand the strengths and weaknesses of such forecasts at a detailed level.

Level of Detail

Before we discuss the specific variables to be used, we need to consider the level of detail at which we will view investment. There are two basic approaches to this issue. One is to work with total investment and ignore questions concerning the details of which the total is composed. The other is to work with estimates of investment at some level of industry detail and sum the details to obtain a total. Either of the two approaches can be followed when evaluating forecasts. We can limit the evaluation to the accuracy of total spending or evaluate the accuracy of forecasts of detailed industries and of total spending.

If we followed the processing of the P&E Survey, we would work at a detailed level. During survey processing, planned spending by detailed industries is estimated and then the details are summed to obtain total planned spending. This allows us to consider both what change is planned for total investment and what the sources of that change may be. However, evaluating the accuracy of detailed forecasts becomes confusing very quickly and is outside the scope of this paper. Therefore, we will limit our evaluation to the accuracy of total spending.

Note that this approach can be misleading. We will compare planned spending with forecasted spending. The forecasts will be made at the total level while the plans were estimated at the detailed level. If details were forecasted and the forecasts summed, a different forecast of the total would result. It might be more or less accurate than the ones cited here. Nonetheless, everything that follows is in terms of total investment in the U.S. The resource requirements of creating detailed forecasts just to provide comparisons are too great. To avoid confusion, we simplify further by using the "All industries" total from the P&E Survey as our measure of both quarterly and annual, survey-based total investment.

Actual Spending

Two estimates of actual investment are available. One is the estimate of actual spending derived from the P&E Survey. The other is the estimate of Nonresidential Fixed Investment (NRFI) produced as part of the national income and product accounts for the U.S. prepared by the Bureau of Economic Analysis². The estimates of planned spending are more closely related to the estimates of actual P&E. However, many data users use the estimates of planned spending as forecasts of NRFI and might prefer that we use NRFI for the evaluation. The difficulty with using NRFI is that it combines an evaluation of the accuracy of planned spending as a forecast with an evaluation of the accuracy of P&E as a proxy for NRFI.

^{1.} A more inclusive total called "Total nonfarm" is actually estimated once a year. Total nonfarm consists of "All industries" plus real estate; professional services; membership organizations and social services; and forestry, fisheries, and agricultural services. The industries estimated only once a year account for approximately 10% of Total nonfarm spending.

^{2.} All data on nonresidential fixed investment were acquired on January 31, 1991. Revisions since that date are not reflected in the tabulations and/or results cited here.

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We will use actual P&E as the variable being forecast by the estimates of planned spending. We will also compare P&E to NRFI for the convenience of those whose primary focus is NRFI. Analysts who want to forecast NRFI are advised to first estimate the relationship between P&E and NRFI; and to then use a forecast of P&E with the estimated relationship to construct a forecast of NRFI. That way if a forecast error does develop, they can identify the source of the error as either the use of planned spending as a forecast of future spending, or the use of P&E as a proxy for NRFI.

Of course, this does not completely settle matters since we can use either levels of spending or percentage changes in spending. In keeping with the majority of studies of this type, we will use the percentage change in spending -- both planned and actual -- as the variable of interest. We will consider percentage changes in current-dollar estimates so any issues of deflation technique and/or the projection of deflators can be avoided.

Planned Spending

The need to use revised estimates of actual spending is widely understood, but the need to use revised estimates of planned spending is less commonly known. To understand the sources of revised estimates of planned spending and why these estimates should be used, we will briefly discuss estimates of actual spending and why they are revised.

Every quarter, survey responses are used to estimate both levels of spending and changes in actual spending. Changes in reported company spending are used to extrapolate previously estimated detailed industry spending. Spending by aggregate industries is estimated by summing the estimates of detailed industries, and estimated changes in aggregate spending are the changes implied in these sums. These estimates are revised each year when updated seasonal adjustment factors are incorporated into the series, and every five years when benchmarking occurs. During benchmarking, both survey and nonsurvey sources are used to construct estimates of spending by detailed industries for a benchmark year. Spending by detailed industries for nonbenchmark years is revised to be consistent with spending during the benchmark year. Quarterly estimates are revised to be consistent with the new annual estimates and new quarterly and annual aggregates are calculated.

Planned spending is estimated each quarter by extrapolating the actual spending estimated that quarter. When the level of actual spending that was the basis for an extrapolation is revised, the planned spending needs to be revised as well. The survey staff tabulate and publish revised estimates of planned spending

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whenever revised estimates of actual spending are introduced. Usually, the ratio of planned spending to actual spending for each quarter is used to revise planned spending at the detailed industry level. When appropriate, planned spending may be revised by using revised estimates of change to re-extrapolate from the revised level of actual spending.

Some loss of intuitive appeal occurs when the originally published plans are not used to evaluate a forecast. But the gain in historical consistency makes the loss necessary. The time series of actual spending over any significant range cannot be consistent unless revised data are used. When the revised estimates of actual spending are used, it is necessary to use revised estimates of planned spending to avoid comparing "apples and oranges." Appendix A uses data from past survey revisions to illustrate this point. We use revised estimates of both actual and planned spending throughout this paper.

