Calculation of Canopy Bidirectional Reflectance Using the Monte Carlo Method

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For a calculation of the plant canopy bidirectional reflectance distribution function (BRDF) the Monte Carlo method is used. The plant architecture is given by a rather universal mathematical model which allows to consider such structural parameters as canopy density and height, the number of leaves per plant, distance between leaves, dimensions and orientations of leaves and stems, etc., and their influence on the shape of the BRDF as a function of solar and view directions. To quantify these effects, a series of numerical experiments has been carried out. The information content of the BRDF about canopy architecture is the largest, if it is determined in the principal plane. The change of the BRDF in the region of "hot spot" characterizes leaf dimensions by increasing the leaf area or by decreasing the distance between leaves, the region of the "hot spot" increases. The change of the BRDF near nadir view direction is influenced by soil brightness and by arrangement of the leaves on the stem. The presence of vertical stems or nonhorizontal mat leaves increases the asymmetry of BRDF relative to nadir; in the opposite side of the sun the canopy reflectance is several times smaller than on the sun side. Adding the effect of multiple scattering to the BRDF changes the shape of BRDF only a little. The BRDF as a function of view directions contains information about canopy architecture and can be used for future progress of the remote sensing technique.

Introduction

In remote sensing studies of vegetation the central task is the solving of the inverse problem, i.e., the determination of agronomical characteristics necessary for crop management from the optical canopy characteristics (e.g., Goel and Strebel, 1983). The theoretical basis for the solving of such a problem is a mathematical model of canopy bidirectional reflectance distribution function (BRDF). During the last decade several canopy reflectance models have been proposed (e.g., Suits, 1972; Goudriaan, 1977; Ross, 1981; Kimes and Kirchner, 1982; Norman, 1984; Verhoef, 1984; Chen, 1985, see the review of models by Goel, 1982). All these models consider the canopy as a turbid plate medium in which leaves are modeled as thin little plates distributed randomly in horizontal layers and oriented in given directions. In these models the canopy architecture which plays a decisive role in canopy reflectance has been taken into account approximately usually through the leaf area index and through the leaf inclination angle distribution function only. Consideration of such important canopy parameters as canopy height, leaf dimensions, effective distance between leaves, nonrandom distribution of leaves, etc., is in principal impossible by means of these models due to the turbid medium concept. An attempt to introduce the hot spot affect on the basis of the turbid medium concepts has been made by Kuusk (1985), who considered the plate medium, in horizontal elemental layer of which the plates have finite dimensions and nonrandom distribution. The use of two-direction indicator function in solving the radiative transfer equation gives the BRDF with hot spot. One of the ways to treat canopy architecture in more detail in calculations of the BRDF

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0034-4257/88/$3.50

is the use of the Monte Carlo simulation method. The Monte Carlo method for the study of the canopy light regime was first used by Japanese scientists (Tanaka, 1969; Oikawa and Saeki, 1972; 1977). Szwarchbaum and Shaviv (1976) used the Monte Carlo method for evaluating the radiation field inside plant canopies. Kanevskii and Ross (1982, 1983) used this method for the calculation of the BRDF of a coniferous tree. Gerstl et al. (1986) proposed a simple three-dimensional Monte Carlo ray tracing model and an analytic two-dimensional model to estimate the angular distribution of the hot spot as a function of the leaf size. Their results show that the brightness distribution and slope of the hot spot change distinctively for different leaf sizes indicating a much more peaked maximum for the smaller leaves. Ross and Marshak (1984) constructed a rather universal model of the plant canopy architecture containing the most essential structural parameters and elaborated the Monte Carlo procedure for computing the BRDF of this model canopy. The aim of the present paper is to present the Monte Carlo computational procedure and to calculate the BRDF using the Ross–Marshak canopy model and to evaluate the role of various parameters affecting the canopy BRDF.

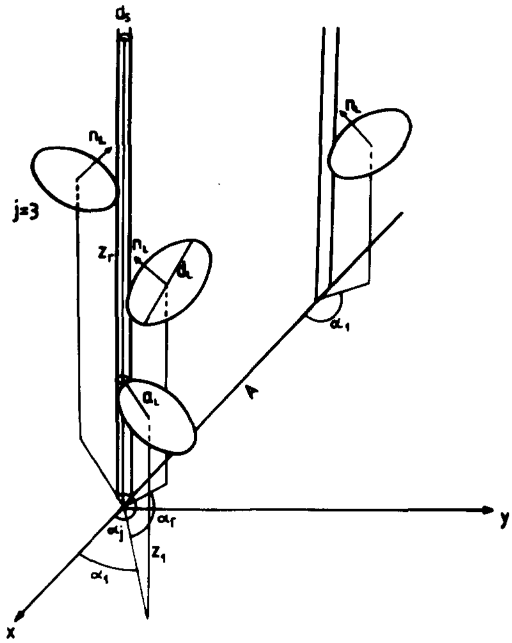

FIGURE 1. A model of the canopy architecture.

The Model of Canopy Architecture

Let our canopy with a height H consist of model plants planted in checkrows and let A be the distance between the plants (Fig. 1) Hence the plant area density is $1/A^2$ plants per m^2. Each plant has a vertical cylindrical stem with a diameter d_s, N_L elliptical leaves with a length d_{L1}, and a width d_{L2}. The height of the first leaf above the soil z_1 is a random quantity given by a normal distribution with a_{z1} and σ_{z1}. The azimuth angle of the first leaf is also a random quantity, α_1 with a uniform distribution. The distance between the neighboring leaves z_r is a constant value. So $N_L = [(H - z_1)/z_r] + 1$, where [ ] is an integer part and the height of the jth leaf is $z_j = z_1 + (j-1)z_r$. The azimuth of the second leaf is $\alpha_2 = \alpha_1 + \alpha_r$, of the jth one $\alpha_j = \alpha_1 + (j-1)\alpha_r$, where α_r is the azimuth angle between the successive leaves on the genetic spiral. All the leaves have a constant inclination angle ϑ_L. The direction of the jth leaf normal is $n_{Lj} = (\vartheta_L, \varphi_{Lj})$. Let the stem and the leaf normal n_{Lj} be placed on the same vertical plane, then $\varphi_{Lj} = \pi + \alpha_j$. The leaf area index of such a model canopy is $\mathrm{LAI} = \pi N_L d_{L1} d_{L2}/4A^2$.

In the following the numerical experiments were carried out with two types of the canopy.

Canopy A. Round horizontal $\vartheta_L = 0$ leaves with the diameter $d_{L1} = d_{L2} =$ 4 cm, LAI = 1.19, $H = 100$ cm, $d_s =$ 0.0 cm, $A = 10$ cm. $z_r = a_{z1} = 10$ cm, $\sigma_{z1} = 3$ cm, $\alpha_r = 2\pi/3$. Of the real canopies the horse bean (*Vicia faba L.*) crop corresponds to this model canopy most of all.

Canopy B. Elliptical leaves with the length $d_{L1} = 16$ cm and with the width $d_{L2} = 1$ cm and with the inclination angle $\vartheta_L = \pi/3$. $A = 5$ cm, the crop density – 400 plants per m^2, $d_s = 0.6$ cm, $z_r = a_{z1}$ $= 8.9$ cm, $\sigma_{z1} = 2.7$ cm, $\alpha_r = \pi$. This model canopy corresponds to the crop of cereals.

The Monte Carlo Method

The Monte Carlo method is the method of the simulation of random variable quantity in order to estimate some of the characteristics of their distribution (mathematical expectation, moments, etc.). If we compute the model of a real process according to the Monte Carlo method, we simulate the random variable connected with this process. However, the direct imitation or the so-called method of a straightforward simulation that traces the history of each photon from "birth" to "death," does not always lead to satisfactory results. For example, this method is acceptable for the estimation of the penetration through the plant canopy, since each photon makes "a contribution" to the estimated function. The situation with the estimation of the BRDF is rather different. If we use straight simulation, some of the photons (often considerably many, depending on optical properties of the phytoelements) are absorbed by the media and do not contribute to the BRDF estimate. Therefore, the computer time spent on the examination of their "fate" is inexpediently used.

Below we propose the algorithm which is free from such defects. First we model without absorption, changing accordingly the "weight" of the photon. Secondly, we use the so-called "fictitious flight." After each interaction the portion of the "weight" of the photon is directed to the receivers that are above the canopy. And each photon multiply makes a contribution to the BRDF estimate. And thirdly, our computed estimates are without shifting although we do not spend computer time for the study of the "fate" of the photon, if its "weight" is small.

We shal describe shortly the algorithm of the Monte Carlo method which in our opinion is most suitable for the estimation of BRDF for such a model.