Models and Statistics

In this section, we discuss the two forecast models to be used for comparisons, the measurement of period-to-period changes, and the summary statistics to be used. The general reader may find some of this material technical in nature but it is helpful material for anyone wishing to choose a forecast to base decisions upon.

Models

There are an infinite number of models that can be used to forecast investment spending. We have chosen two for comparison purposes. The first is a widely cited, simple model based on the well known "Random Walk" assumption. Under this assumption, we use this period's change as the forecast of next period's change. The use of the Random Walk model is usually interpreted as meaning that differences between periods are due to random chance or at least that no better prediction can be made of what they will be.

The second model we will use is based on a combination of the Random Walk assumption and the planned changes from the P&E Survey. Under this approach, we average this period's change with the planned change for next period to get a forecast of next period's change. We will call this the "Average" model. The Average model is based on the rationale that actual changes result from a combination of changes beyond the control of individual companies and planned changes. The changes beyond the control of individual companies are approximated with a Random Walk approach.

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Both of these models are in reduced form. That is, there is assumed to be some underlying model of supply and demand that yields the forecasting equations discussed here. We are interested in the models only as a means of demonstrating the effect of including data on planned spending in a model. We are not concerned here with analyzing the underlying structural form.

We used each of these models to generate a time series of forecasts of changes in spending. Quarterly forecasts were done only one quarter ahead and were based on revised quarterly estimates of actual spending in quarters earlier than the one being forecasted. Annual forecasts were based on revised estimates of annual spending in years earlier than the one being forecasted. The forecasts were then compared to actual spending to obtain the summary statistics cited below.

Period-to-period changes

When discussing planned quarterly changes, we will always refer to changes across one quarter. This needs to be explicitly stated because a two- or three-quarter ahead planned change can be stated as either a change from quarter-to-quarter or a change across several quarters. The following example illustrates the convention adopted here and shows the methodology for calculating errors in quarterly changes.

The fourth quarter survey of 1973 produced the estimates shown below.

Table A. Actual and Planned Spending Estimates: Fourth Quarter, 1973 Survey

Year:Quarter	Actual Spendi	ng Planned Spending	Errors				
	Billions	of Dollars					
1973:IV	31.44	NA	•••				
1974:I	NA	33.09	• • •				
1974:II	NA	33.55					
1974:III	NA	35.29	•••				
Quarterly Percentage Changes							
1973:IV	2.5	NA	•••				
1974:I	NA	5.3	• • •				
1974:II	NA NA	1.4	• • •				
1974:III	NA	5.2	•••				
Actual	Percentage Ch	anges and Associated	Errors				
1973:IV	2.5	NA	NA				
1974:I	3.5	5.3	1.8				
1974:II	6.7	1.4	-5.3				
1974:III	3.0	5.2	2.2				

Summary Statistics

We will summarize period-to-period changes, errors in forecasts of those changes, and differences between the changes in different data series with three commonly used concepts. These are the mean, the mean absolute, and the root mean square. The formulas for these measures are shown in Appendix B. The following abbreviations are used for convenience throughout the paper: mean change (MC), mean error (ME), mean difference (MD), mean absolute change (MAC), mean absolute error (MAE), mean absolute difference (MAD), root mean squared change (RMSC), root mean squared error (RMSE), and root mean squared difference (RMSD).

The mean is a measure of central tendency. Whether a mean -change, -error, or -difference, it reflects the center around which the observations are grouped. When both negative and positive values occur, they cancel out and the mean will be zero if they cancel out exactly. This canceling of negatives and positives can sometimes cause confusion so the mean absolute is also calculated.

The mean absolute removes the canceling of negatives and positives from the mean. It is useful when a series contains large absolute values that are evenly distributed about zero.

The mean of the series may be quite small and the mean absolute is needed to reveal the presence of the large values.

The root mean square emphasizes the presence of large values. When the root mean squared errors associated with two forecasts are compared, any "large" errors in either forecast will be exponentially emphasized and therefore more easily noticed.

Consider the following example.

Table B. Example of Mean, Mean Absolute, and Root Mean Squared

Actual Spending	Forecast	Error	Absolute Error	Squared Error
100	50	- 50	50	2500
80	85	5	5	25
50	100	50	50	2500

Mean Error....1.7

Mean Absolute Error....35.0

Root Mean Squared Error.....40.9

The mean error shows the central tendency but does not reveal the large errors. The mean absolute error reveals the presence of large errors because no canceling occurs. The root mean squared error emphasizes the presence of large errors because the squared error increases more than proportionally when the error increases. Since a single 25% error is often regarded as more serious than five 5% errors, the root mean squared error is frequently used to evaluate forecasts.

All tables of summary statistics will display three sets of results. We will show the results for the entire time period 1970:I to 1989:IV and for the two subperiods, 1970:I-1979:IV and 1980:I-1989:IV. We do this so we can verify that the results being obtained are not sensitive to the specific time period chosen. Whenever we compare statistics, we will verify that the comparison is valid for all three periods considered. Comparisons that yield different results in different periods must be considered less significant than those that are verified for multiple time periods.

Data and Comparisons

In this section, we discuss both the characteristics of the data series under study and the results of comparing planned spending and the forecasts to actual spending. We consider comparisons based on the summary statistics cited above and on forecasting the turning points of investment spending. We end with a comparison of P&E spending with NRFI.