1. Realization of the plant canopy model. We consider the plant canopy to be a horizontally infinite field that consists of identical test areas and each area includes N_p^2 plants. We assume that the photon flies out across the lateral wall and flies into the opposite wall of the next area at the same angles (Oikawa and Saeki, 1972). Naturally a question arises about the choice of N_p. It is clear that if N_p increases, the degree of the periodicity of the canopy decreases; on the other hand, the computer time spent on the modeling of the test area increases. We describe the finding of the optimal N_p in Appendix 1.

The second question is the optimal choice of the number of the photons M whose histories are considered in the constructed system (one realization of the random number that characterizes the model of the plant canopy). This question is discussed in Appendix 2.

2. *Choice of the initial point of the trajectory.* We propose that it is uniformly distributed on the upper side of the test area.

3. *The flight's direction and the "weight" of the photon.* In the case of direct solar radiation the "weight" of the photon $W = 1$ and the flight's direction is (ϑ_s, φ_s). If radiation is diffuse, then $W =$ SKYL (Goel and Thompson, 1984), and the flight's direction is defined according to Appendix 3.

4. *Definition of the length of free flight.* The length of a free flight in the constructed model is defined according to the finding of the point of the intersection of the trajectory with the phytoelement and soil. It is pointed out that an ordinary choice of all phytoelements takes too much computer time. Therefore, this step of the algorithm must be sufficiently optimized by the choice of only those phytoelements that may be intersected with.

5 *"Fictitious flight."* After intersection with phytoelement or soil we send the photon in the direction of each receiver at a solid angle $\Delta\Omega_i$, $i = 1,2,\ldots,L$ ($L =$ is the number of receivers). The "weight" on the photon W changes according to Appendix 4. If the length of the free flight is more than the distance between the point of intersection and the receiver, then the value W is added to the digital count of the corresponding receiver. In the opposite case the "fictitious flight" in the direction of the ith receiver is finished.

6. *Further movement of the photon.* First the type of the interaction is defined (Appendix 5), and then the direction of movement (Appendix 6). The "weight" W changes according to Appendix 7.

7. *Departure from the inner cycle.* The inner cycle is a trajectory of one photon. It contains the procedures 4 (with an escape), 5, 6, and 7. The departure from the cycle is described in Appendix 8.

8. *Departure from the outer cycle.* Takes place if all photons have been accounted for. Their number is taken so that the root-mean-square error E was no more than the given admissible accuracy.

The question of substantiation and estimation of the method's accuracy are discussed by Ross and Marshak (1984).

Results of Numerical Experiments

The BRDF was calculated for the red region and the photosynthetically active region (PAR) of the spectrum, considering only first-order scattering and for the near infrared (NIR) region considering also the multiple scattering. The spectral coefficients of the leaf reflection R_L and transmission T_L, stem reflection R_s and soil reflectance b_s are given in Table 1. The stem transmission $T_s = 0$. For crop B, calculations were carried out for different phenological stages.

Figure 2 demonstrates the behavior of the crop BRDF in the red spectral interval during the growth period. Since in this situation the soil reflectance is higher

TABLE 1 Spectral Coefficients of Vegetation and Soil Used for Model Calculations

Spectral Region	R_L	T_L	R_S	b_S Dry	b_S Wet
Red	0.06	0.03	0.09	0.14	0.06
PAR	0.10	0.05	0.10	0.10	0.06
NIR	0.48	0.45	0.30	0.30	0.13

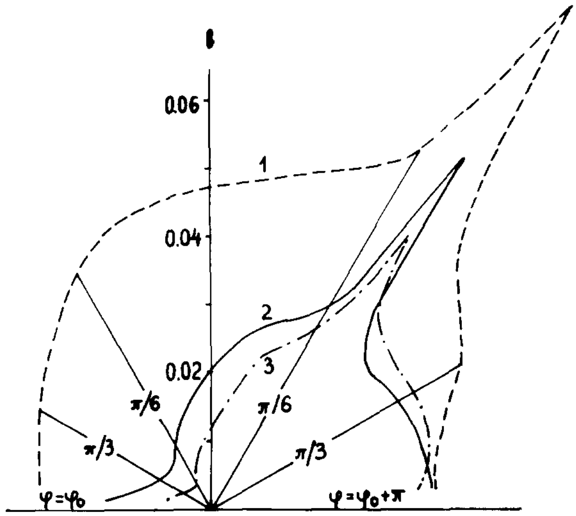

FIGURE 2. Polar diagram of the canopy BRDF on the principal plane at different growth stages. Canopy B; solar zenith angle $\vartheta_0 = \pi/5$; dry soil; red spectral interval.

No. of Curve	Growth Stage	Crop Height (cm)	Leaf Area Index	Stem Area Index
1	tillering	20	0.875	0
2	stem elongation	40	2.0	0
3	heading	80	4.0	1.9

than that of the leaves so by increasing the leaf area index, the crop reflectance decreases. For canopy with nonhorizontal leaves typical is the great asymmetry of the BRDF in relation to the nadir view direction, the reflectance being much on the side of the sun and the presence of stems in fully developed crops increases this asymmetry since the stem transmittance is zero. The contribution to the crop BRDF asymmetry is caused particularly by the fact that leaf reflectance is twice as great as leaf transmittance. Hence the main reason for such asymmetry is as follows: In spite of the uniform azimuthal distribution of the nonhorizontal leaves, the leaves whose normals have been directed to the opposite side of the sun ($\varphi_L = \varphi_0$) are not illuminated by direct solar beams. The main contribution to the BRDF gives the photons reflected from the leaves whose normals are directed to the sun's side ($\varphi_L = \varphi_0 + \pi$). The minimum of the BRDF at $\vartheta = \pi/6$ with $\varphi_L = \varphi_0$ is caused by the abovementioned effect and by the inclination angle of leaves $\vartheta_L = \pi/3$. It is clear that such effect is most pronounced in the case of $\vartheta_0 = \pi/2 - \vartheta_L$. Note that for the canopy with uniform leaf normal distribution the influence of the above mentioned effect decreases.

In Fig. 3 the crop BRDF with horizontal round leaves at different leaf diameters are presented. Increasing the leaf diameter correspondingly decreases the distance A between the plants, the leaf area index being constant, $L_L = 2.69$. The

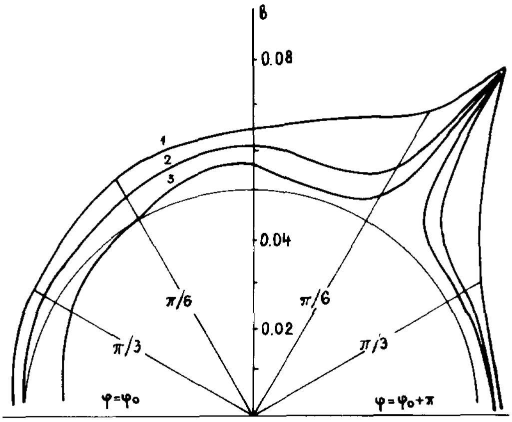

FIGURE 3. Polar diagram of the canopy bidirectional reflectance on the principal plane. Canopy A; spectral region of PAR; LAI = 2.69.

No. of Curve	Leaf Diameter (cm)	Distance between Plants A (cm)
1	16	27
2	4	7
3	2	3

crop with horizontal leaves in comparison with nonhorizontal ones has a much greater degree of symmetry in relation to the nadir view direction. The asymmetry is caused by the hot spot effect and the area of the hot spot around solar direction greatly increases with an increase in the leaf diameter. Thus the degree of a decrease in the reflected intensities around solar direction may be a new characteristic for determining the effective leaf dimensions of the plant.

The soil reflectance has a great influence on the crop BRDF in a sparse crop. It is demonstrated in Fig. 4, the soil reflectance varying from a very dark soil ($b_s = 0$ and 0.1) to a snow cover ($b_S = 0.6$ and 0.8). The sensitivity of the crop BRDF to the soil reflectance is maximal in the directions around nadir and solar beam.