For simplicity, we will use our earlier abbreviations which do not explicitly contain the word "percent." All changes, errors, and differences referred to in abbreviations are really percent changes, percent errors, and percent differences.

Summary Statistics

The table below provides a context for discussing errors associated with forecasting P&E spending. It shows a summary of the quarterly and annual percentage changes that have occurred in actual P&E spending from 1970 to 1989. The changes in actual P&E spending are important because we need to know how large the values associated with the series we're trying to forecast are. If we're forecasting a series with an average value of 1.0, an error of 10.0 is disheartening to say the least. If we're trying to forecast a series with an average value of 1000.0, an error of 10.0 may be quite acceptable. The MAC and RMSC are shown to provide an idea of the range of the values around the trends reflected by the MC.

Table C. Period-to-Period Changes in Plant and Equipment Expenditures

Root Mean Squared Percent Change	Mean Absolute Percent Change	Mean Percent Change
_		
3.3	2.8	2.3
3.7	3.2	2.8
2.9	2.5	1.7
11.6	9.9	9.5
13.1	11.7	11.7
9.8	8.1	7.3
	Squared Percent Change 3.3 3.7 2.9	Squared Percent Percent Change Change 3.3 2.8 3.7 3.2 2.9 2.5 11.6 9.9 13.1 11.7

An important characteristic of the P&E series can be noted. The RMSC's and MAC's are only slightly larger than the MC's. This indicates a general absence of changes that are much larger than the average change. We can note that a series without

large, possibly erratic, variations in size is much easier to forecast and in particular, is a good candidate for the Random Walk model.

Table 1 presents a summary of the results of forecasting P&E spending. Error summaries are shown for one-, two-, and three-quarter ahead plans, for the Random Walk model, and for the Average model. Only one-quarter ahead forecasts are made with the two models. The annual forecast from the Average model uses fourth-quarter year ahead plans since the third quarter plans were not available before 1985. We will first discuss the plans and the Random Walk model. Later we'll return to discuss the Average model.

Since the ME's are not 0.0, we can note that the estimates of planned spending are not unbiased forecasts. The quarterly ME's range from -0.6 for two-quarters ahead plans for 1970:I-79:IV, to 1.1 for three-quarters ahead plans for 1980:I-89:IV. The annual ME's range from -0.1 for 4th quarter plans for 1970-79, to 1.6 for 4th quarter plans for 1980-89.

The mean quarterly errors and the mean annual errors associated with planned spending are similar in magnitude. The mean annual error for 1970-89 is 0.8 while the mean quarterly errors during those years are 0.6 for one-quarter ahead plans, 0.1 for two-quarter ahead plans, and 0.8 for three-quarters ahead plans. If all other things were equal, we would expect to observe larger ME's when forecasting a series with larger MC's. The mean annual change in P&E spending is more than four times the mean quarterly change. The fact that the ME's of these series are similar when the MC's are so different indicates that the annual plans are a more accurate forecast than any of the quarterly plans.

The Random Walk model was used to produce both quarterly and annual forecasts. The ME's associated with these forecasts are usually smaller than those associated with planned spending but the RMSE's and MAE's associated with the forecasts are usually larger than those associated with planned spending. The differences in all three measures seem small when quarterly data are considered but when the annual data are considered the RMSE's and MAE's of the Random Walk forecasts are twice as large as those associated with planned spending. The Random Walk model and planned spending are about equal as quarterly forecasts but planned spending is probably a better annual forecast.

^{1.} These are independent forecasts. That is, the annual forecast is the result of using annual data in the Random Walk model, not the result of summing quarterly forecasts. Similarly, the quarterly forecasts are not dependent upon (or necessarily consistent with) the annual forecasts.

The Average model, which uses information from both the estimates of planned spending and the Random Walk model, is a better forecaster of quarterly changes than either the Random Walk model or planned spending. The ME of the Average model is larger than the ME of the Random Walk in only one case, 1980:I-89:IV. The MAE and RMSE of the Average model are always smaller than those of both planned spending and the Random Walk model. The consistently smaller squared errors associated with the Average model indicate that, in general, the largest errors associated with either planned spending or the Random Walk model are being dampened.

Comparing the results of the two models and planned spending for annual changes gives different results. While none of the three always has smaller ME's, planned spending has smaller RMSE's and MAE's. Thus, we have not specified a forecast model for annual changes that is better than planned spending.

Turning Points

Another approach to forecast accuracy is to examine whether a forecasting procedure correctly "signals" cyclical changes in spending. Table 2 shows the quarterly percentage changes in actual P&E spending for the period 1970:I to 1989:IV. It also shows the planned changes and the forecasted changes for each quarter. We will define a cyclical trough in investment to occur whenever negative quarterly change in actual spending is observed for three consecutive quarters.

On this basis, there have been three cycles since 1970. A peak in 1974:IV was followed by a trough in 1975:III. The next peak came in 1982:I and was followed by a trough ending in 1983:I. Then a peak in 1985:III was followed by a trough in 1987:I.