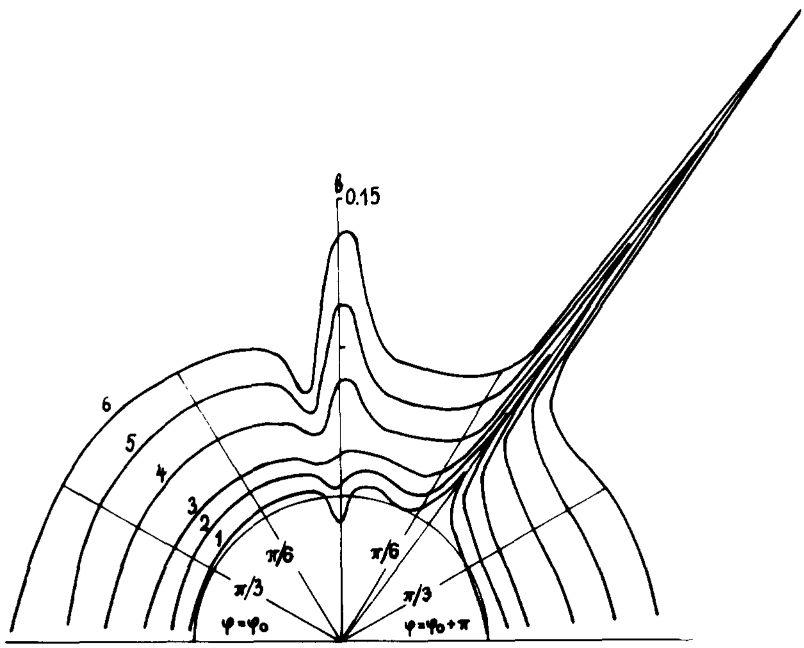

FIGURE 4. Polar diagram of the sparse canopy BRDF on the principal plane at different soil reflectance b_S. Canopy A; spectral region of PAR. b_S: 1) 0; 2) 0.1; 3) 0.2; 4) 0.4; 5) 0.6; 6) 0.8.

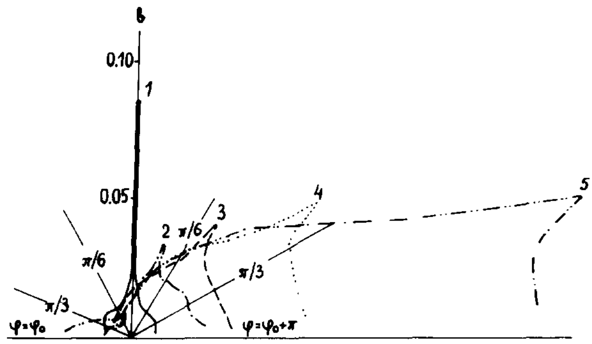

FIGURE 5. Polar diagram of the canopy bidirectional reflectance on the principal plane at different solar zenith angles. Canopy B; red spectral interval; Heading growth stage; $L_L = 4.0$, $L_S = 1.9$. ϑ_0: 1) 0, 2) 0.1π; 3) 0.2π; 4) 0.3π; 5) 0.4π.

In other directions the crop brightness increases monotonically with an increase in the soil reflectance.

In Fig. 5 the dependence of the crop BRDF on solar zenith angle is shown. The total amount of reflected radiation and asymmetry of the BRDF increase with an increase in the solar zenith angle. By increasing the solar zenith angle the role of stems in total reflectance increases. A very high reflectance in the nadir at the zero solar zenith angle is caused by two factors: by the effect of hot spot and by the maximum probability to see bright soil through the canopy.

On the sun's side ($\varphi = \varphi_0 + \pi$) the BRDF increases by increasing the sun's elevation. It is not so on the opposite side of the sun ($\varphi = \varphi_0$) near the view zenith angle $\vartheta = \pi/4$. At the sun zenith angle $\vartheta_0 = 0$ the contribution of the bright soil is maximal. By increasing of ϑ_0 the influence of the soil decreases and leading off with $\vartheta_0 \sim 2\pi/9$, the probability to see

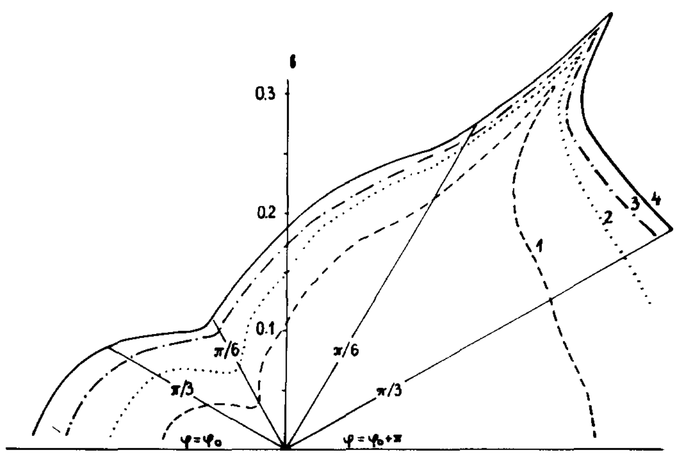

FIGURE 6. Polar diagram of the canopy BRDF on the principal plane considering different orders of scattering. Canopy B; spectral region of near infrared radiation; heading growth stage; $L_L = 4.0$, $L_S = 1.9$. 1) first-order; 2) second-order; 3) third-order; 4) fourth-order.

sunlit soil at $\vartheta = \pi/4$ is near zero. The reflectance of the soil–canopy system is formed by reflection from all canopy layers, the lower ones being shaded by the uppermost ones. With a further increase of the solar zenith angle the contribution of the uppermost layers without shading increases and the total reflectance of the system increases also.

The results presented in Fig. 2–5 were calculated for the visible spectral region, including only first-order scattering. The calculations of the crop BRDF in the NIR spectral regions need to consider the multiple scattering due to high scattering coefficients of plant elements. As shown in Fig. 6, an increase in the order of scattering in calculations essentially increases the reflected radiation although the shape of the crop BRDF changes only a little. The minimum of the BRDF is replaced on the opposite side of the sun at the zenith angle $\vartheta = \pi/6$. With increasing the order of scattering the minimum is less pronounced.

Conclusions

The results presented in this paper demonstrate that the Monte Carlo simulation method is a useful tool for evaluating the influence of the canopy architecture on the BRDF of the soil–canopy system. The method allows us to consider the role of such canopy parameters as leaf dimensions, plant height, distance between leaves etc., which is impossible to estimate by means of the classical theory of radiative transfer for random distributions scatterers.

The BRDF of all types of the canopies considered here has a noticeable hot spot effect. By increasing leaf dimensions or by decreasing the distance between leaves (Fig. 3) the region of the hot spot increases around direction of the solar retro direction. It means that the behavior of the BRDF around the region of the hot spot may be an indicator of leaf dimensions and gives us a key for determining these canopy characteristics by remote sensing. The BRDF as a function of the zenith angle of view direction is moderately symmetrical relative to both sides of nadir, if the canopy has mostly horizontal leaves. In the case of nonhorizontal leaves there arises a great asymmetry and on the opposite side of the sun ($\varphi = \varphi_0$) the values of the BRDF at the same zenith angle are many times smaller than the values on the sun's side ($\varphi = \varphi_0 + \pi$). The asymmetry effect will be especially boosted by cereal crops through the additional effect of vertical stems.

It can be concluded that the degree of asymmetry of the BRDF relative to both sides of the nadir direction may be a useful indicator of the orientation of the phytoelements and gives us a possibility to determine this canopy characteristic by remote sensing.

The influence of the soil brightness is maximal on the direction of nadir. At the zenith angle $\vartheta \sim \pi/4$ this influence is negligible if the leaf area index is not very small. Thereby the difference of values of the bidirectional reflectance between $\pi/4$ and 0 may characterize the difference between leaf and soil brightness (Fig. 4). If this difference is positive, the leaves are brighter than the soil and vice versa. The determination of the sign of this difference is important since on it depends the shape of the BRDF as a function of solar elevation. Our results in case of mat leaves show that the bidirectional reflectance as a function of the view of the zenith angle has four informative regions for the remote sensing problem. The first one is the region of hot spot and the shape of the reflectance function characterizes the leaf dimension (Fig. 3). The second one is the region around the nadir direction, and the shape of the function is influenced by the soil brightness (Fig. 4) and by individual plant geometry. The third one is the region near the view inclination angle about $\pi/4$ on the opposite side of the sun, and the reflectance function characterizes optical properties of leaves and the amount of the canopy leaf area (Fig. 2). The fourth one is the region around the direction of $\vartheta = \vartheta_0$ on the opposite side of the sun. The existence of the minimum of BRDF shows us that the leaf inclination angle $\vartheta_L \approx \pi/2 - \vartheta_0$ (Figs. 2 and 6).

It must be noted that the regularities of the BRDF function was considered above only for the principal plane. The shape of the reflectance function on the plane perpendicular to the solar one ($\varphi = \varphi_0 + \pi/2$ and $\varphi = \varphi_0 + 3\pi/2$) is much more symmetrical relative to the nadir direction and contains less useful information about canopy architecture.

The addition of multiple scattering in the near infrared region does not essentially change the shape of the BRDF as a function of the solar and view zenith angles in comparison with the first-order scattering approximation only.