We will say a forecast has signaled a change when the forecasted percent change goes from positive to negative, or vice versa, and keeps its new sign for two consecutive quarters. That is, when the forecast changes sign and doesn't immediately change back. If a forecast changes sign and then immediately changes back, we will ignore the change. That is, we won't consider the new For example, the first signal sent by the one change a signal. quarter ahead plans is in 1975: I when the series becomes negative and stays negative in 1975:II. Note that we say the signal is in 1975:I although 1) we don't confirm it as a signal until we see that 1975:II is also negative, and 2) the one quarter ahead plan for 1975: I would actually have been reported in 1974: IV. The one quarter ahead series went from negative to positive in 1971:I but we don't count this as a signal because it had only been negative for one quarter.

The table below shows the actual turning points in P&E and the downturn and upturn signals from each series using this definition.

Table D. Turning Points in P&E and Signals From Various Forecasts, 1970:I to 1989:IV

Series	Turning Point or S	ignal
	Downturn	Upturn
Actual P&E	1975:I 1982:II 1985:IV	1975:IV 1983:II 1987:II
One quarter ahead plan	1975:I 1980:II 1982:II	1975:III 1981:I 1983:I
Two quarters ahead plan	1986:III	1987:I
Three quarters ahead plan	1986:II	1987:I
Random Walk model	1971:I 1975:II 1982:III 1986:I	1972:I 1976:I 1983:III 1987:III
Average model	1975:I 1982:III 1986:II	1975:IV 1983:II 1986:IV

There are two things to consider when looking at turning points. First, we ask if the forecast correctly signals future turns (both up and down). The forecast may signal a turn or it may miss the turn. Then, we ask if the forecast sends false signals of turns that don't occur. The forecast may indicate a turn when none occurs. Finally, we "split hairs" by asking when a signal is early or late rather than false. We will count signals that are one quarter early as correct. Signals that are one quarter late will be late. Signals more than one quarter away from a turning point will be false. On this basis, we have the following summary.

Table E. Summary of Correct, Late, and False Signals of Turning Points: 1970:I to 1989:IV

Series	Correct	Late	Miss	False
One quarter ahead plan	4	0	2	2
Two quarters ahead plan	1	0	5	1
Three quarters ahead plan	1	0	5	1
Random Walk model	0	6	0	2
Average model	3	1	2	2

The one-quarter ahead planned change correctly signaled four turning points. It was never late but missed 2 turns completely and sent 2 false signals. The two quarters ahead plans and the three quarters ahead plans were each correct only once and both missed 5 of the 6 turning points that occurred. At least, they each sent only 1 false signal.

The Random Walk model was always late and sent 2 false signals. In general, it is simply announcing what has happened. The Average model was correct 3 times and late once. It missed 2 turns and sent 2 false signals.

In general, the one-quarter ahead plans and the Average model provide the best forecasts of turning points. They both sent 2 false signals and missed 2 turns but the Average model was late once. We can note the two- and three- quarter ahead plans miss most turns but send fewer false signals. With only 6 turns to consider this may not be strongly established, but it appears that when these plans do send a signal that signal should be considered significant.

Nonresidential Fixed Investment

As was discussed earlier, spending on plant and equipment can be used as a proxy for NRFI. Table 3 summarizes the differences between the period-to-period percentage changes in these two series. Note that the MD's are 0.4 or less for quarterly changes, and vary from 0.5 to 1.3 for annual changes. Compare this to the MC's in NRFI which range from 1.2 to 2.8 for quarterly changes, and from 5.6 to 11.7 for annual changes. Similar relationships can be cited between MAD's and MAC's, and between RMSD's and RMSC's. The movement of P&E appears to be a reasonable proxy for the movement of NRFI during any one time period.

Over time, unfortunately, a different situation has developed. The positive mean differences shown indicate that the percent change in P&E is consistently above that in NRFI. In light of this, we could expect the two series to diverge over time. What has happened is that, although NRFI includes all of "Total"

nonfarm business" and "All industries" P&E does not, P&E has gradually increased relative to NRFI and in late 1989 P&E became larger than NRFI for the first time. The meaning of this event is not yet fully understood. It's significance is difficult to judge because the values of both series during 1989 will be revised several times in coming years due to normal benchmarking procedures.

Summary

The error in any forecast must be considered in the context of the size of the variable being forecast as well as the variation of that variable. A 2.0 percentage point error in the forecast of a growth rate whose average size is 10.0 percent is not the same as a 2.0 percentage point error in the forecast of a growth rate whose average size is 1.0 percent.

The table below accounts for these considerations. It shows the root mean squared error of various forecasts divided by the mean change in the series being forecast. It also shows the root mean squared difference between P&E and NRFI divided by the mean change in NRFI. These ratios are shown for the entire period 1970:I-1989:IV in the left column. The other two columns show the periods 1970:I-1979:IV and 1980:I-1989:IV. The two subperiods are shown so we can check for results that are sensitive to the specific time period chosen for analysis. None of our results appear to be sensitive. In the table, smaller numbers are associated with better forecasts and with better proxies.

One-quarter ahead plans and the Random Walk model have quarterly ratios of 1.0 and 1.1. They are equivalent forecasters of quarterly change. The Average model has a quarterly ratio of 0.7. It is a better forecaster of quarterly change. The Random Walk model is the poorest forecaster of annual change with a ratio of 0.9. The year-ahead plans and the Average model have annual ratios of 0.4 and 0.5 which indicate little difference in their forecasting ability.