To quantify the effects of the influence of the parameters of canopy architecture on the BRDF and to use these effects for the determination of the additional canopy structure parameters by the remote sensing techniques, more detailed field BRDF measurements are needed.

Appendix

1. The choice of an optimal N_p

Let x_{N_p} be the weighted average of the estimates of unknown functions if the test area includes N_p^2 plants and ε is the derived accuracy of the calculation. The number N_p is chosen as a minimum n at which the inequality

$$|x_n - x_{n-1}| < \varepsilon$$

is valid.

2. The choice of an optimal M

This choice is made according to the method of splitting (Ermakov and Mikhailov, 1976). In our model it is the following. For one value of the constructed model of plant canopy we change m values of ξ that characterizes the randomness of the trajectory. The optimal value $m = M$ of the values $\xi_1, \ldots, \xi_m$ is defined by minimizing on m of the value $T_m D\rho_m$, where $T_m = t_1 + mt_2$, $D\rho_m$ is the dispersion of a random estimate of unknown functionals, t_1 is the average time of construction of the realization of the canopy model and t_2 is the average time spent on the calculation of a trajectory within a constructed model.

3. The modeling of the direction of diffuse radiation

If the sky is uniformly bright, we may approximate the diffuse radiation by the formula (Ross, 1981)

$$D_1(\theta) = 2\cos\theta\sin\theta, \qquad \theta \in (0, \pi/2),$$

and, in the case of a standard cloudy sky, by the formula

$$D_2(\theta) = 6/7(1 + 2\cos\theta)\cos\theta\sin\theta.$$

It is obvious that

$$\int_0^{\pi/2} D_i(\theta)\, d\theta = 1, \qquad i = 1,2.$$

It follows from this and from the equation

$$\int_0^{\xi_i} D_i(\theta)\, d\theta = \alpha, \qquad i = 1,2,$$

where $\alpha \in (0,1)$ is uniformly distributed such that

$$\cos\xi_1 = \sqrt{1-\alpha}\,, \qquad \sin\xi_1 = \sqrt{\alpha}$$

and the random value $\mu = \cos\xi_2$ satisfies the equation

$$4\mu^3 + 3\mu^2 = 7(1-\alpha).$$

It is not difficult to see that the last equation has a real positive solution μ^*, which may be easily found according to Newton's method, where the initial datum is $\mu_0 = 1$.

4. The "weight" of the photon at "fictitious flight"

1. *From the leaf's surface.* Let $r_L = (\theta_L, \varphi_L)$ be a normal to the leaf and we suppose that the reflection is the Lambert law along θ and uniformly in leaf's plane along φ. Let us define $P(\theta_1, \theta_2, \varphi_1, \varphi_2)$ as the probability that after the interaction the photon reflects in the direction of $r = (\theta, \varphi)$, where $\theta \in (\theta_1, \theta_2)$, $\varphi \in (\varphi_1, \varphi_2)$, and $\theta_1, \theta_2, \varphi_1, \varphi_2$ are given.

Let $\gamma = \cos(r_L, r)$, then the phase function is

$$g(\gamma) = 2\gamma = 2\big[\cos\theta\cos\theta_L + \sin\theta\sin\theta_L\cos(\varphi - \varphi_L)\big]. \qquad (1)$$

We propose that $\gamma > 0$. In the opposite case $\theta_L := \theta_L + \pi$ and reflection takes place on the opposite side. Thus

$$\begin{aligned} P(\theta_1, \theta_2, \varphi_1, \varphi_2) &= c\int_{\theta_1}^{\theta_2}\int_{\varphi_1}^{\varphi_2} g(\gamma)\sin\theta\, d\theta\, d\varphi \\ &\approx c\Delta\varphi\Big[\cos\theta_L(\mu_1^2 - \mu_2^2) \\ &\qquad + \cos(\bar\varphi - \varphi_L)\sin\theta_L \\ &\qquad \times\Big(\arccos\mu_2 - \arccos\mu_1 \\ &\qquad + \mu_1\sqrt{1-\mu_1^2} - \mu_2\sqrt{1-\mu_2^2}\Big)\Big]. \end{aligned}$$

Here $\Delta\varphi = \varphi_2 - \varphi_1$, $\bar\varphi = (\varphi_1 + \varphi_2)/2$, $\mu_1 = \cos\theta_1$, $\mu_2 = \cos\theta_2$, and c is R_L or T_L, depending on its describing a "fictitious flight" reflection or transmission by the leaf.

2. *From the stem's surface.* In this case $r_L = (\pi/2, \varphi_L)$, where φ_L is the azimuth angle of the normal to stem in the point of interaction. If φ_L has the value so that $\gamma = \sin\theta\cos(\varphi - \varphi_L) < 0$, then reflection does not occur, and the constant $c = R_S$.

3. *From the surface of the soil.* For soil surface $r_s = (0,0)$ and $g(\gamma) = 2\cos\theta$. From this

$$P(\theta_1, \theta_2, \varphi_1, \varphi_2) = b_s\Delta\varphi(\mu_1^2 - \mu_2^2).$$

Thus, the "weight" of the photon in "fictitious flight" is $W \cdot P$, where W denotes the "weight" of the photon before interaction.

5. Determination of the type of interaction

We simulate the discrete random number, the distribution of which is given by the table

$$\begin{pmatrix} \text{reflection} & \text{transmission} \\ W_{LR} & W_{LT} \end{pmatrix}$$

where $W_{LR} = P$ (ξ = reflection), $W_{LT} = P$ (ξ = transmission). We write the value of W_{LR} and W_{LT} in the case of interaction of the photon with the upper surface of the leaf (for details see Ross and Marshak, 1984), namely,

$$W_{LR} = \frac{Q_1}{Q_1 + Q_2}, \qquad W_{LT} = 1 - W_{LR},$$

where

$$Q_1 = R_L\left(1 - \frac{\Delta\varphi(\pi - \vartheta_L)}{\pi^2}\right),$$

$$Q_2 = T_L\left(1 - \frac{\Delta\varphi\,\vartheta_L}{\pi^2}\right).$$

If the photon falls on the lower surface of the leaf, then in the equalities for Q_1 and Q_2 coefficients R_L and T_L and indices 1 and 2 change their places.

Then, the type of interaction depends upon the inequality $\alpha < W_{LR}$. If it is valid, then reflection takes place, in the opposite case, it is transmission.

6. Direction of the flight of the photon after interaction

The scattering of the photon of the leaf surface is assumed to be Lambertian, and the probability that the photon will be scattered at a given angle depends only on the orientation of the surface. The phase function g, defined by (1) is normalized by the equality $\int_0^1 g(\gamma)\,d\gamma = 1$ and it is the density of the distribution of a continuous random variable γ. From this according to the rule of simulation of a continuous random variable (Ermakov and Mikhailov, 1976) from the equation

$$\int_0^{\gamma} g(\gamma')\,d\gamma' = \alpha,$$

we obtain that $\cos(r_L, r) = \sqrt{\alpha}$ and then $\sin(r_L, r) = \sqrt{1-\alpha}$, where α is a random number from a uniform distribution on [0, 1]. The outgoing azimuth has a uniform distribution on the plane of the leaf and its functions cos and sin can be simulated as an isotropic vector on this plane (Ermakov and Mikhailov, 1976). Now the outgoing zenith and azimuth angles must be transformed into coordinates relative to the standard $x-y-z$ system using the matrix of transformation (Cooper and Smith, 1985). (Note that for the economy of the computer time we do not use the angles themselves but their sin and cos.)

In the case of scattering of the stem's surface we put $r_L = (\pi/2, \varphi_L)$, where φ_L is the azimuth of the normal to stem in the point of interaction.

7. The change of the "weight" of the photon

Taking into account the "fictitious flight" of the photon, its "weight" after interaction can be calculated by

$$W = cB(1 - \Delta\varphi/\pi),$$

where B is the "weight" of the photon before interaction and c is the constant that equals to R_L, T_L, R_S and b_S, depending on the phytoelements (or the soil) being interacted with. We note that if the photon interacts with the leaf, the surface of the leaf is considered (for details see Ross and Marshak, 1984).

8. Unbiased estimation

When W becomes sufficiently small, it is "not profitable" to continue the calculation of its trajectory. In order not to neglect such a 'weight," we undertake a random cut off of the trajectory (Sobol, 1973).

Let $W < \varepsilon$, then there arises an alternative, either the "weight" of the photon with probability q becomes W/q or its trajectory ends with probability $1 - q$. But the "weight" of the photon remains on an average unchanged as

$$q \cdot (W/q) + 0 \cdot (1-q) = W.$$

(The question of the optimal change of q remains still open.)

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Received 4 September 1986; revised 22 May 1987