Comparing P&E to NRFI gives a quarterly ratio of 1.6 for the entire period and an annual ratio of 0.4. It seems clear that annual changes in P&E are a better proxy for annual changes in NRFI than quarterly changes in P&E are of quarterly changes in NRFI.

^{1.} See footnote 1 on page 5 for discussion of the coverage difference between P&E and NRFI that lead us to expect P&E to be less than NRFI.

Therefore, if one is going to use the estimates of planned changes in spending as a forecast of actual changes in spending, one should place more faith in the annual plans. It is better, however, to use the estimates of planned changes as one input in the construction of a forecast. A forecast that uses the estimates of planned changes as well as other data can predict actual changes better than the planned changes alone.

Table D. Ratios of Root Mean Squared Errors (or Differences) to Mean Changes

	1970:I to	1970:I to	1980:I to
	1989:IV	1979:IV	1989:IV
Quarterly			
1-quarter ahead plan	1.0	0.8	1.3
2-quarters ahead plan	1.3	1.1	1.7
3-quarters ahead plan	1.5	1.1	2.1
Random Walk	1.1	0.9	1.2
Average Model	0.7	0.6	0.8
P&E to NRFI	1.6	1.3	2.4
	1970-89	1970-79	1980-89
Annual			
4th quarter survey	0.4	0.3	0.5
Random Walk	0.9	0.6	1.2
Average Model	0.5	0.4	0.7
P&E to NRFI	0.4	0.3	0.5

Table 1. Root Mean Squared Errors (RMSE), Mean Absolute Errors (MAE) and Mean Errors (ME) in Planned and Forecasted Period-to-Period Percentage Changes

Quarterly Data Survey Plans	RMSE	MAE 70:I-8	ME 9:IV	RMSE 1970	MAE M ::1-79:1		RMSE 1980	MAE 0:I-89	ME IV
1-quarter ahead 2-quarters ahead 3-quarters ahead	2.2 3.0 3.3	1.8 2.4 2.5	0.6 0.1 0.8		1.9 0 2.5 -0 2.3 0	.6	2.2 2.9 3.5	1.8 2.4 2.7	0.9 0.7 1.1
Models Random Walk Average Model	2.4 1.5	1.9	0.0	2.7 1.7	2.2 -0 1.5 0		2.1	1.7	0.1 0.5
Annual Data Survey Plans	19	970-89		19	70-79		1	L980-8	9
4th quarter survey 3rd quarter survey		2.7 availa		3.3 ice 1985	2.9 -0 5)	.1	3.5 3.4	2.6 3.0	1.6
Models Random Walk Average Model	8.2 5.3	6.5 4.4	0.0 -0.1	7.2 4.5	5.7 -0 3.7 -0		9.0 6.0	7.3 5.1	0.6

Notes:

Quarter-to-quarter changes are calculated as follows: For 1-quarter ahead - the planned change is the change from the previous quarter's actual to this quarter's planned level; for 2-quarters ahead - the planned change is the change from the previous quarter's 1-quarter ahead to this quarter's 2-quarters ahead plan; and for 3-quarters ahead - the planned change is the change from the previous quarter's 2-quarter ahead plan to this quarter's 3-quarter ahead plan. In all cases, the planned percentage change is compared to the final actual percentage change. When changes are viewed in this manner, the actual quarter-to-quarter change is the result of a series of revisions from 3-quarter ahead planned change to 2-quarters ahead to 1-quarter ahead to actual.

The Random Walk estimates are constructed by lagging the actual changes one period. Thus the 1-quarter ahead quarterly Random Walk estimate is last quarter's actual change. There are no 2-quarter or 3-quarter ahead Random Walk estimates.

The Average Model estimates are constructed by averaging the planned percentage change for one-period ahead and the Random Walk estimate for the same period. This approach can be said to result from a model that assumes actual changes will be the result of both individual plans and events beyond the control of the individual. The events outside individual control are approximated with a simple Random Walk forecast.

Table 2. Actual, Planned, and Forecasted Percentage Changes in Plant and Equipment Expenditures, 1970:I-89:IV

Actual Year Quarter Changes			−Qua		Ahead	Model Forecasts		
rear		Changes	One		Three	Walk	Average	
1970	_I	1.7	4.5		1.6	1.0	2.8	
	_II	2.3	0.9	2.1	5.2	1.7	1.3	
	III ·			5.5	NA	2.3	1.4	
	IV	-2.6	-0.8	3.5	NA	1.5	0.3	
1971	I	-0.8	0.7	0.5	-2.0	-2.6	-1.0	
	II	2.6	5.1	4.4	5.0	-0.8	2.2	
	III	-0.2	2.7	2.5	NA	2.6	2.6	
	IV	2.9	2.2	-0.7	NA	-0.2	1.0	
1972	I	3.7	7.9	6.7	7.3	2.9	5.4	
	II	1.9	5.5	0.9	4.7	3.7	4.6	
	III ·	2.4	6.3	2.4	NA	1.9	4.1	
	IV	7.0	8.0	0.3	NA	2.4	5.2	
1973	I	2.3	3.8	2.4	2.7	7.0	5.4	
	II	4.9	6.6	3.1	3.1	2.3	4.4	
	III	3.5	5.6	3.2	3.7	4.9	5.2	
	IV	2.5	3.5	0.2	-1.8	3.5	3.5	
1974	I	3.5	5.2	3.4	3.8	2.5	3.9	
	II	6.7	6.1	1.4	4.6	3.5	4.8	
	III	3.0	2.2	2.5	5.2	6.7	4.4	
	IV	2.7	1.1	1.5	2.1	3.0	2.1	
		downturn						
1975	I	-1.8	-5.0	2.8	7.4	2.7	-1.2	
	II	-1.8	-2.7	0.8	2.1	-1.8		
	III	-0.4	0.9	0.3	5.7	-1.8		
~·		upturn						
	IV	1.3	3.5	0.7	2.4	-0.4	1.5	
1976	I	5.1	5.3		6.0	1.3		
	II	2.9	5.4		3.3	5.1	5.2	
	III	4.3		-0.2	2.1	2.9	3.9	
	IV	4.1	6.3		3.1	4.3		
1977	ľ	5.3	4.6		2.1	4.1	4.3	
	II	2.9	3.1	1.1	2.3	5.3	4.2	
	III	4.8	2.4		4.1	2.9		
	IV	-0.0	3.2	5.1	5.2	4.8	4.0	
1978	Ī	4.9	5.1	2.5	-0.7	-0.0	2.5	
	ΙÎ	6.3	2.5		0.6	4.9	3.7	
	III	3.0	0.9	3.8	3.6	6.3	3.6	
	IV	6.5	6.1	2.1	3.4	3.0	4.5	
1979	Ĭ	2.3		-0.1	1.4	6.5		
4313	ıΪ	4.6	2.8	1.4	3.8	2.3	3.6	
	III	4.1	1.1	3.0	4.5		2.5	
	IV	4.2	0.2	3.9	4.5 5.8	4.6	2.8	
	v	7.4		J.7	J.O	4.1	2.1	

continued...

Table 2. Actual, Planned, and Forecasted Percentage Changes in Plant and Equipment Expenditures, 1970:I-89:IV, continued

1980 I 3.6 1.5 5.9 1.6 II 1.9 -2.4 3.4 4.3 III 1.1 -1.0 4.0 5.3 IV 1.1 -1.3 1.8 5.3 1981 I 6.0 5.2 9.7 9.2 II 3.6 1.8 3.7 4.3 III 4.2 2.7 4.0 5.2 IV 0.2 1.3 2.9 4.0 1982 I 1.3 2.0 6.3 8.0	alk 4.2 3.6 1.9 1.1	2.8 0.6 0.4 -0.1 3.2
1980	4.2 3.6 1.9 1.1 1.1 6.0 3.6 4.2	0.6 0.4 -0.1 3.2
II	3.6 1.9 1.1 1.1 6.0 3.6 4.2	0.6 0.4 -0.1 3.2
III 1.1 -1.0 4.0 5.3 IV 1.1 -1.3 1.8 5.3 1981 I 6.0 5.2 9.7 9.2 III 3.6 1.8 3.7 4.3 III 4.2 2.7 4.0 5.2 IV 0.2 1.3 2.9 4.0 1982 I 1.3 2.0 6.3 8.0	1.9 1.1 1.1 6.0 3.6 4.2	0.4 -0.1 3.2
IV 1.1 -1.3 1.8 5.3 1981 I 6.0 5.2 9.7 9.2 II 3.6 1.8 3.7 4.3 III 4.2 2.7 4.0 5.2 IV 0.2 1.3 2.9 4.0 1982 I 1.3 2.0 6.3 8.0	1.1 1.1 6.0 3.6 4.2	-0.1 3.2
1981 I 6.0 5.2 9.7 9.2 III 3.6 1.8 3.7 4.3 IIII 4.2 2.7 4.0 5.2 IV 0.2 1.3 2.9 4.0 1982 I 1.3 2.0 6.3 8.0	1.1 6.0 3.6 4.2	3.2
III 3.6 1.8 3.7 4.3 IIII 4.2 2.7 4.0 5.2 IV 0.2 1.3 2.9 4.0 1982 I 1.3 2.0 6.3 8.0	6.0 3.6 4.2	
III 4.2 2.7 4.0 5.2 IV 0.2 1.3 2.9 4.0 1982 I 1.3 2.0 6.3 8.0	3.6 4.2	2 0
IV 0.2 1.3 2.9 4.0 1982 I 1.3 2.0 6.3 8.0	4.2	3.9 3.1
1982		
II -1.6 -0.7 1.8 3.4 III -3.2 -1.6 1.3 3.8 IV -2.7 -0.4 0.9 1.1 1983 I -2.2 1.0 2.3 3.1		
II		1.1
III	1.3	0.3
IV -2.7 -0.4 0.9 1.1 - 1983 I -2.2 1.0 2.3 3.1 -	1.6	
1983 I -2.2 1.0 2.3 3.1	3.2	
II 1.3 5.3 -0.4 0.8 -1	2.7	-0.9
II 1.3 5.3 -0.4 0.8 -1 III 4.1 7.5 0.7 2.3 IV 4.8 6.3 3.1 3.2 1984 I 5.4 8.7 2.5 3.5 II 3.1 2.9 1.4 1.3 III 3.4 4.5 3.6 4.6 IV 0.5 0.9 -1.3 1.3 1985 I 3.7 6.2 8.3 7.4 II 3.8 2.9 1.7 1.1 III 0.5 0.5 1.9 1.4	,	-0.9
III 4.1 7.5 0.7 2.3 IV 4.8 6.3 3.1 3.2 1984 I 5.4 8.7 2.5 3.5 III 3.1 2.9 1.4 1.3 III 3.4 4.5 3.6 4.6 IV 0.5 0.9 -1.3 1.3 1985 I 3.7 6.2 8.3 7.4 II 3.8 2.9 1.7 1.1 III 0.5 0.5 1.9 1.4	2.2	1.6
IV 4.8 6.3 3.1 3.2 1984 I 5.4 8.7 2.5 3.5 II 3.1 2.9 1.4 1.3 III 3.4 4.5 3.6 4.6 IV 0.5 0.9 -1.3 1.3 1985 I 3.7 6.2 8.3 7.4 II 3.8 2.9 1.7 1.1 III 0.5 0.5 1.9 1.4	1.3	4.4
1984 I 5.4 8.7 2.5 3.5 II 3.1 2.9 1.4 1.3 III 3.4 4.5 3.6 4.6 IV 0.5 0.9 -1.3 1.3 1985 I 3.7 6.2 8.3 7.4 II 3.8 2.9 1.7 1.1 III 0.5 0.5 1.9 1.4	4.1	5.2
II 3.1 2.9 1.4 1.3 III 3.4 4.5 3.6 4.6 IV 0.5 0.9 -1.3 1.3 1985 I 3.7 6.2 8.3 7.4 II 3.8 2.9 1.7 1.1 III 0.5 0.5 1.9 1.4	4.8	6.8
III 3.4 4.5 3.6 4.6 IV 0.5 0.9 -1.3 1.3 1985 I 3.7 6.2 8.3 7.4 II 3.8 2.9 1.7 1.1 III 0.5 0.5 1.9 1.4	5.4	4.2
IV 0.5 0.9 -1.3 1.3 1985 I 3.7 6.2 8.3 7.4 II 3.8 2.9 1.7 1.1 III 0.5 0.5 1.9 1.4 downturn IV -0.3 -2.7 -3.1 -2.0 1986 I -2.2 1.5 7.2 7.6	3.1	3.8
1985 I 3.7 6.2 8.3 7.4 II 3.8 2.9 1.7 1.1 III 0.5 0.5 1.9 1.4 downturn IV -0.3 -2.7 -3.1 -2.0 1986 I -2.2 1.5 7.2 7.6	3.4	2.2
II 3.8 2.9 1.7 1.1 III 0.5 0.5 1.9 1.4 downturn IV -0.3 -2.7 -3.1 -2.0 1986 I -2.2 1.5 7.2 7.6	0.5	3.4
III 0.5 0.5 1.9 1.4 :downturn IV -0.3 -2.7 -3.1 -2.0 : 1986 I -2.2 1.5 7.2 7.6	3.7	3.3
IV -0.3 -2.7 -3.1 -2.0 1986 I -2.2 1.5 7.2 7.6	3.8	2.2
1986 I -2.2 1.5 7.2 7.6 -		
1986 I -2.2 1.5 7.2 7.6 -	0.5	-1.1
		0.6
II -1.6 1.3 1.2 -0.6 -	2.2	-0.4
	1.6	-0.3
	8.0	1.1
****	1.7	1.8
upturn		
II 1.0 5.0 2.7 1.2 -	1.4	1.8
III 3.7 5.1 1.2 0.7	1.0	3.0
	3.7	4.4
1988 I 2.1 5.3 4.7 6.7	3.2	4.2
II 3.7 4.8 1.2 0.4	2.1	3.4
III 1.8 2.8 1.3 1.6	3.7	3.2
IV 1.6 2.2 0.9 0.3	1.8	2.0
1989 I 4.2 6.0 5.2 2.6	1.6	3.8
II 3.0 2.7 1.7 2.0	1.2	3.4
III 2.6 2.2 1.7 1.7		2.6
IV. 0.9 0.6 0.4 1.1	3.0	1.6

Table 3. Summary of Period-to-Period Changes in Monresidential Fixed Investment and Comparisons of That Series With Plant and Equipment Expenditures

Period-to-Period Changes in Nonresidential Fixed Investment

Frequency and Range	Root Mean Squared Percent Change	Mean Absolute Percent Change	Mean Percent Change
Quarterly			
1970:I-89:IV	3.3	2.8	2.0
1970:I-79:IV	3.6	3.1	2.8
1980:I-89:IV	3.0	2.5	1.2
Annual			
1970-89	11.2	9.1	8.6
1970-79	13.5	11.7	11.7
1980-89	8.4	6.6	5.6

Root Mean Squared Differences (RMSD),
Mean Absolute Differences (MAD), and Mean Differences (MD) Between
Percentage Changes in Plant and Equipment Expenditures and
Percentage Changes in Nonresidential Fixed Investment

	RMSD	MAD	MD	RMSD	MAD	MD	RMSD	MAD	MD
	197	0:I-89	9:IV	197	0:1-79	9:IV	198	0:I-8	9: IV
Quarterly	1.8	1.5	0.2	1.8	1.4	0.0	1.9	1.5	0.4
	1970-89		1970-79			1980-89			
Annual	3.1	2.3	0.9	3.4	2.8	0.5	2.6	1.8	1.3

Appendix A. An illustration of revising estimates of actual and planned spending.

The following estimates of spending by Public Utilities were published in June 1990 (all dollar amounts are in billions of seasonally adjusted dollars at annual rates):

	Actual Spending	Planned Spending		
_	90:I	90:II	90:III	90:IV
Public utilities	49.53	54.54	53.97	52.01
Electric	31.94	36.75	35.52	35.26
Gas and other	17.59	17.79	18.44	16.75

A benchmark revision was completed in September 1990 and revised estimates of the levels of spending in the first quarter of 1990 were published. The revised estimates of actual spending and the corresponding revised levels of planned spending are:

	Actual Spending	Planned Spending		
	90:I	90:II	90:III	90:IV
Public utilities	65.72	73.28	70.98	66.05
Electric	43.37	50.41	47.74	45.51
Gas and other	22.34	22.88	23.24	20.54

Any evaluation of survey accuracy that compared the originally published plans to the revised actuals would be seriously flawed. After all, the estimate of \$17.79 billion of planned spending by Gas and Other in the second quarter is an extrapolation of the \$17.59 billion in the first quarter. To change one and not the other would make the plan meaningless.

At first, it might seem plausible to work with percentage changes rather than levels and thereby avoid these difficulties. Unfortunately, unless one works at the most detailed level tabulated, this does not work. When detailed estimates of actual and planned spending are revised, aggregate estimates are revised by re-summing the details. Even though the ratios of planned to actual spending may be preserved at the detailed level, this is frequently not the case at the aggregate level.

More importantly, although period-to-period changes may be preserved at the detailed level, such changes are almost never preserved at the aggregate level. The percentage change in a sum is the weighted average of the percentage changes in the details being summed. The weights will be the proportion of the sum accounted for by each detail. Since a revision in detailed estimates can change the proportions accounted for by each

detail, the percentage change in any aggregate can change even when the percentage changes in the details do not.

In the example above, all of the estimates shown are aggregations of more detailed tabulations. Even in this case, the percentage changes in the aggregate "Public utilities" are sometimes revised more than those of either subaggregate shown.

QUARTER-TO-QUARTER PERCENTAGE CHANGES IN P&E

	Actual Spendi	ng Plan	Planned Spending		
	90:I	90:II	90:III		
	BEFORE REVISION				
Public utilities	-1.92	10.12	-1.05	-3.63	
Electric	-6.17	15.06	-3.35	-0.73	
Gas and other	6.87	1.14	3.65	-9.16	
		AFTER REVIS	TER REVISION		
Public utilities	-0.15	11.50	-3.14	-6.95	
Electric	-2.89	16.23	-5.30	-4.67	
Gas and other	5.63	2.42	1.57	-11.62	
Public utilities	-1.77	DIFFERENCES -1.39	2.09	3.31	
Electric	-3.28	-1.17	1.95	3.94	
Gas and other	1.24	-1.28	2.08	2.45	

Another problem arises when changes in the structure of the economy or in the needs of the survey users lead to changes in the structure of the survey. When the 1977 benchmark revisions were published in 1985, estimates of actual spending were revised back to 1947. In addition, some industries that were formerly surveyed every quarter began to be surveyed only annually. After this revision, quarterly estimates of total planned spending were no longer available for the same set of industries as previously. The new total was tabulated and published for 1947 forward so consistent historical estimates were available for analysis but a comparison of originally published estimates of total planned spending with this new historic series of total actual investment would not be meaningful.

The point here is simply that the revised data should be used in an evaluation. Some loss of intuitive appeal occurs when the originally published plans are not used but the gain in historical consistency make the loss necessary. Comparing originally published planned spending to finally revised actual spending is really "Comparing apples and oranges."

Appendix B. Summary Statistics

Consider two data series whose values are given by x_i , and y_j for $i,j=0,1,2,\ldots,n$ and let f_i be a forecast of x_i for $i=1,2,\ldots,n$. We define:

Mean Change = MC = (1/n) $\sum_{i=1}^{n} (x_i-x_{i-1})$

Mean Error = ME = (1/n) $\sum_{i=1}^{n} (f_i - x_i)$

Mean Difference = MD = (1/n) $\sum_{i=1}^{n} (x_i - y_i)$

Mean Absolute Change = MAC = $(1/n)\sum_{i=1}^{n} |x_i-x_{i-1}|$

Mean Absolute Error = MAE = $(1/n)\sum_{i=1}^{n} |f_i - x_i|$

Mean Absolute Difference = MAD = $(1/n)\sum_{i=1}^{n} |x_i - y_i|$

Root Mean Squared Change = RMSC = $[(1/n)\sum_{i=1}^{n}(x_i-x_{i-1})^2]^{1/2}$

Root Mean Squared Error = RMSE = $[(1/n)\sum_{i=1}^{n}(f_i - x_i)^2]^{1/2}$

Root Mean Squared Difference = RMSD = $[(1/n)\sum_{i=1}^{n}(x_i - y_i)^2]^{1/2}$